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Research Article

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Evaluation of Unit's Performance in Presence of Subunits by Using GDEA

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Abstract

Data Envelopment Analysis (DEA) is a technique that uses all collected observations to measure performance. This method presents no data about how to operate on DMU. The present research attempted to study a unit with all its subunits, if the unit is efficient, it means that all its subunits are efficient too and if it is an inefficient, it shows clearly that which one of the subunits makes inefficiency in order to reach to desired performance by correcting just that submit. Studying the performance by each of these DEA models is time-consuming and long. By using general DEA model (GDEA) we can reach to better speed in evaluation of working with five mentioned DEA methods. The present research attempted to study the unit's performance of general GDEA model in presence of subunits, and a general model illustrated to evaluation of unit performance in presence of decision-making subunits.

Keywords: Data envelopment analysis, performance, Decision-making subunits, General model of Data Envelopment Analysis.

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1. Introduction

The main purpose of DEA is to estimate the border efficiency experimentally based on existence DMU set. DMU is efficient if there is no other unit that can create more output by using less or same usage of input by the name of DMU. Using GDEA can consider five illustrated DEA model in a model by different interoperation that gives to a coefficient α.Each unit contains subunits that their efficiency are effective on performance Of all units. Therefore with GDEA model can make more speed and accuracy in related calculations.

2.Main DEA models for subunits

Suppose that we have n DMU and each DMU has b subunit that are called DMSU. Each DMUj transmites the resources, or the inputs to outputs of production. See figure 1.

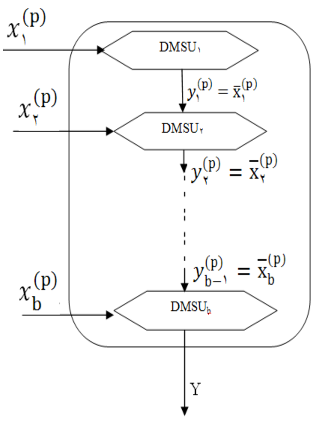

figure1. DMU with its DMSU

Suppose that, j=1,…,b, $y_j^{(p)}$ show the output vectors that are made by jth of DMU_p sub-units, in which $Y_j^{(p)} = \left(y_{j1}^{(p)},\dots,y_{j,k_j}^{(p)}\right)$.

Also we have $x_j^{(p)}$,$\bar{X}_j^{(p)}$, $j = 2,\dots,b$ and $x_j^{(p)}$ shows the I_j, $\acute{I}_j$ vectors of internal and external inputs dimensions for jth sub-DMU of DMU_p, respectively in which:

$$X_j^{(p)} = \left(x_{j1}^{(p)},\dots,x_{j,I_j}^{(p)}\right),$$

$$\bar{X}_j^{(p)} = \left(\bar{x}_{j1}^{(p)},\dots,\bar{x}_{j,\acute{I}_j}^{(p)}\right) = \left(y_{j-1,1}^{(p)},\dots,y_{j-1,\acute{I}_j}^{(p)}\right) \qquad (1\text{-}2)$$

Therefore, evaluation of sum performance of $e_p^{(a)}$ can be presented by the following formula that is called cumulative sub-performance formula:

$$e_p^{(a)} = \left(\mu^{(1)T}y_1^{(p)} + \mu^{(2)T}y_2^{(p)} + \cdots + \mu^{(b)T}y_b^{(p)}\right) / \left(v^{(1)T}x_1^{(p)} + v^{(2)T}x_2^{(p)} + \cdots + v^{(b)T}x_b^{(p)} + \bar{v}^{(1)T}y_1^{(p)} + \dots + \bar{v}^{(b-1)T}y_{b-1}^{(p)}\right)$$

and performance for each sub-unit of DMUp can be represented by:

$$e_p^{(1)} = \frac{\mu^{(1)T}y_1^{(p)}}{v^{(1)T}x_1^{(p)}}$$

$$e_p^{(i)} = \frac{\mu^{(i)T}y_i^{(p)}}{v^{(i)T}x_i^{(p)} + \bar{v}^{(i-1)T}y_{i-1}^{(p)}}, \quad i = 2,\dots,b \qquad (2\text{-}2)$$

Theorem 1-2: sum performance of $e_p^{(a)}$ is a convex combination of performance of its sub-units.

Proof: proof is clear.

Theorem 2-2: DMU_p is efficient if all its sub-units are efficient.

Proof: proof is clear.

Then we have the following mathematical programming problem:

$$\begin{aligned}
&Max\, e_p^{(a)} \\
&s.t. \quad e_j^{(a)} \le 1, \quad j = 1,\dots,n \\
&e_j^{(i)} \le 1, \quad i = 1,\dots,b, \quad j = 1,\dots,n \\
&\mu^{(i)} \in \bar{\Omega}_1, \quad i = 1,\dots,b \\
&\left(v^{(i)}, \bar{v}^{(i)}\right) \in \bar{\Omega}_2, \quad i = 1,\dots,b \qquad (3\text{-}2)
\end{aligned}$$

The sets $\overline{\Omega}_1$ and $\overline{\Omega}_2$ are assurance regions defined by any imposed restrictions on multipliers. The model (3-2) can be expressed in the following form:

$$Max \quad \sum_{i=1}^{b} \mu^{(i)T} y_i^{(p)}$$
$$s.t. \quad \sum_{j=1}^{b} v^{(i)T} x_i^{(p)} + \sum_{j=1}^{b-1} \bar{v}^{(i)T} x_i^{(p)} = 1,$$
$$\sum_{i=1}^{b} \mu^{(i)T} y_i^{(j)} - \sum_{j=1}^{b} v^{(i)T} x_i^{(j)} - \sum_{i=1}^{b-1} \bar{v}^{(i)T} y_i^{(j)} \leq 0 \ ,$$
$$j = 1, \dots, n$$
$$\mu^{(i)T} y_i^{(j)} - v^{(i)T} x_i^{(j)} - \bar{v}^{(i-1)T} y_{i-1}^{(j)} \leq 0,$$
$$i = 2, \dots, b, \qquad j = 1, \dots, n$$
$$\mu^{(i)} \in \Omega_1, \ \ i = 1, \dots, b$$
$$\left(v^{(i)}, \bar{v}^{(i)}\right) \in \Omega_2, \ \ i = 1, \dots, b \qquad (4\text{-}2)$$

The form of Ω_1 and Ω_2 depends upon how 1 and 2 are structured.

FDH model in presence of DMSU is as the follow:

$$Min \quad \theta - \varepsilon\left(1^t \ s_x + 1^t \ s_y + 1^t \ \acute{s}_y\right)$$
$$S.t. \quad \sum_{j=1}^{n} \lambda_j x_i^{(j)} + \sum_{j=1}^{n} \lambda_{ij} x_i^{(j)} - \theta x_i^{(p)} + s_x = 0, \qquad i = 1, \dots, b$$
$$\sum_{j=1}^{n} \lambda_j y_i^{(j)} + \sum_{j=1}^{n} \lambda_{ij} y_i^{(j)} - \theta y_i^{(p)} + \acute{s}_y = 0, \qquad i = 1, \dots, b-1$$
$$\sum_{j=1}^{n} \lambda_j y_i^{(j)} - \sum_{j=1}^{n} \lambda_{ij} y_i^{(j)} - s_y = y_i^{(p)}, \qquad i = 1, \dots, b$$
$$\sum_{j=1}^{n} \lambda_j + \sum_{i=1}^{b} \sum_{j=1}^{n} \lambda_{ij} = 1$$
$$\lambda_j \in \{0, 1\}, \lambda_{ij} \in \{0, 1\} j = 1, \dots, n \qquad (5\text{-}2)$$

3. General model in presence of sub-units

In this section, General model in presence of sub-units is formulated based on massive structure and definition of new performance in this model.[1] .

General model is formulated by using chebyshev function. [3].It can evaluate the performance in some DEA main models. The model is as follow:

$$Max \ \Delta$$
$$s.t \qquad \Delta \leq \tilde{d}_j + \alpha\Big(\sum_{i=1}^{b} \mu^{(i)T}\left(y_i^{(p)} - y_i^{(j)}\right) + \sum_{i=1}^{m} v^{(i)T}\left(-x_i^{(p)} + x_i^{(j)}\right) + \sum_{i=1}^{b-1} \bar{v}^{(i)}\left(-y_i^{(p)} + y_i^{(j)}\right)\Big)$$
$$\sum_{i=1}^{b} \mu^{(i)} + \sum_{i=1}^{b} v^{(i)} + \sum_{i=1}^{b-1} \bar{v}^{(i)} = 1$$
$$\mu^{(i)} \geq 0, v^{(i)} \geq 0, \quad i = 1, \dots, b$$
$$\bar{v}^{(i)} \geq 0, i = 1, \dots, b-1 \qquad (1\text{-}3)$$

That α parameter supposed positive and:

$$\tilde{d}_j = \max_{i=1,\dots,b;t=1,\dots,b-1} \left\{\mu^{(i)T}\left(y_i^{(p)} - y_i^{(j)}\right), \ v^{(i)T}\left(-x_i^{(p)} + x_i^{(j)}\right), \bar{v}^{(i)}\left(-y_i^{(p)} + y_i^{(j)}\right)\right\}$$

Note that when j = p then $\Delta \leq 0$.

Definition 1:(α-efficiency) For a given positive number α, DMUp is defined to be α-efficiency if and only if the optimal value to the problem (1-3) equals to zero.

Otherwise, DMUp is said to be α-inefficiency.

Theorem 1-3: If $\Delta \neq 0$ the existence DMU which dominated DMUp.

Proof: refer to [1].

4. Relationship between general model and CCR, BCC and FDH models in presence of sub-units

This section codified theoretical characteristics of relationship between performance in basic DEA models and general model in presence of sub-units.

Theorem 1-4: DMUp is BCC-efficiency in present sub-units if and only if DMUp is α -efficiency for some sufficiently large positive number α.

Proof: refer to [1].

Theorem 2-4: DMUp is CCR-efficient if and only if DMUp is α-efficient for sufficient large positive α in present sub-units.

Proof: refer to [1].

Theorem 3-4: DMU_p is FDH-efficient in presence of its sub-units if and only if DMUp is α-efficient for some small sufficient positive values of α.

Proof: If DMU_p is efficient FDH, therefore there is no $\hat{\lambda}$ as:

$$\hat{\lambda}z = Z_j \geq Z_P , \hat{\lambda} = \{\lambda | 1^t\lambda = 1 , \lambda_j \in \{0,1\}, j = 1, \dots, n\}$$

$$Z_p - Z_j \nleq \circ \quad (1\text{-}4)$$

$$\textit{that is for each } j$$

(reduction presume) DMU_p is not α-efficient for great and positive α sufficiently, hence, for each positive α there is Δ^*<∘.suppose that $(\mu^*, v^*, \bar{v}^*)$ are optimal answer of GDEA model in presence of sub-units, so we have:

$$\widetilde{d_j} = \max_{i=1,\dots,b;t=1,\dots,b-1} \left\{\mu^{(i)T}\left(y_i^{(p)} - y_i^{(j)}\right), v^{(i)T}\left(-x_i^{(p)} + x_i^{(j)}\right), \bar{v}^{(i)}\left(-y_i^{(p)} + y_i^{(j)}\right)\right\} < \circ.$$

Then for some $j \neq p$

$$\widetilde{d_j} + \alpha(\mu, v, \bar{v}) \begin{bmatrix} Y_i^{(p)} - Y_i^{(j)} \\ -X_i^{(p)} + X_i^{(j)} \\ -\bar{X}_i^{(p)} + \bar{X}_i^{(j)} \end{bmatrix} < \circ$$

That is:

$$\widetilde{d_j} + \alpha(\mu, v, \bar{v})\left(Z_i^{(p)} - Z_i^{(j)}\right) < \circ \quad (2\text{-}4)$$

To make relation of (2-4) for α in small sufficient value $\widetilde{d_j}$ <∘.

Because $\alpha >\circ$ and $\mu^* > \varepsilon$ and relation of (1-4) happens.

According to $\widetilde{d_j}$ definition we have:

$$\widetilde{d_j} = \max_{i=1,\dots,b;t=1,\dots,b-1} \left\{\mu^{(i)T}\left(y_i^{(p)} - y_i^{(j)}\right), v^{(i)T}\left(-x_i^{(p)} + x_i^{(j)}\right), \bar{v}^{(i)}\left(-y_i^{(p)} + y_i^{(j)}\right)\right\} < \circ.$$

$$(\textit{for some of } j \neq p) \quad (3\text{-}4)$$

Necessary and sufficient condition to make (3-4) relation is:

$$\begin{cases} Y_i^{(p)} - Y_i^{(j)} < \circ \\ -X_i^{(p)} + X_i^{(j)} < \circ \\ -\bar{X}_i^{(p)} + \bar{X}_i^{(j)} < \circ \end{cases}$$

the concludes: $Z_p - Z_j < \circ$

And it is contradiction to (1-4) relation.

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