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Data Envelopment Analysis with LINGO Modeling for Technical Educational Group of an Organization

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Abstract

Data Envelopment Analysis (DEA) was developed to help compare the relative performance of decision-making units. It is a non-parametric method for performing frontier analysis. It uses linear programming to estimate the efficiency of multiple decision-making units and it is commonly used in production, management and economics [3]. DEA generates an efficiency score between 0 and 1 for each unit, indicating how effectively they are managing their resources. A compelling feature of DEA is it develops a unique rating system for each unit designed to make them look their best. This should facilitate acceptance of DEA within an organization [2].

LINGO is a comprehensive tool designed to make building and solving mathematical optimization models easier and more efficient. In this paper, we have explained what DEA does? How DEA evaluates efficiency, how DEA identifies paths to improve efficiency using this LINGO software, and how to use DEA. We have considered a group of 3 technical colleges (campuses). We assume that each campus "converts" 2 inputs into 3 outputs. This will enable managers to explore and assess the value of using DEA in their service operations. This paper also shows that Lingo Model can offer more information than the standard LP approach with exclusion of Simulation modeling approach [4]. The advantages of this approach are modesty, flexibility and visualization. The results by comparison of educational campuses were helpful for organization management.

Keywords: Data Envelopment Analysis (DEA); Efficiency; Performance, Reference Units, Weights.

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Introduction

Data Envelopment Analysis, also known as DEA, is a non-parametric method for performing frontier analysis. It uses linear programming to estimate the efficiency of multiple decision-making units and it is commonly used in production, management and economics. The technique was first proposed by Charnes, Cooper and Rhodes in 1978 and since then it became a valuable tool for estimating production frontiers.

DEA was developed to help compare the relative performance of decision-making units [2]. It generates an efficiency score between 0 and 1 for each unit, indicating how effectively they are managing their resources. A compelling feature of DEA is it develops a unique rating system for each unit designed to make them look their best. This should facilitate acceptance of DEA within an organization.

One of the core characteristics of this method is the separation of the record features into two categories: input and output. For example if we measure the efficiency of a car, we could say that the input is the liters of petrol and the output is the number of kilometers that it travels.

What does DEA do?

1. DEA compares service units considering all resources used and services provided, and identifies the most efficient units or best practice units (branches, departments, individuals) and the inefficient units in which real efficiency improvements are possible. This is achieved by comparingthe mix and volume of services provided and the resources used by each unit compared with those of all the other units.

2. DEA calculates the amount and type of cost and resource savings that can be achieved by making each inefficient unit as efficient as the most efficient - best practice – units [9].

3. Specific changes in the inefficient service units are identified, which management can implement to achieve potential savings located with DEA. These changes would make the efficient units performance approach the best practice unit performance. In addition, DEA estimates the amount of additional service an inefficient unit can provide without the need to use additional resources.

4. Management receives information about performance of service units that can be used to help transfer system and managerial expertise from better-managed, relatively efficient units to the inefficient ones. This has resulted in improving the productivity of the inefficient units, reducing operating costs and increasing profitability [8].

Why Data Envelopment Analysis is interesting?

Data Envelopment Analysis is a method that enables us compare and rank records (stores, employees, factories, webpages, marketing campaigns etc.) based on their features (weight, size, cost, revenues and other metrics or KPIs) without making any prior assumptions about the importance or weights of the features. The most interesting part of this technique is that it

allows us to compare records comprised of multiple features that have totally different units of measurement [11]. This means that we can have records with features measured in kilometers, kilograms or monetary units and still be able to compare, rank them and find the best/worst and average performing records.

The description and assumptions of Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a method for evaluating the relative efficiency of production units characterized by multiple inputs and outputs. The standard DEA models are based on exact inputs and outputs.

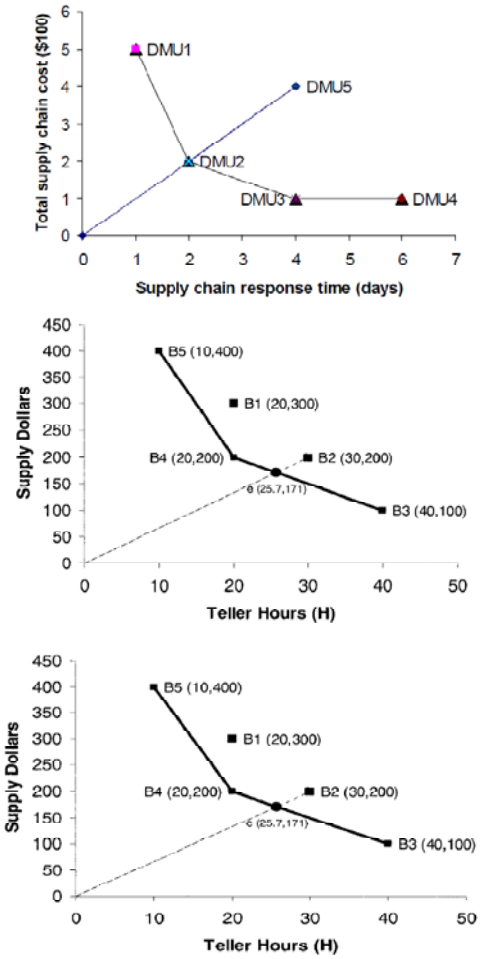

Figure:Graphic Representation of Five Bank Branches

In DEA, all features must be positive and it is assumed that the higher their value, the more their input/output is. Additionally Data Envelopment Analysis assumes that the features can be combined linearly as a weighted sum of non-negative weights and form a ratio between input and output that will measure the efficiency of each record [1]. For a record to be efficient it must give us a "good" output relative to the provided input. The efficiency is measured by the ratio between output and input and then compared to the ratio of the other records.

BASIC EFFICIENCY CONCEPT

Efficiency can be simply defined as the ratio of output to input. More output per unit of input reflects relatively greater efficiency [12]. If the greatest possible output per unit of input is achieved, a state of *absolute* or *optimum efficiency* has been achieved and it is not possible to become more efficient without new technology or other changes in the production process.

In quantitative term, The efficiency ratio of a particular record i with x input and y output (both feature vectors with positive values) is estimated by using the following formula:

Efficiency of record i=

$$\frac{weighted\ sum\ of\ outputs}{weighted\ sum\ of\ inputs} =$$

$$\frac{u_1 y_{1,i} + u_2 y_{2,i} + \cdots + u_s y_{s,i}}{v_1 x_{1,i} + v_2 x_{2,i} + \cdots + v_m x_{m,i}} = \frac{\sum_r^s u_r y_{r,i}}{\sum_p^m v_p x_{p,i}}$$

Where u and v are the weights of each output

and input of the record, s is the number of output features and m is the number of input features.

The problem of finding the best/ideal weights for a particular record i can be formulated as follows:

$$\textit{maximized } h = \frac{\sum_r^s u_r y_{r,i}}{\sum_p^m v_p x_{p,i}}$$

$$\textit{subject to } \frac{\sum_r^s u_r y_{r,j}}{\sum_p^m v_p x_{p,j}} \leq 1 \textit{ for every j record}$$

$$u_r, v_p \geq 0$$

Again the above is just the mathematical way of finding the weights u and v that maximize the efficiency of record i, provided that those weights will not make any of the other records more efficient than 100%.

To solve this problem we must use linear programming. Unfortunately linear programming does not allow us to use fractions and thus we need to transform the formulation of the problem as following:

$$\textit{maximized } h = \sum_r^s u_r y_{r,i}$$

$$\textit{subject to } \sum_r^s u_r y_{r,j} - \sum_p^m v_p x_{p,j} \leq 0 \quad \textit{for every j record}$$

$$\sum_p^m v_p x_{p,i} = 1$$

$$u_r, v_p \geq 0$$

We should stress that the above linear programming problem will gives us the best weights for record i and calculate its efficiency under those optimal weights. The same must be repeated for every record in our dataset. So if we have n records, we have to solve n separate linear problems.

Technical and Scale Efficiency

When we focus on service organizations' we generally cannot determine what the engineered, optimum or absolute efficient output-to-input ratio is. This is in contrast to the auto example where it was possible to determine the efficient engine performance. Consequently we cannot determine whether a service unit is absolutely efficient. We can, however, compare several service unit output-to-input ratios and determine that one unit is more or less efficient than another - benchmarking. The difference in efficiency will be due to the technology or production process used, how well that process is managed, and/or the scale or size of the unit.

Price Efficiency

If inputs and/or outputs are measured in monetary terms rather than physical units, the efficiency differences we observe can be due to price efficiency as well as scale and technical efficiency. If we use both methods, physical and monetary measures, we can begin to segregate price from technical and scale efficiency. To understand which units are inefficient and how to improve them, we need to separately measure all the types of inefficiency present, DEA is one method of separating technical and scale efficiency from price efficiency, enabling it to locate methods of improving profitability of service organizations that already appear relatively

profitable based on accounting measures reflected in an income statement.

Allocative Efficiency

Consider two employees, X and Y, who each own two autos, one of type A and one of type B, and they can use either one or both for business transportation. If they use their respective A and B cars in different proportions, they may have different travel costs. Assume both A autos get 20 MPG, both B autos get 25 MPG, and all gas is purchased at $1 per gallon, so that the As cost 5 cents per mile and the Bs cost 4 cents per mile. If X uses only B and Y uses A and B equally, X's cost is 4 cent per mile while Y's cost is 4.5 cents per mile. Comparing these travel cost ratios, we could conclude that X is less efficient than Y. His costs are 4.5/5 or 90% of Y's and Y is thus only 90% as efficient as X. This is an example of *allocative* inefficiency, which results from an inefficient mix of inputs - cars - used to produce the output - miles traveled.

In other words, when more than one input and/or output are involved in the production process' inefficiencies can also be due to the mix of inputs used to produce the mix of outputs, which is referred to as allocative efficiency.

Another concept is that, Ratios can provide very useful managerial information about efficiency; however, they are incapable of accommodating multiple inputs and outputs when accurate objective relative weights for inputs and outputs are not known.

Limitations of Data Envelopment Analysis

DEA is a great technique but it has its limitations. We must understand that DEA is like a black box. Since the weights that are used in the effectiveness ratio of each record are different, trying to explain how and why each score was calculated is pointless. Usually we focus on the ranking of the records rather than on the actual values of the effectiveness scores. Also note that the existence of extremums can cause the scores to have very low values.

Since DEA uses linear combinations of the features to estimate the ratios. Thus if combining them linearly is not appropriate in our application, we must apply transformations on the features and make them possible to be linearly combined. Another drawback of this technique is that we have to solve as many linear programming problems as the number of records, something that requires a lot of computational resources.

Another problem that DEA faces is that it does not work well with high dimensional data. To use DEA the number of dimensions $d = m + s$ must be significant lower than the number of observations. Running DEA when d is very close or larger than n does not provide useful results since most likely all the records will be found to be optimal.

Finally we should note that in the general form of the algorithm, the weights of the features in DEA are estimated from the data and thus they don't use any prior information about the

importance of features that we might have in our problem (of course it is possible to incorporate this information as constrains in our linear problem). Additionally the efficiency scores that are calculated are actually the upper limit efficiency ratios of each record since they are calculated under "ideal situations". This means that DEA can be a good solution when it is not possible to make any assumptions about the importance of the features but if we do have any prior information or we can quantify their importance then using alternative techniques is advised [13].

The Mathematical Formulations of the Problem under DEA:

Let is consider a group of technical colleges which have 3 campuses. To simplify matters, we assume that each campus "converts" two inputs into three different outputs. The two inputs used by each campuses are:

Input 1: = Capital (Measured by the number of Faculty in different courses)

Input 2: = Staff (Measured in thousands of working hours used during a month)

The output produced by each campus are:

Output-1 = Admission for the new session in course A

Output-2 = Admission for the new session in course B

Output-3 = Admission for the new session in course C

Suppose that the inputs and outputs for the 3 campuses are given in the table-1

Table1: Input and Outputs for each Campuses
(Units in hundreds)

Campus	Inputs		Outputs		
	1	2	1	2	3
1	5	14	12	6	17
2	4	15	8	4	11
3	3	12	7	8	14

The DEA approach uses the following four ideas to determine if a campus is efficient:

1. No campus can be more than 100% efficient. Thus, the efficiency of each campus must be less than or equal to 1.

2. Suppose we are interested in evaluating the efficiency of campus i. we attempt to choose output prices (t1, t2. t3) and inputs costs (w1 and w2) that maximize efficiency. If the efficiency of campus I equal 1, then it is efficient, if the efficiency is less than 1, then it is inefficient.

3. To simplify computations, we may scale the output prices so that the cost of campus I's inputs equal 1. Thus, for campus 2 we add the constraints 8w1 + 15w2 = 1.

4. We must ensure that each output cost and output price is strictly positive. If for example, t1 = 0, then DEA could not detect an inefficiency involving output I; if wj=0, then DEA could bit detect an inefficiency involving input j.

Point (1) – (4) lead to the following LPs for testing the efficiency of each campus.

Campus-1: $\text{Max } z = 12t_1 + 6t_2 + 17t_3$ (1)

s.t. $-12t_1 - 6t_2 - 17t_3 + 5w_1 + 14w_2 \geq 0$ (2)

$-8t_1 - 4t_2 - 11t_3 + 4w_1 + 15w_2 \geq 0$ (3)

$-7t_1 - 8t_2 - 14t_3 + 3w_1 + 12w_2 \geq 0$ (4)

$w_1 + 14w_2 = 1$ (5)

$t_1 \geq 0.0001$ (6)

$t_2 \geq 0.0001$ (7)

$t_3 \geq 0.0001$ (8)

$w_1 \geq 0.0001$ (9)

$w_2 \geq 0.0001$ (10)

Campus-2: Max $z = 8t_1 + 4t_2 + 11t_3$ (1)

s.t. $-12t_1 - 6t_2 - 17t_3 + 5w_1 + 14w_2 \geq 0$ (2)

$-8t_1 - 4t_2 - 11t_3 + 4w_1 + 15w_2 \geq 0$ (3)

$-7t_1 - 8t_2 - 14t_3 + 3w_1 + 12w_2 \geq 0$ (4)

$4w_1 + 15w_2 = 1$ (5)

$t_1 \geq 0.0001$ (6)

$t_2 \geq 0.0001$ (7)

$t_3 \geq 0.0001$ (8)

$w_1 \geq 0.0001$ (9)

$w_2 \geq 0.0001$ (10)

Campus-3: Max $z = 7t_1 + 8t_2 + 14t_3$ (1)

s.t. $-12t_1 - 6t_2 - 17t_3 + 5w_1 + 14w_2 \geq 0$ (2)

$-8t_1 - 4t_2 - 11t_3 + 4w_1 + 15w_2 \geq 0$ (3)

$-7t_1 - 8t_2 - 14t_3 + 3w_1 + 12w_2 \geq 0$ (4)

$3w_1 + 1w_2 = 1$ (5)

$t_1 \geq 0.0001$ (6)

$t_2 \geq 0.0001$ (7)

$t_3 \geq 0.0001$ (8)

$w_1 \geq 0.0001$ (9)

$w_2 \geq 0.0001$ (10)

We can see how the campus 1 LP incorporates points (1) – (4). Point (1) maximizes the efficiency of Campus 1. This is because Constraints (5) implies that the total cost of campus 1 inputs equal 1. Constraints (2) – (4) ensure that no campus is more than 100% efficient. Constraints (6) – (10) ensure that each cost and output price is strictly positive (the 0.0001 right hand side is arbitrary; any small positive number may be used).

LINGO output for these LPS is given in Figure A, B and C [4]. From the optimal objective function value to each LP we find that

Campus1 efficiency = 1

Campus 2 efficiency = 0.773

Campus 3 efficiency = 1

Thus we find that campus 2 is inefficient and campus 1 and 3 are efficient.

LINGO MODEL PRGRAMMES:

Campus-wise:

```
MAX     12 T1 +  6 T2 + 3 T3
SUBJECT TO
   2)  -12 T1 - 6 T2 - 3 T3 + 5 W1 + 14 W2 >= 0
   3)  -8 T1 - 4 T2 - 2 T3 + 4 W1 + 15 W2 >= 0
   4)  -7 T1 - 3 T2 - 4 T3 + 3 W1 + 12 W2 >= 0
   5)   W1 >= 0.0001
   6)   W2 >= 0.0001
   7)   T1 >= 0.0001
   8)   T2 >= 0.0001
   9)   T3 >= 0.0001
   10)  5 W1 + 14 W2 = 1
END

LP OPTIMUM FOUND AT STEP  4
        OBJECTIVE FUNCTION VALUE
1)        1.00000000
VARIABLE        VALUE          REDUCED  COST
T1              .110889          .000000
T2              .000100          .000000
T3              .000100          .000000
W1              .000100          .000000
W2              .071393 .        000000

ROW    SLACK/SURPLUS   DUAL PRICES
2)        .000000 -          -1.000000
3)        .515548             .000000
4)        .411659             .000000
5)        .000000             .000000
6)        .071293             .000000
7)        .110789             .000000
8)        .000000            .000000
9)        .000000             .000000
10)       .000000            1.000000

FIGURE A: FOR CAMPUS I          NO. OF

ITERATION       6
```

```
MAX     8 T1 + 4 T2 + 2 T3
SUBJECT TO
   2)   -12 T1 - 6 T2 - 3 T3 + 5 W1 + 14 W2 >= 0

   3)   -8 T1 - 4 T2 - 2 T3 + 4 W1 + 15 W2 >= 0
   4)   -7 T1 - 3 T2 - 4 T3 + 3 W1 + 12 W2 >= 0
   5)   W1 >= 0.0001
   6)   W2 >= 0.0001
   7)   T1 >= 0.0001
   8)   T2 >= 0.0001
   9)   T3 >= 0.0001
   10)  4 W1 + 15 W2 = 1
END

LP OPTIMUM FOUND AT STEP  0
          OBJECTIVE FUNCTION VALUE
1)        .773030000
VARIABLE       VALUE          REDUCED
COST
T1             .079821          .000000
T2             .053275          .000000
T3             .000100          .000000
W1             .000100          .000000
W2             .066613          .000000

ROW    SLACK/SURPLUS     DUAL PRICES
2)        .000000           -.261538
3)        .226970            .000000
4)        .000000           -.661538
5)        .000000            .773333
6)        .000000            .248206
7)        .066513            .000000
8)        .079721            .000000
9)        .053175            .000000
10)       .000000          -2.784615
```

FIGURE B: FOR CAMPUS II NO. OF ITERATION 0

```
MAX     7 T1 + 3 T2 + 4 T3
SUBJECT TO
   2)   -12 T1 - 6 T2 - 3 T3 + 5 W1 + 14 W2 >= 0
   3)   -8 T1 - 4 T2 - 2 T3 + 4 W1 + 15 W2 >= 0
   4)   -7 T1 - 3 T2 - 4 T3 + 3 W1 + 12 W2 >= 0
   5)   W1 >= 0.0001
   6)   W2 >= 0.0001
   7)   T1 >= 0.0001
   8)   T2 >= 0.0001
   9)   T3 >= 0.0001
   10)…3 W1 + 12 W2 = 1
END

LP OPTIMUM FOUND AT STEP  7
          OBJECTIVE FUNCTION VALUE
1)        1.00000000
VARIABLE       VALUE          REDUCED COST
T1             .099815          .000000
T2             .066605          .000000
T3             .000100          .000000
W1             .000100          .000000
W2             .083275          .000000
ROW    SLACK/SURPLUS     DUAL PRICES
2)        .000000           .000000
3)        .283620           .000000
4)        .000000         -1.000000
5)        .000000           .000000
6)        .083175           .000000
7)        .099715           .000000
8)        .066505           .000000
9)        .000000           .000000
10)       .000000          1.000000
```

FIGURE C: FOR CAMPUS III NO. OF ITERATION 7

MODEL (COMBINED):

```
! Data Envelope Analysis of Decision Maker
Efficiency;
SETS:
DMU/A B C D E F /: ! Six Campuses;
SCORE; ! Each decision making unit has a;
! score to be computed;
FACTOR/COST RICH WRIT SCIN/;
! There is a set of factors, input & output;
DXF( DMU, FACTOR): F; ! F( I, J) = Jth factor
of DMU I;
ENDSETS
DATA:
! Inputs are spending/pupil, % not low income;
! Outputs are Writing score and Science score;
NINPUTS = 2; ! The first NINPUTS factors are
inputs;
! The inputs, the outputs;
F =     5 14 12 6 17
        4 15 8 4 11
        3 12 7 8 14
```

```
        2 10 9 2 2
        6 13 6 5;
ENDDATA
SETS:
! Weights used to compute DMU I's score;
DXFXD(DMU,FACTOR) : W;
ENDSETS
! Try to make everyone's score as high as possible;
MAX = @SUM( DMU: SCORE);
! The LP for each DMU to get its score;
@FOR( DMU( I):
SCORE( I) = @SUM( FACTOR(J)|J #GT# NINPUTS:
F(I, J)* W(I, J));
! Sum of inputs(denominator) = 1;
@SUM( FACTOR( J)| J #LE# NINPUTS:
F( I, J)* W( I, J)) = 1;
! Using DMU I's weights, no DMU can score better than 1;
@FOR( DMU( K):
@SUM( FACTOR( J)| J #GT# NINPUTS:
F( K, J) * W( I, J))
<= @SUM( FACTOR( J)| J #LE# NINPUTS:
F( K, J) * W( I, J))
)
);
! The weights must be greater than zero;
@FOR( DXFXD( I, J): @BND( .00001, X, 100000));
END
```

Window Analysis:

Dual Price: The LINGO solution report also give a dual price figure for each constraints. We can interpret the dual price as the amount that the objective would improve as the right hand side, or constant term, of the constraints is increased by one unit. It is also known as Shadow price.

Dual Prices and DEA:

The Dual Prices section of the LINGO output gives us great insight in to Campus 2's (or which is inefficient by DEA) inefficiency [5]. Consider all the campuses whose efficiency constraints have nonzero dual prices in the campus II (i.e B as mention in LINGO Program) LP figure B. If we average the output vectors and input vectors for these campuses (using the absolute value of the dual price for each campus as the weight) we obtain the following

Average Output Vector

$$.261538\begin{bmatrix}12\\6\\17\end{bmatrix}+.661538\begin{bmatrix}7\\8\\14\end{bmatrix}=\begin{bmatrix}7.76\\6.86\\13.71\end{bmatrix}$$

Averaged Input Vector

$$.261538\begin{bmatrix}5\\15\end{bmatrix}+.661538\begin{bmatrix}3\\12\end{bmatrix}=\begin{bmatrix}3.29\\11.86\end{bmatrix}$$

In terms of composite analysis of campuses. The averaged output vector tells us that the composite campuses produces the same amount of outputs I and II as campus III, but the composite campus produces 13.71-11 = 3.71 more of output 3. From the averaged input vector for the composite campus, we find that composite campus uses less of each input than does campus 2. So we can see exactly where campus 2 is inefficient.

The objective function value of .7730 for the campus 2 LP implies that the more efficient composite campus produces its superior outputs by using at most 77.30% as much of each input. Similar analysis can be made for other campuses whether they are efficient or inefficient.

Conclusions

The paper shows that the Lingo based DEA techniques can offer more information that standard LP approach. The experiments show that this approach can be an appropriate instrument for analysing DEA problems with imprecise data and can produce interesting results in comparison with other approaches. Advantages of this approach are modesty, flexibility and visualisation. The results by comparison of technical campuses of group of organisation were helpful for campus management.

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