Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 14, No. 1, 2022 Article ID IJIM-1397, 14 pages
DOI: http://dx.doi.org/10.30495/ijim.2021.18999
Research Article

Science and Research Branch (IAU)

Congestion-Based Benchmarking and Accessing Sustainability in Network DEA, Case study: Nine Iranian Tomato Paste Supply Chains

E. Mollaeian [*], F. Hosseinzadeh Lotfi [†‡], A. Toloie-Eshlaghy [§], M. Rostamy-Malkhalifeh [¶], M. A. Afshar Kazemi [‖]

Received Date: 2019-12-11 Revised Date: 2020-01-14 Accepted Date: 2020-02-02

Abstract

This paper deals with benchmarking for a three-stage DMU. After investigating and eliminating congestion, adjustment of intermediate products and initial inputs have achieved by utilizing an Inverse DEA model, which dominates the current vectors and serve as a benchmark that is to the best of this paper knowledge. This process has two cases. First, the overall efficiency stays fixed, and the second case, is corresponded to the overall efficiency improvement.

Keywords : Network DEA; Benchmark; Congestion; Inverse DEA; Sustainable development.

1 Introduction

Data Envelopment Analysis (DEA) is an excellent approach to measure the efficiency of Decision Making Units (DMUs) with multiple inputs and outputs. This method was first introduced by Charnes et al. [4] and named CCR (Charnes, Cooper and Rhodes) model. After that many researchers have extended their models in more different production technology. Although in many cases DMUs may consist of network structure with intermediate activity, traditional DEA models treat each DMU as a black box and by only considering initial inputs and final outputs, ignore intermediate measures. Therefore, those models cannot be useful to make the correct decision. To overcome such drawback, several authors developed traditional DEA models and provided network DEA models (Lothgren and Tambour [23], Prito and Zofio [28] and Yu [35]). Moreover, the conventional DEA models were modified by taking into account the series relationship of the two sub-processes within the whole process (Kao and Hwang [18]). Under their framework, the efficiency of the whole process can be decomposed into the product of the efficiencies

[*]Department of Industrial Management, Science and Research Branch, Islamic Azad University, Tehran, Iran.
[†]Corresponding author. farhad@hosseinzadeh.ir, Tel:+98(912)303 4649.
[‡]Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran Tehran, Iran.
[§]Department of Industrial Management, Science and Research Branch, Islamic Azad University, Tehran, Iran.
[¶]Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran Tehran, Iran.
[‖]Department of Industrial Management, Central Tehran Branch, Islamic Azad University, Tehran, Iran.

of the two sub-processes. After that, the additive decomposition approach version of Kao and Hwang's model was introduced by Chen et al. [5], in two-stage network structure which can be applied under both Constant Return to Scale (CRS) and Variable Return to Scale (VRS) assumptions. Chen and Yan [6] constructed an alternative network DEA model to evaluate performance supply chain. A review of network DEA models was performed to treat the structures of DMU by Liu et al. [24]).

Despite the fact that there are more studies in DEA and supply chain management (SCM) which focus on economic factors and ignore social and environmental criteria, during past decades decision makers have considered sustainable features in evaluating performance of the units. Sustainable development involves the design of integrated approaches that are capable of addressing environmental sustainability and waste while ensuring social and economic prosperity at the national or even global level, implying a macroeconomic scope (Khalili and Duecker [21]). There are many applications of NDEA model such as evaluating performance supply chain and sustainable supply chain management (SSCM). Linton et al. [25] assessed sustainability of supply chains by focusing on environmental and social factors. Mirhedayatian et al. [27] accessed green supply chains through NDEA. Khodakarami et al. [22] developed two-stage DEA models for evaluating sustainability of supply chains. Izadikhah and Farzipoor Saen [14] evaluated sustainability of supply chain by two-stage range directional measure in the presence of negative data.

Apart from computing relative efficiency scores, DEA is also useful because it provides a target efficient operating point, i.e. an efficient projection/benchmark, for each DMU. Although the computed targets depend on the specific DEA model used, they are almost always chosen so that they dominate the DMU being projected. There are many methods for DEA target setting. In particular, all approaches that compute efficiency scores also compute targets. Krger [20] presented a two-stage procedure for finding realistic benchmarks for non-parametric efficiency analysis. With this approach, there is no need to use linear combinations of existing units as benchmarks which may not be achievable in reality and also no need to accept slacks which are not reflected by the efficiency measure. Ruiz and Sirvent [31] developed a common framework for benchmarking and ranking units with DEA. Their approach identified a common best practice frontier as the facet of the DEA efficient frontier spanned by the technically efficient DMUs in a common reference group. A model developed which allows to deal not only with the setting of targets, but also with the measurement of efficiency, because it is possible to define efficiency scores of the DMUs by using the common set of weights (CSW) it provides. In the last decade there are more studies to deal with different issues related to target setting and benchmarking: in Ramn et al. [29] to deal with models including weight restrictions, in Ruiz and Sirvent [32] for developing a common benchmarking framework, in Aparicio et al. [2] it is extended to oriented models, in Cook et al. [9] for the benchmarking of DMUs classified in groups, in Ramn et al. [30] , which propose a sequential approach for the benchmarking and in Cook et al. [10], for target setting in pay-for performance incentive plans.

Performance evaluation and target setting are powerful tools to develop the decision making unit. It is necessary to have accurate information and proper target for a decision maker, in order to maintain in the global competition marketing. Benchmarking is the systematic comparison of the performance of one unit against other units as well as achieving a benchmark is a requirement to calculate or obtain relative efficiency. One of the weaknesses of a DMU, which leads to inefficiency, is its congestion and many sources have a negative impact on efficiency when there is a large amount of surplus. Congestion is a type of inefficiency caused by the accumulation of inputs. Therefore, congestion detection can help management reduce inputs, which is cost-effective, in order to both reduce costs and increase output. It is important to reduce all or some of the inputs. Although, usually an increase in input causes increase in output, but there are situations where the increase of one or more input leads to a reduction in one or more output. In this case there is congestion in the input vectors of unit. Not only does congestion lead to increase the cost, but also it alters a DMU to inefficient one. Accordingly, there are many DEA researches on the concept of congestion. In the other words, due to the costly properties of the congestion, because of congestion refers to the inputs of units, eliminating it leads to performance improvement and it may be achieve a benchmark for the evaluated DMU. By noting the drawback of congestion, we have considered this concept in a network structure unit to find a benchmark for the unit. It was first introduced by Fare and Sevensoon [11] and then Fare et al. [12] presented a method to identify congestion by DEA models called FGL. Jahanshahloo and Khodabakhshi [15] provided a model to calculate the congestion which was presented as two models and needed to solve three linear programming problems. We have considered a three-stage DMU, as a network DMU, in this paper. Its an extension of a unit that has three sub-processes connected in series. The output of previous sub-unit is in turn input to the next stage and output of the last sub-unit which is used to produce the unit output. This DMU diagram is depicted in Fig. 2. The overall efficiency is measured by using a novel network DEA model under input-oriented. To the best of our knowledge, few papers have looked at the congestion phenomenon in benchmarking, in network DMU, especially. What almost papers concerned is to achieve benchmark by depicting under evaluated DMU on the production possibility set and by using the slack variable find a dominated unit. As such, this paper serves to bridge these gaps in literature through the well extension of the basic DEA approach in network DEA model and benchmarking. The purpose of this paper is to discuss how we can achieve a benchmark for a NDMU by identifying and eliminating congestion of this activity, which may be a good suggestion for the decision maker to determine of the resources of the performance inefficiency.

Among lots DEA models, inverse DEA first proposed by Wei et al. [34], is an important research direction. Contrary to the traditional DEA method which is used to measure efficiency, inverse DEA is used to deal with two types of problems. The first type is related to the number of additional outputs that a particular DMU from a group of DMUs could produce for the given inputs, assuming that this DMU maintains its current efficiency value with respect to the rest. The second type is related to how much more input should that DMU be provided for the given outputs under the previous assumption. Lertworasirikul et al. [26] studied an inverse DEA model for the case of variable return to scale, i.e., inverse BCC model, and developed a linear programming model to solve this model for a Pareto-efficient solution. Jahanshahloo et al. [17] developed an inverse DEA model under an intertemporal dependence assumption and introduced the concept of periodic weak Pareto optimality to solve this model. Ghiyasi [13] pointed out that some important insight can be gained if price information is available in the classical efficiency analysis.

They considered the inverse DEA problem when price information is available and provided the theoretical foundation of the problem. Thier Proposed models guarantee not only fixed technical efficiency but also unchanged cost efficiency while process of input estimation associated with a perturbed output. A real world data empirical illustration shows pertinence and future applicability of proposed approaches. Amin et al. [1] suggested a novel method to deal with target setting in mergers using goal programming (GP) and inverse data envelopment analysis (InvDEA). However, the inverse DEA aims to identify the quantities of inputs and outputs when efficiency score is given as a target. Their study provided an effective method that allows decision makers to incorporate their preference in target setting of a merger for saving specific input(s) or producing certain output(s) as much as possible. Wegener and Amin [33] suggested a novel inverse problem in environmental efficiency. They used an inverse DEA model to minimize the overall greenhouse gas GHG emissions generated by a set of decision making units (DMUs) for producing a certain level of outputs, given that the DMUs maintain at least their existing performance status. The usefulness of the proposed method is demonstrated through an application in the oil and gas industry. Kalantary and Farzipoor [19] after developed a network dynamic SBM model to assess sustainability of supply chains, proposed an inverse DEA model with network and dynamic structure. A

new inverse DEA model was proposed by Chen et al. [7] with undesirable outputs to make several scientifically based investment schemes from different perspectives. By this method, decision maker can forecast the specific amount of investment based on their actual sustainable development objectives.
In many situations, in supply chain performance evaluation, there is no unique decision maker (DM) for all sub-units and each sub-unit might be measured by different DM. In our proposed model each sub-unit is controlled by one unique decision maker and each one might follow different production technology. We have assumed that the first and second stage follow Constant Return to Scale (CRS) and the last stage is Variable Return to Scale (VRS). It is emphasized that since congestion is definded in VRS production technology, at least one of the sub-units must be follow VRS . As finding a benchmark can be useful to performance improvement, after evaluating overall efficiency score of DMU and determining efficient and inefficient unit, we are able to identify the resource of inefficiency, that refers to congestion concept or management weakness in both cases. According to the definition of congestion, it will just occur in VRS technology, so in this paper, it has investigated in the inputs of the last stage. For this, we have used common congestion method by Cooper et al. [8] and the new vector is obtained by omitting this value from the input vector. Then, adjustment of all intermediate products and initial inputs are done by utilizing an Inverse DEA (IDEA) model without changes in the overall efficiency. Finally, efficiency improvement of this unit is executed by proposing an Inverse DEA model. In other words, this paper has determined the best level of DMU's factors which need adjustment while maintaining or improving NDMU's overall efficiency level. Both states might make a benchmark for the evaluated unit and may be a good suggestion to the decision maker for making better decision.
The rest of this paper is organized as; The preliminary of DEA is given in Section 2. In Section 3, a network DEA model has proposed to evaluate performance of NDMU and congestion concept in NDMU. Moreover, an Inverse NDEA model is introduced to adjust initial input and intermediate factor level, in both improving and maintaining efficiency score of DMU. A numerical example is illustrated in Section 4 to demonstrate the application of the proposed method. The proposed ideas are used in a case study where nine Iranian supply chain producing tomato pastes are measured in term of sustainability. Finally, in Section 5 the conclusion of this paper is presented.

2 Preliminaries

2.1 BCC model

Since the very beginning of DEA studies, various extensions of the CCR model have been proposed, among which the BCC (Banker, Charnes and Cooper) model is represented in 1984 [1]. The BCC model has its production frontiers spanned by the convex hull of the existing DMUs. The Production Possibility Set (PPS) corresponding to BCC model is as:

$$T_{BCC} = \left\{ \begin{array}{l} (X,Y)|X \geq \sum_{j=1}^{n} \lambda_j x_{ij}, \\ Y \leq \sum_{j=1}^{n} \lambda_j y_{rj}, \sum_{j=1}^{n} \lambda_j = 1, \lambda \geq 0 \end{array} \right\} \tag{2.1}$$

Suppose we have a set of n DMUs ; DMU_1 ,..., DMU_n. Let $x_{io}, i = 1,, m$ and $y_{ro}, r = 1, s$ denote the input and output vectors of Oth DMU. The efficiency of DMU_o can be evaluated by BCC model. This model under output-oriented which assumed Variable Return to Scale (VRS) to obtain the efficiency of each DMU is formulated as follows:

$[BCC]$

$$\begin{aligned} &\max \;\; \phi \\ &\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ip} \qquad i = 1, ..., m \qquad (1) \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \geq \phi y_{rp} \qquad r = 1, ..., s \\ &\sum_{j=1}^{n} \lambda_j = 1 \\ &\lambda \geq 0, \end{aligned}$$

If the optimal solution of model 1 be $^*_o = 1$, evaluated DMU is BCC-efficient otherwise is said BCC-inefficient.

2.2 Network DEA

Now, suppose a production process is composed of some sub-processes. The whole process uses m inputs $x_{ij}, i = 1,, m$ to produce s outputs $y_{rj}, r = 1, s.$ The convnetional model fails to properly characterize the performance of DMU, since it only considers the initial inputs and the final outputs of the DMU and ignores intermediate products associated with DMU members. Chen and Yan' [6] model is considered in this section. In their model, an alternative network DEA model is constructed that embodies the internal structure for DMU performance evaluation and taken the perspective of organization mechanism to deal with the complex interactions in DMU. Three different network DEA models are introduced under the concept of centralized, decentralized and mixed organization mechanisms, respectively, which here the decentralized mechanism is pointed out. Fig 1. depicts a two stage suppliermanufacturer chain as a two-stage DMU where S and M represent the supplier and the manufacturer, respectively. X is the input vector of supplier (S) and Y^1 and Y^2 are its output vectors which are also input vectors to the manufacturer stage. z^1 andz^2 are the output vectors corresponding to manufacturer (M1) and manufacturer (M2), respectively. In a decentralized

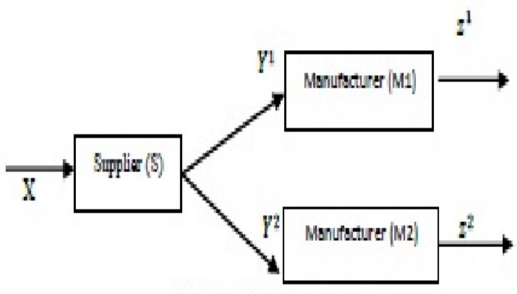

Figure 1: Supplier manufacturer chain

control organization, there is no such a super decision maker to control all divisions and each of the division has its own incentive and strategies. The production possibility set corresponding to decentralized control as follows:

$$T_{Decentral} = \left\{ \begin{array}{l} (X, Z^1, Z^2)|X \geq \sum_{j=1}^{n} \lambda_j^1 x_j, \\ \sum_{j=1}^{n} \lambda_j^1 y_j^1 \geq \sum_{j=1}^{n} \lambda_j^2 y_j^1 \\ , \sum_{j=1}^{n} \lambda_j^1 y_j^2 \geq \sum_{j=1}^{n} \lambda_j^3 y_j^2, \\ \sum_{j=1}^{n} \lambda_j^2 y_j^1 \leq y^1, \\ \sum_{j=1}^{n} \lambda_j^3 y_j^2 \leq \ y^2, \\ \sum_{j=1}^{n} \lambda_j^2 z_j^1 \geq z^1, \\ \sum_{j=1}^{n} \lambda_j^3 z_j^2 \geq \ z^2, \\ \lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0. \end{array} \right\} \quad (2.2)$$

Accordingly, its DEA model is expressed as model (2). The first, the fifth and the seventh inequalities in constraint set represent minimizing the initial input X at given level of final outputs Z_o^1 and Z_o^2. The second, the third, the fourth and the sixth inequalities in constraint set are corresponding to intermediate products. For division $M1$, the adjustment toY^1 should be no larger than its current level Y_o^1, since otherwise, division M1 would not agree to proceed. That is the fourth inequality in constraint set. The same is to division M2 as represented in the sixth inequality of model (decentral). [6]

$$\begin{array}{l} min \ \theta_{Decentral} \\ \sum_{j=1}^{n} \lambda_j^1 x_j \leq \theta_{Decentral} \ \ x_p \qquad (2) \\ \sum_{j=1}^{n} \lambda_j^1 y_j^1 \geq \sum_{j=1}^{n} \lambda_j^2 y_j^1 \\ \sum_{j=1}^{n} \lambda_j^1 y_j^2 \geq \sum_{j=1}^{n} \lambda_j^3 y_j^2 \end{array}$$

$$\sum_{j=1}^{n} \lambda_j^2 y_j^1 \leq y_p^1$$
$$\sum_{j=1}^{n} \lambda_j^2 z_j^1 \geq z_p^1$$
$$\sum_{j=1}^{n} \lambda_j^3 y_j^2 \leq y_p^2$$
$$\sum_{j=1}^{n} \lambda_j^3 z_j^2 \geq z_p^2$$
$$\lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0, \quad .$$

2.3 Congestion

The congestion concept is more investigated in both DEA and economics. Due to the fact that not only existence congestion leads to higher cost, because of its input role, but also congestion guides a DMU to the inefficient one. Therefore, eliminating this factor leads to efficiency improvement and target setting for under evaluated unit. Several authors have proposed different models to identify congestion in the input vector of each DMU. So far we deal with situations in which increasing inputs were allowed such as T_{BCC}. Unlimited input of $X \geq \lambda X$ is used to produce output of Y, but there are some cases where an increase in one or more input is caused to deterioration of one or more outputs which such situations are called congestion. To deal with such situations we should consider the product possibility set as follows:

$$T_{Convex} = \left\{ \begin{array}{l} (X,Y)|X = \sum_{j=1}^{n} \lambda_j x_j, \\ Y \leq \sum_{j=1}^{n} \lambda_j y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda \geq 0 \end{array} \right\} \tag{2.3}$$

Definition 2.1. *The economic concept of congestion is a widely observed phenomenon in which inefficiency is identified in such a manner that a reduction in an input(s) results in an increase in a maximum possible output(s) without worsening other inputs and outputs. ([8])*

3 Proposed Method

3.1 Novel Model

Assume a three-stage DMU as Fig. 2 as a network DMU. $X = (x_1, , x_m)$ is the initial input vector of DMU and $I = (i_1, , i_l)$,$Y = (y_1, , y_s)$ are the intermediate products, of the first stage to the second and the second to the last stage, respectively. $Z = (z_1, , z_k)$ is the final output of DMU. Let n same DMU denoted by $(DMU_1, , DMU_n)$. To evaluate the efficiency of such DMU we have

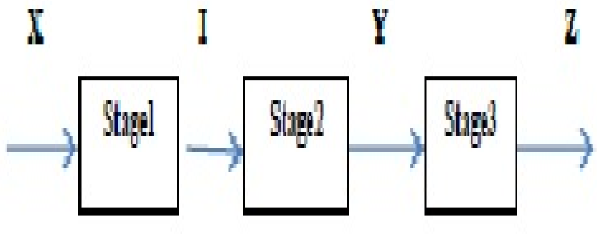

Figure 2: Three stage DMU

introduced following network DEA model, in which the first and second stage follow CCR technology and the last stage is VRS. In this paper, we have presumed each division has its own strategies and there is no one decision maker to control all divisions of the DMU. Also, by taking CCR for the first and second stage and VRS for the last, the Production Possibility Set (PPS) corresponding to this assumption is as:

$$T_{Decentral} = \left\{ \begin{array}{l} (X,I,Y,Z)|X \geq \sum_{j=1}^{n} \lambda_j^1 x_j, \\ \sum_{j=1}^{n} \lambda_j^1 I_j \geq \sum_{j=1}^{n} \lambda_j^2 Ij, \\ \sum_{j=1}^{n} \lambda_j^2 I_j \leq \ I, \\ \sum_{j=1}^{n} \lambda_j^2 y_j \geq \sum_{j=1}^{n} \lambda_j^3 y_j, \\ \sum_{j=1}^{n} \lambda_j^3 y_j \leq \ y, \sum_{j=1}^{n} \lambda_j^3 z_j \geq \ z, \\ \sum_{j=1}^{n} \lambda_j^3 = 1, \lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0. \end{array} \right\} \tag{3.4}$$

According to the above PPS, its DEA model is expressed:

$$\begin{aligned}
& min \ \ \theta_{Decentral} \\
& \sum_{j=1}^{n} \lambda_j^1 x_j \leq \theta_{Decentral} \ \ x_p \qquad (3) \\
& \sum_{j=1}^{n} \lambda_j^1 I_j \geq \sum_{j=1}^{n} \lambda_j^2 I_j \\
& \sum_{j=1}^{n} \lambda_j^2 I_j \leq \ I_p \\
& \sum_{j=1}^{n} \lambda_j^2 Y_j \geq \sum_{j=1}^{n} \lambda_j^3 Y_j \\
& \sum_{j=1}^{n} \lambda_j^3 Y_j \leq \ Y_p \\
& \sum_{j=1}^{n} \lambda_j^3 Z_j \geq \ Z_p \\
& \sum_{j=1}^{n} \lambda_j^3 = 1 \\
& \lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0,
\end{aligned}$$

In model (3), the first and the sixth inequalities represent minimizing the initial input X at given level of final output Z. The second, the third, the fourth and the fifth inequalities are corresponding to intermediate products. It is noteworthy that, the adjustment to I should be no longer than its current level I_o, in the second stage, otherwise, this step would not pursue to proceed. That is clear in the third inequality in constraint set. The same is the third stage as representation in the fifth inequality of the model. Thus the selfishness of stage two and three are reacted in the fourth and seventh inequality in constraint set.

Lemma 3.1. *Model (3) is feasible and its optimal solution is limited and* $0 < \theta^* \leq 1$.

Proof. Let $theta = 1$ and $lambda_o^1 = lambda_o^2 = lambda_o^3 = 1, jO, lambda_o^1 = lambda_o^2 = lambda_o^3 = 0$. That would be a feasible solution of model (3). This solution implies $\theta^* \leq 1$. The optimal solutionθ^* yields an efficiency score for a particular DMU. The process is repeated for each DMUj. Moreover,θ is positive in any feasible solution. If $\theta^- = 0$, it comes from the first constraint, ,$\lambda^1 = 0$ so according to the second constraint $\lambda^2 = 0$ and then$\lambda^3 = 0$ due to the fifth constraint. Finally, as for the eighth constraint, we have $Z_o \leq 0$ and it leads to $Z_o = 0$. That's a contradiction. Therefore, in every feasible solution and especially in optimal solution we have $\theta^* > 0$ and there is a limited optimal solution in model (3). □

3.2 Congestion in Network DEA

Let us present a definition about congestion in Network DMU structure and then continue our method.

Definition 3.1. *Congestion phenomenon is occurred in network structure DMU, if there is at least in one of the sub-DMUs.*

In order to identify congestion, taking above definition, we have to measure each of the sub-units independently. Assume that there is congestion phenomenon in inputs of the last stage which follows VRS. As we know, these input vectors are the outputs of previous stage too. In order to decrease these outputs, first, we need to use an IDEA model and reduce its corresponding inputs. Therefore, we have introduced an IDEA model based on the proposed model by Jahanshahloo et al. [16]. Initially, we have identified congestion of component input vector for final stage by common method and reduced them in input vector in order to be able to obtain $\beta = Y - C$ (In this phrase "C" is the congestion amount of Y). Note that β is the input, without congestion, for last stage and the output of previous stage one, thus to determine modified inputs of this sub-unit the following model is proposed. This model is considered $\theta^*_{decentral}$ as an optimal value of model (3) to fix the efficiency of DMUs.

$$\begin{aligned}
& min \ \ \alpha_p \\
& \sum_{j=1}^{n} \lambda_j^1 x_j \leq \theta^*_{Decentral} \ \ x_p \qquad (4) \\
& \sum_{j=1}^{n} \lambda_j^1 I_j \geq \sum_{j=1}^{n} \lambda_j^2 I_j \\
& \sum_{j=1}^{n} \lambda_j^2 I_j \leq \alpha_p
\end{aligned}$$

$$\sum_{j=1}^{n} \lambda_j^2 \beta_j \geq \sum_{j=1}^{n} \lambda_j^3 \beta_j$$
$$\sum_{j=1}^{n} \lambda_j^3 \beta_j \leq \ \beta_p$$
$$\sum_{j=1}^{n} \lambda_j^3 Z_j \geq \ Z_p$$
$$\sum_{j=1}^{n} \lambda_j^3 = 1$$
$$\alpha_p \leq \ I_p$$
$$\lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0, \quad , j = 1, ..., n.$$

By solving the model (4), we are able to diminish input of the second stage (vector I) toα. According to the fourth condition, it is clear that, this model for less value ofI_p is feasible. The eighth constraint states that not only the adjusted α must not be less than the current value, but it may be equal to the current I. Model (4) is a multiple objective programming problem and to solve this model, it needs to be turned into a single objective programming problem. We consider $C_p > 0$ as the weight of l^{th} inputs for$l = 1, , L$. Thus we will have the following model:

$$min \ \sum_{l=1}^{L} C_l \alpha_{lp}$$
$$\sum_{j=1}^{n} \lambda_j^1 x_j \leq \theta^*_{Decentral} \ \ x_p \qquad (5)$$
$$\sum_{j=1}^{n} \lambda_j^1 I_j \geq \sum_{j=1}^{n} \lambda_j^2 I_j$$
$$\sum_{j=1}^{n} \lambda_j^2 I_j \leq \alpha_p$$
$$\sum_{j=1}^{n} \lambda_j^2 \beta_j \geq \sum_{j=1}^{n} \lambda_j^3 \beta_j$$
$$\sum_{j=1}^{n} \lambda_j^3 \beta_j \leq \ \beta_p$$
$$\sum_{j=1}^{n} \lambda_j^3 Z_j \geq \ Z_p$$
$$\sum_{j=1}^{n} \lambda_j^3 = 1$$
$$\alpha_p \leq \ I_p$$
$$\lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0, \quad , j = 1, ..., n.$$

Lemma 3.2. *Model (5) is feasible.*

Proof. Let $\theta = 1$ and $\alpha_p = I_p$ and $\lambda_p^1 = \lambda_p^2 = \lambda_p^3 = 1, jO, \lambda_p^1 = lambda_p^2 = lambda_p^3 = 0$. That would be a feasible solution of model (5). □

Note that α is the output of the first stage. Therefore, we again need to use IDEA model to modify its input to produce α. Like previous step, we have the following model to estimate initial input of DMU. A vector less, or equal to, than X can be obtained by solving this model that $X_i \geq \gamma_i$ for $i = 1, ..., m$. Notice that γ can produceα as input for stage 2 and it again produces intermediate product β as its input without congestion for last sub-unit of DMU. To estimate the initial input level of DMU, when I is decreased to α and Y is reduced to β, the following model is introduced:

$$min \ \ \gamma_p$$
$$\sum_{j=1}^{n} \lambda_j^1 x_j \leq \theta^*_{Decentral} \ \ \gamma_p \qquad (6)$$
$$\sum_{j=1}^{n} \lambda_j^1 \alpha_j \geq \sum_{j=1}^{n} \lambda_j^2 \alpha_j$$
$$\sum_{j=1}^{n} \lambda_j^2 \alpha_j \leq \alpha_p$$
$$\sum_{j=1}^{n} \lambda_j^2 \beta_j \geq \sum_{j=1}^{n} \lambda_j^3 \beta_j$$
$$\sum_{j=1}^{n} \lambda_j^3 \beta_j \leq \ \beta_p$$
$$\sum_{j=1}^{n} \lambda_j^3 Z_j \geq \ Z_p$$
$$\sum_{j=1}^{n} \lambda_j^3 = 1$$
$$\gamma_p \leq \ X_p$$
$$\lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0, \quad , j = 1, ..., n.$$

A multiple objective programming problem can be solved by converting it to a single objective programming one. For this aim, we replace its objective function to $min \ \ \sum_{i=1}^{m} C_i \gamma_{ip}$ Where C_i

is the weight of inputs and $C_i > 0$, $i = 1, ..., m$. Note that this model for less value of X is feasible. By solving model (6) we obtain at least initial input of DMU. In this case, the efficiency of DMU with (x, I, Y, Z) and $(\gamma, \alpha, \beta, Z)$ is equal and (X, I, Y, Z) is dominated by $(\gamma, \alpha, \beta, Z)$. Finally, at the end of this section, we would like to come back to the definition of congestion which was mentioned first. If we implement congestion' model for the third sub-DMU, that follows VRS technology, there will be no congestion in it. Therefore, there is no congestion in network structure DMU and if the network DMU is inefficient, it relates to management weakness. On the other hand, obtained vector of inputs-intermediate measures and outputs might be a benchmark for the evaluated DMU.

4 Improving Efficiency

In this section we would like to estimate initial inputof DMU according to improve its efficiency. The amount of recovery of its efficiency is $\eta - percent$ of θ_p^*, so the following model is presented by considering omit the congestion:

$$
\begin{aligned}
& min \;\; \alpha_p \\
& \sum_{j=1}^{n} \lambda_j^1 x_j \leq \; (1 + \frac{\eta}{100})\theta^*_{Decentral} \;\; x_p \qquad (7) \\
& \sum_{j=1}^{n} \lambda_j^1 I_j \geq \sum_{j=1}^{n} \lambda_j^2 I_j \\
& \sum_{j=1}^{n} \lambda_j^2 I_j \leq \alpha_p \\
& \sum_{j=1}^{n} \lambda_j^2 \beta_j \geq \sum_{j=1}^{n} \lambda_j^3 \beta_j \\
& \sum_{j=1}^{n} \lambda_j^3 \beta_j \leq \; \beta_p \\
& \sum_{j=1}^{n} \lambda_j^3 Z_j \geq \; Z_p \\
& \sum_{j=1}^{n} \lambda_j^3 = 1 \\
& \alpha_p \leq \; I_p \\
& \lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0, \;\; , j = 1, ..., n.
\end{aligned}
$$

where $0 \leq \eta \leq \frac{100(1-\theta_p^*)}{\theta_p^*}$. According to model (7) it is a multi-objective programming problem model. In order to solve this model, it must be turned into a single objective programming problem model by changing its objective function. Thus $\alpha_p = \sum_{l=1}^{L} C_l \alpha_{lp}$.
Now it is time to estimate the initial input levels of DMU. The following model is about the improvement of efficiency as $\eta - percent$ of θ_p^* is assumed.

$$
\begin{aligned}
& min \;\; \gamma_p \\
& \sum_{j=1}^{n} \lambda_j^1 x_j \leq \; (1 + \frac{\eta}{100})\theta^*_p \;\; \gamma_p \qquad (8) \\
& \sum_{j=1}^{n} \lambda_j^1 \alpha_j \geq \sum_{j=1}^{n} \lambda_j^2 \alpha_j \\
& \sum_{j=1}^{n} \lambda_j^2 \alpha_j \leq \alpha_p \\
& \sum_{j=1}^{n} \lambda_j^2 \beta_j \geq \sum_{j=1}^{n} \lambda_j^3 \beta_j \\
& \sum_{j=1}^{n} \lambda_j^3 \beta_j \leq \; \beta_p \\
& \sum_{j=1}^{n} \lambda_j^3 Z_j \geq \; Z_p \\
& \sum_{j=1}^{n} \lambda_j^3 = 1 \\
& \gamma_p \leq \; X_p \\
& \lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0, \;\; , j = 1, ..., n \\
& 0 \leq \eta \leq 100\frac{(1 - \theta_p^*)}{\theta_p^*}
\end{aligned}
$$

Model (8) is a multiple objective programing problem which can be solved by transforming its objective function to $\gamma_p = \sum_{i=1}^{m} C_i \gamma_p$ (such as previous steps) and get a single objective programming problem.

5 Numerical Example

In this section, we have presented a case study to exhibit the efficacy of the procedures. Also,

Table 1: Factor Used in sustainable SCM.

Stage	Notation	Definition
Supplier	x_j^1	material purchasing cost
	x_j^2	welfare cost
	x_j^2	environmental cost
Distributor	z_j^1	Number of Delivered products
	z_j^2	Revenue
Intermediate	I_j	Number of products from supplier tomanufacturer
input/output	Y_j	Number of green products.

Table 2: Factor used in sustainable SCM.

DMU	Supply Chains	Input	Input	Input	Intermediate products	Intermediate products	Output	Output	Efficiency score
DMU_1	Oila	400	30	10	320	315	1500	1100	0.52
DMU_2	Daland	360	60	12	295	390	2000	1280	0.66
DMU_3	Sahar	330	55	16	290	282	2200	980	0.51
DMU_4	Kambiz	455	25	20	310	312	2700	1200	0.70
DMU_5	Mohsen	370	37	19	280	270	1900	840	0.47
DMU_6	Urum-Ada	332	80	17	210	200	1890	965	0.43
DMU_7	Rojin	355	87	9	235	220	1995	1115	0.58
DMU_8	Mahram	300	95	8	255	235	1650	700	0.62
DMU_9	Chin Chin	295	50	15	315	318	3000	1400	0.76

this example demonstrates the applicability of the proposed method in performance evaluation of a supply chain management (SCM) problem in terms of sustainability. For this, nine Iranian supply chains which produce tomato paste are assessed (Izadikhah et al. [14]). Table 1 depicts the used factors in the concept of sustainability development. The factors are obtained by interviews with managers and experts in supply chains. The supply chain includes three divisions, i.e., supplier (stage 1), manufacturer (stage 2), and distributor (stage 3). The structure is depicted in Fig. 3. Inputs of supplier stage are material purchasing cost (economic factor), staff welfare cost (social factor), and environmental cost (environmental factor). As we have been shown in Table 1 and Fig. 3, inputs vector that enters to the first stage are displayed by x_j^1 ,x_j^2 and x_j^3. I_j is output vector that leaves stage 1 and enters to the second stage as an input. Y_j is output vector that exits stage 2 and enters as an input to the final stage as well as z_j^1 and z_j^2 are the outputs of the last stage. Dataset dates backs to 2014 which are shown in Table 2. Environmental costs

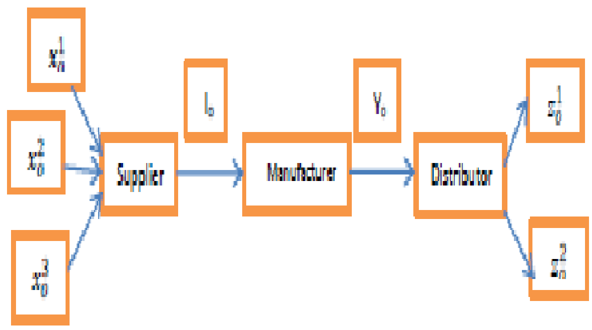

Figure 3: Structure of supply chain

are defined as costs related to actual or potential damage of natural valuable items through economic activities. Environmental costs are defined as costs related to actual or potential damage of natural valuable items through economic activities. Environmental costs are considered as input. Intermediate measures include two parts. The intermediate inputs/outputs in manufacturer stage are the number of products from supplier to manufacturer (economic factor) and intermediate inputs/outputs in distributor are the number of green products (economic factor). The outputs of distributor' stage are revenue (economic factor) and number of delivered products (economic factor). To access efficiency of sustainable SCM, each supply chain is treated as a DMU. The overall efficiency scores for each supply chain are obtained by model (3) and the result is given in the last column of the table. Now it is time to answer how we can obtain a benchmark for under evaluation unit by removing the congestion. Therefore, congestion is assumed in the last stage inputs. Due to compute the amount of congestion and obtained β, we have used a common method. The amount of congestion of DMU_2 is obtained to 72, so we can easily achieve $\beta = 318$ by decreasing from Y. To determine the new inputs of the second stage, by solving the corresponding linear programing problem (model 5) we will get $\alpha = 209.32$. To estimate the initial inputs, it is time to solve the model (6) and obtain $(\gamma_1, \gamma_2, \gamma_3) = (317.37, 53.79, 12)$. In this case, not only the efficiency of DMU with (X, I, Y, Z) and $(\gamma, \alpha, \beta, Z)$ is equal, but also (X, I, Y, Z) is dominated by $(\gamma, \alpha, \beta, Z)$. Now the efficiency of DMU_O can be improved by solving model (7) and (8). Suppose $\beta = 318$ the efficiency of DMU_2 intend to improve as $10\%\theta_2^* = 10\%(0.66)$. Thus, by solving model (7), the new input level for the second sub-unit is determined and $\alpha = 215.65$. Besides, by setting these values (α and β) and solving the model (8), the new initial input levels of $DMU_O$is identified. As a result, when the intermediate products decrease to and also its efficiency increase to $10\%\theta_0^*$ then its initial input levels is obtained by model (8). Therefore, the new initial input will be $(\gamma_1, \gamma_2, \gamma_3) = (302.24, 60, 12)$.

6 Conclusion

The current paper analyses the overall efficiency of three-stage DMU by proposing a network DEA model. In this model the first and second stage are under Constant Return to Scale (CCR), as well as the last stage is under variable return to scale (VRS) production technology. Although in order to find a benchmark, most of the studies depict under evaluated unit onto the production possibility set, this paper use the congestion concept to find the dominate DMU. That is the best of this paper knowledge. For this aim, the congestion is investigated in the stages which follow Variable Return to Scale technology. It is emphasized that since congestion is definded in VRS production technology, at least one of the subunits must be follow VRS. Then, after eliminating congestion, the adjustment all intermediate products and initial inputs are achieved, by utilizing an Inverse DEA model, which dominates the current vectors so can serve as a benchmark. In all proposed model, there are constraints which state that not only the adjusted vectors must not be less than the current value, but it may be equal to the current one. This process is done in two cases. First, subject to the requirement that the overall efficiency of under evaluated DMU stays fixed and the second is corresponded to the overall efficiency improvement. On the other hand, applying this model, the best level of initial inputs of DMU can be estimated. Moreover, the obtained vector by IDEA model, which is congestion-free, might be a benchmark for the evaluated DMU because by decreasing the initial input level a dominated DMU is achieved. This activity may be a good suggestion for the decision maker to determine of the resources of the performance inefficiency. In this paper, the relationship between the DMU components or sub-unit and their effect on the overall efficiency is assumed which will therefore lead to the system more accurate evaluation. Finally, as nowadays evaluation of sustainable supply chain has gained significant attention, we have used nine Iranian tomato paste supply chains by considering the sustainable development factors to demonstrate the application of the proposed model.

References

[1] Gh R.Amin, S. Al-Muharrami, M. Toloo, A combined goal programming and inverse DEA method for target setting in mergers, *Expert Systems with Applications* 115 (2019) 412-417.

[2] J. Aparicio, J. M. Cordero, J. T. Pastor, The determination of the least distance to the strongly efficient frontier in Data En-

velopment Analysis oriented models: Modelling and computational aspects, *Omega* 71 (2017) 1-10.

[3] R. D. Banker, A. Charnes, W. W. Cooper, Some models for estimating technical and scale inefficiencies in data envelopment analysis, *Management Science* 30 (1984) 1078-1092.

[4] A. Charnes, W. W. Cooper, E. Rhodes, Measuring the efficiency of decision-making, *European Journal Operation Research* 2 (1978) 429-444.

[5] Y. Chen, W. Cook, N. Li, J. Zhu, Additive efficiency decomposition in two-stage DEA, *European Journal of Operational Research* 196 (2009) 1170-1176.

[6] C. Chen, H. Yan, Network DEA model for supply chain performance evaluation, *European Journal Operation Research* 213 (2011) 147-155.

[7] L. Chen, Y .Wang, F .Lai, F. Feng, An investment analysis for China's sustainable development based on inverse data envelopment analysis, *Journal of Cleaner Productivity* 2 (2016) 1-12.

[8] W. W. Cooper, H. Deng, Z. M. Huang, S. X. Li, A one-model approach to congestion in DEA, *Socio-Economic Planning Sciences* 36 (2002) 231-238.

[9] W. D. Cook, J. L. Ruiz, I. Sirvent, J. Zhu, Within-group common benchmarking using DEA, *European Journal of Operational Research* 256 (2017) 901-910.

[10] W. D. Cook, N. Ramn, J. L. Ruiz, I. Sirvent, J. Zhu, DEA-based benchmarking for performance evaluation in pay-for-performance incentive plans, *Omega*, to appear (2018).

[11] R. Fare, L. Svensoon, Congestion on production factors, *Econometrica* 48 (1980) 1745-1753.

[12] R. Fare, S. Grosskopf, C. A. K. Lovell, The Measurement of Efficiency of Production, *Kluwer Nijhoff, Boston*, (1985).

[13] M. Ghiyasi, Inverse DEA based on cost and revenue efficiency, *Computers and industrial Engineering* 114 (2017) 258-263.

[14] M. Izadikhah, R.Farzipoor Saen, Evaluating sustainability of supply chains by two-stage range directional measure in the presence of negative data, *Transportation Research Part D* 49 (2016) 110-126.

[15] G. R. Jahanshahloo, M. Khodabakhshi, Suitable combination of improving output in DEA with determining input congestion considering textile industry of china, *Applied Mathematics and computation* 151 (2004) 263-273.

[16] G. R. Jahanshahloo, F. Hosseynzadeh Lotfi, N. Shoja, G. Tohidi, S. Razavyan, Input estimation and Identification of extra inputs in Invers DEA models, *Applied Mathematics and Computation* 156 (2004) 427-437.

[17] G. R. Jahanshahloo, M. Soleimani-damaneh, Inverse DEA under intertemporal dependence using multiple-objective programming, *Eur. J. Operational Res.* 240 (2015) 447-456.

[18] Ch. Kao, Sh. Hwang, Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan, *European Journal of Operational Research* 185 (2008) 418-429.

[19] M. Kalantary, R. Farzipoor Saen, Assessing sustainability of supply chains: An inverse network dynamic DEA model, *Computers and industrial Engineering* 135 (2019) 1224-1238.

[20] J. Krer, Direct targeting of efficient DMUs for benchmarking, *International Journal of Production Economics* 18 (2018) 5250-5273.

[21] R. N. Khalili, S. Duecker, From cleaner production to sustainable development: the role of academia, *Journal Clean. Product.* 96 (2015) 30-43.

[22] M. khodakarami, M. Shabani, R. Farzipoor-Saen, M. Azadi, Developing distinctive two-stage data envelopment analysis model: an

application in evaluating the sustainability of supply chain management, *Measurement* 70 (2015) 62-74.

[23] M. Lothgren, M. Tambour, Productivity and customer satisfaction in Swedish pharmacies: A network model, *European Journal of Operation Research* 115 (1999) 449-458.

[24] J. S. Liu, L. Y. Lu, W. M. Lu, Research fronts in data envelopment analysis, *Omega-Int J Manag Sci* 41 (2016) 3-15.

[25] J. D. Linton, R. Kelassen, V. Jayaraman, Sustainable supply chain : an introduction, *Journal of Operation Management* 25 (2007) 1075-1082.

[26] S. Lertworasirikul, P. Charnsethikul, Inverse data envelopment analysis model to preserve relative efficiency values: the case of variable returns to scale, *Comput. Industrial Eng.* 61 (2011) 1017-1023.

[27] S. M. Mirhedayatiyan, M. Azadi, R. Farzipoor-Saen, A novel network DEA model for evaluating green supply chain, *International Journal Production Economic* 147 (2014) 544-554.

[28] A. M. Prito, J. L. Zofio, Network DEA efficiency in input-output models: With an application to OECD countries, *European Journal of Operation Research* 178 (2007) 292-304.

[29] N. Ramn, J. L. Ruiz, I. Sirvent, On the use of DEA models with weigh restrictions for benchmarking and target setting, in Advances in efficiency and productivity (J. Aparicio, C. A. K. Lovell and J. T. Pastor, eds.), *Springer* (2016).

[30] N. Ram, J. L. Ruiz, I. Sirvent, Two-step benchmarking: setting more realistically achievable targets in DEA, *Expert Systems with Applications* 92 (2018) 124-131.

[31] J. L. Ruiz, I. Sirvent, Common benchmarking and ranking of units with DEA, *Omega* 65 (2016) 1-9.

[32] J. L. Ruiz, I. Sirvent, Performance evaluation through DEA benchmarking adjusted to goals, *Omega* 15 (2018) 305-483.

[33] M. Wegener, Gh. R. Amin, Minimizing greenhouse gas emissions using inverse DEA with an application in oil and gas, *Expert Systems with Applications* 122 (2019) 369-375.

[34] Q. L. Wei, J. Zhang, X. Zhang, An inverse DEA model for input/output estimate, *Eur. J. Operation Res.* 121 (2000) 151-163.

[35] M. M. Yu, Measuring the efficiency of multimode bus transit- evidence form Taiwans bus system, *Applied Economics Letters* 15 (2007) 647-653.

Elahe Mollaeian is a Ph.D candidate in Industrial Management at Science and Research Branch, Islamic Azad University, Tehran, Iran. She has held her master degree in DEA from Research and Science Branch, Islamic Azad University, Tehran, Iran. Her studies of DEA are Benchmarking, Network DEA decision making in industry and economic, supply chain and its scopes. Her research interest includes; Linear programming, Artificial and Intelligence

Farhad Hosseinzadeh-Lotfi is a full professor in Applied Mathematics at Science and Research Branch, Islamic Azad University, Tehran, Iran. In 1991, he received his undergraduate degree in Mathematics at Yazd Univer-sity, Yazd, Iran. He received his MSc in Operations Research at IAU, Lahijan, Iran in 1995 and PhD in Applied Mathematics (O.R.) at IAU, Science and Research Branch, Tehran, Iran in 1999. His major research interests are, Linear Programing, operations research and data envelopment analysis. He has been Advisor and Co-advisor of 46 and 31 Ph.D dissertations, respectively.

Abbas Toloei Eshlaghy is a full professor in Industrial Management at Science and Research Branch, Islamic Azad University, Tehran, Iran. He received his MSc and PhD in Industrial Management at IAU, Tehran, Iran in at IAU, Science and Research Branch, Tehran, Iran. His major research interests are System Dynamic, Simulation, Data science Artificial and Intelligence. He was selected as the country's top researcher in the humanities in 2012 and has more than 200 articles in international journals and conferences and 23 volumes of books

Mohsen Rostamy-Malkhalifeh is an associated professor from Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran. He has received his PHD in DEA from, Science and Research Branch, Islamic Azad University, Tehran, Iran. Currently he is as supervisor of DEA courses at Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran. His major research interests are DEA and its application, Mathematic Modeling and Operation Research.

Mohammad Ali Afshar-Kazemi is an associated professor in Industrial Management at Science and Research Branch, Islamic Azad University, Tehran, Iran. He received his PhD in Industrial Management at IAU, Tehran, Iran in at IAU, Science and Research Branch, Tehran, Iran. His major research interests are Information Technology, Simulation, Data science, Fuzzy system, Operation Research, Artificial and Intelligence.