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Research Article

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A Fuzzy DEA Approach for Project Selection Utilizing Analyze Desirable and Undesirable Risk

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Abstract

This paper proposes a DEA-based model for analyze the fuzzy risk in project selection. We used concept semi-variance for measure upper and downside risk and a DEA model for classification desirable and undesirable risk. Firstly, the proposed model includes new desirable and undesirable risk-return indexes. Thus a novel DEA model is presented for evaluation and classification desirable and undesirable risks and finally, is extend to fuzzy DEA model for project portfolio selection. An applied example is used to explain the proposed approach and usefulness and applicability of this approach have been illustrated using the 37 available projects.

Keywords : Fuzzy data envelopment analysis; Project portfolio selection; Downside risk; Upper Risk.

1 Introduction

PRoject portfolio selection is an importance issue for organizations that to make the best decisions for selecting an optimal subset of projects, especially, if the decision-makers are challenged with limited resources. The selection is difficult because of the existence of uncertainty in Success of projects. Obtaining an optimal subset of projects which achieve organizations earnings target and avoid risk is main objective in modeling project portfolio selection. Moreover, avoiding risk dont should to deprive organization of opportunities. As a result, a model should be designed to consider the risk and opportunities in project selection. So far, much research has been done on the selection of research projects, including Shi et al. [?] used DEA and fuzzy theory to calculate risk in 2010 Asian Games construction projects. To select an optimal subset of projects and risk assessment, G. Hall et al. [?] proposed a discrete nonlinear optimization model using Banders analysis method. Cheaitoua et al. [?] presented a combined approach of multi-criteria decision making (MCDM) and fuzzy theory for selecting the best construction contractor for construction projects. They have used DEA to evaluate contractors as well as the fuzzy theory to

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assess risk. Kettunen and Salo [?] presented a calibration framework by using Monte Carlo simulation for estimation of downside risks in project portfolio selection. Dandage et al. [?] provided a TOPSIS method for ranking the risk categories according to their importance. TO select project portfolio, Tavana et al. [?] proposed a two-stage hybrid model using FAHP method and Fuzzy Inference System (FIS). They tried Maximizes project benefits and minimizes project risks. Although variance is used as a risk measure, but, the variance considers higher and lower returns equally undesirable, Li and Qin [?], Mashayekhi and Omrani [?] and Qin et al. [?]. Vercher et al. [?] presented two fuzzy portfolio selection models where the objective is to minimize the downside risk constrained by a given expected return and finally, formulated the portfolio selection problem as a linear program. To evaluate the portfolio efficiency, Chen et al. [?] used of DEA based fuzzy portfolio estimation models in different risk measures, i.e., possibilistic variance, possibilistic semi-variance, and possibilistic semi- absolute deviation. Similarly, Speranza [?] and Vercher and Bermdez [?] introduced the possibilistic semi-absolute deviation as in addition to semi-variance. To assess a project's NPV and its impact on a firm's expected profitability and down-side operational risk, Paquin et al. [?] used of a stationary stochastic model. Also, Chen et al. [?] proposed a model for multi-objective portfolio selection in fuzzy environment by mean-semivariance model and data envelopment analysis cross-efficiency model. To rebalancing portfolio and efficiency evaluation, Zhou et al. [?] a DEA frontier improvement approach under the mean-variance and Gopta et al. [?] used a credibilistic fuzzy DEA approach. To select the IT process as well as improve the development strategy in small and medium-sized enterprises, Yamami et al. [?] introduced a multi-objective method. To select the project portfolio under prioritization between criteria, uncertainty and interdependence between projects, Jafarzadeh et al. [?] proposed a combination of DEA and fuzzy theory. Toloo and Mirblouki [?] was obtained the average efficiency with different weights for each project using a linear programming method. They note that Cook and Green [?] considers the hybrid project as $(n+1)^{th}$ project under evaluation, and finds the optimal set of common weights. To prioritize the improvement of highway safety projects considering limited resources, Dadashi and Mirbaha [?] used the Monte Carlo-based DEA method. Zhang et al. [?] considered projects historical performances and using EVIKOR method, evaluated the performances of projects. To select a feasible portfolio based on the political goal and the annual budget in the selection smart city project, Wu and Chen [?] used the modified Delphi method to determine the elements of the decision considering panel members for their opinions, as well as prioritized each alternative according to the goal of the decision using AHP method. Ebrahimnejad and Amani [?] introduced a novel approach for solving FDEA models in the presence of undesirable outputs using Ideal and anti-ideal points, as well as used a lexicographic approach to find the best and the worst fuzzy efficiencies of ideal and anti-ideal points, respectively. To evaluate the efficiency of decision-making units in fuzzy data envelopment analysis with undesirable outputs, Kachouei et al. [?] used a common-weights approach. They considered the fuzzy additive DEA model as a linear programing problem and then obtained fuzzy efficiency based on the common set of weights. Arteaga et al. [?] presented a model for solving intuitionistic fuzzy data envelopment analysis problems. Hatami-Marbini et al. [?] used a lexicographic multi-objective linear programming (MOLP) approach to solve FDEA models. Peykani et al. [?] introduced a FDEA model based on adjustable the optimistic-pessimistic parameters, the model considered preference of decision makers. And also Peykani et al. [?] presented a paper to review some FDEA models based on applied possibility, necessity, credibility, general fuzzy measures and chance-constrained programming (CCP) to deal with data ambiguity. To assess of efficiency under uncertainty, Omrani et al. [?] proposed a robust credibility DEA model in which a fuzzy credibility approach was used for constructing fuzzy sets and a robust optimization approach for uncertainty in fuzzy sets. Chen and Ming [?] proposed a combined approach of rough-fuzzy and BWM-DEA method for selection smart product service module. The bestworst method (BWM) construct a practical multi-criteria framework with pairwise relations. To analyzing the effects of uncertainty and risk, Liu et al. [?] extended the R-numbers method to BWM and presented an R-BWM approach in the R&D project selection. Amirteimoori et al. [?] proposed a combined fuzzy nondiscretionary DEA (FNDEA) model and artificial immune system (AIS) to predict and find the optimal values of DMUs. Nasseri et al. [?] presented fuzzy stochastic DEA model with undesirable outputs for hybrid uncertain environment. They considered three fuzzy DEA models with respect to probabilitypossibility, probability necessity and probabilitycredibility constraints.

The main contributions of this paper are as follows:

1- Providing a novel project portfolio selection model in fuzzy environment, the model is a single objective linear programming and formulated based on upper, downside risk and the possibilistic mean value.

2- Classification risk into desirable and undesirable outputs using DEA.

3-introduct new index of risk-opportunity in project selection, because dont deprive organization of opportunities due to avoiding of risk.

4- The model is capable to consider resource constraints.

The rest of the paper is organized as follows. Section 2 provides background involving the DEA method and fuzzy upper and downside risk measure. The proposed model is presented in Section ?? an applied example and its results are presented in Section ??. Finally, the conclusion of the paper is summarized Section ??.

2 Preliminaries

In this section, we briefly review some definitions triangular fuzzy numbers which will be used in the following sections.

2.1 Fuzzy numbers

Let $\widetilde{\mathrm{A}} = (\alpha, a, \beta)_{LR}$ be a triangular fuzzy number with tolerance interval $[\alpha, \beta]$, left width $\alpha > 0$ and right width $\beta > 0$, if its membership function determines the following form:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \frac{a-x}{\alpha}, & a-\alpha \leq x \leq a, \\ 1 - \frac{x-a}{\beta}, & a \leq x \leq a-\beta, \\ 0, & \text{otherwise.} \end{cases} \tag{2.1}$$

The γ-level set of $[A]^\gamma = [a-(1-\gamma)\alpha, a+(1-\gamma)\beta], \forall \gamma \epsilon [0,1]$.

Carlsson and Fuller [?] presented the possibilistic mean value and variance of A as

$$\begin{aligned} \mathrm{E(A)} = \int_0^1 \gamma[a-(1-\gamma)\alpha + a \\ + (1-\gamma)\beta] d\gamma = a + \frac{\beta-\alpha}{6} \end{aligned} \tag{2.2}$$

$$\begin{aligned} \mathrm{Var}(A) &= \int_0^1 \gamma[\beta(\gamma) - \alpha(\gamma)]^2 d\gamma \\ &= \frac{(\beta-\alpha)^2}{24}. \end{aligned} \tag{2.3}$$

According to Saeidifar and Pasha [?], and Zhang et al. [?] the upper possibilistic semivariance and the lower possibilistic semivariance of A defined, respectively

$$\begin{aligned} \mathrm{Var}^+(A) &= \int_0^1 2\gamma(E(A) - \beta(\gamma))^2 d\gamma \\ &= \left(\frac{\alpha+\beta}{6}\right)^2 + \frac{\beta^2}{18} \end{aligned} \tag{2.4}$$

$$\begin{aligned} \mathrm{Var}^-(A) &= \int_0^1 2\gamma(E(A) - \alpha(\gamma))^2 d\gamma \\ &= \left(\frac{\alpha+\beta}{6}\right)^2 + \frac{\alpha^2}{18} \end{aligned} \tag{2.5}$$

Easily seen that if $\tilde{Y}_{rk} = (\alpha_{rk}, y_{rk}, \beta_{rk})$ is a triangular fuzzy number then

$$\mathrm{E}\left(Y_{rk}\right)=\int_0^1 \gamma[y_{rk}-(1-\gamma)\alpha_{rk}+y_{rk}+(1-\gamma)\beta_{rk}]d\gamma=y_{rk}+\frac{\beta_{rk}-\alpha_{rk}}{6} \tag{2.6}$$

$$\mathrm{Var}^{+}\left(Y_{rk}\right)=\int_0^1 2\gamma\left(E\left(Y_{rk}\right)-\beta_{rk}(\gamma)\right)^2 d\gamma=\left(\frac{\alpha_{rk}+\beta_{rk}}{6}\right)^2+\frac{\beta_{rk}^2}{18} \tag{2.7}$$

$$\mathrm{Var}^{-}\left(Y_{rk}\right)=\int_0^1 2\gamma\left(E\left(Y_{rk}\right)-\alpha_{rk}(\gamma)\right)^2 d\gamma=\left(\frac{\alpha_{rk}+\beta_{rk}}{6}\right)^2+\frac{\alpha_{rk}^2}{18} \tag{2.8}$$

2.2 Non-linear Mean - variance model

In the last paper, we have proposed a model based on mean -variance using DEA for selection of project. Sadeghiyan et al. [?] used direction vectors for increasing the expected return and decreasing the risk of project portfolio. Let $\mathrm{P}=\{\mathrm{p}_1,\ldots,\mathrm{p}_\mathrm{k},\ldots,\mathrm{p}_\mathrm{n}\}$ be the set of independent projects, $\mathrm{I}=\left\{i_1,\ldots,i_k,\ldots,i_{|I|}\right\}$ be the set of inputs and $\mathrm{O}=\left\{\mathrm{o}_1,\ldots,\mathrm{o}_\mathrm{r},\ldots,\mathrm{o}_{|\mathrm{o}|}\right\}$ be the set of outputs. And also, let $d'_{pj}=\max\left\{\mathrm{E}\left(y_{kj}\right)\right\}-\mathrm{E}\left(y_{pj}\right), d'_{pj}=\sigma^2\left(y_{pj}\right)-\min\left\{\sigma^2\left(y_{kj}\right)\right\}$, and $d'''_{pi}=x_{pi}-\min\left\{x_{ki}\right\}$ be the direction vectors for the mean, the variance of outputs and inputs, respectively. Thus the model as follow as

$$
\begin{aligned}
&\max \mathrm{Z}=\sum_{\mathrm{p}=1}^{\mathrm{n}} \mathrm{c_p}\alpha_\mathrm{p} && && (2.9)\\
&S.t \quad \sum_{\mathrm{k}=1}^{\mathrm{n}} \lambda_{\mathrm{pk}} E\left(\mathrm{y_{kj}}\right) \geq E\left(\mathrm{y_{pj}}\right)+\alpha_\mathrm{p}\mathrm{d}'_{\mathrm{pj}}, && \mathrm{j}\in\mathrm{O}, \quad \mathrm{p}\in\mathrm{P} && (2.9a)\\
&\sum_{\mathrm{k}=1}^{\mathrm{n}} \lambda_{\mathrm{pk}}\sigma^2\left(\mathrm{y_{kj}}\right) \leq \sigma^2\left(\mathrm{y_{pj}}\right)-\alpha_\mathrm{p}\mathrm{d}''_{\mathrm{pj}}, && \mathrm{j}\in\mathrm{O}, \quad \mathrm{p}\in\mathrm{P} && (2.9b)\\
&\sum_{k=1}^{n} \lambda_{pk}x_{ki} \leq x_{pi}-\alpha_p d''_{pi}, && i\in\mathrm{I}, \quad \mathrm{p}\in\mathrm{P} && (2.9c)\\
&\sum_{k=1}^{n} \lambda_{pk}=1, && \mathrm{p}\in\mathrm{P} && (2.9d)\\
&\sum_{k=1}^{n} c_k x_{ki}+b_i=B_i, && i\in\mathrm{I} && (2.9e)\\
&\left(1-c_k\right)x_{ki}+Mc_k+Md_{ki} \geq b_i+\frac{1}{M}, && \mathrm{k}\in\mathrm{P}, \quad i\in\mathrm{I} && (2.9f)\\
&\sum_{i\in I} d_{ki} \leq |I|-1, && \mathrm{k}\in\mathrm{P} && (2.9g)\\
&c_k, d_{ki}\in\{0,1\} && \mathrm{k}\in\mathrm{P}, \quad i\in\mathrm{I} &&\\
&\lambda_{\mathrm{pk}} \geq 0, && \mathrm{k}\in\mathrm{P}, \quad \mathrm{p}\in\mathrm{P} &&\\
&b_i \geq 0, && i\in\mathrm{I} &&\\
&M \gg 0 && &&
\end{aligned}
$$

The first constraints set (2.9a)-(??) increase the expected return as well as decrease the risk of project portfolio. And also, this model remains feasible despite the presence of negative data due to the directional vectors. The constrains (??)-(??) are for controlling the resources.

3 Modeling

In this section, we proposed a novel project portfolio selection model in fuzzy environment that is capable to consider different risks.

3.1 Project selection model

Firstly, we present a project selection model with limited resources based on risk oriented DEA. In order to improve the performance of project selection model, we classified risk in to upper and downside risks, which the downside risk (undesirable output) and inputs should be decreased and also, the upper risk (desirable output) with the possibilistic mean value should be increased. Because, it is necessary to consider the risks of not selecting the project as well as the risks of selecting it. In this paper, we employ the lower possibilistic semivariance and the upper possibilistic semivariance to measure the undesirable risk and desirable risk of model selection of projects, respectively. We will have

$$
\begin{aligned}
& \min \theta && (3.10)\\
& S.t \sum_{k=1}^{n} \lambda_k \delta_{DO}^{r}(y_{rk}) \geq \delta_{DO}^{r}(y_{rp}), \quad \forall r \\
& \sum_{k=1}^{n} \lambda_k \delta_{UDO}^{r}(y_{rk}) \leq \delta_{UDO}^{r}(y_{rp}), \forall r \\
& \sum_{k=1}^{n} \lambda_k x_{ik} \leq \theta x_{ip}, \forall i \\
& h_k \geq 0, \forall k
\end{aligned}
$$

By integrating model (2.5) into binary programming framework, the project portfolio selection model will be defined in the sequel.

$$
\begin{aligned}
& \min \sum_{p=1}^{n} c_p \theta_p && && (??)\\
& \text{S.t} \sum_{k=1}^{n} \lambda_{pk} \delta_{DO}^{r}(y_{rk}) \geq \delta_{DO}^{r}(y_{rp}), && \forall r, \forall p && (3.11a)\\
& \sum_{K=1} \lambda_{pk} E(y_{rk}) \geq E(y_{rp}), && \forall r, \forall p && (3.11b)\\
& \sum_{k=1}^{n} \lambda_{pk} \delta_{UDO}^{r}(y_{rk}) \leq \delta_{UDO}^{r}(y_{rp}), && \forall r && (3.11c)\\
& \sum_{k=1}^{n} \lambda_{pk} x_{ik} \leq \theta_p x_{ip}, && \forall i && (3.11d)\\
& \sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{c_k} x_{\mathrm{ki}} + L_i = \mathrm{B_i}, && \forall i && (3.11e)\\
& (1 - c_{\mathrm{k}}) x_{\mathrm{ki}} + \mathrm{Mc_k} + \mathrm{Md_{ki}} \geq L_i + \frac{1}{\mathrm{M}}, && \forall \mathrm{k}, \forall \mathrm{i} && (3.11f)\\
& \sum_{i} d_{ki} \leq |I| - 1, && \forall k && (3.11g)\\
& c_{\mathrm{k}}, \mathrm{d_{ki}} \in \{0, 1\} && \forall \mathrm{k}, \forall \mathrm{i} \\
& \lambda_{\mathrm{pk}} \geq 0, && \forall \mathrm{p}, \forall \mathrm{k} \\
& L_i \geq 0, && \forall \mathrm{i} \\
& M \gg 0
\end{aligned}
$$

where c_k is a binary variable and M is a sufficiently large number. If $c_k = 1$, then project k is included in the portfolio s^*, and 0 otherwise. The optimal objective function value of model (??) is the highest aggregate-efficiency score of selected feasible portfolio. The constraint (??) shows that the sum of used resources of projects in the portfolio s^* cannot exceed available resource $\mathrm{B_i}, \mathrm{b_i}$ is the slack in resource i and indicates the remaining amount of resource i. The constraints set (??)-(??) do not let a project be considered which violate resource constrains. Let $\mathrm{c_k} = 1$. Therefore, the constraint (??) is obviously satisfied because M is a sufficiently large number. Now, let $c_k = 0$, $\mathrm{d_{ki}}$ be a binary variable and the constraint (??) guarantees that at least one of the variables $\mathrm{d_{k1}},\ \mathrm{d_{k2}}, \ldots \mathrm{d_{k|I}}$ remain at zero. If $\mathrm{d_{ki}} = 0$, then $\mathrm{x_{ki}} \geq \mathrm{b_i} + \frac{1}{\mathrm{M}}$. Thus the remaining amount of resource i, is not sufficient for support the project k, similar to Cook and Green model (2000).

3.2 Fuzzy project portfolio selection model

The evaluation of risks is hard, accurately. Therefore, it is necessary to consider the uncertainty on inputs and outputs. Hence, we construct the project portfolio selection model in fuzzy environment. In the following, we use of equations (2.6), (2.7) and (2.8) for measure fuzzy parameters the lower possibilistic semivariance, the upper possibilistic semivariance and the possibilistic mean value in the purposed model. Based on model (??), we will have

$$
\begin{aligned}
&\min \sum_{p=1}^{n} c_p\theta_p && (3.12)\\
&S.t \sum_{k=1}^{n} \lambda_{pk}\left(\left(\frac{\alpha_{rk}+\beta_{rk}}{6}\right)^2 + \frac{{\beta_{rk}}^2}{18}\right) \geq \left(\frac{\alpha_{rp}+\beta_{rp}}{6}\right)^2 + \frac{{\beta_{rp}}^2}{18}, && \forall r, p \in K\\
&\sum_{K=1} \lambda_{pk}\left(y_{rk} + \frac{\beta_{rk}-\alpha_{rk}}{6}\right) \geq y_{rp} + \frac{\beta_{rp}-\alpha_{rp}}{6}, && \forall r, \quad p \in K\\
&\sum_{k=1}^{n} \lambda_{pk}\left(\left(\frac{\alpha_{rk}+\beta_{rk}}{6}\right)^2 + \frac{{\alpha_{rk}}^2}{18}\right) \leq \left(\left(\frac{\alpha_{rp}+\beta_{rp}}{6}\right)^2 + \frac{{\alpha_{rp}}^2}{18}\right), && \forall r, \quad p \in K
\end{aligned}
$$

4 Definition

Let A with $[A]^\gamma = [a(\gamma), \bar{a}(\gamma)]$ and B with $[B]^\gamma = [b(\gamma), \bar{b}(\gamma)]$ be two fuzzy numbers for all $\gamma \in [0, 1]$ Then $\mathrm{A} \leq \mathrm{B}$ if and only if $\bar{a}(\gamma) \leq \bar{b}(\gamma)$ and $a(\gamma) \leq \underline{b}(\gamma)$.

Based on above Definition, let $X_{ik} = (\alpha_{ik}, x_{ik}, \beta_{ik})$ and $B_i = (\alpha_i, b_i, \beta_i)$ be two triangular fuzzy numbers, and the γ-level set of $[X_{ik}]^\gamma = [x_{ik} - (1-\gamma)\alpha_{ik}, x_{ik} + (1-\gamma)\beta_{ik}]$ and $[B_i]^\gamma = [b_i - (1-\gamma)\alpha_i, b_i + (1-\gamma)\beta_i], \forall\gamma\epsilon[0, 1]$. Then the constraint $\sum_{\mathrm{k=1}}^{\mathrm{n}} \mathrm{h_{pk}}\tilde{\mathrm{x}}_{\mathrm{ik}} \leq \theta_{\mathrm{p}}\tilde{\mathrm{x}}_{\mathrm{ip}}$ transfer into two constraint following

$$\sum_{\mathrm{k=1}}^{\mathrm{n}} \lambda_{\mathrm{pk}}\left(x_{ik} - (1-\gamma)\alpha_{ik}\right) \leq \theta_{\mathrm{p}}\left(x_{ip} - (1-\gamma)\alpha_{ip}\right)$$

$$\sum_{\mathrm{k=1}}^{\mathrm{n}} \lambda_{\mathrm{pk}}\left(x_{ik} + (1-\gamma)\alpha_{ik}\right) \leq \theta_{\mathrm{p}}\left(x_{ip} + (1-\gamma)\beta_{ip}\right)$$

And the constraint $\sum_{\mathrm{k=1}}^{\mathrm{n}} \mathrm{c_k}\tilde{\mathrm{x}}_{\mathrm{ik}} + \tilde{L}_i = \widetilde{B}_l$ transfer into two constraint following

$$\sum_{\mathrm{k=1}}^{\mathrm{n}} \mathrm{c_k}\left(x_{ik} - (1-\gamma)\alpha_{ik}\right) + \left(m_i - (1-\gamma)l_i\right) = b_i - (1-\gamma)\alpha_i,$$

$$\sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{c_k}\left(x_{ik}+(1-\gamma)\alpha_{ik}\right)+\left(m_i+(1-\gamma)R_i\right) = b_i+(1-\gamma)\beta_i,$$

Where $L_i = (l_i, m_i, R_i)$ and $[L_i]^\gamma = [m_i-(1-\gamma)l_i, m_i+(1-\gamma)R_i], \forall\gamma\epsilon[0,1]$. And also the constraint $(1-\mathrm{c_k})\,\tilde{x}_{\mathrm{ki}}+\mathrm{Mc_k}+\mathrm{Md_{ki}} \geq \tilde{L}_i+\frac{1}{\mathrm{M}}$ into

$$(1-\mathrm{c_k})\left(x_{ik}-(1-\gamma)\alpha_{ik}\right)+\mathrm{Mc_k}+\mathrm{Md_{ki}} \geq m_i-(1-\gamma)l_i+\frac{1}{\mathrm{M'}}$$

$$(1-\mathrm{c_k})\left(x_{ik}+(1-\gamma)\beta_{ik}\right)+\mathrm{Mc_k}+\mathrm{Md_{ki}} \geq m_i+(1-\gamma)R_i+\frac{1}{\mathrm{M}}$$

Therefore, we present fuzzy model following

$$\begin{aligned}
&\min \sum_{p=1}^{n} c_p\theta_p && (4.13)\\
&\text{S.t} \sum_{k=1}^{n} \lambda_{pk}\left(\left(\frac{\alpha_{rk}+\beta_{rk}}{6}\right)^2+\frac{\beta_{rk}^2}{18}\right) \geq \left(\frac{\alpha_{rp}+\beta_{rp}}{6}\right)^2+\frac{\beta_{rp}^2}{18}, && \forall r, \forall p\\
&\sum_{K=1} \lambda_{pk}\left(y_{rk}+\frac{\beta_{rk}-\alpha_{rk}}{6}\right) \geq y_{rp}+\frac{\beta_{rp}-\alpha_{rp}}{6}, && \forall r, \forall p\\
&\sum_{k=1}^{n} \lambda_{pk}\left(\left(\frac{\alpha_{rk}+\beta_{rk}}{6}\right)^2+\frac{\alpha_{rk}^2}{18}\right) \leq \left(\left(\frac{\alpha_{rp}+\beta_{rp}}{6}\right)^2+\frac{\alpha_{rp}^2}{18}\right), && \forall r\\
&\sum_{\mathrm{k}=1}^{\mathrm{n}} \lambda_{\mathrm{pk}}\left(x_{ik}-(1-\gamma)\alpha_{ik}\right) \leq \theta_{\mathrm{p}}\left(x_{ip}-(1-\gamma)\alpha_{ip}\right), && \forall i, \forall p\\
&\sum_{\mathrm{k}=1}^{\mathrm{n}} \lambda_{\mathrm{pk}}\left(x_{ik}+(1-\gamma)\alpha_{ik}\right) \leq \theta_{\mathrm{p}}\left(x_{ip}+(1-\gamma)\beta_{ip}\right), \quad \forall \mathrm{i}, && \forall i, \forall p\\
&\sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{c_k}\left(x_{ik}-(1-\gamma)\alpha_{ik}\right)+\left(m_i-(1-\gamma)l_i\right)=b_i-(1-\gamma)\alpha_i, && \forall \mathrm{i}\\
&\sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{c_k}\left(x_{ik}+(1-\gamma)\beta_{ik}\right)+\left(m_i+(1-\gamma)R_i\right)=b_i+(1-\gamma)\beta_i, && \forall \mathrm{i}\\
&(1-\mathrm{c_k})\left(x_{ik}-(1-\gamma)\alpha_{ik}\right)+\mathrm{Mc_k}+\mathrm{Md_{ki}} \geq m_i-(1-\gamma)l_i+\frac{1}{\mathrm{M}}, && \forall \mathrm{k}, \forall \mathrm{i}\\
&(1-\mathrm{c_k})\left(x_{ik}+(1-\gamma)\beta_{ik}\right)+\mathrm{Mc_k}+\mathrm{Md_{ki}} \geq m_i+(1-\gamma)R_i+\frac{1}{\mathrm{M}}, && \forall \mathrm{k}, \forall \mathrm{i}\\
&\sum_{\mathrm{i}} \mathrm{d_{ki}} \leq |\mathrm{I}|-1, && \forall \mathrm{k}\\
&\mathrm{c_k}, \mathrm{d_{ki}} \in \{0,1\} && \forall \mathrm{k}, \forall \mathrm{i}\\
&\lambda_{\mathrm{pk}} \geq 0, && \forall \mathrm{k}, \forall p\\
&l_i \leq m_i \leq R_i, && \forall i\\
&l_i, m_i, R_i \geq 0, && \forall \mathrm{i}\\
&\mathrm{M} \gg 0
\end{aligned}$$

Where the variable θ_{p} is accurate. Whilst software is capable to solve model (**??**), it can be linearized as follows: $0 \leq \mathrm{a}_p = c_p\theta_p \leq 1$

$$\mathrm{a}_p \leq c_p$$

$\mathrm{a}_p \leq \theta_p$

$\mathrm{a}_p \geq c_p + \theta_p - 1$ Then model (??) becomes:

$$\min \sum_{p=1}^{n} \mathrm{a}_p \qquad (??)$$

$$\text{S.t} \sum_{k=1}^{n} \lambda_{pk} \left(\left(\frac{\alpha_{rk}+\beta_{rk}}{6} \right)^2 + \frac{{\beta_{rk}}^2}{18} \right) \geq \left(\frac{\alpha_{rp}+\beta_{rp}}{6} \right)^2 + \frac{{\beta_{rp}}^2}{18}, \qquad \forall r, \forall p \qquad (4.14a)$$

$$\sum_{K=1} \lambda_{pk} \left(y_{rk} + \frac{\beta_{rk}-\alpha_{rk}}{6} \right) \geq y_{rp} + \frac{\beta_{rp}-\alpha_{rp}}{6}, \qquad \forall r, \forall p \qquad (4.14b)$$

$$\sum_{k=1}^{n} \lambda_{pk} \left(\left(\frac{\alpha_{rk}+\beta_{rk}}{6} \right)^2 + \frac{{\alpha_{rk}}^2}{18} \right) \leq \left(\left(\frac{\alpha_{rp}+\beta_{rp}}{6} \right)^2 + \frac{{\alpha_{rp}}^2}{18} \right), \qquad \forall r \qquad (4.14c)$$

$$\sum_{\mathrm{k}=1}^{\mathrm{n}} \lambda_{\mathrm{pk}} \left(x_{ik} - (1-\gamma)\alpha_{ik} \right) \leq \theta_{\mathrm{p}} \left(x_{ip} - (1-\gamma)\alpha_{ip} \right), \qquad \forall i, \forall p \qquad (4.14d)$$

$$\sum_{\mathrm{k}=1}^{\mathrm{n}} \lambda_{\mathrm{pk}} \left(x_{ik} + (1-\gamma)\alpha_{ik} \right) \leq \theta_{\mathrm{p}} \left(x_{ip} + (1-\gamma)\beta_{ip} \right), \qquad \forall \mathrm{i}, \forall p \qquad (4.14e)$$

$$\sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{c}_{\mathrm{k}} \left(x_{ik} - (1-\gamma)\alpha_{ik} \right) + (m_i - (1-\gamma)l_i) = b_i - (1-\gamma)\alpha_i, \qquad \forall \mathrm{i} \qquad (4.14f)$$

$$\sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{c}_{\mathrm{k}} \left(x_{ik} + (1-\gamma)\beta_{ik} \right) + (m_i + (1-\gamma)R_i) = b_i + (1-\gamma)\beta_i, \qquad \forall \mathrm{i} \qquad (4.14g)$$

$$(1-\mathrm{c}_{\mathrm{k}}) \left(x_{ik} - (1-\gamma)\alpha_{ik} \right) + \mathrm{Mc}_{\mathrm{k}} + \mathrm{Md}_{\mathrm{ki}} \geq m_i - (1-\gamma)l_i + \frac{1}{\mathrm{M}}, \qquad \forall \mathrm{k}, \forall \mathrm{i} \qquad (4.14h)$$

$$(1-\mathrm{c}_{\mathrm{k}}) \left(x_{ik} + (1-\gamma)\beta_{ik} \right) + \mathrm{Mc}_{\mathrm{k}} + \mathrm{Md}_{\mathrm{ki}} \geq m_i + (1-\gamma)R_i + \frac{1}{\mathrm{M}}, \qquad \forall \mathrm{k}, \forall \mathrm{i} \qquad (4.14i)$$

$$\sum_{\mathrm{i}} \mathrm{d}_{\mathrm{ki}} \leq |\mathrm{I}|-1 \qquad \forall k \qquad (4.14j)$$

$$\mathrm{a}_p \leq c_p, \qquad \forall p$$

$$\mathrm{a}_p \leq \theta_p, \qquad \forall p$$

$$\mathrm{a}_p \geq c_p + \theta_p - 1, \qquad \forall p$$

$$\mathrm{c}_{\mathrm{k}}, \mathrm{d}_{\mathrm{ki}} \in \{0,1\} \qquad \forall k, \forall i$$

$$\lambda_{\mathrm{pk}} \geq 0, \qquad \forall k, \forall p$$

$$l_i \leq m_i \leq R_i, \qquad \forall i$$

$$l_i, m_i, R_i \geq 0, \qquad \forall i$$

$$\mathrm{M} \gg 0$$

Let $c_p = 0$, then from the last constraints set we have $a_p = 0$ which shows the p^{th} project is excluded from the evaluation. Otherwise, it can achieve non-negative values. The constraints set (??)-(??) are for feasibility of selected projects, similar to the model (??) that explained in section ??. In this model, the selected projects evaluates according to three index the lower possibilistic semivariance, the upper possibilistic semivariance and the possibilistic mean value. The constraints set (??)-(??) increases the upper possibilistic semivariance and the possibilistic mean value of the project portfolio, and the constraint (**??**) prevents the lower possibilistic semivariance of the portfolio be increased. That is, any project entered into the project portfolio must increase the desirable risk of the portfolio and vice versa, decreases the undesirable risk. Therefore, projects selects in terms of better performance as much as possible.

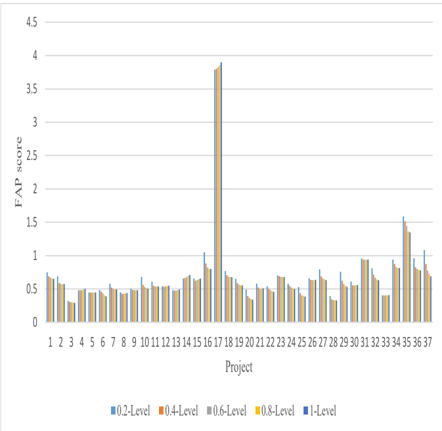

Figure 1: FAP results and ranking of all Projects

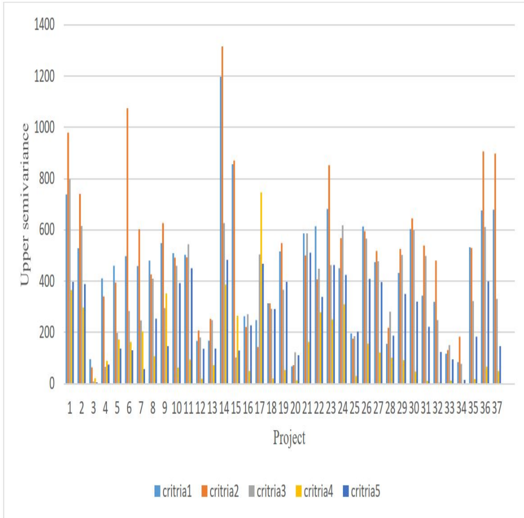

Figure 2: Upper semivariance

Theorem 4.1. *The model* (**??**) *is feasible if* $\exists t; t \in P \quad \& \widetilde{X_t} \leq \tilde{B}$.

Proof. Let $X = (\alpha, x, \beta)_{LR}$ and $B = (\alpha, b, \beta)_{LR}$ be two triangular fuzzy numbers, then

$$\exists t; t \in P \quad \& x_t - (1-\gamma)\alpha_t \leq b - (1-\gamma)\alpha,$$
$$\exists t; t \in P \& x_t + (1-\gamma)\beta_t \leq b + (1-\gamma)\beta.$$

Without loss of generality, suppose that $t = j_1$ and $P_1 = P - \{\mathrm{j}_1\}$. We consider the following two cases:

Case A: If $\sim \left(\exists \mathrm{j}; \mathrm{j} \in \mathrm{P}_1 \& \widetilde{\mathrm{X}}_\mathrm{j} \leq \widetilde{\mathrm{B}} - \widetilde{\mathrm{X}}_{\mathrm{j}_1}\right)$. then $\forall \mathrm{j}\left(\mathrm{j} \in \mathrm{P}_1 \Rightarrow \widetilde{\mathrm{X}}_\mathrm{j} \not\leq \overleftarrow{\ } \mathrm{B} - \widetilde{\mathrm{X}}_{\mathrm{j}_1}\right)$, and hence $\forall \mathrm{j}(\mathrm{j} \in \mathrm{P}_1 \Rightarrow \exists \mathrm{i}; \quad x_{ij} - (1-\gamma)\alpha_{ij} > (b_i - (1-\gamma)\alpha_i) - \left(\mathrm{x}_{\mathrm{ij}_1} + (1-\gamma)\alpha_{\mathrm{ij}_1}\right)$), and $x_{ij} + (1-\ \gamma)\beta_{ij} > (b_i + (1-\gamma)\beta_i) - \left(\mathrm{x}_{\mathrm{ij}_1} - (1-\gamma)\beta_{\mathrm{ij}_1}\right)$.

If $c_{\mathrm{j}_1} = 1, c_k = 0\,(k \neq \mathrm{j}_1)$, we will have $\forall j(j \in \mathrm{P}_1 \Rightarrow \exists \mathrm{i}; x_{ij} - (1-\gamma)\alpha_{ij} > m_i - (1-\gamma)l_i$ & $x_{ij} + (1-\gamma)\beta_{ij} > m_i + (1-\gamma)R_i)$

If $i = i_j$ and $c_{j_1} = 1 \& c_k = 0, k \neq \mathrm{j}_1$ then $\lambda_{kp} = \begin{cases} 1, & \mathrm{k} = p \\ 0, & \mathrm{k} \neq p \end{cases}$, $\mathrm{a_p} = 0, \forall p$ And $\forall i, d_{j_1 i} = 0$ & $d_{ki} = \begin{cases} 1, & i = i_j \\ 0, & i \neq i_j \end{cases}$, $j \in \mathrm{P}_1$ is a feasible solution.

Case B: $\exists \mathrm{j}; \mathrm{j} \in \mathrm{P}_1 \& \widetilde{\mathrm{X}}_\mathrm{j} \leq \widetilde{\mathrm{B}} - \widetilde{\mathrm{X}}_{\mathrm{j}_1}$. without loss of generality, let that $\mathrm{j} = \mathrm{j}_2$ and $\mathrm{P}_2 = \mathrm{P}_1 - \{\mathrm{j}_2\}$. Then following two cases are considered:

Case B1: Suppose $\sim (\exists \mathrm{j}; j \in P_2 \& \tilde{X}_j \leq \tilde{B} - \tilde{X}_{j_1} - \tilde{X}_{j_2})$. then $\forall j(j \in P_2 \Rightarrow \tilde{X}_j \neq \tilde{B} - \tilde{X}_{\mathrm{j}_1} - \tilde{X}_{\mathrm{j}_2})$, so

$\forall \mathrm{j}(j \in P_2 \Rightarrow \exists i, x_{ij} - (1-\gamma)\alpha_{ij} > (b_i - (1-\gamma)\alpha_i) - (x_{ij_1} + (1-\gamma)\alpha_{ij_1}) - (x_{ij_2} + (1-\gamma)\alpha_{ij_2})$

and

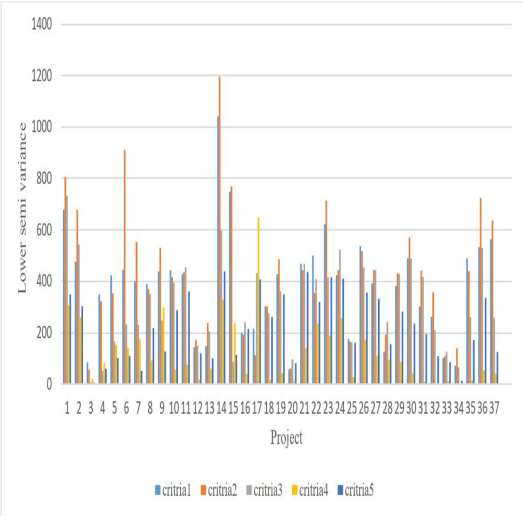

Figure 3: lower semivariance

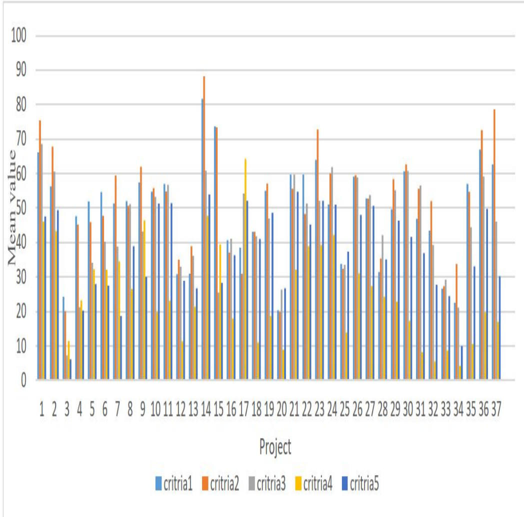

Figure 4: Mean value

$\forall$j$(j \in P_2 \Rightarrow \exists i, x_{ij} + (1-\gamma)\beta_{ij} > (b_i + (1-\gamma)\beta_i) - (x_{ij_1} - (1-\gamma)\beta_{ij_1}) - (x_{ij_2} - (1-\gamma)\beta_{ij_2})$.

If $c_{\mathrm{j}_1} = c_{\mathrm{j}_2} = 1, c_k = 0(k \neq \mathrm{j}_1$ and $k \neq \mathrm{j}_2)$, then for all j we will have

$$(j \in \mathrm{P}_2 \Rightarrow \exists \mathrm{i}; x_{ij} + (1-\gamma)\beta_{ij} > m_i + (1-\gamma)R_i).$$

Now if $i = i_j$ and $\mathrm{c}_{\mathrm{j}_1} = c_{\mathrm{j}_2} = 1\&c_k = 0(\mathrm{k} \neq \mathrm{j}_1$ and $\mathrm{k} \neq \mathrm{j}_2)$ then $\lambda_{\mathrm{kp}} = \begin{cases} 1, & \mathrm{k} = p \\ 0, & \mathrm{k} \neq p \end{cases}, \mathrm{a}_p = 0, \forall p$

And

$$\forall i, d_{\mathrm{j}_1 i} = d_{\mathrm{j}_2 i} = 0 \ \& \ d_{ki} = \begin{cases} 1, & i = i_j \\ 0, & i \neq i_j \end{cases}, j \in P_2$$

is a feasible solution model (**??**) and completes the proof.

CaseB2: $\exists \mathrm{j}, \mathrm{j} \in \mathrm{P}_2 \& \widetilde{\mathrm{X}}_{\mathrm{j}} \leq \widetilde{\mathrm{B}} - \widetilde{\mathrm{X}}_{\mathrm{j}_1} - \widetilde{\mathrm{X}}_{\mathrm{j}_2}$. without loss of generality, let $\mathrm{j} = \mathrm{j}_3$ and $\mathrm{P}_3 = \mathrm{P}_2 - \{j_3\}$. Then we consider the following two cases:

Case1: $\sim \left(\exists j; j \in P_3 \& \widetilde{X}_j \leq \widetilde{B} - \widetilde{X}_{j_1} - \widetilde{X}_{j_2} - \widetilde{X}_{j_3}\right)$.

By continuing in the same way, we can find the feasible solution. □

Lemma 4.1. *For all the optimal solution of the model* (**??**)

$$\forall p H_p^* = \left(\lambda_{p1}^*, \cdots, \lambda_{pn}^*\right) \neq 0$$

Proof. (reductio ad absurdum), Assume that there is an optimal solution $H_p^* = 0$.

From the first set of constraints, we have $\left(\frac{\alpha_{rp}+\beta_{rp}}{6}\right)^2 + \frac{\beta_{rp}^2}{18} \leq 0$ that is impassible. Thus for each the optimal solution of the model (**??**)

$\forall$p $\exists$t; $\lambda_{pt}^* > 0$. □

Lemma 4.2. *For each the optimal solution of the model* (**??**)

$$\forall p \exists i \sum_{k=1}^{n} \lambda_{pk}^* \left(x_{ik} - (1-\gamma)\alpha_{ik}\right) = \theta_p^* \left(x_{ip} - (1-\gamma)\alpha_{ip}\right)$$

or

$$\forall p \exists i \sum_{k=1}^{n} \lambda_{pk}^* \left(x_{ik} + (1-\gamma)\alpha_{ik}\right) = \theta_p^* \left(x_{ip} + (1-\gamma)\alpha_{ip}\right)$$

Table 1: Fuzzy data set for 37 projects

Project	Critria1	Critria2	Critria3
1	(61.003, 64.809, 69.354)	(62.099, 71.806, 83.708)	(63.422, 67.126, 72.049)
2	(50.600, 54.909, 59.163)	(60.823, 66.397, 69.556)	(53.446, 58.779, 64.329)
3	(21.107, 23.504, 25.536)	(16.776, 19.391, 21.194)	(05.544, 06.734, 08.695)
4	(41.672, 45.734, 53.526)	(42.673, 44.557, 46.497)	(15.091, 20.066, 22.210)
5	(48.165, 50.841, 54.714)	(43.269, 44.557, 51.355)	(28.779, 32.750, 37.334)
6	(48.644, 53.138, 57.636)	(67.433, 44.557, 86.558)	(33.347, 38.325, 45.074)
7	(45.420, 49.552, 55.919)	(55.079, 58.174, 62.621)	(35.939, 38.215, 39.852)
8	(42.867, 49.314, 58.941)	(43.515, 48.880, 54.013)	(42.092, 49.260, 53.298)
9	(44.992, 54.337, 63.355)	(51.377, 59.490, 66.191)	(34.919, 41.390, 45.643)
10	(47.794, 52.843, 58.956)	(45.473, 53.554, 58.619)	(44.591, 51.253, 56.464)
11	(46.268, 54.841, 59.153)	(47.827, 53.153, 57.585)	(47.111, 54.274, 62.027)
12	(26.951, 29.679, 34.010)	(28.740, 33.434, 38.480)	(26.733, 31.470, 35.930)
13	(27.810, 30.051, 33.735)	(36.652, 38.403, 40.089)	(31.410, 34.351, 42.089)
14	(73.200, 78.811, 90.397)	(80.489, 86.166, 92.952)	(58.453, 60.115, 62.736)
15	(62.289, 71.367, 76.268)	(63.440, 71.231, 76.607)	(20.636, 24.494, 26.949)
16	(29.264, 38.070, 44.828)	(31.361, 35.805, 38.998)	(35.607, 39.942, 42.720)
17	(33.372, 37.181, 41.194)	(22.364, 29.239, 32.673)	(46.664, 52.132, 59.061)
18	(41.755, 42.708, 44.221)	(41.755, 42.708, 44.221)	(39.555, 41.200, 43.071)
19	(45.493, 52.520, 60.587)	(50.526, 55.374, 60.735)	(46.109, 46.788, 47.329)
20	(16.912, 19.557, 21.781)	(17.265, 19.014, 22.713)	(20.891, 24.833, 30.249)
21	(46.456, 56.550, 65.529)	(48.251, 54.024, 58.004)	(46.456, 56.550, 65.529)
22	(48.648, 56.738, 66.519)	(42.729, 46.603, 52.737)	(47.295, 50.233, 54.124)
23	(58.157, 62.495, 66.830)	(59.239, 69.723, 77.487)	(47.167, 50.758, 55.443)
24	(48.867, 50.274, 53.561)	(44.480, 56.614, 65.019)	(51.017, 59.473, 65.723)
25	(30.677, 32.871, 36.158)	(30.496, 31.868, 33.565)	(29.260, 32.529, 35.365)
26	(52.801, 57.149, 64.523)	(51.635, 57.481, 63.748)	(45.786, 55.863, 64.313)
27	(43.523, 50.355, 58.124)	(47.514, 50.712, 59.810)	(49.576, 52.777, 55.505)
28	(24.307, 29.890, 33.595)	(31.815, 34.267, 38.329)	(35.145, 40.669, 43.926)
29	(44.723, 48.103, 53.971)	(45.367, 55.747, 61.382)	(46.321, 53.047, 59.087)
30	(48.076, 57.672, 66.003)	(54.752, 60.814, 65.864)	(48.190, 57.859, 65.686)
31	(39.657, 45.465, 48.227)	(45.993, 52.896, 62.148)	(45.185, 54.163, 59.352)
32	(35.399, 41.366, 47.850)	(38.563, 48.321, 60.852)	(32.382, 37.708, 41.686)
33	(22.941, 25.760, 28.291)	(23.158, 26.122, 30.584)	(24.866, 27.942, 32.606)
34	(19.106, 21.719, 24.212)	(24.485, 31.662, 37.337)	(18.009, 20.222, 23.485)
35	(51.809, 55.842, 58.844)	(46.297, 52.265, 61.262)	(34.812, 42.165, 48.418)
36	(49.182, 63.388, 70.649)	(57.886, 68.713, 81.386)	(52.113, 57.054, 64.714)
37	(52.358, 59.816, 69.397)	(49.579, 72.834, 84.585)	(34.108, 43.517, 49.621)

Proof. (reductio ad absurdum), Assume that this constraints be strict in an optimal solution (for a constant p '), then

$$\forall i, \theta_p^* > \frac{\sum_{k=1}^{n} \lambda_{pk}^* (x_{ik} + (1-\gamma)\alpha_{ik})}{x_{ip} + (1-\gamma)\alpha'_{ip}}, \quad \text{and}$$

$$\forall i, \theta_p^* > \frac{\sum_{k=1}^{n} \lambda_{pk}^* (x_{ik} - (1-\gamma)\alpha_{ik})}{x_{ip} - (1-\gamma)\alpha_{ip}}.$$

We consider

$\bar{\theta}_p = \max\left\{\max_i \left\{\frac{\sum_{k=1}^{n} \lambda_{pk}^*(x_{ik}+(1-\gamma)\alpha_{ik})}{x_{i\dot{p}}+(1-\gamma)\alpha_{i\dot{p}}}\right\}, \max_i \left\{\frac{\sum_{k=1}^{n} \lambda_{pk}^*(x_{ik}-(1-\gamma)\alpha_{ik})}{x_{i\dot{p}}-(1-\gamma)\alpha_{i\dot{p}}}\right\}\right\}$. It is obviously $\bar{\theta}_{\dot{p}} < \theta_p^*$. we define $f = \theta_p^* - \bar{\theta}_{\dot{p}} > 0$, obviously, $\bar{a}'_p = a_p^* - f < a_p^*$ is a feasible solution. Assuming that all components of the previous optimal solution are constant except θ_p^* which is changed to $\bar{\theta}_{\dot{p}}$ and also, a_p^* to $\bar{a}_{\dot{p}}$.

$\bar{a}_{\dot{p}} < a_{\dot{p}}^* \leq c_{\dot{p}}^*, \quad \bar{a}_{\dot{p}} \leq \overline{\theta_p} \Leftrightarrow a_{\dot{p}}^* - f \leq \theta_{\dot{p}}^* - f \Leftrightarrow$

Continued of table **??**

Project	Critria4	Critria5
1	(38.717, 44.055, 50.812)	(42.629, 46.035, 51.94)
2	(36.611, 41.904, 45.049)	(36.833, 46.547, 53.782)
3	(10.516, 11.194, 12.202)	(05.159, 05.915, 06.544)
4	(21.067, 22.693, 24.274)	(16.729, 19.185, 23.415)
5	(27.996, 31.240, 34.334)	(20.386, 25.862, 32.776)
6	(26.614, 30.935, 33.624)	(23.161, 26.334, 30.463)
7	(30.019, 33.333, 37.424)	(16.538, 18.182, 19.686)
8	(21.441, 25.538, 27.335)	(33.224, 37.510, 41.909)
9	(38.405, 44.564, 49.717)	(25.411, 28.938, 31.892)
10	(17.371, 19.382, 20.584)	(34.232, 47.803, 55.315)
11	(18.533, 21.879, 26.268)	(40.789, 48.608, 57.426)
12	(9.266, 10.987, 11.77)	(25.124, 27.999, 30.398)
13	(17.054, 20.443, 22.839)	(20.067, 24.609, 32.884)
14	(40.589, 45.871, 51.928)	(48.810, 52.718, 56.348)
15	(35.516, 38.335, 42.132)	(24.194, 27.358, 29.755)
16	(13.924, 17.043, 18.951)	(34.717, 35.783, 38.093)
17	(57.700, 61.968, 71.445)	(45.824, 50.373, 56.476)
18	(09.494, 10.558, 12.098)	(37.261, 39.917, 44.115)
19	(13.959, 17.748, 19.901)	(42.597, 47.079, 51.931)
20	(07.812, 08.657, 09.555)	(18.457, 24.946, 29.352)
21	(26.276, 30.894, 33.767)	(46.819, 52.602, 59.600)
22	(34.039, 37.243, 44.279)	(42.460, 44.585, 46.410)
23	(28.378, 36.740, 43.829)	(47.167, 50.758, 55.443)
24	(35.253, 40.201, 46.952)	(48.772, 50.578, 51.333)
25	(12.167, 13.553, 14.263)	(27.213, 35.497, 38.659)
26	(33.734, 31.842, 29.115)	(42.757, 46.370, 52.874)
27	(24.449, 26.720, 28.385)	(40.375, 48.544, 52.832)
28	(23.130, 23.854, 25.595)	(27.543, 33.579, 36.450)
29	(21.986, 22.496, 24.266)	(36.240, 44.002, 50.522)
30	(14.666, 16.765, 18.063)	(30.901, 38.503, 49.984)
31	(06.730, 07.785, 09.367)	(31.867, 35.760, 38.772)
32	(05.020, 05.373, 05.908)	(23.801, 26.941, 28.963)
33	(08.143, 08.567, 08.827)	(21.551, 23.841, 25.106)
34	(03.311, 03.985, 04.561)	(08.414 09.643, 10.208)
35	(09.252, 10.247, 11.031)	(30.954, 32.478, 34.402)
36	(15.805, 18.835, 21.875)	(40.663, 47.725, 52.994)
37	(13.888, 16.213, 18.999)	(25.073, 29.036, 31.994)

$a^*_{\dot{p}} \leq \theta^*_{\dot{p}}$,

$\bar{a}_{\dot{p}} \leq c^*_{\dot{p}} + \overline{\theta_p} - 1 \Leftrightarrow a^*_{\dot{p}} - f \leq c^*_{\dot{p}} + \theta^*_{\dot{p}} - f - 1 \Leftrightarrow a^*_{\dot{p}} \leq c^*_p + \theta^*_{\dot{p}}$, so $\bar{a}_{\dot{p}}$ is the feasible solution which $\bar{a}^*_p < a^*_p$ then $\sum_{p=1}^{n} a^*_p + \bar{a}^*_p < \sum_{p=1}^{n} a^*_p$. This contradicts the optimality of a^*. □

5 An applied example

In this section, we consider fuzzy data set for 37 projects under five criteria with $(889.650, 1023.569, 1123.017)$ units of accessible resources, which are seen in Table **??**. In addition, the cost of each project is listed in Table **??**. The data were generated for each project based on real data using software [**?**]. The proposed model is implemented using CPLEX

Table 2: Fuzzy data set for 37 projects

Project	Cost
1	(78.962, 83.578, 90.500)
2	(83.130, 88.054, 96.686)
3	(50.051, 53.755, 57.247)
4	(52.725, 65.804, 73.368)
5	(67.962, 77.904, 89.209)
6	(85.400, 93.316, 108.04)
7	(85.483, 88.450, 92.285)
8	(71.805 83.671, 96.716)
9	(84.633, 96.467, 105.637)
10	(72.105, 80.947, 95.529)
11	(66.771, 76.148, 88.229)
12	(40.000, 45.689, 56.209)
13	(49.995, 57.876, 66.438)
14	(79.541, 90.292, 113. 035)
15	(62.745, 80.938, 93.699)
16	(33.155, 35.912, 37.932)
17	(25.028, 30.160, 34.693)
18	(46.540, 49.133, 54.132)
19	(72.152, 75.746, 80.053)
20	(43.878, 50.427, 60.108)
21	(73.469, 84.100, 97.111)
22	(77.981, 85.978, 89.727)
23	(64.099, 76.239, 87.662)
24	(73.230, 87.817, 95.449)
25	(61.778, 68.889, 77.589)
26	(53.768, 67.188, 74.211)
27	(57.881, 63.199, 66.746)
28	(76.782, 85.049, 102.071)
29	(73.897, 76.763, 81.123)
30	(69.613, 80.007, 88.174)
31	(35.501, 42.683, 48.746)
32	(50.490, 52.349, 54.644)
33	(43.969, 49.835, 59.291)
34	(21.623 26.468, 33.420)
35	(33.808, 35.651, 38.406)
36	(50.884, 62.204, 68.806)
37	(68.804, 72.340, 76.710)

solver in Gams software version 25.1.2. All data sets considered non-symmetric triangular fuzzy number. Tables **??**, **??** and **??** show the lower possibilistic semivariance, upper possibilistic semivariance and possibilistic mean value, which are obtained from (2.6), (2.7) and (2.8), respectively. Moreover, it can be clearly observed in Figures **??**, **??** and **??**. The proposed model is solved at five different γ - levels and the selected projects are shown in Table **??**. The decision makers can choose the satisfying portfolio based on the preferences between different γ - levels. The model according to each γ - level, increases desirable risk and return, and also decreases undesirable risk. Additionally, note that the obtained portfolios are welldiversified. With diverse portfolios, an unexpected bad outcome for one project will likely be offset by a good outcome on another. For instance, in Table **??** under $\gamma = 0.2$, projects selected are in form of

Table 3: Lower possibilistic semivariance

project	critria1	critria2	critria3	critria4	critria5
1	678.769	804.785	733.253	305.929	349.382
2	476.907	677.71	543.997	259.697	303.456
3	085.183	55.683	007.339	020.480	005.283
4	348.216	322.035	051.301	081.762	060.313
5	422.884	352.725	167.428	151.461	101.594
6	445.22	911.324	232.608	140.145	109.678
7	399.876	553.352	231.319	176.412	051.644
8	390.001	369.412	351.187	091.626	218.129
9	438.545	530.595	248.025	297.649	127.085
10	443.447	415.854	394.134	056.780	287.842
11	427.64	435.737	454.167	074.835	360.380
12	143.582	171.403	148.777	017.062	120.698
13	148.183	238.220	204.869	060.365	100.255
14	1041.124	1195.521	597.786	329.287	439.529
15	748.830	768.401	086.556	237.555	113.366
16	200.066	192.150	240.857	040.792	214.218
17	216.319	111.927	431.468	648.250	407.360
18	302.19	302.190	276.563	017.958	261.078
19	427.561	485.688	360.632	042.672	349.015
20	057.477	060.956	096.894	011.769	082.417
21	468.249	442.957	468.249	138.501	436.362
22	499.908	354.591	409.985	234.750	319.544
23	621.840	714.237	416.063	189.569	416.063
24	424.096	442.971	523.158	256.756	410.512
25	176.363	165.662	163.575	027.628	161.673
26	537.245	517.933	453.181	172.943	355.600
27	392.239	445.378	443.266	110.748	331.884
28	125.953	192.905	242.293	095.670	155.898
29	381.689	430.880	427.837	086.278	282.064
30	489.906	570.661	489.230	041.705	234.781
31	301.915	442.366	416.982	009.714	195.024
32	262.127	357.154	210.646	004.717	108.806
33	102.147	110.022	126.102	011.683	086.271
34	72.404	139.472	065.844	002.330	013.566
35	489.234	440.438	259.750	016.183	171.881
36	533.256	724.952	530.002	053.316	335.516
37	564.083	636.559	259.368	040.759	125.388

$\{6, 8, 9, 13, 14, 15, 21, 22, 23, 24, 26, 30\}, z = 12$. It can be found that among the projects selected, projects $\{14, 6, 15, 23, 30, 26\}$ are risky, respectively. The project 14 has the higher undesirable risk in criteria 1 and 2, but note that the project has the higher desirable risk and return in comparison to other projects. And also project 13 is a riskless project. Projects $\{8, 9, 21, 22, 24\}$ have fewer risks in comparison to other projects. Success a riskless project is valuable, because of advancing one or more organization objectives. With selecting different of γ-levels, we will have different portfolios. The selected projects could be different in each γ - level and it was due to values of γ and resource constraints. According to the γ-level, the model uses of riskless projects for decreasing risk of portfolio. In the $\gamma = 0.8$, projects selected listed in form of $\{1, 2, 3, 4, 6, 7, 8, 9, 14, 21, 22, 24\}$. It can be easily observed that projects $\{1, 2, 6, 14\}$ are risky in comparison to

Table 4: Upper possibilistic semivariance

project	critria1	critria2	critria3	critria4	critria5
1	739.247	979.826	798.180	366.090	398.301
2	529.123	740.965	615.210	297.980	388.780
3	096.660	065.003	009.832	022.608	006.184
4	410.909	340.979	066.054	089.841	075.224
5	460.315	395.233	198.850	173.410	138.187
6	498.313	1074.94	283.700	163.610	131.431
7	458.985	602.669	247.800	204.160	057.979
8	480.915	426.292	410.570	107.600	254.380
9	549.077	627.354	296.020	353.030	147.718
10	509.644	491.876	460.790	063.555	392.727
11	503.104	492.882	544.610	094.087	451.158
12	167.489	207.776	180.790	019.988	136.966
13	168.441	252.873	248.470	073.186	137.959
14	1197.42	1315.61	626.620	387.570	483.567
15	856.435	870.845	103.250	266.100	130.034
16	264.131	222.002	271.810	049.974	227.874
17	248.722	143.448	504.280	746.870	467.899
18	313.969	313.969	292.700	021.082	292.064
19	516.515	548.791	366.970	053.850	398.033
20	067.944	073.056	123.480	013.450	111.355
21	586.909	500.529	586.910	163.490	511.926
22	614.250	407.671	448.460	279.310	339.046
23	682.062	852.846	463.240	251.550	463.241
24	450.807	567.916	618.540	310.190	424.755
25	196.714	176.584	185.490	030.706	203.560
26	613.649	595.579	566.500	156.820	409.350
27	474.692	518.692	477.880	122.300	396.388
28	155.830	218.289	280.870	102.340	187.564
29	432.396	525.857	502.590	092.137	350.905
30	603.523	645.121	599.920	047.881	320.533
31	343.758	539.423	499.260	012.072	222.122
32	319.712	480.258	248.930	005.256	123.938
33	117.374	132.193	150.820	012.328	095.486
34	84.691	183.613	078.468	002.877	015.422
35	532.481	529.861	322.660	018.188	184.400
36	676.168	906.779	611.790	066.023	399.677
37	679.338	897.478	331.530	050.097	147.330

other projects as well as the project 3 has riskless among 37 projects (see Table **??** and figure**??**). That is, the project 3 has the minimum of downside risk among accessible projects. In the $\gamma = 1$, projects selected are in form of $\{2, 3, 5, 6, 7, 9, 14, 21, 22, 23, 24, 29\}$. We can see that, in all cases, portfolios are including riskless projects and risky, and the model is committed to its presentation goals. Using riskless project, it can offset greedy project selection.that is, whit diversifiable risks, an unexpected bad outcome for one project will be offset by a good outcome on another. Also, it can be observed the FAP model of ranking results in figure **??**.

Figure **??** shows results of the FAP model for ranking in different of γ- levels. As can be seen, project 17 have high ranking, but in comparison to other projects, it havent the highest score the upper semivariance and the mean value (See figures **??**, **??** and **??**). For example, project 14 have

Table 5: Possibilistic mean value

project	critria1	critria2	critria3	critria4	critria5
1	66.201	75.407	68.564	46.071	47.587
2	56.336	67.853	60.593	43.310	49.372
3	24.242	20.127	07.259	11.475	06.146
4	47.710	45.194	21.252	23.228	20.299
5	51.933	45.905	34.176	32.296	27.927
6	54.637	47.745	40.279	32.103	27.551
7	51.302	59.431	38.867	34.567	18.707
8	51.993	50.630	51.128	26.520	38.957
9	57.398	61.959	43.177	46.449	30.018
10	54.703	55.745	53.232	19.918	51.317
11	56.989	54.779	56.760	23.168	51.381
12	30.855	35.057	33.003	11.404	28.878
13	31.038	38.976	36.131	21.407	26.745
14	81.677	88.243	60.829	47.761	53.974
15	73.697	73.425	25.546	39.438	28.285
16	40.664	37.078	41.127	17.881	36.346
17	38.485	30.957	54.198	64.259	52.148
18	43.119	43.119	41.786	10.992	41.059
19	55.036	57.076	46.991	18.738	48.635
20	20.368	19.922	26.393	08.947	26.762
21	59.729	55.65	59.729	32.142	54.732
22	59.716	48.271	51.371	38.950	45.243
23	63.941	72.764	52.137	39.315	52.137
24	51.056	60.037	61.924	42.151	51.005
25	33.785	32.380	33.547	13.902	37.405
26	59.103	59.500	58.951	31.072	48.056
27	52.788	52.761	53.765	27.376	50.620
28	31.438	35.353	42.132	24.265	35.064
29	49.644	58.416	55.175	22.876	46.382
30	60.66	62.666	60.775	17.331	41.684
31	46.893	55.589	56.524	08.225	36.911
32	43.441	52.036	39.259	05.521	27.801
33	26.652	27.360	29.232	08.681	24.434
34	22.57	33.804	21.135	04.193	09.942
35	57.014	54.759	44.433	10.543	33.053
36	66.966	72.630	59.154	19.847	49.780
37	62.656	78.668	46.103	17.065	30.190

high upper semivariance and the mean value in criteria 1, 2, 3 and 5.

Figure ?? shows results of the upper semivariance projects under five criteria.

Figure ?? shows results of the lower semivariance projects under five criteria.

Figure ?? shows results of the mean value projects under five criteria.

6 Conclusion

In this paper, a novel fuzzy project selection model based on risk oriented DEA is proposed. Indeed, a model of project evaluation as well as a model of project portfolio selection were presented. In this approach, proposed a new index as an opportunity factor and desirable risk. Also, introduces downside risk as an undesirable risk. In other words, the proposed model includes down-

Table 6: Results of model (14) with γ Level set

project	0.2	0.4	0.6	0.8	1
1	0	1	0	1	0
2	0	0	1	1	1
3	0	0	0	1	1
4	0	0	1	1	0
5	0	1	0	0	1
6	1	1	1	1	1
7	0	0	1	1	1
8	1	1	1	1	0
9	1	1	1	1	1
10	0	0	1	0	0
11	0	0	0	0	0
12	0	0	0	0	0
13	1	0	0	0	0
14	1	0	0	1	1
15	1	1	1	0	0
16	0	0	0	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	1	0	0
20	0	0	0	0	0
21	1	1	0	1	1
22	1	1	1	1	1
23	1	1	0	0	1
24	1	1	1	1	1
25	0	1	0	0	0
26	1	0	0	0	0
27	0	0	0	0	0
28	0	1	0	0	0
29	0	0	0	0	1
30	1	0	1	0	0
31	0	0	0	0	0
32	0	0	0	0	0
33	0	0	0	0	0
34	0	0	0	0	0
35	0	0	0	0	0
36	0	0	0	0	0
37	0	0	0	0	0

side risk-return index and upper risk-return index. To comprehend the impact of risk on project selection, it is necessary to consider the risk of selecting the project and the risk of not doing it. Because, avoiding risk dont should to deprive organization of opportunities. Therefore, decision makers are able to get more information of the project portfolio selection and can to make better decisions. In the existing literature, there are no studies in this field. Also, the proposed model can be employed as an expert and analyzer system for project portfolio selection. It should be noted that project selection is a problem of decision making under uncertainty. Hence, the model considers uncertain factors. Moreover, the model was presented in the form of fuzzy linear programing. The proposed method provides a general framework, in order to increasing, decreasing indices as well as controlling the resource, unlike the existing models, there is no need for separate modeling or multi- objective. For further research, the proposed model could be applied in decision oriented systems and stock markets that could be an interesting research direction.

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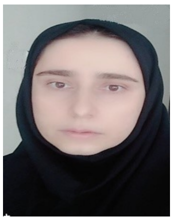

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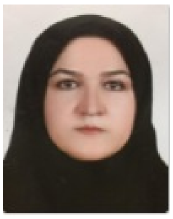

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