Available online at https://sanad.iau.ir/journal/ijim/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 15, No. 4, 2023 Article ID IJIM-1502, 5 pages
Research Article

Science and Research Branch (IAU)

Vibration Analysis of a Current-Carrying Wire in a Magnetic Field Apllying Variational Iteration Method

M. Hosseini *†

Submission Date: 2021/03/08 Revised Date: 2021/06/17 Date of Acceptance: 2021/06/27

Abstract

In the present investigation, Variational Iteration Method (VIM) is applied to solve the dynamic oscillation of a current-carrying wire in a magnetic field which is generated by a fixed current-carrying conductor parallel to the wire. Two linear springs are considered to restrict the wire to a rigid wall. In a special case, the periodic solution of the problem is obtained by VIM and compared with numerical solutions for different parameters. Results indicate high accuracy of this method which can be easily extended to solve other non-linear vibration equations, therefore can be applicable in other engineering problems.

Keywords : Current-carrying wire; Non-linear equation; linear elastic springs; Variational Iteration Method.

1 Introduction

Nonlinear oscillator models arise in many areas of physics and engineering and are important in mechanical dynamics because in theoretical and applied physics and mechanics, there are many nonlinear systems involving oscillation, wave propagation, electromagnetic systems, etc. These scientific problems and phenomena are modeled by ordinary or partial differential equations. In most cases, these problems do not admit analytical solution, so these equations should be solved using special techniques. In recent years, much attention has been devoted to the newly developed methods to construct an analytic solution of equation, such methods include the Adomian decomposition method [1]-[3], the Homotopy Perturbation Method (HPM) [4]-[9], and the Variational Iteration Method (VIM) [10]-[15]. Perturbation techniques are too strongly dependent upon the so-called small parameters [16]-[17]. Thus, it is worthwhile developing some new analytic techniques independent upon small parameters. The Variational Iteration method (VIM) is one of the well-known methods to solve the nonlinear equations. This method established by He [10]-[11]. The method has been used by many authors in [12]-[15] and the references therein to handle a wide variety of scientific and engineering applications: linear and nonlinear, and homogeneous and inhomogeneous as well. It was shown by many authors that this

*Corresponding author. aimirhossini˙54@yahoo.com, Tel:+98(912)5916726.
†Department of Mathematics, Islamic Azad University, Qhemshahr Branch, Qhaemshahr, Iran.

method provides improvements over existing numerical techniques. In this Letter, the basic idea of the VIM is introduced and then the problem of a special case of dynamic oscillation of a current-carrying wire in a magnetic field is solved through VIM and numerical methods. This method gives rapidly convergent series with specific significant features for each scheme.

2 Analysis of the Variational Iteration Method

To illustrate the basic concept of the Variational Iteration Method, we consider the following differential equation [10]-[11]:

$$Lu + Nu = g(t) \tag{2.1}$$

where L is a linear operator, N a nonlinear operator and $g(t)$ an inhomogeneous term. According to the Variational Iteration Method, we can construct a correct functional as follows:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda(t)(Lu_n(\tau) + N\tilde{u}_n(\tau) - g(\tau))d\tau \tag{2.2}$$

where $\lambda(t)$ is a general Lagrangian multiplier which can be identified optimally via the variational theory. The subscript n indicates the n^{th} approximation and $\tilde{u}_n$ is considered as a restricted variation $\delta\tilde{u}_n = 0$. Therefore, we first determine the Lagrange multiplier $\lambda(t)$ that will be identified optimally via integration by parts. The successive approximations $u_{n+1}(t) \geq 0$, $n \geq 0$ of the solution $u(t) \geq 0$ will be readily obtained upon using the obtained Lagrange multiplier and by using any selective function $u(t)$. With $\lambda(t)$ determined, then several approximations $u_j(t), j \geq 0$, follow immediately. Consequently, the exact solution may be obtained by using:

$$u = \lim_{n \to \infty} u_n \tag{2.3}$$

3 Problem statement and mathematical formulation

Consider the problem of current-carrying wire-conductors in parallel where the wire which is restrained to a fixed wall by linear elastic springs. The physical model of this problem is presented in Fig. (1). The distance between wire and conductor is b, parallel to one another for a length l, and carrying currents i_1 and i_2, respectively. Let $\bar{x}$ be the displacement of the wire, k be the stiffness coefficient of the springs and m be the mass of the wire. In dimensionless form, the equation of motion for the current-carrying wire in the magnetic field generated by the fixed conductor is [18]:

$$\frac{d^2x}{dt} + x - \frac{\Lambda}{1-x} = 0, \quad x(0) = \hat{A}, \quad \frac{dx}{dt}(0) = 0 \tag{3.4}$$

where $x = \frac{\bar{x}}{b}$, $t = w_0\bar{t}$, $w_0^2 = \frac{k}{m}$ and $\Lambda = \frac{2i_1i_2l}{kb^2}$ and $\hat{A}$ is the initial value of $x(0)$ it is an example of a conservation nonlinear oscillatory system having a rational form for the restring force. To construct a analytical solution, Sun et al. [18] introduced a variable as:

$$u = x - \alpha \tag{3.5}$$

Where α is one of the stable equilibrium points. Substituting Eq. (3.5) into Eq. (3.4), we will have:

$$\frac{d^2u}{dt} + \alpha + u - \frac{\Lambda}{1-u-\alpha} = 0, \quad u(0) = A, \quad \frac{du}{dt}(0) = 0 \tag{3.6}$$

where $A = \hat{A} - \alpha$. The corresponding potential energy function is:

$$V(u) = \frac{1}{2}(\alpha + u)^2 + \Lambda.ln|1 - u - \alpha|. \tag{3.7}$$

To solve Eq. (3.6) via an analytical method, a special case of the problem is considered as follows[18]:

$$u'' + \alpha_1 u + \alpha_2 u^2 + \alpha_3 u^3 = 0 \tag{3.8}$$

where primes denote differentiation with respect to t and:

$$\alpha_1 = 1 - \Lambda(1-\alpha)^{-2} \tag{3.9}$$

$$\alpha_2 = -\Lambda(1-\alpha)^{-3} \tag{3.10}$$

$$\alpha_3 = -\Lambda(1-\alpha)^{-4} \tag{3.11}$$

and the boundary conditions of the Eq. (3.8) are:

$$u(0) = A, \quad u'(0) = 0 \tag{3.12}$$

Eq. (3.8) with the boundary conditions (3.12) are solved by Sun et al. [18] for different values of parameters Λ, A and α. In this article, we reconsider this equation and solve it by Hes analytical method called Variational Iteration Method (VIM).

4 Solution with Variational Iteration Method

The Variational Iteration Method is an analytical method that has been used to solve effectively, easily, and accurately a large class of linear and nonlinear, ordinary or partial, deterministic or stochastic differential equations with approximate which converge rapidly to accurate solutions. In this section, we will apply the VIM to solve differential equation (3.8). In order to solve Eq. (3.8) using VIM, we construct a correction functional, as follows:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda(t)(\tau\frac{d^2u}{d\tau^2} + \alpha_1 u(\tau) + \alpha_2 u^2(\tau) + \alpha_3 u^3(\tau))d\tau \tag{4.13}$$

where $\lambda(t)$ is Lagrange multiplier, which can be identified optimally via the Variational theory. Its stationary conditions can be obtained as follows:

$$\lambda^{"}(\tau)+\lambda(\tau)|_{\tau=t}= 0,\ 1-\lambda^{'}(\tau)|_{\tau=t}= 0,\ \lambda(\tau)|_{\tau=t}= 0 \tag{4.14}$$

The Lagrange multiplier can be identified as $\lambda(\tau) = sin(\tau - t)$ and the following Variational iteration formula can be obtained:

$$u_{n+1}(t) = u_n(t) + \int_0^t sin(\tau - t)(\tau\frac{d^2u}{d\tau^2} + \alpha_1 u(\tau) + \alpha_2 u^2(\tau) + \alpha_3 u^3(\tau))d\tau \tag{4.15}$$

We begin with an arbitrary initial approximation as:

$$\theta_0 = Acos\sqrt{\alpha_1 t} \tag{4.16}$$

Using (4.15) we obtain the 1^{th}-order approximations. For comparison with existing studies, for one case, we set $\Lambda = -1.5$, $\alpha = -3.5$ and $A = 1$. Therefore, we obtain the following 1^{th}-order approximate solution as follows:

$$u^{[1]}(t) = 1,292968cos(1,038327t) - 0,276801cos(t) + 0.876627E-3cos(3,114983t) + 0.737028E-2cos(2,0766559t) - 0,024414 \tag{4.17}$$

In the same manner, for the different values of Λ, α and A, the solutions of equation (3.8) have been obtained for 2^{th}-order approximate and presented in 1 and Figs. (2)-(7).

5 Conclusion

In this paper, we have successfully developed VIM to obtain the exact solutions of a special case of dynamic oscillation of a current-carrying wire in a magnetic field. Also, governing equation is solved by the numerical method (Runge-Kutta method of order four) using the software MAPLE whose results and the consequent results of the two different methods of VIM and NM are compared in Table ((1) and Figs. (2)-(7). It is apparently seen that this method is very powerful and efficient techniques for solving different kinds of problems arising in various fields of science and engineering and present a rapid convergence for the solutions.

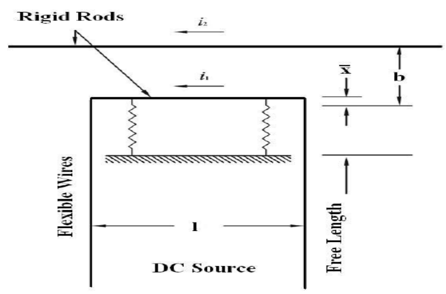

Figure 1: Schematic diagram of the physical system

References

[1] G. Adomian, A review of the decomposition method and some recent results for nonlinear equation, *Math. Comput. Model.* 13 (1992) 17-27.

Table 1: The results of VIM and NM methods for $\Lambda = -2.5$, $\alpha = -4.5$ and $A = 0.5$.

t	u_{VIM}	u_{NM}	$\lvert u_{NM} - u_{VIM} \rvert$
0.0	0.499999959	0.500000000	0.000000041
0.5	0.428876328	0.428876353	0.000000025
1.0	0.236384181	0.236383134	0.000001048
1.5	-0.022170280	-0.022178694	0.000008414
2.0	-0.274822062	-0.274852547	0.000030485
2.5	-0.453153720	-0.453226771	0.000073051
3.0	-0.509609482	-0.509745933	0.000136451
3.5	-0.429207185	-0.429420929	0.000213744
4.0	-0.233273800	-0.233563471	0.000289671
4.5	0.025857842	0.025513008	0.000344834
5.0	0.277733888	0.277375955	0.000357933
5.5	0.451830940	0.451530960	0.000299980
6.0	0. 497829879	0. 497694099	0. 000135781

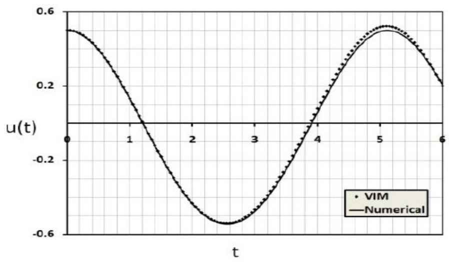

Figure 2: Comparison of the VIM solution with the numerical result for the case that $\Lambda = -2.3$, $\alpha = -1.5$ and $A = 0.5$

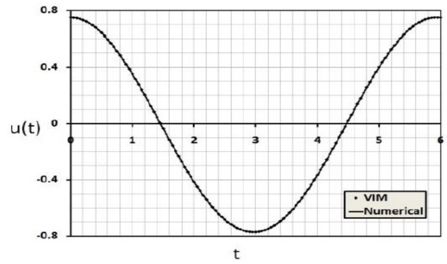

Figure 4: Comparison of the VIM solution with the numerical result for the case that $\Lambda = -1.3$, $\alpha = -3.4$ and $A = 0.75$

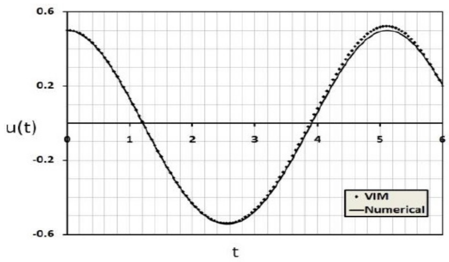

Figure 3: Comparison of the VIM solution with the numerical result for the case that $\Lambda = -2.5$, $\alpha = -4.5$ and $A = 0.5$

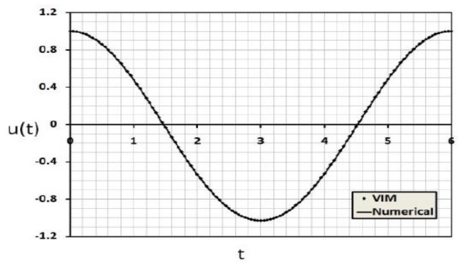

Figure 5: Comparison of the VIM solution with the numerical result for the case that $\Lambda = -1.5$, $\alpha = -3.5$ and $A = 1.0$

[2] M. M. Hosseini, Adomian decomposition method with Chebyshev polynomials, *Appl. Math. Comput.* 175 (2006) 1685-1693.

[3] A. M. Wazwaz, A new method for solving singular initial value problems in the second-order ordinary differential equations,*Appl. Math. Comput.* 128 (2002) 45-57.

[4] J. H. He, Homotopy perturbation technique,*J. Comput. Math. Appl. Mech. Eng.* 17 (1999) 257-262.

[5] J. H. He, Approximate analytical solution for seepage flow with fractional derivatives in porous media,*J. Comput. Math. Appl. Mech. Eng.* 167 (1998) 57-68.

[6] J. H. He, A review on some new recently developed nonlinear analytical techniques,*Int.*

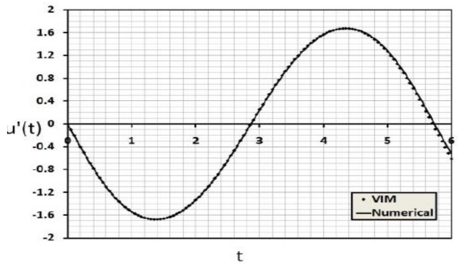

Figure 6: Comparison of the VIM solution with the numerical result for the case that $\Lambda = -1.7$, $\alpha = -2.9$ and $A = 1.5$

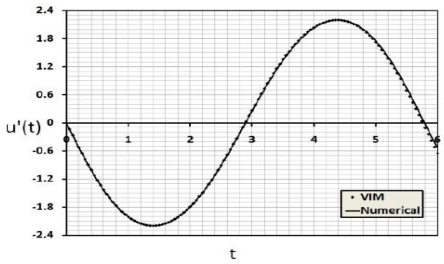

Figure 7: Comparison of the VIM solution with the numerical result for the case that $\Lambda = -1.10$, $\alpha = -4.15$ and $A = 2.0$

J. Non-linear Sci Numer Simul. 12 (2000) 51-70.

[7] J. H. He, Homotopy perturbation method for bifurcation on nonlinear problems, *Int. J. Non-linear Sci. Numer. Simul.* 13 (2005) 7-18.

[8] L. N. Zhang, J. H. He, Homotopy perturbation method for the solution of the electrostatic potential differential equation, *Mathematical Problems in Engineering, Art.* 8 (2006).

[9] P. D. Ariel, T. Hayat, S. Asghar, Homotopy perturbation method and axisymmetric Flow over a stretching sheet, *Internat J. Nonlinear Sci. Numer. Simul.* 4 (2006) 399-406.

[10] J. H. He, Variational iteration methoda kind of non-linear analytical technique: some examples, *Internat. J. Nonlinear Mech.* 34 (1999) 699-708.

[11] J. H. He, Variational iteration method for autonomous ordinary differential systems, *Applied Mathematics and Computation* 114 (2000) 115-123.

[12] M. A. Abdou, A.A. Soliman, Variational iteration method for solving Burgers and coupled Burgers equations, *J. Comput. Appl. Math.* 181 (2005) 245-251.

[13] A. M. Wazwaz, A comparison between the variational iteration method and Adomian decomposition method, *J. Comput. Appl. Math.* (2006), in press.

[14] M. A. Abdou, A. A. Soliman, Variational iteration method for solving Burgers and coupled Burgers equations, *J. Comput. Appl. Math.* 181 (2005) 245-251.

[15] A. A. Soliman, Numerical simulation of the generalized regularized long wave equation by Hes variational iteration method, *Math. Comput. Simulation* 70 (2005) 119-124.

[16] A. H. Nayfeh, Problems in Perturbation, *2nd Edition, Wiley, New York* (1993).

[17] A. H. Nayfeh, Perturbation Methods, *Wiley, New York,* (2000).

[18] W. P. Sun, C. W. Lim, B. S. Wu, C. Wang, Analytical approximate solutions to oscillation of a current-carrying wire in a magnetic field, *Nonlinear Analysis: Real World Applications* 10 (2009) 1882-1890.

Mirolla Hosseini, he has born in the mazandaran, babol in 1976. Got B.Sc and M.Sc degrees in mechanical engineering field from Islamic Azad University and PHD degree in mechanical engineering field from Noshirvani University of Technology Babol. Main research interest include CFD, Analytical methods, Nanofluid.