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Research Article

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Provide a Model for Reallocating Resources in the Structure of Pasargadae Bank Branches With Emphasis on Efficiency and Productivity

M. Bakhtiari [a], M. Rostamy-Malkhalifeh [b*]

[(a)] *Department of athematic, Science and Research Branch, Islamic Azad University, Tehran, Iran.*
[(b)] *Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.*

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Abstract
Data envelopment analysis (DEA) creates many opportunities for collaboration between analyst and decision-maker. There are, however, situations in which all of the decision-making units (DMUs) fall under the umbrella of a centralized decision maker that oversees them. Many organizations such as bank branches, chain stores, … can do this. This centralized decision maker unit expect that resource allocation and revenue efficiency be in a way that DMUs not separately but in a group and simultaneously projected onto the efficiency frontier; as a result, it won't be possible based on current DEA models. Therefore, centralized resource allocation or institutional allocation was formulated. There are situations in which centralized method presented in a central decision maker unit to allocate resources based on revenue efficiency. However, in reality value and rate are not often observed for all of the undesirable and desirable output units, which poses a problem in determining the revenue efficiency. Therefore, the best solution in these cases is to divide the outputs into two categories of known and unknown prices, which will be a more valid criterion for determining the revenue efficiency. In this paper, based on these methods, the revenue efficiency in branches of Pasargadae Bank will be analyzed and a comprehensive ranking will be made on these branches.

Keywords: DEA, revenue efficiency, resource allocation, undesirable output, desirable output.

*.Corresponding author Email: mohsen_rostamy@yahoo.com

1. Introduction

Since Charnes *et al* (1978), data envelopment analysis (DEA) as a non-parametric method has been widely applied to measure the relative efficiency of a set of decision-making units (DMUs) with multiple inputs and outputs, through the use of linear and math planning. Korhonen and Syrjanen (2004) formulated a method based on DEA and multiple-criteria decision analysis (MOLP) for centralized allocation of resources. The proposed method then used for 25 chain stores under the supervision of a central manager. Indeed, the chain store manager's main purpose is to allocate resources between all stores, proportionally, to maximize the output of all stores at a time. Diou *et al* (2004) developed the Korhonen and Syrjanen method. Their purpose is to maximize all outputs and minimize all inputs of DMUs at the same time. Lozano and Villa (2004) proposed BCC and CRA-BCC models for centralized allocation of resources. First (second) model decrease inputs of all DMUs in the radial (non-radial) form, simultaneously. Mar-Molinero *et al* (2012) by proposing a simpler model, developed the CRA-BCC model of Lozano and Villa. Hosseinzadeh Lotfi *et al* (2010) formulated a non-radial model based on Russel's model for centralized resource allocation. In 2016, Fang proposed centralized resource allocation models to determine the revenue efficiency. Cook *et al* (2004) stated that efficient units form a certain linear boundary which can be considered as the best boundary in modelling conditions. In 2008, Thanassoulis *et al* stated that in modeling we usually move from efficient technical points to point with global performance. Adler *et al* in 2013, analyzed the application of modelling in flight and airline industry. Then in 2013 he ranked the airports. Cook *et al* in 2014, analyzed the application of modelling in banking. In 2014, Aparicio *et al* achieved modelling by minimizing the distance of efficient pareto boundary to production possibility set. Jose *et al* in 2015, proposed a common criterion for modelling and ranking decision-making units in DEA.

2. Previous methods

Lei Fang studied centralized DEA models for resource allocation based on revenue efficiency and under limited information. As value and cost of all outputs are not always equal, therefore, Lei Fang designed his model accordingly and showed that Lozano model is a special case of proposed model in which the market prices for all the outputs are equal to their absolute shadow prices. Lozano *et al* stated that the centralized DEA models that allow the reallocation of certain inputs should generally produce better results, but they could not clearly identify the reason. Whereas, Lei Fang in this paper, decomposed the aggregate shadow revenue efficiency into three components to uncover the resources of this total output increase.

2.1. Resource allocation model with partial price data

Assume that there are n DMUs and each DMUs uses m inputs to produce s outputs. For each DMU_j $(j = 1, \dots, n)$ we denote the input and the output vectors as $(\mathrm{X_j}, \mathrm{Y_j})$, respectively, where

$\mathrm{X_j} = (\mathrm{x_{1j}}, \mathrm{x_{2j}}, \dots, \mathrm{x_{mj}})^{\mathrm{t}} \geq 0, \mathrm{X_j} \neq 0, \mathrm{j} = 1,2, \dots, \mathrm{n}$,

$\mathrm{Y_j} = (\mathrm{y_{1j}}, \mathrm{y_{2j}}, \dots, \mathrm{y_{sj}})^{\mathrm{t}} \geq 0, \mathrm{Y_j} \neq 0, \mathrm{j} = 1,2, \dots, \mathrm{n}$

Assuming that all DMUs are under the control of the centralized decision maker, the centralized DM aims to maximize the average percentage increase in total outputs given the resources available to each unit. The centralized DEA model proposed by Lozano *et al* (2011) can be formulated using a non-radial Russel aggregate output measure of technical efficiency as follows:

$$Max\ \frac{1}{s}\sum_{k=1}^{s}\gamma_k$$

$$\text{s.t}\ \sum_{j=1}^{n}\lambda_{jr}x_{ij} \le \hat{x}_{ir} \quad i = 1,..,m\,;r = 1,..,n$$

$$\hat{x}_{ir} \le x_{ir} \quad i \notin I^{realloc};r = 1,..,n$$

$$\sum_{r=1}^{n}\hat{x}_{ir} \le \sum_{r=1}^{n}x_{ir} \quad i \in I^{realloc} \qquad (1)$$

$$\sum_{j=1}^{n}\lambda_{jr}y_{kj} \ge \hat{y}_{kr} \quad k = 1,..,s\,;r = 1,..,n$$

$$\hat{y}_{kr} \ge y_{kr} \quad k = 1,..,s;r = 1,..,n$$

$$\sum_{r=1}^{n}\hat{y}_{kr} \ge \gamma_k\sum_{r=1}^{n}y_{kr} \quad k = 1,..,s$$

$$\sum_{j=1}^{n}\lambda_{jr} = 1 \quad r = 1,\dots,n$$

$$\lambda_{rj} \ge 0 \quad r = 1,\dots,n;j = 1,\dots,n$$

Where I^{realloc} represents the subset of inputs that can be reallocated among the units and $\hat{x}_{ir}$ and $\hat{y}_{kr}$ are the targets for input *i* and output *k*, respectively, of unit *r*.

Let γ_k^* be the optimal solution to model (1) and let $\hat{y}_k^* = \sum_{r=1}^{n}\gamma_k^* y_{kr}$.

The corresponding production set denoted by as can be represented as follows:

$$T_1 = \left\{\begin{array}{c}(\hat{x}_{ir},\hat{y}_{kr})|\sum_{j=1}^{n}\lambda_{jr}x_{ij} \le \hat{x}_{ir}\ ;\hat{x}_{ir} \le x_{ir} \quad i \notin I^{realloc} \\ \sum_{r=1}^{n}\hat{x}_{ir} \le \sum_{r=1}^{n}x_{ir} \quad i \in I^{realloc};\sum_{j=1}^{n}\lambda_{jr}y_{kj} \ge \hat{y}_{kr}\,; \\ \hat{y}_{kr} \ge y_{kr}\ ;\sum_{r=1}^{n}\hat{y}_{kr} \ge \gamma_k\sum_{r=1}^{n}y_{kr}\ ;\sum_{j=1}^{n}\lambda_{jr} = 1; \\ \lambda_{rj} \ge 0\ ;\quad r = 1,\dots,n \\ j = 1,\dots,n;k = 1,\dots,s\end{array}\right\}$$

In the following section we present a centralized approach for allocating resources based on revenue efficiency in a decision making environment. In practice, the prices are often not observed for some outputs. We assume that the outputs can be categorized as outputs with known prices ($k \in O_p$) and outputs with unknown prices ($k \in O_s$). For simplicity and without loss of generality, we assume that some inputs are resources to be allocated.

To allocate the input resources to a set of existing units so that the total output revenue will be maximized, the resource allocation model based on revenue efficiency within the original production possibility set can be formulated as follows:

$$Max\ \Psi = \sum_{k\in o_p}\sum_{r=1}^{n}p_k\bar{y}_{kr} + \sum_{k\in o_s}\sum_{r=1}^{n}\tilde{p}_k\bar{y}_{kr}$$

$$\text{s.t}\ \sum_{j=1}^{n}\lambda_{jr}x_{ij} \le \bar{x}_{ir} \quad i = 1,..,m\,;r = 1,..,n$$

$$\bar{x}_{ir} \le x_{ir} \quad i \notin I^{realloc};r = 1,..,n$$

$$\sum_{r=1}^{n}\bar{x}_{ir} \le \sum_{r=1}^{n}x_{ir} \quad i \in I^{realloc} \qquad (2)$$

$$\sum_{j=1}^{n}\lambda_{jr}y_{kj} \ge \bar{y}_{kr} \quad k = 1,..,s\,;r = 1,..,n$$

$$\bar{y}_{kr} \ge y_{kr} \quad k = 1,..,s;r = 1,..,n$$

$$\sum_{j=1}^{n}\lambda_{jr} = 1 \quad r = 1,\dots,n$$

$$\lambda_{rj} \ge 0 \quad r = 1,\dots,n;j = 1,\dots,n$$

Where p_k denotes the price for the outputs with known prices, that is, ($k \in O_p$), $\tilde{p}_k$ denotes the shadow price for the outputs without known prices, that is, ($k \in O_s$), and $\bar{x}_{ir}$ and $\bar{y}_{kr}$ are the targets for input *i* and output *k*, respectively, of unit *r*.

3. Proposed method

In some real world applications, outputs are not always desirable, means in some cases the system faces undesirable outputs which its aim is to reduce them. In this paper, based on Lozano *et al* model (2016) we determine the revenue efficiency in branches of Pasargadae Bank in Tehran. In this model, as outputs involves both desirable and undesirable ones, the aim is to increase desirable outputs and decrease undesirable outputs in order to achieve the maximum profitability. Accordingly, the model is presented as follows:

3.1. Resource allocation model with desirable and undesirable data

Assuming that all DMUs are under the control of the centralized decision maker, the centralized DM aims to maximize the average percentage increase in total outputs given the resources available to each unit. In the centralized DEA model, using a non-radial method and considering desirable and undesirable outputs, technical efficiency can be formulated as follows:

$$Max\ \frac{1}{s}\sum_{k=1}^{s}\gamma_k-\frac{1}{q}\sum_{l=1}^{q}\gamma'_l$$

$$\begin{aligned}
\text{s.t}\ &\sum_{j=1}^{n}\lambda_{jr}x_{ij}\le\hat{x}_{ir} && i=1,..,m\,;r=1,..,n\\
&\hat{x}_{ir}\le x_{ir} && i\notin I^{realloc};r=1,..,n\\
&\sum_{r=1}^{n}\hat{x}_{ir}\le\sum_{r=1}^{n}x_{ir} && i\in I^{realloc} && (3)\\
&\sum_{j=1}^{n}\lambda_{jr}y_{kj}\ge\hat{y}_{kr} && k=1,..,s\,;r=1,..,n\\
&\hat{y}_{kr}\ge y_{kr} && k=1,..,s;r=1,..,n\\
&\sum_{r=1}^{n}\hat{y}_{kr}\ge\gamma_k\sum_{r=1}^{n}y_{kr} && k=1,..,s\\
&\sum_{j=1}^{n}\lambda_{jr}g_{lj}=\hat{g}_{lr} && l=1,..,q\,;r=1,..,n\\
&\hat{g}_{lr}\le g_{lr} && l=1,..,q;r=1,..,n\\
&\sum_{r=1}^{n}\hat{g}_{lr}\le\gamma'_l\sum_{r=1}^{n}g_{lr} && l=1,..,q\\
&\sum_{j=1}^{n}\lambda_{jr}=1 && r=1,\dots,n\\
&\lambda_{rj}\ge 0 && r=1,\dots,n;j=1,\dots,n
\end{aligned}$$

Where $I^{realloc}$ represents the subset of inputs that can be reallocated among the units and $\hat{x}_{ir}$, $\hat{y}_{kr}$ and $\bar{g}_{lr}$ are the targets for input i and desirable output k and undesirable output l of unit r.

So far, a centralized resource allocation model based on revenue efficiency in centralized decision making and in presence of desirable and undesirable outputs has been proposed. However, in reality the value of majority of outputs is not determined precisely. Therefore, outputs are divided into two categories of known prices ($k\in O_p$) and unknown prices ($k\in O_s$). To allocate the input resources to a set of existing units so that the total output revenue will be maximized for desirable outputs and minimized for undesirable outputs, the resource allocation model based on revenue efficiency in presence of desirable and undesirable outputs within the original production possibility set can be formulated as follows:

$$Max\ \Psi=\sum_{k}\sum_{r=1}^{n}p_k\bar{y}_{kr}-\sum_{l}\sum_{r=1}^{n}\tilde{p}_l\bar{g}_{lr}$$

$$\begin{aligned}
\text{s.t}\ &\sum_{j=1}^{n}\lambda_{jr}x_{ij}\le\bar{x}_{ir} && i=1,..,m\,;r=1,..,n\\
&\bar{x}_{ir}\le x_{ir} && i\notin I^{realloc};r=1,..,n\\
&\sum_{r=1}^{n}\bar{x}_{ir}\le\sum_{r=1}^{n}x_{ir} && i\in I^{realloc} && (4)\\
&\sum_{j=1}^{n}\lambda_{jr}y_{kj}\ge\bar{y}_{kr} && k=1,..,s\,;r=1,..,n\\
&\bar{y}_{kr}\ge y_{kr} && k=1,..,s;r=1,..,n\\
&\sum_{j=1}^{n}\lambda_{jr}g_{lj}=\bar{g}_{lr} && l=1,..,q\,;r=1,..,n\\
&\bar{g}_{lr}\le g_{lr} && l=1,..,q;r=1,..,n\\
&\sum_{j=1}^{n}\lambda_{jr}=1 && r=1,\dots,n\\
&\lambda_{rj}\ge 0 && r=1,\dots,n;j=1,\dots,n
\end{aligned}$$

Where p_k denotes the price for desirable outputs, and $\tilde{p}_k$ denotes the price for undesirable outputs and $\bar{x}_{ir}$, $\bar{y}_{kr}$ and $\bar{g}_{lr}$ are the targets for input i and desirable output k and undesirable output l.

4. Illustrative examples

Now we analyze proposed models of previous section for 18 branches of Pasargadae Bank in Tehran metropolis. In this example, three types of inputs involve concession facilities, deposit received, and warranty profit, two desirable outputs involve profit received and payment received, and an undesirable output involves profit paid have been used (unit of all data is million Rials), their information is presented in table 1:

Table 1. inputs and outputs

	Input 1	Input 2	Input 3	Desirable output 1	Desirable output 2	Undesirable output
1	653640	4369768	1172377	385951	21031	783183
2	3414100	2412669	53877	581119	654	350803
3	17711	835433	31566	6258	276	140879
4	5959869	2600965	103682	1949549	1500	4429102
5	141245	1631590	17085	37385	153	234780
6	52862	2730464	59489	27989	827	371962
7	8995	654427	3561	2143	16	90784
8	178055	1478011	59565	119756	831	238754
9	1093092	1805003	1376960	914956	24656	290821
10	39936	966818	4792	45183	36	159005
11	14188	959469	253275	8148	4592	147478
12	192641	989709	28197	56509	41	134575
13	12476	698185	137267	3541	2099	103533
14	80076	683408	23008	40555	335	106105
15	1368245	1798316	140641	350100	1036	305505
16	67191	548403	16208	20472	194	62354
17	7625	2629956	14142	2246	231	442793
18	91748	1374525	188351	15782	1940	83809

As outputs are divided into desirable and undesirable categories, therefore models number 3 and 4 are used to determine their efficiency. By applying model number 3, efficiency amount of total bank branches is determined to be equal to zero. It shows that generalization of Lozano *et al* model (2004) for desirable and undesirable outputs won't provide the decision maker with a correct analysis of a system. As a result, model number 4 is used to determine revenue efficiency of collection of Pasargadae Banks which their activity is analyzed as a single system. In this model according to manager's opinion, vectors $p = (10000{,}10000)$ and $\tilde{p} = (1000)$ are considered. Consequently, target of each unit for all inputs, desirable and undesirable outputs is the value determined in table 1. It shows that all bank branches are at the maximum profitability situation. Moreover, the aggregate revenue efficiency is estimated as 0.4180468E+11. In other words, it is considered that Pasargadae Bank, as an independent bank, by granting bank facilities and respective warranties and making accurate management decisions, has the highest profitability for customers, which shows the top efficiency and customer's satisfaction, with the provided services, of this complex.

5. Conclusion

Assume all DMUs are under the control of one centralized decision maker. This occurs when all units belong to a specific organization which provide required inputs for each unit. Many organizations, such as bank branches, chain stores and etc. can act like that. This centralized decision maker unit expect that resource allocation and revenue efficiency be in a way that DMUs not separately but in a group and simultaneously projected onto the efficiency frontier. According to this definition it sometimes happens that all units doesn't have desirable outputs. In this article, a method is presented to determine target units and aggregate revenue efficiency of this centralized decision maker unit, in a system with desirable and undesirable outputs, and to resolve inability of previous models in presence of undesirable outputs.

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