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Research Article

Anti-Synchronization of Complex Chaotic T-System Via Optimal Adaptive Sliding-Mode and Its Applications In Secure Communication

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Abstract

In this paper, an optimal adaptive sliding mode controller is proposed for anti-synchronization of two identical hyperchaotic systems. We use hyperchaotic complex T-system for master and slave systems with unknown parameters in the slave system. To construct the optimal adaptive sliding mode controller, first a simple sliding surface is designed. Then, the optimal adaptive sliding mode controller is derived to guarantee the occurrence of the sliding motion. Also, suitable update laws are designed to estimate the unknown parameters. The optimality and stability of the proposed method are proved using Hamilton-Jacobi-Bellman(HJB) technique and Barbalates lemma, respectively. Finally, anti-synchronization result is applied to secure communication via masking method. Numerical simulations illustrate the ability and effectiveness of proposed method.

Keywords : Anti-synchronization; Optimal control; Sliding mode; Barbalate's lemma; Secure communication.

1 Introduction

Chaos is a very interesting nonlinear effect which has been intensively studied during the last four decades. Chaotic systems, as nonlinear deterministic systems, display complex and unpredictable behaviors. The most important characteristic of these systems is their sensitivity to initial conditions. Chaotic phenomena can be found in many scientific and engineering fields such as biological systems, electronic circuits, power converters, chemical systems. The chaotic systems have many important applications in nonlinear sciences, such as laser physics, secure communications, nonlinear circuits, neural networks, and active wave propagation [22, 25, 40, 29, 39, 3, 26, 27, 19, 24, 4].

Synchronization of dynamical systems is an important field of nonlinear dynamics. The idea of synchronizing two identical chaotic systems has been introduced by Pecora and Carroll [29] in 1990. Recently, the synchronization of chaotic systems widely investigated in many fields, such as physics, chemistry, ecological sciences, and se-

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cure communications [15, 6, 39]. Till now, different methods for synchronization have been proposed to achieve chaos synchronization and anti-synchronization, such as adaptive control, active control, sliding mode, nonlinear and linear control [2, 18, 36, 12, 19]. Fortunately, some existing synchronization methods can be generalized to anti-synchronization of chaotic systems. To address the anti-synchronization problem, we consider a chaotic system, the master (or transmitter), together with the slave (or receiver), such that the anti-synchronization is to make the output of the slave system follows the symmetry of output of the master system.

Most methods mentioned in pervious paragraph synchronize or anti-synchronize two chaotic systems using robust methods. Obviously, The roust methods differ from the optimal methods and in general, the sliding mode or adaptive synchronization does not need to satisfy the optimality conditions. The problem of minimal control synthesis algorithm for the control and synchronization of chaotic systems was studied in [14] and the optimal control for the chaos synchronization of Rössler systems with complete uncertain parameters was discussed in [13]. Recently the optimal adaptive synchronization of chaotic complex systems was studied in [27].

In this paper, we discuss the anti-synchronization of a new chaotic system [20] via optimal adaptive sliding-mode method. The optimal adaptive sliding-mode control is used as a robust and optimal method for anti-synchronization.

Secure communication was developed in 1992 based on the synchronization of chaotic dynamical systems. The general idea for transmitting information via chaotic systems is that an information signal is embedded in a transmitter system which produces a chaotic signal. The sending information signal is then recovered by the chaotic receiver.

Chaotic communication techniques include chaos masking, chaos modulation, and chaos shift keying. In chaos masking, the information signal is added directly to the transmitter. Chaos modulation is based on the master-slave synchronization, where the information signal is injected into the transmitter as a nonlinear filter. In chaos shift keying, the information signal is a binary signal mapped into the transmitter and the receiver. In these three cases, the information signal can be recovered by a receiver, if the transmitter and the receiver are synchronized [21, 7, 28].

In 1993, Cuomo et al. [10] developed the additive chaos masking approach. Dedieu et al. [11] presented the chaotic shift keying or the chaotic switching approach in 1993. Then in 1996, Yang and Chua [39] introduced the chaotic parameter modulation method, where the information signal is used to modulate the chaotic system parameters in the transmitter. In this scheme, the information signal is encrypted by an encryption rule with a key generated from the chaotic system in the transmitter. In the last decade, several new chaotic systems have been introduced and the synchronization of these systems via schemes such as adaptive, sliding-mode, and nonlinear feedback control have been discussed. Some manuscripts used these systems for secure communication [34, 38, 37, 5, 23, 41, 42, 16, 33, 26]. Recently, we used the synchronization of hyperchaotic complex T-system for secure communication [27].

The sensitivity of chaotic and hyperchaotic system with respect to initial condition caused it most attended in secure communication. Secure communication of information in the world is of utmost importance. Chaotic (especially hyperchaotic) systems is a very powerful tool for this purpose [30, 31]. Recently, chaotic lorenz system is used for monitoring of nuclear reactors[1].

Some properties of hyperchaotic systems have directed attentions respect to chaotic systems. Hyperchaotic systems exhibit more than one positive Lyapunov exponent which means that their dynamics can expand in more than one direction. Compared with chaotic systems, hyperchaotic systems with more complexity dynamics characteristics can provide tighter security for secure communication. Also, the synchronization and anti-synchronization of such systems are leading to control problems [30, 31].

Motivated by the above discussion, in this paper, the new complex hyperchaotic system introduced by Kheiri and Naderi [20] is used for secure communication based on anti-synchronization. We present the optimal adaptive sliding-mode scheme for anti-synchronization of two identical hyperchaotic complex T-systems, with three unknown parameters. We use the result of anti-synchronization for secure communications via the chaotic masking method. By applying this process, we can achieve chaos anti-synchronization of master and slave systems, identify the unknown parameters, and mask and unmask the message signals simultaneously. In secure communication, a parameter and a state of master system are used for masking the information, such that a chaotic signal is added to the information. The anti-synchronized states and estimation of unknown parameters of the slave system are used to unmask the information. Based on the Barbalate's lemma and Hamilton-Jacobi-Bellman(HJB) technique, an optimal adaptive sliding-mod controller with parameters estimation rules is designed to anti-synchronize complex chaotic T-systems asymptotically.

The rest of this paper is organized as follows: Section 2 briefly introduces hyperchaotic complex T-systems. In Section 3, the optimal adaptive sliding-mode anti-synchronization of two identical complex T-systems and numerical simulations are addressed. In section 4, secure communication based on anti-synchronization and masking method is proposed, and the effectiveness of the proposed scheme is evaluated with simulations for digital images and continuous signals. Finally, concluding remarks are presented in Section 5.

2 Complex chaotic T-system

In 2005, G. Tigen [35] introduced a new real chaotic nonlinear system called T-system, presented as follows:

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = (c-a)x - axz \\ \dot{z} = xy - bz, \end{cases} \tag{2.1}$$

where x, y and z are the state variables and a, b and c are real positive parameters.

In [20], a new complex hyperchaotic system is introduced and its dynamical properties and synchronization are discussed. This system is expressed as:

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = (c-a)x - axz \\ \dot{z} = \frac{1}{2}(\bar{x}y + x\bar{y}) - bz, \end{cases} \tag{2.2}$$

where $x = u_1 + iu_2$, $y = u_3 + iu_4$ are complex variables, $i = \sqrt{-1}$ and $z = u_5$ is a real variable; dots represent derivatives with respect to time and the overbars in $\bar{x}$ and $\bar{y}$ denote the complex conjugate of x and y, respectively. The real version of (2.2) is expressed as follows:

$$\begin{cases} \dot{u_1} = a(u_3 - u_1) \\ \dot{u_2} = a(u_4 - u_2) \\ \dot{u_3} = (c-a)u_1 - au_1u_5 \\ \dot{u_4} = (c-a)u_2 - au_2u_5 \\ \dot{u_5} = u_1u_3 + u_2u_4 - bu_5. \end{cases} \tag{2.3}$$

By choosing $a = 2.1$, $b = 0.6$, and some value of $0 < c < 40$, the Lyapunov exponents in Figure 1 show that the system (2.3) is a hyperchaotic system because at least two Lyapunov exponents of system are positive [20]. Also this system is a dissipative system, since sum of its Lyapunov exponents is negative. The attractors of a chaotic system are bounded but they are not a fixed point or limit cycle. It is a property of chaotic systems [8]. Figure 2 displays the attractors of the complex T-system. Anti-synchronization of this system can be used for the cryptography and decryption of data in secure communication.

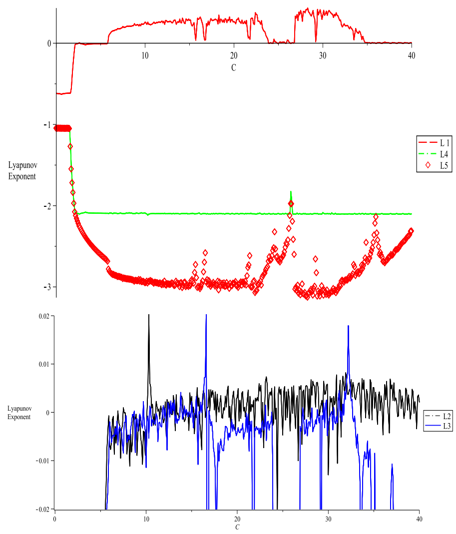

Figure 1: Lyapunov exponents of (2.3) for $a = 2.1$, $b = 0.6$ and $0 < c < 40$.

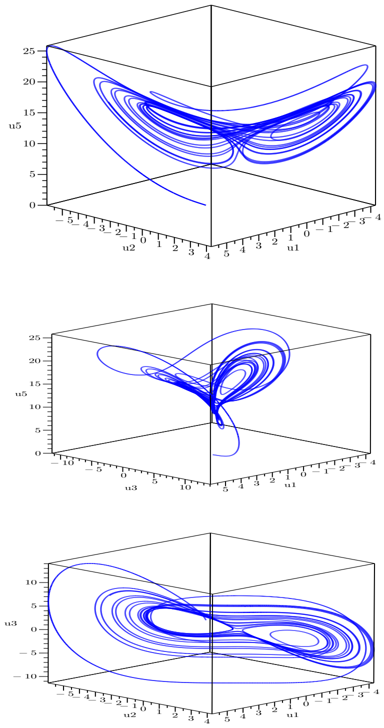

Figure 2: Attractors of (2.3) for $a = 2.1$, $b = 0.6$ and $c = 30$ with initial conditions: $u_1(0) = 1, u_2(0) = -1, u_3(0) = 2, u_4(0) = -2, u_5(0) = 0$.

3 Anti-synchronization with optimal adaptive sliding-mode control method

For anti-synchronization of two identical chaotic systems, assume that the master and slave systems are defined as follows:

$$\dot{x} = f(x) \tag{3.4}$$

$$\dot{y} = f(y) + \omega, \tag{3.5}$$

where $x,\ y \in R^n$ are the state vectors of the systems (3.4) and (3.5), $f : R^n \longrightarrow R^n$ and $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is control signal. The anti-synchronization error is $e = y + x$. Then, the dynamical system error between master and slave systems is calculated as follows:

$$\begin{aligned} \dot{e} &= \dot{y} + \dot{x} = f(y) + f(x) + \omega \\ &= f(e - x) + f(x) + \omega. \end{aligned} \tag{3.6}$$

The goal is to design an appropriate controller ω such that for any initial condition y_0 and x_0, we have:

$$\lim_{t \longrightarrow \infty} \|e\| = \lim_{t \longrightarrow \infty} \|y(t) + x(t)\| = 0,$$

where $\|.\|$ is the Euclidean norm.

We assume the master system and the slave system with unknown parameters are defined respectively, as follows:

$$\begin{cases} \dot{u_{1m}} = a(u_{3m} - u_{1m}) \\ \dot{u_{2m}} = a(u_{4m} - u_{2m}) \\ \dot{u_{3m}} = (c - a)u_{1m} - au_{1m}u_{5m} \\ \dot{u_{4m}} = (c - a)u_{2m} - au_{2m}u_{5m} \\ \dot{u_{5m}} = u_{1m}u_{3m} + u_{2m}u_{4m} - bu_{5m} \end{cases} \tag{3.7}$$

and

$$\begin{cases} \dot{u}_{1s} = \hat{a}(u_{3s} - u_{1s}) + \omega_1 \\ \dot{u_{2s}} = \hat{a}(u_{4s} - u_{2s}) + \omega_2 \\ \dot{u_{3s}} = (\hat{c} - \hat{a})u_{1s} - \hat{a}u_{1s}u_{5s} + \omega_3 \\ \dot{u_{4s}} = (\hat{c} - \hat{a})u_{2s} - \hat{a}u_{2s}u_{5s} + \omega_4 \\ \dot{u_{5s}} = u_{1s}u_{3s} + u_{2s}u_{4s} - \hat{b}u_{5s} + \omega_5, \end{cases} \tag{3.8}$$

where $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unknown parameters, which need to be estimated in the slave system. In order to obtain the complex error system, we sum (3.7) and (3.8), then we have:

$$\begin{cases} \dot{e}_1 = & -\tilde{a}(u_{3m} - u_{1m}) \\ & +\hat{a}(e_3 - e_1) + \omega_1 \\ \dot{e}_2 = & -\tilde{a}(u_{4m} - u_{2m}) \\ & +\hat{a}(e_4 - e_2) + \omega_2 \\ \dot{e}_3 = & -\tilde{c}u_{1m} + \tilde{a}u_{1m} + (\hat{c} - \hat{a})e_1 \\ & -\hat{a}(e_1e_5 - e_1u_{5m} - e_5u_{1m}) \\ & -(\hat{a} + a)u_{1m}u_{5m} + \omega_3 \\ \dot{e}_4 = & -\tilde{c}u_{2m} + \tilde{a}u_{2m} + (\hat{c} - \hat{a})e_2 \\ & -\hat{a}(e_2e_5 + e_2u_{5m} + e_5u_{2m}) \\ & -(\hat{a} + a)u_{2m}u_{5m} + \omega_4 \\ \dot{e}_5 = & \tilde{b}u_{5m} + e_1e_3 + e_2e_4 \\ & -e_1u_{3m} - u_{1m}e_3 \\ & +2u_{1m}u_{3m} + 2u_{2m}u_{4m} \\ & -e_2u_{4m} - e_4u_{2m} \\ & -\hat{b}e_5 + \omega_5, \end{cases} \tag{3.9}$$

where $e_i = u_{is} + u_{im}$, $\tilde{a} = \hat{a} - a$, $\tilde{b} = \hat{b} - b$ and $\tilde{c} = \hat{c} - c$.

The sliding mode control is an excellent strategy in the area of robust control [17, 32]. The main feature of the sliding mode control is its capability for total rejection of matched uncertainties. In the sliding mode control for practical realization, the discontinuous feedback law can be replaced with a high-gain linear feedback if the system state is close to the switching manifold. To this end, we rewrite (3.9) as the equation below:

$$\begin{cases} \dot{e}_1 = -\tilde{a}(u_{3m} - u_{1m}) + W_1 - \xi_1 sign(e_1) \\ \dot{e}_2 = -\tilde{a}(u_{4m} - u_{2m}) + W_2 - \xi_2 sign(e_2) \\ \dot{e}_3 = -\tilde{c}u_{1m} + \tilde{a}u_{1m} + W_3 - \xi_3 sign(e_3) \\ \dot{e}_4 = -\tilde{c}u_{2m} + \tilde{a}u_{2m} + W_4 - \xi_4 sign(e_4) \\ \dot{e}_5 = \tilde{b}u_{5m} + W_5 - \xi_5 sign(e_5), \end{cases} \tag{3.10}$$

such that

$$\begin{cases} W_1 - \xi_1 sign(e_1) = \omega_1 + \hat{a}(e_3 - e_1) \\ W_2 - \xi_2 sign(e_2) = \omega_2 + \hat{a}(e_4 - e_2) \\ W_3 - \xi_3 sign(e_3) = \omega_3 + (\hat{c}\hat{a})e_1 \\ -\hat{a}(e_1e_5 - e_1u_{5m} - e_5u_{1m}) \\ -(\hat{a}+a)u_{1m}u_{5m} \\ W_4 - \xi_4 sign(e_4) = \omega_4 + (\hat{c}-\hat{a})e_2 \\ -\hat{a}(e_2e_5 + e_2u_{5m} + e_5u_{2m}) \\ -(\hat{a}+a)u_{2m}u_{5m} \\ W_5 - \xi_5 sign(e_5) = \omega_5 + e_1e_3 \\ +e_2e_4 - e_1u_{3m} - u_{1m}e_3 \\ +2u_{1m}u_{3m} + 2u_{2m}u_{4m} \\ -e_2u_{4m} - e_4u_{2m} - \hat{b}e_5. \end{cases} \tag{3.11}$$

The first step taken to design sliding mode control is to select an appropriate sliding surface [17, 32]. The sliding surface can be designed as follow:

$$S(t) = [S_e(t), S_p(t)], \tag{3.12}$$

where $S_e(t) = [\lambda_1 e_1, \lambda_2 e_2, \lambda_3 e_3, \lambda_4 e_4, \lambda_5 e_5]$ and $S_p(t) = [\lambda_6\tilde{a}, \lambda_7\tilde{b}, \lambda_8\tilde{c}]$ such that surface parameters λ_i are positive constants.

The next step is to determine an optimal input signal $W(t)$ to guarantee that the error system trajectories reach the sliding surface $S(t) = 0$.

The following theorem discusses the optimal controllers, dynamic of unknown parameters, and stability of error system in sliding surface.

Theorem 3.1 *Let the parameter estimation dynamic is as follow*

$$\begin{cases} \dot{\hat{a}} = & e_1(u_{3m} - u_{1m}) + e_2(u_{4m} - u_{2m}) \\ & -e_3u_{1m} - e_4u_{2m} - k_1(\hat{a} - a) \\ \dot{\hat{b}} = & -e_5u_{5m} - k_2(\hat{b} - b) \\ \dot{\hat{c}} = & e_3u_{1m} + e_4u_{2m} - k_3\hat{c} - c) \end{cases} \tag{3.13}$$

and $W_i = -\frac{e_i}{2\eta_i}$, where $k_i \geq 0$ $(i = 1, 2, 3)$ and $\eta_i > 0$ $(i = 1, ..., 5)$. Then $\omega = (\omega_1, ..., \omega_5)$ in relation (3.11) with $\xi_i = |e_i|$ is the optimal sliding-mode control in the sliding surface (3.12) and minimizes the cost function:

$$\begin{aligned} I &= \int_0^\infty \Omega(e_1, e_2, e_3, e_4, e_5, \hat{a}, \hat{b}, \hat{c}, \vec{W})dt \\ &= \int_0^\infty \{k_1(\hat{a} - a)^2 + k_2(\hat{b} - b)^2 \\ &+ k_3(\hat{c} - c)^2 + \textstyle\sum_1^5 (c_i e_i^2 + \eta_i W_i^2)\}dt, \end{aligned} \tag{3.14}$$

where c_i, is real positive constant. Also error dynamical system (3.10) and parameter estimation low (3.13) converge to equilibriums, $e_i = 0$ $(i = 1, ..., 5)$, $\hat{a} = a$, $\hat{b} = b$ and $\hat{c} = c$ respectively.

Hence, the master system (3.7) and slave system (3.8) are asymptotically anti-synchronize.

Proof. Assume that the minimum of (3.14) is obtained with $W = W^* = (W_1^*, ..., W_5^*)$, then we have:

$$\begin{aligned} & V(e_1, e_2, e_3, e_4, e_5, \hat{a}, \hat{b}, \hat{c}, W^*, t) = \\ & \min_W \int_0^\infty \Omega(e_1, \ldots, e_5, \hat{a}, \hat{b}, \hat{c}, W)dt. \end{aligned} \tag{3.15}$$

We shall show that V may be treated as the Lyapunov function in our system.

Now, we find the optimal controller W^* such that the systems (3.10)and (3.13) stabilize in equilibrium points and the integral (3.14) is minimum. This requirement can be met by applying HJB technique of dynamic programming problem [17, 13]; that is

$$\begin{aligned} & \tfrac{\partial V}{\partial e_1}\dot{e_1} + \tfrac{\partial V}{\partial e_2}\dot{e_2} + \tfrac{\partial V}{\partial e_3}\dot{e_3} + \tfrac{\partial V}{\partial e_4}\dot{e_4} + \tfrac{\partial V}{\partial e_5}\dot{e_5} \\ & + \tfrac{\partial V}{\partial \hat{a}}\dot{\hat{a}} + \tfrac{\partial V}{\partial \hat{b}}\dot{\hat{b}} + \tfrac{\partial V}{\partial \hat{c}}\dot{\hat{c}} + k_1(\hat{a} - a)^2 \\ & + k_2(\hat{b} - b)^2 + k_3(\hat{c} - c)^2 \\ & + \textstyle\sum_1^5 (c_i e_i^2 + \eta_i W_i^{*2}) = 0. \end{aligned} \tag{3.16}$$

By using (3.10) in (3.16), we get:

$$\begin{aligned}
&\tfrac{\partial V}{\partial e_1}[-(\hat{a}-a)(u_{3m}-u_{1m})\\
&+W_1-\xi_1 sign(e_1)]+\\
&\tfrac{\partial V}{\partial e_2}[-(\hat{a}-a)(u_{4m}-u_{2m})\\
&+W_2-\xi_2 sign(e_2)]+\\
&\tfrac{\partial V}{\partial e_3}[-(\hat{c}-c)u_{1m}+(\hat{a}-a)u_{1m}\\
&+W_3-\xi_3 sign(e_3)]+\\
&\tfrac{\partial V}{\partial e_4}[-(\hat{c}-c)u_{2m}+(\hat{a}-a)u_{2m}\\
&+W_4-\xi_4 sign(e_4)]+\\
&\tfrac{\partial V}{\partial e_5}[(\hat{b}-b)u_{5m}+W_5-\xi_5 sign(e_5)]\\
&+\tfrac{\partial V}{\partial \hat{a}}\dot{\hat{a}}+\tfrac{\partial V}{\partial \hat{b}}\dot{\hat{b}}+\tfrac{\partial V}{\partial \hat{c}}\dot{\hat{c}}+\\
&k_1(\hat{a}-a)^2+k_2(\hat{b}-b)^2+k_3(\hat{c}-c)^2\\
&+\textstyle\sum_1^5(c_ie_i^2+\eta_iW_i^{*2})=0.
\end{aligned} \tag{3.17}$$

Minimizing (3.17) with respect to W^*, gives the optimal controllers as follows:

$$\begin{aligned}
&\tfrac{\partial V}{\partial e_i}+2\eta_iW_i^*=0\\
&\Rightarrow W_i^*=\tfrac{-1}{2\eta_i}\tfrac{\partial V}{\partial e_i}(i=1,...,5).
\end{aligned} \tag{3.18}$$

By substituting the equations (3.18) in (3.17), we have:

$$\begin{aligned}
&\tfrac{\partial V}{\partial e_1}[-(\hat{a}-a)(u_{3m}-u_{1m})-\xi_1 sign(e_1)]+\\
&\tfrac{\partial V}{\partial e_2}[-(\hat{a}-a)(u_{4m}-u_{2m})-\xi_2 sign(e_2)]+\\
&\tfrac{\partial V}{\partial e_3}[-(\hat{c}-c)u_{1m}+(\hat{a}-a)u_{1m}\\
&-\xi_3 sign(e_3)]+\\
&\tfrac{\partial V}{\partial e_4}[-(\hat{c}-c)u_{2m}+(\hat{a}-a)u_{2m}\\
&-\xi_4 sign(e_4)]+\\
&\tfrac{\partial V}{\partial e_5}[(\hat{b}-b)u_{5m}-\xi_5 sign(e_5)]\\
&+\tfrac{\partial V}{\partial \hat{a}}\dot{\hat{a}}+\tfrac{\partial V}{\partial \hat{b}}\dot{\hat{b}}+\tfrac{\partial V}{\partial \hat{c}}\dot{\hat{c}}+\\
&k_1(\hat{a}-a)^2+k_2(\hat{b}-b)^2+k_3(\hat{c}-c)^2\\
&+\textstyle\sum_1^5(c_ie_i^2-\frac{1}{4\eta_i}(\frac{\partial V}{\partial e_i})^2)=0.
\end{aligned} \tag{3.19}$$

Now, we choose the Lyaponuv function in the sliding surface (3.12) as follows:

$$\begin{aligned}
&\phi(e_1,\dots,e_5,\hat{a},\hat{b},\hat{c})=SAS^T\\
&=\tfrac{1}{2}[\textstyle\sum_1^5 e_i^2+(\hat{a}-a)^2+(\hat{b}-b)^2+(\hat{c}-c)^2],
\end{aligned} \tag{3.20}$$

where $A=diag[\frac{1}{2\lambda_i^2}]\in R^{8\times 8}$. The HJB equation (3.19) is satisfied for $V=\phi$, $\xi_i=|e_i|$ and $\eta_i=\frac{1}{4(c_i-1)}(c_i\neq 1)$. Then, ϕ is a solution of partial differential equation (3.19). Hence, by substituting the (3.20) in (3.18), the optimal controllers can be derived as follows:

$$W_i^*=-\frac{e_i}{2\eta_i}(i=1,..,5).$$

Also, from (3.20) we have:

$$\dot{\phi}=-\left[k_1\tilde{a}^2+k_2\tilde{b}^2+k_3\tilde{c}^2+\sum_1^5 2c_ie_i^2\right]<0. \tag{3.21}$$

Equation (3.21) can be rewritten as follow:

$$\begin{aligned}
&\dot{\phi}=-(\tfrac{k_1\lambda_6^2}{\lambda_6^2}\tilde{a}^2+\tfrac{k_2\lambda_7^2}{\lambda_7^2}\tilde{b}^2+\tfrac{k_3\lambda_8^2}{\lambda_8^2}\tilde{c}^2\\
&+\textstyle\sum_1^5 2\frac{c_i\lambda_i^2}{\lambda_i^2}e_i^2)<0.
\end{aligned} \tag{3.22}$$

Let

$$L=\min\{\frac{2c_1}{\lambda_1^2},\frac{2c_2}{\lambda_2^2},\frac{2c_3}{\lambda_3^2},\frac{2c_4}{\lambda_4^2},\frac{2c_5}{\lambda_5^2},\frac{k_1}{\lambda_6^2},\frac{k_2}{\lambda_7^2},\frac{k_3}{\lambda_8^2}\}$$

and $|S|^2$ is Euclidean norm of (3.12). Then

$$\dot{\phi}<-L|S|^2=-h(t)<0, \tag{3.23}$$

where $L|S|^2=h(t)$. Integrating equation (3.23) from zero to t yields:

$$\begin{cases}\phi(0)-\phi(t)\geq 0\\ \phi(0)-\phi(t)\geq\int_0^t h(\tau)d\tau.\end{cases} \tag{3.24}$$

Hence $\lim\limits_{t\to\infty}\int_0^t h(\tau)d\tau\geq 0$. Thus, according to the Barbalate's lemma [17], we have:

$$\lim_{t\to\infty}h(t)=\lim_{t\to\infty}L|S|^2=0. \tag{3.25}$$

Since L is greater than zero, (3.25) implies $\lim\limits_{t\to\infty}S(t)=0$. This shows the solutions (3.10) and (3.13) are asymptotically stable via optimal adaptive sliding-mode control [17].Then the unknown parameters of slave system are successfully estimated and the master-slave system is anti-synchronized via the optimal adaptive sliding mode control.

3.1 Numerical simulation of anti-synchronization

To demonstrate the validity of the proposed method, we present and discuss the numerical simulation for anti-synchronization of the hyperchaotic complex T-system. For anti-synchronizing, systems (3.7), (3.8) and (3.13) with obtained controllers are solved numerically. In the numerical simulation, the fourth-order Runge-Kutta method is used.

By assuming $a = 2.1$, $b = 0.6$, and $c = 30$, the system (3.7) is turned into a hyperchaotic system [20]. To anti-synchronize the master and slave systems, we choose initial conditions as follows:

$$\begin{cases} u_m(0) = (0.5, 0.5, 1, 1, -0.5) \\ u_s(0) = (2, 0.25, 0.5, -0.4, -0.2, -2) \\ \hat{a}(0) = 0.2, \hat{b}(0) = 1, \hat{c}(0) = 29. \end{cases} \tag{3.26}$$

Also, suppose the constant parameters in optimal adaptive sliding-mode controller and parameters estimation rules are:

$$\begin{array}{lll} \eta_1 = 1, & \eta_2 = 5, & \eta_3 = 1, \\ \eta_4 = 0.3, & \eta_5 = 0.3 & k_1 = 5, \\ k_2 = 10, & k_3 = 10. & \end{array} \tag{3.27}$$

The results of chaotic anti-synchronization of two identical hyperchaotic T-systems (3.7) and (3.8), via optimal adaptive sliding-mode controller are shown in Figure 3.

Clearly, the slave and master systems are anti-synchronized after a short time. The anti-synchronization errors are plotted in Figure 4. Figure 5 shows the estimates $\hat{a}(t)$, $\hat{b}(t)$, and $\hat{c}(t)$ of the unknown parameters converge to $a = 2.1$, $b = 0.6$, and $c = 30$, respectively. As expected from in the above analytical considerations, the anti-synchronization errors e_i and error of parameter estimations are converged to zero as $t \longrightarrow \infty$.

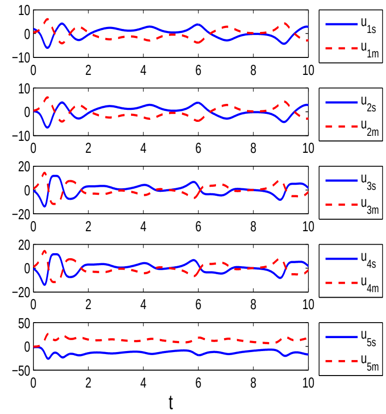

Figure 3: State trajectories for anti-synchronization via optimal sliding-mode controllers.

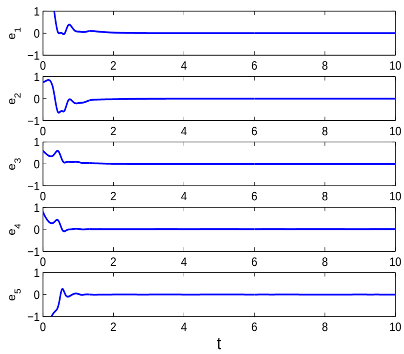

Figure 4: The error due to anti-synchronization.

4 Secure communication

This section addresses a new secure communication scheme based on anti-synchronization of two complex chaotic T-systems. According to Theorem 3.1, the architecture of the proposed secure communication scheme is established by two identical complex chaotic systems with uncertain parameters as shown in Figure 6. At the transmitter side, system (3.7) with state variables $u_m(t)$ and parameter $P = (a, b, c)$ are used as the master system, while at the receiver side, system (3.8) has the same structure as system (3.7). However, with different state variables $u_s(t)$, parameter $\hat{P}(t) = (\hat{a}(t), \hat{b}(t), \hat{c}(t))$, and an optimal adaptive sliding mode controller is employed to anti-synchronize the two systems. At the transmitter side, the original message $M(t)$ is masked by the chaotic signal $P_i u_{\iota m}(t)$. Let $T(t)$ is the masked message as shown as follows:

$$T(t) = M(t) + p_i u_{\iota m}(t), \tag{4.28}$$

where $i = 1, 2$ or 3 and $\iota = 1, 2, 3, 4$ or 5. In the proposed chaotic masking secure communication scheme, one of the known parameters of master

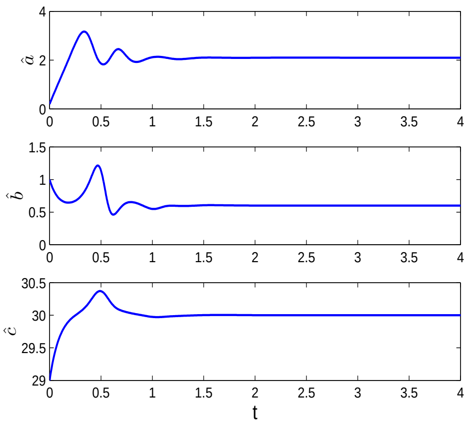

Figure 5: Estimation of unknown parameters.

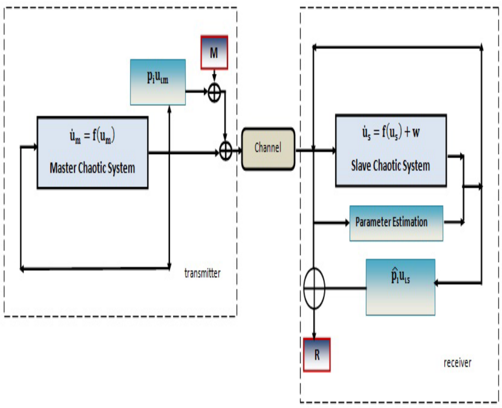

Figure 6: Diagram of secure communication

system is employed as a encryption key for masking, so the estimation of this parameter in the slave system must be used as a decryption key for unmasking. Therefore, this scheme has more security compared than similar work [33, 42]

Also, $M(t)$ must be well chosen in a way that it can be successfully masked by $P_i u_{\iota m}(t)$. Otherwise, the original message $M(t)$ is multiplied by a scaling factor [9] is used for resizing the original message. The resulting signal $T(t)$ from the transmitter is sent to the receiver via a public channel. By theorem 3.1, the chaotic anti-synchronization will be achieved by the optimal adaptive sliding mode controller after some time T_s (T_s is the anti-synchronization time). If T_c is a time greater than T_s, it will be suitable time for transfer and recovery. The received signal by the receiver is recoverable with following equation:

$$R(t) = T(t) + \hat{p_i}(t) u_{\iota s}(t) \cong M(t). \tag{4.29}$$

Because, according to the concept of anti-synchronization and parameter estimation rule, we have:

$$\begin{aligned} R(t) &= M(t) + p_i u_{\iota m}(t) + \hat{p_i}(t) u_{\iota s}(t) \\ &\cong M(t). \end{aligned} \tag{4.30}$$

During the communication process, the transmitter can use k_e as the encryption key, while k_d is used as the decryption key by the receiver, such that

$$k_e = \{u_{1m}(0), \ldots, u_{5m}(0), a, b, c, T_c\}$$

and

$$k_d = \{u_{1s}(0), \ldots, u_{5s}(0), \hat{a}, \hat{b}, \hat{c}, T_c\}.$$

The encrypted signal is transmitted through the public channel which is open to any intruder. Nevertheless, intruders are unable to decrypt the message even if they have knowledge of the structure of the chaotic system at the transmitter. Since the initial conditions, the system parameters and the anti-synchronized time T_c are completely unknown, it would be extremely difficult for an intruder to recover the masked messages from the system without the correct encryption or decryption keys. It is also noted that even an extremely small deviation of the correct keys will lead to a completely different result due to the systems chaotic nature, thus increasing the difficulty for intruders to recover the original message.

In addition, the proposed communication scheme consists of four phases:

- Design an optimal adaptive sliding mode controller for anti-synchronization of two coupled complex chaotic T-systems

- Design estimation rule of unknown parameters.

- Masking and transmitting the information signal

- Recovering the signal by using the concept of anti-synchronization and the unmasking scheme.

In subsection *3.1*, the numerical simulation of anti-synchronization of complex chaotic system via optimal sliding mode method was discussed. To evaluate the feasibility of the proposed secure communication scheme, numerical simulations are given in following subsection.

4.1 Numerical simulation of secure communication

To test the validity of the proposed scheme, we present and discuss the simulations results for the encryption and decryption of continuous signal and digital images. All the simulations are carried out by the MATLAB software and for solving the dynamical systems the fourth-order Runge-Kutaa numerical method is used. We assume the initial conditions and requirement parameters similar to those in the subsection *3.1*. Encryption and decryption occur after anti-synchronization time T_s. In simulations that are different from a one-dimensional continuous signal, the digital image must be rearranged in an image data set $S = \{s_1, s_2, ..., s_d\}$, in which each element has a value with a *double* format (d is the number pixel of the digital image). For example, the presented method is tested on the continues signal $M(t) = 0.8\sin(200\pi t)$, black-white and color digital images. Simulation results are shown in Figures 7-9 with encryption and decryption rules (4.28) and (4.29) for $\iota = 2$ and $i = 3$.

5 Conclusions

In this study, a new optimal adaptive sliding-mode controller was designed for the anti-synchronization of two identical complex chaotic T-systems with unknown parameters. Our obtained optimal control laws and parameter estimation rules were satisfied in Barbalet's lemma and the HJB technique. So, the obtained controller are robust and optimal. Anti-synchronization results showed that the errors and the parameter estimation are asymptotically convergent. The results of anti-synchronization were used to secure communication via masking method. In the presented method, the state variables, the known parameters and the estimation of unknown parameters were used as encoding and decoding keys. Secure communication based on optimal adaptive sliding-mode anti-synchronization decrypt the continuous signals or digital images easily. The simulation results showed the effectiveness of the proposed method for anti-synchronization and secure communication purposes.

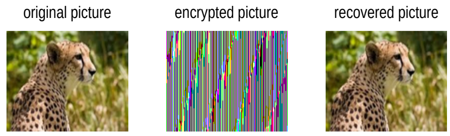

Figure 7: Secure communication based on anti-synchronization for color digital image (Iranian cheetah image with $d = 104 \times 156 \times 3$)

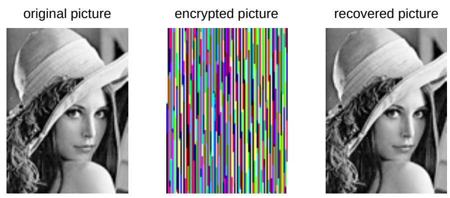

Figure 8: Secure communication based on anti-synchronization for black-white digital image (Lena image with $d = 120 \times 86 \times 3$)

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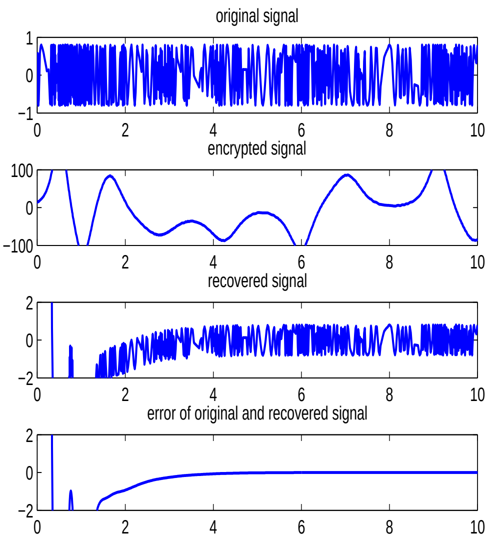

Figure 9: Secure communication based on anti-synchronization for continues signal

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