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Research Article

A Method for Calculating the Marginal Rate in the FDH Model

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Abstract

The concept of trade-off reviews the rate of marginal substitution of inputs and outputs onto the efficient frontier; moreover, marginal rates of substitution (MRS) are important quantities for analysts and managers. In the current study, by changing Asmilds (2004) method, an algorithm has been found that can be used to calculate the marginal rate of the free disposal hull (FDH) model. An empirical study on 20 Iranian bank branches has been presented to illustrate the proposed approach.

Keywords : Data envelopment analysis (DEA); Free disposal hull (FDH); Marginal rate.

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1 Introduction

To Manage an organization, having a knowledge of trade-offs among activities in that organization is necessary; as an instance, to increase the production rate of a specific output in an organization, officials at the organization should calculate the amount of additional necessary input and estimate how much of an input (or output) should increase when the corresponding input (or output) decreases. Another important point not to forget is that in mathematical economics, it is very convenient for in mathematical economics, it is very convenient for mathematicians to operate in terms of efficient surfaces, support hyperplanes, pareto-efficient facets, reference sets, and production correspondences [4]. As a short definition, data envelopment analysis (DEA) provides a mathematical programming method for estimating the best practice frontiers and evaluating the relative efficiency of different entities. In fact, DEA is a tool for assessing past performances as part of the management control function and has been widely used to measure and analyze the relative efficiencies of various types of decision making units, such as banks and university departments, which possess shared functional goals with incommensurate inputs and outputs [10].

One research issue that has attracted considerable attention in DEA is the problem of determining the marginal rates of the substitution (MRS) of inputs and outputs. With this end in view, the current study discusses how to calculate MRS for efficient decision making units (DMUs) in an extreme point in FDH models. Looking at literature of DEA gives us fascinating results; for example, Huang, et al. [6] proposed a general method for

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calculating the rates of the change of the outputs to the inputs along the efficient surface of the DEA production set. Rosen et al. [12] studied the problem of MRS on efficient frontier and presented a general framework for the calculation of the trade-offs between two variables in DEA.
Cooper et al. [4] proposed an algorithm to search through the extreme points for cost improvements in the case of given prices or relative weights and constant outputs and thus considered non-marginal trade-offs as well.
Asmild et al. [3] applied the envelopment form of the BCC (Banker, Charnes and Cooper) model to calculate the marginal rates of substitution. They also developed some methods for evaluating larger trade-offs or non-marginal rates of substitution between variables in DEA. Khoshandam et al. [9] proposed a production function in which a group of variables were changed in a given direction, and then they calculated the effect of this change on some throughput.
Mirzaei et al. [11] showed that the binding efficient supporting surfaces of an efficient point may be used to define the alternative marginal rates of substitutions. Having these studies in mind, the aim of the current paper is to present a technique that enables DEA practitioners to determine rates of substitution and transformation in FDH models. This research discusses how to calculate the MRS for efficient DMUs in an extreme point in FDH models. With this end in view, the researchers generalize the work of Asmild et al. [3] to conduct the analysis of MRS in FDH models.
The rest of the paper is structured as follows: Next section provides the original FDH model and its linearization. Section 2 introduces our proposed method to obtain marginal rates of substitution in the FDH model and the trade-off analyses which are illustrated by a simple DEA problem. In Section 3, the marginal rate of substitution of one input and one output for 20 Iranian bank branches (DMUs) will be conducted with the use of the proposed method of the current paper. Conclusions will be presented in Section 4.

2 Free Disposal Hull (FDH) Model and Linearization

2.1 The FDH model

The figure of the production possibility set plays an important role in determining the model that we want to use. We know the frontier of the production possibility set approximates the production function. We also know that the production function shows relationship between the productive resources used by an organizations production (input) and the products or services obtained (output) at the same timeregardless of the price. By accepting the postulates, including observation, convexity, monotonicity and minimum extrapolation [c.f. Banker and Thrall (1992)], production possibility set with constant returns to scale can be defined as follows:

$$T_c = \{(x, y) : x \geq \sum_{j=1}^{n} \lambda_j x_j, y \leq \sum_{j=1}^{n} \lambda_j y_j, \lambda_j \geq 0, j = 1, 2, ..., n\} \tag{2.1}$$

But in some issues, the convex combination of decision making units does not make sense, because the reference units in the CCR (Charnes, Cooper and Rhodes) and BCC models are the convex combination of several decision making units. The main motivation for the appearance of the FDH step model is to make sure that the efficiency will be achieved only on the basis of real observations. So, each of the units must be either involved in the construction of the virtual unitλ_j (share of unit j)or one or not involved in that construction,λ_j is zeroin other words, $\lambda_j = 0$ or 1.

The explanation of the discussion above is similar to the computation of the efficiency of different aircraft engines. In the CCR and BCC models, a set of assessment is compared to the virtual unit. For example, the expression $\frac{1}{2}A + \frac{1}{2}B$ on engine efficiency and many real life issues is meaningless. So, the difference between the FDH model and classical models is that technology does not limit itself to the convexity principle. This stance of these models looks attractive, because it can be more in tune with the realities of everyday life.

The FDH model was first introduced in 1984 by Deprin, Simar and Tulkenz. In this model, for the construction of production possibility set, the convex combination of two production possibilities does not necessarily belong to the production possibility set. So accepting the postulates, including observation, monotonicity, possibility and minimum extrapolation, the production possibility set of theFDH_c model is defined as follows:

$$PPS_{FDH_c} = \bigcup_{j=1}^{n}\{(x,y): x \geq \lambda_j^i x_j, \\ y \leq \lambda_j^i y_j, \lambda_j^i \geq 0\} \tag{2.2}$$

The output-oriented FDH model can be compactly written as follows:

$$\begin{aligned} & Max\theta \\ s.t. & \sum_{j=1}^{n} \lambda_j x_j \leq x_k \\ & \sum_{j=1}^{n} \lambda_j y_j \geq \theta y_k \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \in \{0,1\}, j = 1,2,...,n \end{aligned} \tag{2.3}$$

where, λ is the intensity Boolean vector, and θ is the free variable and is continuous.

The efficient surface is a staircase based on those given DMUs that are not dominated by other given DMUs. Thus, the efficiency analysis is done relative to the other given DMUs instead of a hypothetical efficiency frontier. This has an advantage that the achievement goal for an inefficient DMU given by its efficient reference point will be more credible than in the cases of CCR and the BCC models. The reference point will simply be one of the already existing operating DMUs.

2.2 Linearization

We know that the above model is a mixed integer programming one, and it is difficult to use. Although there is no need to solve a linear programming problem for solving the FDH model, only a paired comparison is the optimal solution. But it has its own drawbacks including the assumption of strictly positive input and output vectorsa very difficult assumption in practical issues. Moreover, among these models none comes to the slack variables, and this is a non-linear model. In order to avoid these problems, the linear reformulation of the output-oriented FDH model (2.3) is used as given in [1] as follows:

$$\begin{aligned} & Max \sum_{j=1}^{n} \theta_j \\ s.t.\ & \theta_j y_{ro} - y_{rj}\lambda_j + s_{rj}^{+} = 0, \forall r, \forall j, \\ & (x_{ij} - x_{io})\lambda_j + s_{ij}^{-} = 0, \forall i, \forall j, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \geq 0, s_{rj}^{+} \geq 0, s_{ij}^{-} \geq 0 \end{aligned} \tag{2.4}$$

A DMU_0 is FDH efficient if $\theta_0^* = 1$. It is clear that the FDH model is a special case of DEA models. With the FDH formulation, each DMU is evaluated by comparing it to the other DMUs on a one-to-one basis [5].

3 Marginal Rates Substitution in the FDH Model

Consider a general process, where a vector of outputs $Y = (y_1, y_2, ..., y_s) \in R_+^s$ is produced by a vector of inputs $X = (x_1, x_2, ..., x_m) \in R_+^m$. To simplify the developments, let us write the inputs and outputs as a single vector of throughputs

$$Z = (z_1, ..., z_n) = (-X, Y)^T = \begin{pmatrix} -x_1, ..., -x_n \\ y_1, ..., y_n \end{pmatrix}$$

. We assume that the production frontier, given by the graph $\{Z \,|F(z) = 0\}$ is monotonic, continuous, piecewise, differentiable, and concave. Note that frontiers constructed with DEA satisfy these conditions in general; in fact, the production frontier in DEA may be represented as follows:

$$T = \left\{Z \,|Z\lambda \geq z,\ 1^T\lambda = 1\right\} \tag{3.5}$$

where $\lambda$is n-dimensional intensity vector and **1** is n-dimensional unity vector.
The marginal substitution rate of throughput j to throughput k at a point on the frontier$z_0$is defined as partial derivative as

$$MRS_{jk}(z_0) = \frac{\partial z_{0j}}{\partial z_{0k}}|_{z_0} = -\frac{\partial F/\partial z_{0k}}{\partial F/\partial z_{0j}}|_{z_0}, j \neq k \tag{3.6}$$

This partial derivative gives the increase in throughput j when throughput k is increased by one unit and all the other throughputs are kept constant. When the throughputs correspond to two inputs, the rate (3.6) is usually referred to as the technical rate of substitution (marginal rate of technical substitution); when they correspond to two outputs, it is called the marginal rate of transformation; and when z_j and z_k refer to an output and an input, it is often called the marginal productivity (marginal impact), which is in accordance with what is said in microeconomics.

The marginal rates of substitution of throughput j to throughput k, at the point z_0 on the FDH frontier can be calculated with the use of the following procedure:

1- Determine a small increment h for the kth throughput.

2- Obtain the optimum value of $\sum_{j=1}^{n} z_{0j}^{*}$by solving the linear programming problem below, which is resulted from increasing (or decreasing) the kth throughput to h.

$$Opt \sum_{j=1}^{n} z_{0j}^{*}$$

$$s.t.\ Z\lambda \geq z_0^{*}$$

$$1^T\lambda = 1$$

$$z_{0l}^{*} = z_{0l}, l \neq j, k$$

$$z_{0k}^{*} = z_{0k} + h$$

$$\lambda \geq 0 \qquad (3.7)$$

In fact, applying one of the following linear programming problems for the FDH model in terms of what trade-offs to be done with input and output gives us:

$$Max \sum_{i=1}^{n} y_{i0}^{*}$$

$$s.t.\ (x_{io} + h - x_{ij})\lambda_j \geq 0, \forall i, \forall j,$$

$$y_{rj}\lambda_j \geq y_{i0}^{*}, \forall r, \forall j,$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$\lambda_j \geq 0, \forall j.$$

$$Min \sum_{i=1}^{n} x_{i0}^{*}$$

$$s.t.\ (y_{rj} - y_{i0} - h)\lambda_j \geq 0, \forall r, \forall j$$

,

$$-x_{ij}\lambda_j \geq -x_{i0}^{*}, \forall i, \forall j,$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$\lambda_j \geq 0, \forall j.$$

These linear programming problems are solved for the jth component of the throughput vector when the kth component is increased (decreased) by the small quantity h, such that it still remains on the FDH frontier.

3- Compute the finite marginal rate of substitution from the right:

$$MRS_{jk}^{+}(z_0) = \frac{\sum_{j=1}^{n} z_{0j}^{*} - z_{0j}}{h}$$

If h is replaced by $-h'$ and steps two and three are repeated, the marginal rate of substitution will be obtained from the left. Note that we do not have $|h'| = h$, because the FDH model is a special case of interval data model, and it is not necessary for the increments of left and right to be equal.

3.1 Illustration of Marginal Changes

This approach for calculating the marginal rates of substitution is illustrated by a simple numerical example with four DMUs; each unit has one input and one output:

Fig. 1:

Suppose that $X = (6, 8, 14, 10)$ and $Y = (2, 4, 5, 3)$ and consider $k = 2$ for an output-oriented FDH model; then we will have:

$$Max\ \theta + \varepsilon(s^{+} + s^{-})$$

$$s.t.\ 2\lambda_1 + 4\lambda_2 + 5\lambda_3 + 3\lambda_4 - s^{+} = 4\theta,$$

$$6\lambda_1 + 8\lambda_2 + 14\lambda_3 + 10\lambda_4 + s^{-} = 8,$$

$$\lambda_i \in \{0, 1\}, \theta free, s^{+} \geq 0, s^{-} \geq 0,$$

The linearization equivalent ([1]) will, in this case, be like what follows:

$$Max \sum_{h=1}^{4} \theta_h + \varepsilon \sum_{h=1}^{4} (s_h^+ + s_h^-)$$

$$s.t.\ 2\lambda_1 - s_1^+ - 4\theta_1 = 0,$$
$$4\lambda_2 - s_2^+ - 4\theta_2 = 0,$$
$$5\lambda_3 - s_3^+ - 4\theta_3 = 0,$$
$$3\lambda_4 - s_4^+ - 4\theta_4 = 0,$$
$$(6-8)\lambda_1 + s_1^- = 0,$$
$$(8-8)\lambda_2 + s_2^- = 0,$$
$$(14-8)\lambda_3 + s_3^- = 0,$$
$$(10-8)\lambda_4 + s_4^- = 0,$$
$$\sum_{h=1}^{4} \lambda_h = 1,$$
$$\lambda, s_h^+, s_h^- \geq 0, \theta_h free.$$

We have to solve this problem; $(\lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*) = (0,1,0,0)$ and$(\theta_1^*, \theta_2^*, \theta_3^*, \theta_4^*)$ $= (0,1,0,0)$. That means that the second unit is efficient. Similarly, it can be shown that the 1th and 3th units are as efficient and inefficient as the 4th unit.

Now suppose that $Z_0 = (y_{i0}^*, 0)^T$ and

$$Z = [Y_h, -(X_h - x_k)]^T = [Y_h, -(X_h - 8)]^T$$

$$= \begin{pmatrix} 2 & 4 & 5 & 3 \\ -(6-8) & -(8-8) & -(14-8) & -(10-8) \end{pmatrix}$$

In fact, $(-x_0, y_0)^T = (-8, 4)^T$, and consider the rate of substitution $MRS_{(y)(-x)}$, that is, the change in y results from a marginal change in $-x$.

1. Let $h = 2$.

2. The value of $\sum_{i=1}^{4} y_{i0}^*$ resulted from increasing$-x_0$ by h to -6 (i.e. $Z_0^* = (\sum_{i=1}^{4} y_{i0}^*, -6)^T$) is found by

$$Max \sum_{i=1}^{4} y_{i0}^*$$

$$s.t.\ \begin{array}{l} 2\lambda_1 \geq y_{10}^* \\ 4\lambda_2 \geq y_{20}^* \\ 5\lambda_3 \geq y_{30}^* \\ 3\lambda_4 \geq y_{40}^* \end{array}$$

$$\begin{array}{l} -(6-6)\lambda_1 \geq 0 \\ -(8-6)\lambda_2 \geq 0 \\ -(14-6)\lambda_3 \geq 0 \\ -(10-6)\lambda_4 \geq 0 \end{array}$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1$$

$$\lambda_i \geq 0, y_{i0}^* \, free, i = 1,2,3,4.$$

which has the solution $(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$= $(1,0,0,0)$, $\sum_{i=1}^{4} y_{i0}^* = 2$ and $z_0^* = (2, -6)^T$.

3. The finite marginal rate of substitution from the right:

$$MRS^+_{(y)(-x)}(-8,4) = \frac{2-4}{2} = -1$$

So the marginal rate of substitution from the right between $-x_0$ and y_0 is -1.

In other words, increasing $-x$ with 2- decreasing x with 2- results in decreasing y with 1. Now, in order to get the marginal rate of substitution from the left, the steps above are repeated for $h' = -6$

$$Max \sum_{i=1}^{4} y_{i0}^*$$

$$s.t.\ \begin{array}{l} 2\lambda_1 \geq y_{10}^* \\ 4\lambda_2 \geq y_{20}^* \\ 5\lambda_3 \geq y_{30}^* \\ 3\lambda_4 \geq y_{40}^* \end{array}$$

$$\begin{array}{l} -(6-14)\lambda_1 \geq 0 \\ -(8-14)\lambda_2 \geq 0 \\ -(14-14)\lambda_3 \geq 0 \\ -(10-14)\lambda_4 \geq 0 \end{array}$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1$$

$$\lambda_i \geq 0, y_{i0}^* \, free, i = 1,2,3,4.$$

which has the solution$(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = (0,0,1,0)$, $\sum_{i=1}^{4} y_{i0}^* = 5$ and $z_0^* = (2, -14)^T$.

So the finite marginal rate of substitution from the left is

$$MRS^-_{(y)(-x)}(-8,4) = \frac{5-4}{-6} = -1/6$$

Therefore, to the left of z_0, which is when $-x$ decreases, the change in y has the opposite sign (i.e.y increases when x increases) and the magnitude of the change in y is one sixth of that of x.

Table 1: The data for 20 Iranian bank branches.

DMU	I1	I2	I3	O1	O2	O3	FDH efficiency
1	0.95	0.7	0.155	0.19	0.521	0.293	1
2	0.796	0.6	1	0.227	0.627	0.462	1
3	0.798	0.75	0.513	0.228	0.97	0.261	1
4	0.865	0.55	0.21	0.193	0.632	1	1
5	0.815	0.85	0.268	0.233	0.722	0.246	1
6	0.842	0.65	0.5	0.207	0.603	0.569	1
7	0.719	0.6	0.35	0.182	0.9	0.716	1
8	0.785	0.75	0.12	0.125	0.234	0.298	1
9	0.476	0.6	0.135	0.08	0.364	0.244	1
10	0.678	0.55	0.51	0.082	0.184	0.049	1.341
11	0.711	1	0.305	0.212	0.318	0.403	1
12	0.811	0.65	0.255	0.123	0.923	0.628	1
13	0.659	0.85	0.34	0.176	0.645	0.261	1
14	0.076	0.8	0.54	0.144	0.514	0.243	1.264
15	0.685	0.95	0.45	1	0.262	0.098	1
16	0.613	0.9	0.525	0.115	0.402	0.464	1
17	1	0.6	0.205	0.09	1	0.161	1
18	0.634	0.65	0.235	0.059	0.346	0.068	1.043
19	0.372	0.7	0.238	0.039	0.19	0.111	1
20	0.583	0.55	0.5	0.11	0.615	0.764	1

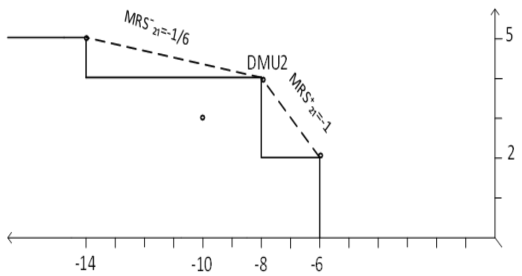

Figure 1: Illustration of marginal rates of substitution in FDH.

4 Empirical Example

To apply the proposed method on real data, we employ the above mentioned method and algorithm on the empirical example used in [11] and [13]. As it can be seen in Table 1, the data set consists of 20 DMUs with 3 inputs and 3 outputs. The data are originally reported by Amirteimoori et al. [13] which consist of 20 Iranian bank branches or DMUs in 2005. The three outputs include deposits, loans and charges. The three inputs include staff, computer terminals and space. In Table 2, we can see the results of the output-oriented FDH model. It can be found that thirteen bank branches are efficient (All units are efficient except for DMUs 10, 14 and 18).

Based on model (3.5), for example, if the staff of $DMU13$ increases with 0.1, the marginal rate of substitution from the right between staff and deposits is 0.60. In other words, increasing Staff with 0.1, results in increasing deposits with 0.06. Also to the left, which is when staff decreases with 0.1, it results in increasing deposit with 0.96. As another example, if the staff of $DMU3$ increases with 0.1, the marginal rate of substitution from the right between staff and deposits is 0.05. In other words, increasing staff with 0.1, results in increasing deposits with 0.05. Also to the left, which is when staff decreases with 0.1, it results in increasing Deposit with 1.48. These results are compiled in Table 2.

5 Conclusion

This paper has presented a method for the calculation of general trade-offs as developed from the point where the work of Asmild et al. [3] on the calculation of marginal rate of substitution in free disposal hull (FDH) models started. Then this method was applied to calculate the trade-offs between staff and deposit in twenty branches of an Iranian bank for efficient units.

Table 2: Staff and Deposit marginal rates of substitution for $h = 0.1, h' = -0.1$

DMU	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
$MRS^{+}_{x_1y_1}$	0.03	0	0.05	0	0	0.21	0	0.68	0	-	0	0.7	0.06	-	0	0.97	1.03	-	0	0
$MRS^{-}_{x_1y_1}$	i.f	i.f	1.48	i.f	1.53	i.f	i.f	i.f	i.f	-	1.32	i.f	0.96	-	1.32	0.35	i.f	-	i.f	i.f

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