Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 13, No. 3, 2021 Article ID IJIM-1304, 9 pages
DOR: http://dorl.net/dor/20.1001.1.20085621.2021.13.4.9.8
Research Article

Science and Research Branch (IAU)

Efficient Median and Center Problems: a Bi-Objective Mixed Integer Programming Approach

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Received Date: 2019-03-14 Revised Date: 2020-08-16 Accepted Date: 2021-07-15

Abstract

This paper proposes a novel problem on the efficiency in the location problem. We consider the problem of finding optimal location of an efficient facility in the plane. This yields to a novel combination of the location problem and efficiency measurement. We describe the difference between our model and the principle existing models in the literature. We then present a bi-objective programming models for the efficient median and center problems.

Keywords : Median problem; Center problem; Efficiency; Data envelopment analysis.

1 Introduction

Finding the location of facilities in order to serve the clients in an optimal criteria play a fundamental rule in the most models of location theory. Median and center problems are two basic problems in location theory. The median problem asks to find the location of a facility such that the sum of weighted distances between this facility and the clients are minimized. The goal in the center problem is finding the location of a facility such that the maximum weighted distances from this facility to the clients is minimized. For more details in median, center and facility location problems [3]. In the many type of location models the goal is finding the optimal location of facilities with respect to different criteria that consider the time, cost and distances between the clients and facilities. However, the efficiency of the facilities which concerns to the location of facilities,has been considered by a few authors.

In general, term of efficiency can be thought as a type of trade off that what we gain by consuming our resources. Data envelopment analysis (DEA) is a well-known methodology, based on mathematical programming models that can be used to assess the relative efficiency of a number of similar processing units, commonly called decision making units (DMUs). This methodology needs data about the amount of each input that is consumed by each unit and the amount of each output that is produced by each unit. In DEA, we seek to find a virtual DMU which produces output equal to the output of the DMU under evaluation, with

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[13] and [8] are the first and the main attempts for considering efficiency concept in the location problem. Other theoretical and application are fulfilled inspired by these two researches.Considering the efficiency of facilities which should be located for serving the clients has been interested in recent years. [13] considered the combination of obnoxious facility location and DEA models. They presented two approaches. In the first approach they find the optimal location of facilities, then these optimal facilities are used as the input of the DEA model. If the efficiency of DEA model is unity, then the optimal solution is found. Otherwise, the optimal location of new facilities should be found. This method continues until all facilities are considered or all DEA scores are efficient. In the second approach the DEA model and location problem have been considered as a single objective linear programming model to maximize the efficiency of those facilities that are going to be opened.[8] used the DEA concept for finding the efficient location of facilities. They presented two bi-objective linear programming models for capacitated and uncapacitated facility location models which combined with DEA models. They consider optimization of both spatial interactions between facilities and the customers they serve and the efficiency of facilities at the selected locations simultaneously. Inspired by [13] and [8] other research are performed in the literature for combining location problem and efficiency concept. [6] utilized fuzzy DEA for location analysis of finding an ideal location for establishing a Japanese-style rotisserie in a metropolis in China. [11] utilized DEA models for measuring relative attractiveness of locations. He proposed an extension of Huff model capable of dealing with multiple factors using DEA models.The multi-criteria DEA model has been used for location problems in fuzzy environment by [10]. In another study on DEA and location, [9] developed a method for evaluating the efficiency of existing facilities and finding location of the new facilities. In fact, they utilized DEA and location methods separately in two different stages. [7] presented the new formulation combines facility location and DEA to support the decision maker with more realistic solutions based on the optimal location-allocation decisions. In fact, they set the outputs of the model as a function dependent on the allocation variables. [4] proposed a two phasemethod for incorporating DEA and location. In the first phase they estimated the efficiency of potential location and then in the second phase efficiency measures are assumed as a goal in the location problem. The second phase used goal programming model. [2] deal with a multiple objective programing forlocal reliability-based maximum expected covering location problem, taking efficiency concept into account that was done by DEA model. [5] incorporated the efficiency concept to the set covering, packing and partitioning problems. Their attempt yielded to a series of multiple objective programming. [12] proposed a joint DEA-Location problem considering an incapacitated single- source multi-product facility location problem. This model differs from our model fundamentally. In the first place, our model are median and center problem while [12] proposed an incapacitated single- source multi-product facility model. In fact, we investigate to find an efficient median an efficient center and the number and location of facilitates are fixed while [12] aim to find the location and number of facilities. Moreover, facilities and candidates are homogenous in our model. Think of a bank brand which seeks for a head office in a discrete. The head branch is a facility and other branches

are candidates. Secondly, all branches including facilities and candidates are assumed as DMUs in our models while in [12] the links between facilities (factories) and candidates (customers) are assumed as DMUs. This is another fundamental difference of our models compared with model of [12].

In this paper, weconsider the concept of efficiency for the facilities using DEA concept. Suppose that each candidate site for location facility has its own used inputs and produced outputs. We considerfacilities asDMUs, and the comparable efficiencies of facilities are calculated as the ratio of the total weighted outputs to the total weighted inputs. Finding the location of facility with concern to both minimum transportation costs and maximumefficiency is the main idea that we are intended to consider in this paper.As the transportation costs we consider the objective functions of two basic models, median and center problems. We call these two combination problems, Efficient Median Problem (EMP) and Efficient Center Problem (ECP), respectively.

Note that, our models are different from the main and basic location-DEA models, that are, [13] and [8]. The difference between our models and [13] is rather obvious by definition and the research question in both models.Compared with [8], in their models the DMUs are defined on the links of clients to the facilities,any potential allocation link is considered as a DMU, and the objectives indicates the total DEA score assigned to the open links. But in our models the facilities are DMUs themselves, and objective expresses the maximum efficiency on selected facility. Moreover, the candidate locations of facilities are differing from the location of clients in their models, whereas in our models we deal with selecting the facility among the clients (think about selecting a hospital to be a large and general hospital serving other small general and specialized hospitals).Furthermore, they consider the capacitated and un-capacitated facility location models, but our models are concern with median and center problems.

In what follows the median, center and DEA models are given Sections 2. Then in Sections 3, the new models of efficient median and center problems are presented. Finally, the numerical examples are given in Section 4 to illustrate proposed models.

2 Preliminaries

2.1 Facility location problems

There are many kind of facility location models that have been considered by the authors. Among them, the median and center problems are two important classic facility location models that play a basic rule in the location theory. These problems ask to find the location of a facility among the set of given points, such that respectively the sum and maximum weighted distances from clients on the existing points to the facility is minimized.

Let n points $A_i = (a_i, b_i)$, $i = 1, \ldots, n$ be given in the plane. Each point A_i has a nonnegative weight w_i. One of this existing points should be selected as the median, there for we set

$$e_j = \begin{cases} 1 & \text{if } A_j \text{ is selected as the median} \\ 0 & \text{otherwise} \end{cases}$$

Let $d(A, B)$ be the distance between points A and B. Then the model of1-median problem can be written as the following linear programming model.

$$Min \sum_{j=1}^{n} \sum_{i=1}^{n} w_i d(A_i, A_j) e_j \tag{2.1}$$

$$\begin{aligned} s.t. \quad & \sum_{j=1}^{n} e_j = 1 \\ & e_j \in \{0, 1\} \; j = 1, 2, \ldots, n. \end{aligned} \tag{2.2}$$

In this model, the objective function (2.1) indicates that the sum of weighted distances from existing points to the selected facility is minimized. The constraint (2.2), indicates that only one points should be selected as the median.

The model of 1-center problem is also as follow

$$Min \, max_{i=1,\ldots,n} w_i \sum_{j=1}^{n} d(A_i, A_j) e_j \tag{2.3}$$

$$\begin{aligned} s.t. \quad & \sum_{j=1}^{n} e_j = 1 \\ & e_j \in \{0, 1\} \; j = 1, 2, \ldots, n. \end{aligned} \tag{2.4}$$

Table 1: Data requirements of each points.

Output (O_{j1}, O_{j2})	acility (A_j)	Coordinate (a_j, b_j)	Weight (w_j)	Inputs (I_{j1}, I_{j2})
A_1	(1,12)	10	(4,17)	(1,3)
A_2	(4,15)	43	(5,9)	(10,19)
A_3	(14,15)	3	(11,7)	(3,2)
A_4	(8,19)	30	(6,8)	(8,5)
A_5	(10,5)	15	(13,14)	(4,1.5)
A_6	(2,3)	21	(12,9)	(3.5,6)
A_7	(3,3)	9	(10,12)	(2,3)
A_8	(7,3)	7	(10,3)	(1,2)
A_9	(8,10)	11	(3,1)	(10,9)
A_{10}	(2,13)	2	(10,2)	(3.5,1)
A_{11}	(6,1)	38	(4,5)	(3,2)
A_{12}	(9,5)	10	(15,1)	(4,5)
A_{13}	(9,1)	12	(8,4)	(15,14)
A_{14}	(1,16)	3	(11,12)	(0.5,8)
A_{15}	(5,0)	20	(6,3)	(1,5)

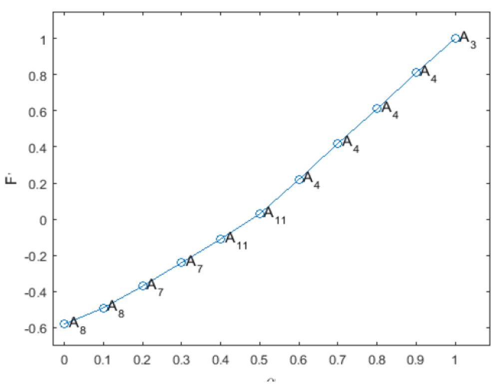

Figure 1: Trade-off between $\alpha = 1$ and objective function F.

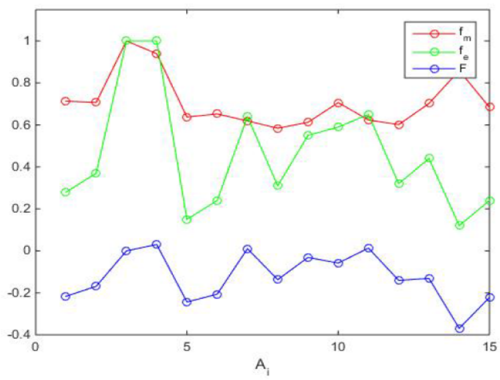

Figure 2: The value of f_e, f_m and F respect to the given points for $\alpha = 0.5$.

Where e_j and constraint (2.4) is the same as 1-median model. The objective function (2.3), indicates the maximum weighted distance between existing points and the selected facility is minimized.

2.2 DEA Models

Assume there are n DMUs which can be banks, schools, ministries, etc. Each DMU uses m inputs to produce s outputs which may be product service etc. For $j = 1, \ldots, n$, let's denote O_{rj}, $r = 1, \ldots, m$, outputs of j-th DMU and I_{tj}, $r = 1, \ldots, s$, inputs of j-th DMU. The following CCR model introduced by (Charnes, Cooper, Rousseau, & Semple, 1987) can be used for assessing the efficiency of k-th DMU, $k \in \{1, 2, \ldots, n\}$.

$$Min\ \theta_k \tag{2.5}$$

$$s.t. \sum_{j=1}^{n} \lambda_j I_{tj} \leq \theta_k I_{tk} \quad t = 1, \ldots, m \tag{2.6}$$

$$\sum_{j=1}^{n} \lambda_j O_{rj} \geq O_{rk} \quad r = 1, \ldots, s \tag{2.7}$$

$$\lambda_j \geq 0 \qquad j = 1, 2, \ldots, n.$$

The above model is known as envelopment form of CCR model. If we transfer to the dual space

Table 2: The efficiency of facilities.

Facility	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
Efficiency	0.28	0.37	1	1	0.15	0.24	0.64	0.31	0.55
$\frac{f_m}{M}$	0.7188	0.7071	1	0.9398	0.6374	0.6524	0.6191	0.5844	0.6124
Facility	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}			
Efficiency	0.59	0.65	0.32	0.44	0.12	0.24			
$\frac{f_m}{M}$	0.7063	0.6237	0.6015	0.7022	0.8587	0.684			

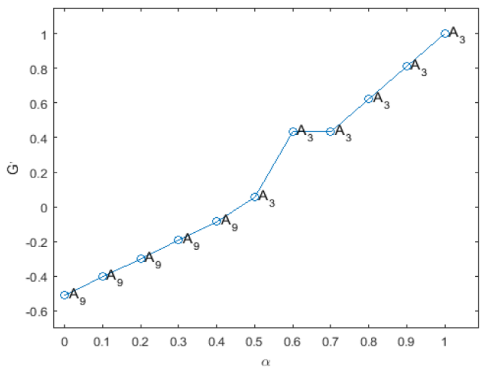

Figure 3: The trade-off between α and objective function G.

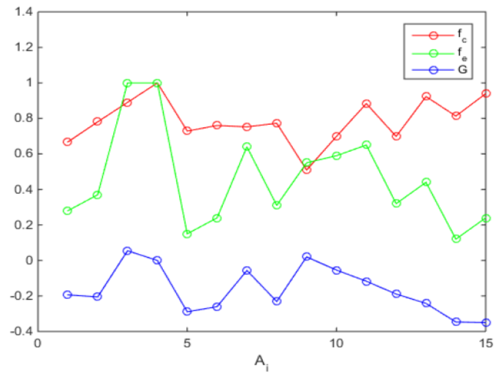

Figure 4: The value of f_e, f_c and G respect to the given points for $\alpha = 0.5$.

we get the following model called multiplier form of CCR model.

$$Max\ E_k = \sum_{r=1}^{s} u_r O_{rk} \tag{2.8}$$

$$s.t.\ \sum_{t=1}^{m} v_t I_{tk} = 1 \tag{2.9}$$

$$\sum_{r=1}^{s} u_r O_{rj} - \sum_{t=1}^{m} v_t I_{tj} \leq 0 \quad j = 1, \ldots, n \tag{2.10}$$

$$v_t, u_r \geq 0 \qquad \forall t, r.$$

In fact above model finds the optimal input weights (v_t) and output weights (u_r) that yield the maximum potential output using the current level of input for DMU under evaluation.

3 New location models dealing with efficiency

3.1 Efficient median problem (EMP)

Consider then given DMU with coordinate $A_j = (a_j, b_j)$ and non negative weight $w_j, j = 1, \ldots, n$, in the plane. Also we assume that the j-th DMU has a set of m inputs $I_{j1}, I_{j2}, \ldots, I_{jm}$ and a set of s outputs $O_{j1}, O_{j2}, , O_{js}$. In the efficient median problem (EMP) we want to select a DMU among $A_1, \ldots, A_n$ as the facility such that the selected point has maximum efficiency and the sum of weighted distances from this point to all other points is minimized. Actually, we consider a bi-objective model. The first objective function is finding the facility with maximum efficiency and the second one is finding the median point. To model this problem, let

$$e_j = \begin{cases} 1 & \text{if } A_j \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

and $v_t, t = 1, 2, \ldots, m$ and $u_r, r = 1, 2, \ldots, s$ be the t-th input coefficient and r-th output coeffi-

Table 3: The optimal points for EMP with varying values of α.

α	Optimal point	Normalized of optimal point($\frac{f_m}{M}$)	Efficiency of optimal point(f_e)	F
1	A_3	1	1	1
0.9	A_4	0.9398	1	0.860
0.8	A_4	0.9398	1	0.6120
0.7	A_4	0.9398	1	0.4180
0.6	A_4	0.9398	1	0.2241
0.5	A_{11}	0.6237	0.65	0.0301
0.4	$A_1 1$	0.6237	0.65	-0.1142
0.3	A_7	0.6191	0.64	-0.2414
0.2	A_7	0.6191	0.64	-0.3673
0.1	A_8v	0.5844	0.31	-0.4932
0.0	A_8	0.5844	0.31	-0.5844

cient, respectively. Let be the inefficiency level of j-th unit, that is, $1 - \sum_{r=1}^{s} u_r O_{rj} = d_j, j = 1, \ldots, n$. Then the bi-objective model of EMP can be written as follows.

$$Min\ f_m = \sum_{j=1}^{n} \sum_{k=1}^{n} w_k d(A_k, A_j) e_j \tag{3.11}$$

$$Min\ f_m = \sum_{j=1}^{n} \sum_{k=1}^{n} w_k d(A_k, A_j) e_j \tag{3.12}$$

$$Max\ f_e = \sum_{j=1}^{n} (1 - d_j) e_j \tag{3.13}$$

$$s.t. \sum_{t=1}^{m} v_t I_{tj} = e_j \qquad j = 1, \ldots, n \tag{3.14}$$

$$\sum_{r=1}^{s} u_r O_{rj} + d_j = e_j \qquad j = 1, \ldots, n \tag{3.15}$$

$$\sum_{r=1}^{s} u_r O_{rj} - \sum_{t=1}^{m} v_t I_{tj} \leq 0 \quad j = 1, \ldots, n \tag{3.16}$$

$$\sum_{j=1}^{n} e_j = 1 \tag{3.17}$$

$$e_j \in \{0, 1\} \quad j = 1, \ldots, n$$

$$v_t, u_r \geq 0 \quad \forall t, r.$$

In this model the first objective function and the constraint (3.16) guarantee to find the median point. The second objective function with constraints (3.13), (3.14) and (3.15) find the facility with maximum efficiency. To solve this model we use the following weighted sum objective function instead of proposed bi-objective model.

$$max\ F = \alpha f_e - \frac{1 - \alpha}{M} f_m \tag{3.18}$$

where $0 \leq \alpha \leq 1$. Since $0 \leq f_e \leq 1$ and f_m is very larger than f_e, then to balance the effectiveness of f_e and f_m in the objective function, we used M, which is a normalized coefficient for the median objective function. In fact, M can be set as the value of objective function of the following problem.

$$M = max \sum_{j=1}^{n} \sum_{i=1}^{n} w_k d(A_k, A_j) e_j \tag{3.19}$$

$$s.t. \sum_{j=1}^{n} e_j = 1 \tag{3.20}$$

$$e_j \in \{0, 1\} \quad j = 1, \ldots, n.$$

This model find the location of a facility among existing points, such that the weighted distances from existing points to this selected facility is maximized. The optimal solution called maxian point.

Table 4: The optimal points of ECP for varying values of α.

α	Optimal point	Normalized of optimal point($\frac{f_c}{M_c}$)	Efficiency of optimal point(f_e)	G
1	A_3	0.8903	1	1
0.9	A_3	0.8903	1	0.811
0.8	A_3	0.8903	1	0.6219
0.7	A_3	0.8903	1	0.4329
0.6	A_3	0.8903	1	0.2438
0.5	A_3	0.8903	1	0.0548
0.4	A_9	0.5091	0.55	-0.0854
0.3	A_9	0.5091	0.55	-0.1913
0.2	A_9	0.5091	0.55	-0.2972
0.1	A_9	0.5091	0.55	-0.4031
0.0	A_9	0.5091	0.55	-0.5091

3.2 Efficient center problem(ECP)

With the same notation in EMP, in the efficient center problem (ECP) we want to find a DMU among $A_1, \ldots, A_n$ such that the selected point has maximum efficiency and the maximum weighted distance from this point to other points is minimized. Therefore, the model of ECP can be written as follows.

$$Min\ f_c = max_{k=1,\ldots,m} w_k \sum_{j=1}^{n} d(A_k, A_j) e_j \tag{3.21}$$

$$Max\ f_e = \sum_{j=1}^{n} (1 - d_j) e_j \tag{3.22}$$

$$s.t. \sum_{t=1}^{m} v_t I_{tj} = e_j \qquad j = 1, \ldots, n \tag{3.23}$$

$$\sum_{r=1}^{s} u_r O_{rj} + d_j = e_j \qquad j = 1, \ldots, n \tag{3.24}$$

$$\sum_{r=1}^{s} u_r O_{rj} - \sum_{t=1}^{m} v_t I_{tj} \leq 0 \quad j = 1, \ldots, n \tag{3.25}$$

$$\sum_{j=1}^{n} e_j = 1 \tag{3.26}$$

$$e_j \in \{0, 1\} \quad j = 1, \ldots, n \tag{3.27}$$

$$v_t, u_r \geq 0 \quad \forall t, r.$$

In this model the objective function (3.20) with constraint (3.25) guarantee the finding center point. For solving this bi-objective model we replace the objective functions by the following weighted sum objective function.

$$max\ G = \alpha f_e - \frac{1 - \alpha}{M_c} f_c \tag{3.28}$$

where $0 \leq \alpha \leq 1$. With the same reason to EMP model, we set a normal coefficient M_c, tobalance the objective function. The value of M_c can be found by solving the following model.

$$M_c = max_{j,k=1,\ldots,n} w_k d(A_k, A_j) \tag{3.29}$$

$$s.t. \sum_{j=1}^{n} e_j = 1 \tag{3.30}$$

$$e_j \in \{0, 1\} \quad j = 1, \ldots, n.$$

In this model we should find the location of a facility with maximum weighted distance to all existing points.
Note that M and M_c can be found in $O(n^2)$ time. Also, the two presented models for EMP and ECP can be solved by mixed integer linear programming methods.

4 Illustrative example and computations

In this section we provide numerical experiment to test the presented models for EMP and ECP. An example with 15 DMU, each with a given weight and two inputs and two outputs, is considered. The coordinates, weights, inputs and outputs of given points are presented in Table 1. We consider the Euclidian norm to measure the distances between given points in the plane. The models are solved by Lingo software.

The median point is A_8 with value of objective function $f_m = 1720.147$. The point in which has the maximum weighted sum of distances to the other points, i.e. maxian point, is A_3 with the value of objective function $M = 2943.587$. The efficiency of the given DMUs and the ratio of $\frac{f_m}{M}$ for median objective function are presented in Table 2. Table 3 reports the results of EMP for varying amounts of α.

The curve in Figure 1 illustrate the trade-off between α and objective function F.This figure indicates that in the case $\alpha = 0$, the solution of EMP is the point A_8 which is the solution of 1-median problem. However, this point has a low efficiency. Then by increasing the value of α, the efficiency of optimal point also increased and effectiveness of median objective function reduced. Until for the case $\alpha = 1$, we obtain the point A_3 with maximum efficiency.

Figure 2 reports the objective functions f_e, f_m and $F = \alpha f_e - \frac{(1-\alpha)}{M} f_m$ respect to the given points for the case $\alpha = 0.5$. This figure indicates that in this case the optimal solution is the point A_{11} with the value of objective function equal to 0.301. This point neither is median nor has maximum efficiency, however its value of objective function for both objective are very near to the optimal solutions.
The center point of given points is A_9 with the value of objective function $f_c = 350.343$. The point with maximum weighted distances to the other points is A_4 with value of objective function $M_c = 688.209$. The results of ECP for varying values of are given Table 4. The curve in Figure 3 shows the trade-off between and objective function G. For the cases $\alpha \in \{0, 0.1, 0.2, 0.3, 0.4\}$ the optimal solution is point A_9 which is also the solution of 1-center problem. For the other cases, A_3 is the optimal solution which is a point with maximum efficiency.

Figure 4 shows the objective functions f_e, f_c and G respect to the given points for the case $\alpha = 0.5$. This figure indicates that for this case the optimal solution is point A_3, which has maximum efficiency and a mean value of objective function for center problem.

5 Conclusion

This paper deals with the combinations of median and center problems with efficiency in data envelope analysis in a new perspective. A facility which is efficient and located optimal respect to objective function of median and center problems is called efficient median and efficient center, respectively. Two bi-objective mixed integer programming models are presented for the efficient median and center problems. Finally, some numerical examples are given to compare the results for varying cases of coefficients on efficiency and location optimality.

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