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Measuring a Dynamic Efficiency Based on MONLP Model under DEA Control

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Abstract
Data envelopment analysis (DEA) is a common technique in measuring the relative efficiency of a set of decision making units (DMUs) with multiple inputs and multiple outputs. Standard DEA models are quite limited models, in the sense that they do not consider a DMU at different times. To resolve this problem, DEA models with dynamic structures have been proposed. In a recent paper byafarian-Moghaddam and Ghoseiri [Jafarian-Moghaddam, A.R., Ghoseiri k., 2011. Fuzzy dynamic multi-objective Data Envelopment Analysis model. *Expert Systems with Applications*, 38 (1), 850-855.] they contribute to an interesting topic by presenting a fuzzy dynamic multi-objective DEA model to evaluate DMUs in which data are changing with time. However, this paper finds that their approach has some problems in the proposed models. In this paper, we first stress the present shortcomings in their modeling and then we propose a new DEA method for improving fuzzy dynamic multi-objective DEA model. The proposed model is a multi-objective non-linear programming (MONLP) problem and there are several methods for solving it; We use the goal programming (GP) method. The proposed model calculates the efficiency scores of DMUs by solving only one linear programming problem. Finally, we present an example with ten DMUs at three times to illustrate the applicability the proposed model.

Keywords: Data Envelopment Analysis; Decision Making Unit; Multi-Objective Programming Problem; Goal Programming.

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1. Introduction

Data envelopment analysis (DEA) is a non-parametric tool to measure the relative efficiency of decision making units (DMUs) that consume multiple inputs to produce multiple outputs; units such as banks manufactories, universities and so on. At first, Charnes et al. [5] proposed the CCR model which considers the constant returns to scale assumption of the production technology. The ratio DEA model obtains the efficiency of DMU as the maximum of the ratio of weighted outputs to weighted inputs subject to the ratio of other DMUs being less than or equal to unity. Thus, DMU_o is efficient if and only if the objective value of the model is one, otherwise DMU_o is called inefficient DMU.

Standard DEA models have two types of framework: envelopment and multiplier. Envelopment and multiplier models can be determine the benchmark of under evaluation DMU and constitutive hyperplanes of the production possibility set, respectively. In the conventional CCR-multiplier model calculate best multiplier in respect of the under evaluation DMU compared to the other DMUs. Indeed, the optimal multipliers corresponding to each under evaluation DMU are new and separate from other them and solve 'n' (the number of DMUs) time this model. Different multipliers provide by solving different models and may not be rational and acceptable to compare and decide management. The concept of common set of weight applies to remove this disadvantage of conventional multiplier CCR model. There are many articles using the common-weight approach. At first Cook et al. [10] and Roll et al. [32] introduced common set of weights models based on DEA approach. These models used to evaluate highway maintenance DMUs. Hosseinzadeh Lotfi et al. [22] represented a non-linear common set of weight model in which evaluated DMUs by solving one

model replace of solving 'n' (the number of DMUs) linear models. They finding common set of weights model by multiple objective programming to calculate the efficiency score of all DMUs, simultaneously. In a similar manner, Jahanshahloo et al. [20] introduced new common-weight DEA method to evaluate efficincy of DMUs by a non-linear program. Moreover, they ranked the efficient DMUs in the two-step procedure based on common-weight concept. Amin and Toloo [1] introduced a common set of weights DEA model to find the most efficient DMUs via an improved integrated model. Liu and Hsuan Peng [29] ranked efficient DMUs by using a common set of weight approach. Jahanshahloo et al. [19] presented new DEA method to evaluate DMUs and rank frontier DMUs based on an ideal line for determining a common set of weights corresponding to inputs and outputs of all DMUs. Wang and Chin [38] introduced a neutral DEA model to measure cross-efficiency by using the common set of weights. They showed this common-weight cross-efficiency model are neutral and are not aggressive and benevolent. Davoodi and Zhiani Rezai [11] proposed a linear programming problem with common set of weights structure based on DEA approach and evaluated the efficiency of the DMUs with respect to the multi-objective model.

Sometime organization decides to analyze under evaluation system in the several points simultaneously such as minimize cost, maximize profit and so on. For this aim, many papers have been published multiple objective linear programming (MOLP) based on DEA approach. The first author to introduce a multiple objective DEA model was Golany[15]. Thanassoulis and Dyson [37] proposed some DEA models to obtain preferred input-output target. Lins et al. [28] Introduced a multi-objective method in which calculated bases by way of

individual projections of each input-output variable as objective function and target extreme-efficient unit on the efficient frontier. Chen [8] obtained a relationship between the efficiency in DEA and the pareto optimality under MOLP models. Joro et al. [21] considered the similarities in structure between DEA and MOLP, and their approach is called the reference point approach. Li and Reeves, [27] proposed a multiple criteria method to find more efficient DMUs based on different criteria. Lozano and Villa [30] introduced two multiple objective DEA methods for finding a target. They used interactive and lexicographic approaches, and then employed the analytic hierarchy process to show preference information of the decision maker. Wong et al. [41] and Yang et al. [42] have presented models that establish the equivalence between DEA and MOLP models, indicated how DEA problems can transform into MOLP models, and then solved the MOLP problem. Hosseinzadeh et al. [23] have proposed a model that establishes the equivalence between DEA and MOLP, and have shown how a DEA problem can be solved interactively by transforming it into MOLP formulation. Also, Hosseinzadeh et al. [24] have used Zionts-Wallenius's (Z-W) method to reflect the decision maker's (DM) preferences in the process of assessing efficiency in the general combined-orientation CCR model.

In data envelopment analysis, there are several real world problems that data changes over time and have dynamic structure. So, for this purpose there exist many methods for evaluating efficiency changes over time, such as the malmquist-index and the window-analysis. Anyway, several these methods focus on the separate time period that each period is independent to other period without considering carry-over activities among two consecutive terms. Some of them can investigate the effect of change time. In addition to, there are many business and investment planning problems in during the along time where conventional DEA model is not valuable to evaluate units. So, many authors proposed suitable method for dynamic performance evaluation that some of them considered carry-over activities and the other not considered. At the beginning, dynamic DEA model was introduced by Färe and Grosskopf [13]. Sengupta [34] decided to assess a set of units that capital inputs varying during the times because of the varying input prices over time based on DEA approach. For this aim, the DEA model introduced that minimized an aggregate of total input costs for each DMU. Sueyoshi and Sekitani [36] calculated the RTS classifications into the dynamic problem based on DEA approach. They treated the reciprocal dependency between sequential periods by dynamic-DEA concept. Emrouznejad and Thanassoulis [12] introduced method to measuring the relative efficiency of set units with inter-temporally dependent assumption in input and output values that observed in time periods. Capital stock is one cause of this characteristic that output levels have effect on many production periods. This method solved the problem of inter-temporally dependent in input–output and showed with application that the achievement results of this model better than static and traditional DEA-model. Amirteimoori [2] proposed a new model to dynamic revenue efficiency based on DEA approach. This model can used to measure the efficiency score of the whole periods and the efficiency score for each of the periods that the whole efficiency score is a convex combination of the periods. This model have some drawback that modified by Färe and Grosskopf [14] such that the aim of the

new model is to more directly allow the effects in the present time to have impacts on the future time. In addition to previous work, Wang and Huang [39] introduced a sequence of four systematic methods to assess the economic efficiency of commercial banks persists and to reveal the potential dynamic link among bank performance and various financial indicators in Taiwan. Chen and Dalen [9] measured efficiency of DMUs by dynamic-DEA model where considered inter temporal effects in efficiency scores. The proposed model has some applicability in static DEA problems such as examining longitudinal firm presentation and variation in productivity. Wang et al. [40] computed the energy and environmental efficiency of 29 administrative regions of China throughout the period of 2000–2008 based dynamic-DEA evaluation. The DEA approach is utilized to measure the efficiency in cross-sectional when data varying during the time. Jafarian-Moghaddam and Ghoseiri [18] represented a fuzzy dynamic multi-objective DEA model to evaluate a set of DMUs that variation during the time. Their model structured based on fuzzy theory that efficiency scores of each DMU changed within interval [0,1] as well as membership function in fuzzy set. In the other word, they have presented a fuzzy dynamic multi-objective DEA model to evaluate DMUs with inputs and outputs of different types, variable inputs/outputs and links as inputs/outputs.

In this paper, we express some drawbacks of the model introduced by Jafarian-Moghaddam and Ghoseiri [18] and improve the model for evaluating DMUs. The proposed model is a multi-objective non-linear programming (MONLP) problem and there are several methods for solving it; We use the goal programming method in this paper. The proposed model calculates the efficiency score of DMUs by solving only one linear programming problem. We proceed as follows: In section 2, the fuzzy dynamic multi-objective DEA model introduced by Jafarian-Moghaddam and Ghoseiri [18] is given. The improved dynamic multiple objective DEA model is presented in section 3. Section 4 illustrates the new model by an example, and the conclusion will be given in section 5.

2. Fuzzy Dynamic Multi-Objective DEA Model

Jafarian-Moghaddam and Ghoseiri [18] have introduced a fuzzy dynamic multiple objective DEA model in which DMUs use two types of inputs (variable inputs and links as inputs) and produce two types of outputs (variable outputs and links as outputs). In the dynamic space, it is assumed that there are n DMUs at time t, $t = 1, \ldots, T$. The aim is evaluating, measuring efficency of DMUs in T times simultaneously. For this purpose, the authors considered the following model for calculating efficiency score of DMUs where it is a multi-objective fractional programming (MOFP) problem.

$$\max \quad z_1^t = \frac{\sum_{r=1}^{s} u_r^t y_{r1}^t + \sum_{l=1}^{L} \rho_l^t k_{l1}^t}{\sum_{i=1}^{m} y_i^t x_{i1}^t + \sum_{l=1}^{L} \beta_l^{t-1} k_{l1}^{t-1}},$$

$$t = 1, \ldots, T,$$

$$\vdots \qquad\qquad\qquad\qquad (1)$$

$$\max \quad z_n^t = \frac{\sum_{r=1}^{s} u_r^t y_{rn}^t + \sum_{l=1}^{L} \rho_l^t k_{ln}^t}{\sum_{i=1}^{m} y_i^t x_{in}^t + \sum_{l=1}^{L} \beta_l^{t-1} k_{ln}^{t-1}},$$

$$t = 1, \ldots, T,$$

$$s\, t \quad \frac{\sum_{r=1}^{s} u_r^t y_{rj}^t + \sum_{l=1}^{L} \rho_l^t k_{lj}^t}{\sum_{i=1}^{m} y_i^t x_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1} k_{lj}^{t-1}} \le 1,$$

$$j = 1, \ldots, n, \ t = 1, \ldots, T,$$

$$u_r^t, v_i^t, \beta_l^{t-1}, \rho_l^t \ge \varepsilon > 0,$$

$$t = 1,\dots,T, r = 1,\dots,s,$$

$$i = 1,\dots,m, l = 1,\dots,L.$$

As can be seen from model (1), there exist $T \times n$ objective functions where T and n denote the numbers of time terms and DMUs, respectively. There is the link among two consecutive time terms (t and t+1) corresponding to the dynamic structure of problem. So, the DMUs Consume two types of inputs in any time t. One is variable input corresponding to time t, and the other is concern to the link between two consecutive time terms. In fact, it yield from previous time, i.e., t-1 and set as input to current time, i.e., t. Also, we have two types of outputs at time t, the variable output that leave the DMU and the output that is used at time t+1. On the other hand, at time t, the output is considered as the input at time

t+1 without any changes that state the stability in the link. (See figure (1)).
Thus, as you can see from model, two types' inputs and outputs are influence to calculating efficiency score of all DMUs in any time. So, the efficiency score of each DMU calculate as following equation:

Efficency score =

$$\frac{\textit{sum of weighted variable outputs} + \textit{sum of weighted outputs of link}}{\textit{sum of weighted current time inputs} + \textit{sum of weighted inputs of link}}$$

The aim of above mentioned model is calculating efficiency score of all DMUs at T times simultaneously. Note that the efficiency score of each DMU at any times is not greater than one, so the constraints for the efficiency score, less or equal one is necessary condition. The details variables and weights of model (1) represented in the below table ,soconsider the following denotations:

Figure 1. Execution of DEA Model in Dynamic Framework.
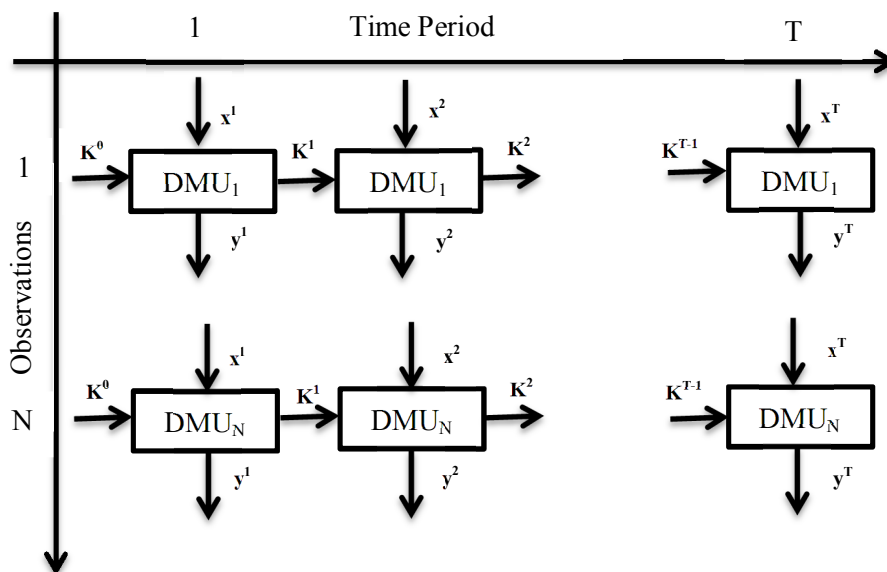
n	number of DMUs,
T	number of times for each DMU,
L	number of links between DMUs at two times t and $t+1$,
m	number of inputs,
s	number of outputs,
j= 1,..., n	index for DMUs,
t= 1,...,T	index for times,
l=1,...,L	index for links,
x_j^t	input of DMU $_j$ at time t,
y_j^t	output of DMU $_j$ at time t,
k_j^t	output of DMU $_j$ at time t, and input of DMU $_j$ at time $t+1$,
u^t	weight variable of variable output at time t,
v^t	weight variable of variable input at time t,
β^t	weight variable of link as input at time t,
ρ^t	weight variable of link as output at time t

As well as above model is a dynamic-multi-objective DEA programming. Thus, there are some methods for solving it. They utilized the fuzzy multiple objective linear programming approach introduced by Zimmermann [43]. In fact, the mentioned method convert the problem with multi-objective functions to the problem with one objective function by preserving the fact that the efficiency score of each DMU belongs to the (0,1), based on the fuzzy membership function concept.

The approach summarized in three steps as below:

Step1: Defining membership function as $\mu_j^t(z_j^t) = \alpha$ such that z_j is the convex composition of upper bound (z_j^R) and lower bound (z_j^L) of DMU$_j$ efficency score.

Step2: By using the concept of Min Max in fuzzy membership function, MOFP model (1) is converted to a model that objective function is as $Max\{Min\, \mu_j^t(z_j^t)\}, t=1,...,T, j=1,...,n,$ the constrains are same as constraints of model (1).

Step3: By setting $Min\, \mu_j^t(z_j^t) = \alpha\, z_j^{tR} + (1-\alpha)\, z_j^{tL}$, and let upper bound and lower bound of objective functions, one and zero, respectively corresponding to the efficency score concept, so, the model in step2 replace with following model:

$$\max\ \alpha$$

$$s.t\quad \sum_{r=1}^{s} u_r^t y_{rj}^t + \sum_{l=1}^{L} \rho_l^t k_{lj}^t - (\sum_{i=1}^{m} v_i^t x_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1} k_{lj}^{t-1}) \le 0,$$

$$j=1,...,n, t=1,...,T,$$

$$\sum_{r=1}^{s} u_r^t y_{rj}^t + \sum_{l=1}^{L} \rho_l^t k_{lj}^t - \alpha(\sum_{i=1}^{m} v_i^t x_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1} k_{lj}^{t-1}) \ge 0,$$

$$j=1,...,n, t=1,...,T,$$

$$0 \le \alpha \le 1, u_r^t, v_i^t, \beta_l^{t-1}, \rho_l^t \ge \varepsilon > 0,$$

$$t=1,...,T, r=1,...,s,$$

$$i=1,...,m, l=1,...,L. \tag{2}$$

Model (2) is a non-linear programming model and can be solved with software like Gams, Lingo-Lindo and etc.

3. Improved Dynamic Multiple Objective DEA Model

In the previous section, we reviewed the model that was proposed by Jafarian-

Moghaddam and Ghoseiri [18] to calculating efficiency score of DMUs in problems with dynamic structure. But, this model has some drawbacks that in this section, we will represent some of these and resolve them. The shortcomings of Model (2) can be expressed as follows:

I. In Model (1), different weights have been assigned to links as inputs and outputs, in which it is not logical to use β^{t-1}, $t=1,\ldots,T$, as the input coefficient for k_j^{t-1}, $j=1,\ldots,n, t=1,\ldots,T$, which is to be minimized, and ρ^{t}, $t=1,\ldots,T$, as the output coefficient for k_j^{t}, $j=1,\ldots,n, t=1,\ldots,T$, which is to be maximized. In other words, the continuity of links has not been considered.

II. As shown in Fig. 1, DMU$_j$, $j=1,\ldots,n$, has only one type of output at time T, that is y_j^T, $j=1,\ldots,n$, while in Model (2) there are two types of output considered for DMU$_j$, $j=1,\ldots,n$, that is y_j^T, $j=1,\ldots,n$, and the link k_j^T, $j=1,\ldots,n$.

III. Model (2) is a non-linear programming model and is, hence, not easy to solve.

It is mentionable that the indicator corresponding to the link between the two times t and $t+1$ is considered as an output at time t, and as an input at time $t+1$. So, this indicator has the same value at uninterrupted times. If we consider different weights for links then, in fact, the continuity of the links between the two times t and $t+1$ is disregarded. Hence, we introduce the common weight w_j^t, $j=1,\ldots,n, t=1,\ldots,T$, to replace β_j^{t-1}, $j=1,\ldots,n, t=1,\ldots,T$, and ρ_j^t, $j=1,\ldots,n, t=1,\ldots,T$, for links as inputs and outputs, respectively. Therefore, the continuity of links is ensured

and deficiency I is resolved.
To remove shortcoming II, we calculate the efficiency scores of DMU$_j$, $j=1,\ldots,n$, at time T reseparately. This means the objective functions of Model (1) have changed into following model, where it is a MOFP model for calculating efficiency scores such that maximize the efficency score of all DMUs in all time, simultaneously.

$$\max \quad z_j^t = \frac{\sum_{r=1}^{s} u_r^t y_{rj}^t + \sum_{l=1}^{L} w_l^t k_{lj}^t}{\sum_{i=1}^{m} v_i^t x_{ij}^t + \sum_{l=1}^{L} w_l^{t-1} k_{lj}^{t-1}},$$
$$j=1,\ldots,n, t=1,\ldots,T-1,$$

$$\max \quad z_j^T = \frac{\sum_{r=1}^{s} u_r^T y_{rj}^T}{\sum_{i=1}^{m} v_i^T x_{ij}^T + \sum_{l=1}^{L} w_l^{T-1} k_{lj}^{T-1}},$$
$$j=1,\ldots,n,$$

$$s\,t. \quad (a)\,\frac{\sum_{r=1}^{s} u_r^t y_{rj}^t + \sum_{l=1}^{L} w_l^t k_{lj}^t}{\sum_{i=1}^{m} v_i^t x_{ij}^t + \sum_{l=1}^{L} w_l^{t-1} k_{lj}^{t-1}} \le 1,$$
$$j=1,\ldots,n, t=1,\ldots,T-1,$$ \quad (3)

$$(b)\,\frac{\sum_{r=1}^{s} u_r^T y_{rj}^T}{\sum_{i=1}^{m} v_i^T x_{ij}^T + \sum_{l=1}^{L} w_l^{T-1} k_{lj}^{T-1}} \le 1,$$
$$j=1,\ldots,n,$$
$$(c)\,u_r^t, v_i^t, w_l^t \ge \varepsilon > 0,$$
$$t=1,\ldots,T, r=1,\ldots,s,$$
$$i=1,\ldots,m, l=1,\ldots,L.$$

We could improve drawbacks I and II, by considering model (3) replace to model (1). It is obvious that Model (3) is a MOFP problem and there are several methods for solving it; (you can see: Hwang and Masud [25]; Chankong and Haimes [3]; Sawaragi et al. [33]; Steuer

[35]; Marler and Arora [31]). Goal programming (GP) method is one of the methods that able to solve multiple criteria decision model (MCDM) as well as MOFP model. The GP method at first utilized by Charnes et al. [7] and then developed by Charnes and Cooper [6], Lee [26], Charnes and Cooper [4], and Ignizio [16] and [17]. Indeed, the GP method characterizes goals for constraints and objective functions which are defined by the managements. Due to this fact that always achieve to characterized goal by management is not possible so, we consider deviation variables for the goals. Whereas these variables denote amount of deviation from goals, therefor it is obvious that the less deviation variable, are better than the large variables.

So, we use the goal programming method as follows:

$$\min \quad \sum_{t=1}^{T}\sum_{j=1}^{n}(\delta_j^{-t} + \delta_j^{+t})$$

$$s.t. \quad \frac{\sum_{r=1}^{s}u_r^t y_{rj}^t + \sum_{l=1}^{L}w_l^t k_{lj}^t}{\sum_{i=1}^{m}v_i^t x_{ij}^t + \sum_{l=1}^{L}w_l^{t-1}k_{lj}^{t-1}} \leq 1,$$

$$j = 1,...,n, t = 1,...,T-1,$$

$$\frac{\sum_{r=1}^{s}u_r^T y_{rj}^T}{\sum_{i=1}^{m}v_i^T x_{ij}^T + \sum_{l=1}^{L}w_l^{T-1}k_{lj}^{T-1}} \leq 1,$$

$$j = 1,...,n, \qquad\qquad (4)$$

$$\frac{\sum_{r=1}^{s}u_r^t y_{rj}^t + \sum_{l=1}^{L}w_l^t k_{lj}^t}{\sum_{i=1}^{m}v_i^t x_{ij}^t + \sum_{l=1}^{L}w_l^{t-1}k_{lj}^{t-1}} + \delta_j^{-t} - \delta_j^{+t} = 1,$$

$$j = 1,...,n, t = 1,...,T-1,$$

$$\frac{\sum_{r=1}^{s}u_r^T y_{rj}^T}{\sum_{i=1}^{m}v_i^T x_{ij}^T + \sum_{l=1}^{L}w_l^{T-1}k_{lj}^{T-1}} + \delta_j^{-T} - \delta_j^{+T} = 1,$$

$$j = 1,...,n,$$

$$u_r^t, v_i^t, w_l^t \geq \varepsilon > 0,$$

$$t = 1,...,T, r = 1,...,s,$$

$$i = 1,...,m, l = 1,...,L,$$

$$\delta_j^{-t}, \delta_j^{+t} \geq 0,$$

$$t = 1,...,T, j = 1,...,n.$$

In the above model, δ_j^{-t} and δ_j^{+t} are deviating variables of the jth goal at time t, $t = 1,...,T$, which are called the under-achievement and over-achievement, respectively. Note that in the model (4), goals for all objective functions is consider as one, because of, the aim corresponding to efficency score is be equal to one. Objective function of model (4) is single such that it calculates the minimum value of the deviating variables from goals. Also, this model is NLP problem. So, for this reason that LP form is easy in solving we use the deviation variables in order to convert it to the LP form. Then, we obtain the following model:

$$\min \quad \sum_{t=1}^{T}\sum_{j=1}^{n}(\delta_j^{-t} + \delta_j^{+t})$$

$$s.t. \quad \frac{\sum_{r=1}^{s}u_r^t y_{rj}^t + \sum_{l=1}^{L}w_l^t k_{lj}^t}{\sum_{i=1}^{m}v_i^t x_{ij}^t + \sum_{l=1}^{L}w_l^{t-1}k_{lj}^{t-1}} \leq 1,$$

$$j = 1,...,n, t = 1,...,T-1, \qquad (5)$$

$$\frac{\sum_{r=1}^{s}u_r^T y_{rj}^T}{\sum_{i=1}^{m}v_i^T x_{ij}^T + \sum_{l=1}^{L}w_l^{T-1}k_{lj}^{T-1}} \leq 1,$$

$$j = 1,...,n,$$

$$\frac{\sum_{r=1}^{s}u_r^t y_{rj}^t + \sum_{l=1}^{L}w_l^t k_{lj}^t + \delta_j^{+t}}{\sum_{i=1}^{m}v_i^t x_{ij}^t + \sum_{l=1}^{L}w_l^{t-1}k_{lj}^{t-1} - \delta_j^{-t}} = 1,$$

$$j = 1,...,n, t = 1,...,T-1,$$

$$\frac{\sum\limits_{r=1}^{s} u_r^T y_{rj}^T + \delta_j^{+T}}{\sum\limits_{i=1}^{m} v_i^T x_{ij}^T + \sum\limits_{l=1}^{L} w_l^{T-1} k_{lj}^{T-1} - \delta_j^{-T}} = 1,$$

$$j = 1,...,n,$$

$$u_r^t, v_i^t, w_l^t \ge \varepsilon > 0, \quad t = 1,...,T,$$

$$r = 1,...,s, i = 1,...,m, l = 1,...,L,$$

$$\delta_j^{-t}, \delta_j^{+t} \ge 0, \ t = 1,...,T, j = 1,...,n.$$

The above model is still NLP problem. So, it can be written in the LP form as follows:

min

$$\sum_{t=1}^{T}\sum_{j=1}^{n}(\delta_j^{-t} + \delta_j^{+t})$$

$s t.$

$$\sum_{r=1}^{s} u_r^t y_{rj}^t + \sum_{l=1}^{L} w_l^t k_{lj}^t - (\sum_{i=1}^{m} v_i^t x_{ij}^t + \sum_{l=1}^{L} w_l^{t-1} k_{lj}^{t-1}) \le 0,$$

$$j = 1,...,n, t = 1,...,T-1,$$

$$\sum_{r=1}^{s} u_r^T y_{rj}^T - (\sum_{i=1}^{m} v_i^T x_{ij}^T + \sum_{l=1}^{L} w_l^{T-1} k_{lj}^{T-1}) \le 0,$$

$$j = 1,...,n,$$

$$\sum_{r=1}^{s} u_r^t y_{rj}^t + \sum_{l=1}^{L} w_l^t k_{lj}^t - (\sum_{i=1}^{m} v_i^t x_{ij}^t + \sum_{l=1}^{L} w_l^{t-1} k_{lj}^{t-1})$$

$$+\delta_j^{-t} + \delta_j^{+t} = 0, \tag{6}$$

$$j = 1,...,n, t = 1,...,T-1,$$

$$\sum_{r=1}^{s} u_r^T y_{rj}^T - (\sum_{i=1}^{m} v_i^T x_{ij}^T + \sum_{l=1}^{L} w_l^{T-1} k_{lj}^{T-1}) + \delta_j^{-T} + \delta_j^{+T} = 0,$$

$$j = 1,...,n,$$

$$u_r^t, v_i^t, w_l^t \ge \varepsilon,$$

$$t = 1,..,T, r = 1,..,s, i = 1,..,m, l = 1,..,L,$$

$$\delta_j^{-t}, \delta_j^{+t} \ge 0, \ t = 1,..,T, j = 1,...,n.$$

Model (6) can be simplified to the following LP model by setting $\delta_j^t, t = 1,...,T, j = 1,...,n,$ substituting $\delta_j^{-t} + \delta_j^{+t}, t = 1,...,T, j = 1,...,n$:

min

$$\sum_{t=1}^{T}\sum_{j=1}^{n}(\delta_j^t)$$

$s t.$

$$(a) \sum_{r=1}^{s} u_r^t y_{rj}^t + \sum_{l=1}^{L} w_l^t k_{lj}^t - (\sum_{i=1}^{m} v_i^t x_{ij}^t + \sum_{l=1}^{L} w_l^{t-1} k_{lj}^{t-1}) \le 0,$$

$$j = 1,...,n, t = 1,..,T-1,$$

$$(b) \sum_{r=1}^{s} u_r^T y_{rj}^T - (\sum_{i=1}^{m} v_i^T x_{ij}^T + \sum_{l=1}^{L} w_l^{T-1} k_{lj}^{T-1}) \le 0, \tag{7}$$

$$j = 1,...,n,$$

$$(c) \sum_{r=1}^{s} u_r^t y_{rj}^t + \sum_{l=1}^{L} w_l^t k_{lj}^t - (\sum_{i=1}^{m} v_i^t x_{ij}^t + \sum_{l=1}^{L} w_l^{t-1} k_{lj}^{t-1}) + \delta_j^t = 0,$$

$$j = 1,...,n, t = 1,..,T-1,$$

$$(d) \sum_{r=1}^{s} u_r^T y_{rj}^T - (\sum_{i=1}^{m} v_i^T x_{ij}^T + \sum_{l=1}^{L} w_l^{T-1} k_{lj}^{T-1}) + \delta_j^T = 0,$$

$$j = 1,...,n,$$

$$u_r^t, v_i^t, w_l^t \ge \varepsilon,$$

$$t = 1,..,T, r = 1,..,s, i = 1,..,m, l = 1,..,L,$$

$$\delta_j^t \ge 0, \ t = 1,..,T, j = 1,...,n.$$

Clearly, constraints (7a) and (7b) are redundant because of constraints (7c) and (7d), respectively. On the other hands, the constraints (7a) and (7b) can be converted to equality constraints by adding non-negative slaks such as constraints (7c) and (7d).

Definition 1: DMU $_j$ at time t, $j = 1,...,n, t = 1,...,T$, is non-dominated if and only if $\delta_j^t = 0, \ j = 1,...,n, t = 1,...,T,$ in Model (6). We can calculate the efficiency scores of DMU $_j$ at time t, $j = 1,...,n, t = 1,...,T.$ Suppose that $(u^{*t}, v^{*t}, w^{*t}, \delta_j^{*t}), j = 1,...,n, t = 1,...,T,$ be the optimal solutions of Model (6) so, we have:

$$\frac{\sum\limits_{r=1}^{s} u_r^{*t} y_{rj}^t + \sum\limits_{l=1}^{L} w_l^{*t} k_{lj}^t}{\sum\limits_{i=1}^{m} v_i^{*t} x_{ij}^t + \sum\limits_{l=1}^{L} w_l^{*t-1} k_{lj}^{t-1}} =$$

$$1 - \frac{\delta_j^{*t}}{\sum_{i=1}^{m} y_i^{*t} x_{ij}^t + \sum_{l=1}^{L} w_l^{*t-1} k_{lj}^{t-1}},$$

$$j = 1,...,n, t = 1,...,T-1,$$

$$\frac{\sum_{r=1}^{s} u_r^{*T} y_{rj}^T}{\sum_{i=1}^{m} y_i^{*T} x_{ij}^T + \sum_{l=1}^{L} w_l^{*T-1} k_{lj}^{T-1}} = \quad (8)$$

$$1 - \frac{\delta_j^{*T}}{\sum_{i=1}^{m} y_i^{*T} x_{ij}^T + \sum_{l=1}^{L} w_l^{*T-1} k_{lj}^{T-1}},$$

$$j = 1,...,n,$$

onsequentlyC, we denote efficency scores of DMU$_j$ by $E_j^t, j=1,...,n, t=1,...,T,$ as follow:

$$E_j^t = \frac{\sum_{r=1}^{s} u_r^{*t} y_{rj}^t + \sum_{l=1}^{L} w_l^{*t} k_{lj}^t}{\sum_{i=1}^{m} y_i^{*t} x_{ij}^t + \sum_{l=1}^{L} w_l^{*t-1} k_{lj}^{t-1}},$$

$$j = 1,...,n, t = 1,...,T-1, \quad (9)$$

$$E_j^T = \frac{\sum_{r=1}^{s} u_r^{*T} y_{rj}^T}{\sum_{i=1}^{m} y_i^{*T} x_{ij}^T + \sum_{l=1}^{L} w_l^{*T-1} k_{lj}^{T-1}},$$

$$j = 1,...,n.$$

Definition 2: DMU$_j$ at time t,

$j = 1,...,n, t = 1,...,T,$ is non-dominated if and only if $E_j^t = 0, \quad j = 1,...,n, t = 1,...,T,$ in Equation (9).

4. Numerical Example

We apply the outcomes bring forward in the perevios sections to illustrate the applicability of the proposed model in this section. For this, we decide to evaluate the performance of ten DMUs where each DMUs consume two types of different inputs to produce one output at three different times by the proposed model. In this numerical example, in addition to input and output values of ten DMUs we consider some links between two consecutive times. The input and output values in three time presented in the Table 1 and the vales of links are shown in Table 2. The details input-output values and link values of this sample represented by the following denotations:

• (x_{ij}^t) Denote the value of ith input of DMU$_j$ in time t, where $i = 1,2, \ t = 1,2,3, \ j = 1,...,10.$

• (y_{rj}^t) Denote the value of rth output of DMU$_j$ in time t, where $r = 1, \ t = 1,2,3, \ j = 1,...,10.$

• (k_{hj}^t) Denote the value of hth link of DMU$_j$ between time t and t+1, where $h = 1,2, \ t = 0,1,2, \ j = 1,...,10.$

Table 1. The Inputs and Outputs of DMUs at Three Times

DMU	x_{1j}^1	x_{2j}^1	y_{1j}^1	x_{1j}^2	x_{2j}^2	y_{1j}^2	x_{1j}^3	x_{2j}^3	y_{1j}^3
1	5	3	10	6	2	12	8	5	15
2	10	14	2	12	12	4	14	10	6
3	6	7	5	6	6	6	8	7	8
4	4	8	4	8	7	7	7	6	5
5	3	2	5	3	2	8	5	3	10
6	1	6	7	2	6	7	4	6	5
7	7	6	2	8	5	5	10	5	15
8	15	5	10	17	4	10	20	2	8
9	8	4	10	9	4	7	19	2	5
10	7	6	9	8	3	10	18	1	11

Table 2. The Data of the Links of DMUs at Three Times

DMU	k_{1j}^0	k_{2j}^1	k_{1j}^2
1	2	1	3
2	4	3	6
3	6	5	9
4	7	7	12
5	8	9	15
6	10	11	18
7	12	13	3
8	13	15	6
9	5	17	9
10	7	10	10

According to the proposed model to evaluate the performance of ten DMUs, at first we must be solve model (7) to determine the optimal deviating variables (δ_j^{t*}, $t = 1,2,3$, $j = 1,...,10$) and then calculate the efficiency scores of each DMU at various times (E_j^t, $t = 1,2,3$, $j = 1,...,10$) by using equation (8) or (9). The results of the deviating variables and efficiency score of all DMUs at three time represent in the Table 3 and Table 4, respectively.

Table 3. The deviating variables of DMUs at each time.

DMU	δ_j^1	δ_j^2	δ_j^3
1	0.1574	0.0000	0.0000
2	3.2108	2.9706	2.3600
3	1.4179	1.0824	1.5467
4	1.3122	1.5000	1.9667
5	0.0000	0.0000	1.2333
6	0.2459	0.3529	2.2667
7	1.3595	3.4235	0.2000
8	1.0377	4.5588	1.9467
9	0.0000	3.3941	2.4667
10	0.2843	1.5118	1.7267

Table 4. The Efficiency Scores of DMUs at Each Time.

DMU	E_j^1	E_j^2	E_j^3
1	0.8808	1.0000	1.0000
2	0.1815	0.2518	0.2133
3	0.4883	0.5808	0.3556
4	0.5486	0.5588	0.2133
5	1.0000	1.0000	0.4638
6	0.9129	0.8763	0.1907
7	0.6401	0.1894	0.8889
8	0.9160	0.2598	0.3048
9	1.0000	0.3204	0.1778
10	0.9016	0.5695	0.4046

According to the proposed model to evaluate the performance of ten DMUs, at first we must be solve model (7) to determine the optimal deviating variables $(\delta_j^{t*}, \quad t=1,2,3, \ j=1,...,10)$ and then calculate the efficiency scores of each DMU at various times $(E_j^t, t=1,2,3, \ j=1,...,10)$ by using equation (8) or (9). The results of the deviating variables and efficiency score of all DMUs at three time represent in the Table 3 and Table 4, respectively.

As can be observed from Table 3, optimal deviating variables of DMU_5 and DMU_9 are zero $(\delta_5^{1*}, \delta_9^{1*} = 0)$. This means that deviance from the efficiency score 1 (goal) is zero for DMU_5 and DMU_9 at time 1and are efficient at time 1. This is true for DMU_1 and DMU_5 at time 2, and DMU_1 at time 3. In fact, by calculating the optimal solution of the deviating variables, we can categorize all DMUs into efficient and inefficient classifications. Also, the efficiency score of each DMU can be obtained by said Equation. From Tables 3 and 4, it is obvious that the DMUs whose deviating variables are equal to zero have an efficiency score of one.

5. Conclusion
DEA is a non-parametric tool to measure the relative efficiency of DMU that use multiple inputs to produce multiple outputs. Standard DEA models are quite limited models, in the sense that they do not consider a DMU at different times. These designs are changed to models with dynamic structures. Also, multi-objective DEA models constitute an important method for evaluating DMUs, because these models can calculate the efficiency scores of DMUs by solving only one model. Jafarian-Moghaddam and Ghoseiri [18] have presented a fuzzy dynamic multi-objective DEA model to evaluate DMUs in which data are changing

sequently. Some comments have been written on their model. In this paper, we stated some of the shortcomings of their model and improved the model. The proposed model is a multi-objective non-linear programming (MONLP) problem and there are several methods for solving it; We used the goal programming method in this paper. The proposed model calculates the efficiency scores of DMUs by solving a linear programming problem. Whereas, the proposed model has MOFP form and optimal solutions gives part of the stability region and it is disadvantage of all MOFP models. Removing this disadvantage and extended this method to network and daynamic-network systems can be considered as future research.

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