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Cross Efficiency Evaluation with Negative Data in Selecting the Best of Portfolio Using OWA Operator Weights

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Abstract

The present study is an attempt toward evaluating the performance of portfolios and asset selection using cross-efficiency evaluation. Cross-efficiency evaluation is an effective way of ranking decision making units (DMUs) in data envelopment analysis (DEA). Conventional DEA models assume non-negative values for inputs and outputs. However, we know that unlike return and skewness, variance is the only variable in the model that takes non-negative values. This paper focuses on the evaluation process of the efficiencies in the cross-efficiency matrix with negative data and proposes the use of ordered weighted averaging (OWA) operator weights for cross-efficiency evaluation. The problem consists of choosing an optimal set of assets in order to minimize the risk and maximize return. This method is illustrated by application in Iranian stock companies and extremely weights are obtained via OWA operator in cross efficiency for making the best portfolio. The finding could be used for constructing the best portfolio in stock companies, in various finance organization and public and private sector companies.

Keywords: Portfolio; Data Envelopment Analysis (DEA); Cross-efficiency evaluation; Negative data; Ordered Weighted Averaging (OWA) Operator.

1. Introduction

In financial literature, a portfolio is an appropriate mix investments held by an institution or private individuals. Evaluation of portfolio performance has created a large interest among employees also academic researchers because of huge amount of money are being invested in financial markets. The theory of mean – variance, Markowitz [12] is considered the basis of many current models and this theory is widely used to select portfolios. This model is due to the nature of the variance in quadratic form. Other problem in Markowitz model is that increasing the number of assets will be developed the covariance matrix of asset returns and will be added to the content calculation. Due to these problems sharp one- factor model is proposed by Sharp [18]. This method reduces the number of calculations required information for the decision. Data envelopment analysis (DEA) has proved the efficiency for assessing the relative efficiency of Decision Making Units (DMUs) that employing multiple inputs to produce multiple outputs (Charnes et al. [2]). Morey and Morey [13] proposed mean – variance framework based on Data Envelopment Analysis, which the variance of the portfolios is used as an input to the DEA and expected return is the output. Joro and Na [8] introduced mean - variance – skewness framework and skewness of returns are also considered as an output. Conventional DEA models assume non-negative values for inputs and outputs. However, we know that unlike return and skewness, variance is the only variable in the model that takes non-negative values. The portfolio optimization problem is a well-known problem in financial real world. The investor’s objective is to get the maximum possible return on an investment with the minimum possible risk. Since there are a large number of assets to invest in, this objective leads to select the best assets via cross-efficiency matrix in presence of negative data by using OWA operator weighted. Cross-efficiency evaluation, proposed by Sexton et al. [17] is effective way of ranking decision making units (DMUs). It allows the overall efficiencies of the DMUs to be evaluated through self- and peer-evaluations. The self-evaluation allows the efficiencies of the DMUs to be evaluated with the most favorable weights so that each of them can achieve its best possible relative efficiency, whereas the peer-evaluation requires the efficiency of each DMU to be evaluated with the weights determined by the other DMUs. The self-evaluated efficiency and peer-evaluated efficiencies of each DMU are then averaged as the overall efficiency of the DMU. Since, its remarkable discrimination power, the cross-efficiency evaluation has found significant number of applications in a wide variety of areas such as preference voting and project ranking [7,22], economic-environmental performance assessment [10,11], Olympic ranking and benchmarking [23-25], etc. Besides a large number of applications, theoretical research has also been conducted on the cross-efficiency evaluation. For example, Doyle and Green [4,5] presented mathematical formulations for possible implementations of aggressive and benevolent cross efficiencies. Liang et al. [9] suggested the concept of game cross-efficiency and developed a game cross-efficiency model which treats each DMU as a player that seeks to maximize own efficiency under the condition that the cross efficiency of each of the other DMUs dose not deteriorate. Wu et al. [25] extended the game cross-efficiency model to variable returns to scale later. Conventional DEA models assume non-negative values for inputs and outputs. These models cannot be used for the case in which DMUs include both negative and positive inputs and/or outputs. Poltera et al. [15] consider a DEA model which can be applied in the cases where input/ output data take positive and negative values. The other models solve negative data such as Modified slacks-based measure model (MSBM) [16], semi-oriented radial measure (SORM) [6] and etc.

In our work, the use of equal weights for cross-efficiency model in presence negative data has a significant problem. That is self-evaluated efficiencies are much less weighted than peer-evaluated efficiencies. This is because each DMU has only one self-evaluated efficiency value, but multiple peer-evaluated efficiency values. When they are simply averaged together, the weight assigned to the self-evaluated efficiency is only $\frac{1}{n}$ if there are n DMUs to be evaluated, whereas the remaining weights $\frac{(n-1)}{n}$ are all given to those peer-evaluated efficiencies. To overcome this problem, the use of ordered weighted averaging (OWA) operator weights are stated for assets cross- efficiencies in presence negative data. The use of OWA operator weights for the assets cross-efficiency allows the weights to be reasonably allocated between self-and peer- evaluated efficiencies by investor's control [20]. The OWA operator weights are generated by the minimax disparity approach and allow the decision maker (DM) or investors to select the best assets that are characterized by an orness degree [21]. The method consists of choosing an optimal set of assets in order to minimize the risk and maximize return in cross efficiency using OWA operator. Since there are a large number of assets to invest in, the best assets are chosen via cross-efficiency evaluation by using OWA weighted by control investors.

The rest of the paper is organized as follows: Section 2 briefly reviews the portfolio performance literature, cross-efficiency in presence of negative data, OWA operators and their weight determination methods. Section 3 develops a proposed method for selecting the best of portfolio. Section 4 presents computational results using Iranian stock companies and finally conclusions are given in section 5.

1. Background

2.1 Portfolio performance literature

Portfolio theory to investing is published by Markowitz (1952). This approach starts by assuming that an investor has a given sum of money to invest at the present time. This money will be invested for a time as the investor's holding period. The end of the holding period, the investor will sell all of the assets that were bought at the beginning of the period and then either consume or reinvest. Since portfolio is a collection of assets, it is better that to select an optimal portfolio from a set of possible portfolios. Hence the investor should recognize the returns (and portfolio returns), expected (mean) return and standard deviation of return. This means that the investor wants to both maximize expected return and minimize uncertainty (risk). Rate of return (or simply the return) of the investor's wealth from the beginning to the end of the period is calculated as follows:

$$\text{Return} = \frac{(\text{end}-\text{of}-\text{period wealth})-(\text{beginning}-\text{of}-\text{period wealth})}{\text{beginning}-\text{of}-\text{period wealth}} \tag{2.1}$$

Since Portfolio is a collection of assets, its return r_p can be calculated in a similar manner. Thus according to Markowitz, the investor should view the rate of return associated to any one of these portfolios as what is called in statistics a random variable. These variables can be described expected return (mean or $\bar{r}_p$) and standard deviation of return. Expected return and deviation standard of return are calculated as follows:

$$\bar{r}_p = \sum_{i=1}^{n} \lambda_i \bar{r}_i$$

$$\sigma_p = \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \sigma_{ij} \right]^{1/2}$$

Where:

n=the number of assets in the portfolio

$\bar{r}_p$ =The expected return of the portfolio

λ_i =The proportion of the portfolio's initial value invested in asset i

$\bar{r}_i$ =The expected return of asset i

σ_p = The deviation standard of the portfolio

σ_{ij} = The covariance of the returns between asset i and asset j

In the above, optimal portfolio from the set of portfolios will be chosen that maximum expected return for varying levels of risk and minimum risk for varying levels of expected return(Sharp 1985 [19]). Data Envelopment Analysis is a nonparametric method for evaluating the efficiency of systems with multiple inputs and multiple outputs. In this section we present some basic definitions, models and concepts that will be used in other sections in DEA. They will not be discussed in details. Consider DMU_j, ($j=1,...,n$) where each DMU consumes m inputs to produce s outputs. Suppose that the observed input and output vectors of DMU_j are $X_j=(x_{1j},...,x_{mj})$ and $Y_j=(y_{1j},...,y_{sj})$ respectively, and let $X_j \geq 0$ and $X_j \neq 0$, $Y_j \geq 0$ and $Y_j \neq 0$. A basic DEA formulation in input orientation is as follows:

$$
\begin{aligned}
\min\ & \theta-\varepsilon\left(\sum_{r=1}^{s} s_r^{+}+\sum_{i=1}^{m} s_i^{-}\right)\\
s.t.\ & \sum_{j=1}^{n}\lambda_j x_{ij}+s_i^{-}=\theta x_{io} \quad i=1,...,m,\\
& \sum_{j=1}^{n}\lambda_j y_{rj}+s_r^{+}=y_{ro} \quad r=1,...,s,\\
& \lambda\in\Lambda,\\
& s^{+},s^{-}\geq 0, \qquad (2.4)\\
& \varepsilon\geq 0
\end{aligned}
$$

Where λ is a n-vector of λ variables, s^{+} as-vector of output slacks, s^{-} an m-vector of input slacks and set Λ is defined as follows:

$$
\Lambda=\begin{cases}\{\lambda\in R_{+}^{n} & \text{with constant returns to scale,}\\ \{\lambda\in R_{+}^{n},1\lambda\leq 1\} & \text{with non-increasing returns to scale,}\\ \{\lambda\in R_{+}^{n},1\lambda=1\} & \text{with variable returns to scale}\end{cases} \qquad (2.5)
$$

Note that subscript ‘o’ refers to the unit under the evaluation. A DMU is efficient if $\theta=1$ and all slack variables s^{-},s^{+} equal zero; otherwise it is inefficient (Charnes et al. [3]). In the DEA formulation above, the left –hand sides in the constraints define an efficient portfolio. θ is a multiplier defines the distance from the efficient frontier. The slack variables are used to ensure that the efficient point is fully efficient. This model is used for asset selection. The portfolio performance evaluation literature is vast. In recent years these models have been used to evaluate the portfolio efficiency. Also in the Markowitz theory, it is required to characterize the whole efficient frontier but the proposed models by Joro and Na do not need to characterize the whole efficient frontier but only the projection points. The distance between the asset and its projection which means the ratio between the variance of the projection point and the variance of the asset is considered as an efficiency measure (θ) [8].

2.2 The cross-efficiency evaluation

Consider n DMUs that are to be evaluated with m inputs and s output. Denote by $x_{ij}(1,...,m)$ and $y_{rj}(r=1,...,s)$ the input and output values of $DMU_j\,(j=1,...,n)$. The efficiencies of the n DMUs can then be computed by solving the following CCR model for each of the n DMUs, respectively [2]:

$$
\begin{aligned}
& max\ \theta_{kk} = \sum_{r=1}^{s} u_{rk} y_{rk}, \\
& \text{s.t.} \quad \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \qquad (2.6) \\
& \quad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{r=1}^{s} v_{ik} x_{ij} \le 0,\ j = 1,\dots,n, \\
& \quad u_{rk}, v_{ik} \ge 0, \quad r = 1,\dots,s,\ i = 1,\dots,m.
\end{aligned}
$$

Where DMU_k is the DMU under evaluation, $v_{ik}\left(i=1,\dots,m\right)$ and $u_{rk}\left(r=1,\dots,s\right)$ are input and output weights. Let $u_{rk}^*\left(r=1,\dots,s\right)$ and $v_{ik}^*\left(i=1,\dots,m\right)$ be the optimal solution to the above CCR model. Then, $\theta_{kk}^* = \sum_{r=1}^{s} u_{rk}^* y_{rk}$ is referred to as the CCR-efficiency of DMU_k, which is the best relative efficiency of DMU_k by self-evaluation. If $\theta_{kk}^* = 1$, DMU_k is said to be CCR-efficient; otherwise, it is said to be non-CCR-efficient.

$\theta_{jk} = \sum_{r=1}^{s} u_{rk}^* y_{rj} \Big/ \sum_{i=1}^{m} v_{ik}^* x_{ij}$ is referred to as the cross-efficiency of DMU_k to DMU_j by peer-evaluation, where $j = 1,\dots,n;\ j \ne k.$

Model (2.6) is solved n times, each time for one particular DMU. As a result, we can get one CCR-efficiency value and $\left(n-1\right)$ cross-efficiency values for each DMU. The n efficiency values constitute a cross-efficiency matrix, as shown in table (2.1), where $\theta_{kk}\left(k=1,\dots,n\right)$ are the CCR-efficiency values of the n DMUs, i.e. $\theta_{kk} = \theta_{kk}^*$. The n efficiency values of each DMU are then simply averaged as its overall performance, which is called average cross-efficiency value. Based on these overall performance values, the n DMUs can be compared or fully ranked.

Table 2.1 cross-efficiency matrix for n DMUs

DMU	Target DMU				average crosses efficiency
	1	2	…	n	
1	θ_{11}	θ_{12}	…	θ_{1n}	$(\frac{1}{n})\sum_{k=1}^{n}\theta_{1k}$
2	θ_{21}	θ_{22}	…	θ_{2n}	$(\frac{1}{n})\sum_{k=1}^{n}\theta_{2k}$
. . .	. . .	. . .	…		. . .
n	θ_{n1}	θ_{n2}	…	θ_{nn}	$(\frac{1}{n})\sum_{k=1}^{n}\theta_{nk}$

The above approach about cross-efficiency value in CCR efficiencies or constant returns to scale (CRS) DEA model was extended to the variable returns to scale (VRS) DEA model [25]. The VRS DEA model can generate negative cross-efficiency scores.

The VRS DEA model is as follows [1]:

$$
\begin{aligned}
& \max \quad \sum_{r=1}^{s} u_{rk} y_{rk} - u_0 \\
& s.t. \quad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} - u_0 \le 0, \quad j = 1,\dots,n \\
& \quad \sum_{i=1}^{m} v_{ik} x_{ik} = 1 \qquad (2.7) \\
& \quad \mathrm{u}_{rk} \ge 0, \qquad r = 1,\dots,s \\
& \quad v_{ik} \ge 0, \qquad i = 1,\dots,m \\
& \quad u_0 \ge 0
\end{aligned}
$$

For each $DMU_k (k = 1,\dots,n)$ under evaluation in model (2.7) we obtain a set of optimal weights (u_{rk}^*, v_{rk}^*). Using this set of weights, the DMU$_k$-based cross efficiency for any $DMU_j \left(j=1,\dots,n \right)$ is calculated as

$$E_{kj} = \frac{\sum_{r=1}^{s} u_{rk}^* y_{rj} - u_0}{\sum_{i=1}^{m} v_{ik}^* x_{ij}}$$

$$\text{k}, j = 1,2,\dots,n$$

$$\textit{The average of all } E_{kj} \ (k = 1,\dots,n)$$

$$\bar{E}_j = \frac{1}{n}\sum_{k=1}^{n} E_{kj}$$

Is used as the cross-efficiency score for $DMU_j \left(j=1,\dots,n \right)$.

Note that the cross-efficiency score obtained in the above manner can be negative. This subject is presented by a simple numerical example involving five DMUs, with two input and single output [25].

The negative VRS cross-efficiency score is due to the fact that $\sum_{r=1}^{s} u_{rk} y_{rj} - u_0 < 0$ for some DMU_j, i.e., some DMU_j will have negative efficiency ratios when they use a set of optimal weights obtained when DMU_k is under evaluation. Naturally, we want every output-input efficiency ratio be positive regardless of the chosen weights. Therefore, adding $\sum_{r=1}^{s} u_{rk} y_{rj} - u_0 \geq 0$ into the VRS model is proposed when calculating the cross-efficiency scores [25]. This will also guarantee non-negativity of both VRS cross-efficiency scores and VRS efficiency ratios.

Therefore the following modified VRS DEA model is used for model (2.7) development and application:

$$\max \quad \sum_{r=1}^{s} u_{rk} y_{rk} - u_0 \qquad (2.8)$$

$$\begin{aligned}
s.t. \quad & \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} - u_0 \leq 0, \quad && j = 1,\dots,n \\
& \sum_{i=1}^{m} v_{ik} x_{ik} = 1 && \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - u_0 \geq 0, \quad (2.9) && j = 1,2,..,n \\
& u_{rk} \geq 0, && r = 1,\dots,s \\
& v_{ik} \geq 0, && i = 1,\dots,m \\
& u_0 \geq 0 &&
\end{aligned}$$

2.3 Cross-efficiency in the presence of negative data

In the conventional DEA models, each $DMU_j (j=1,\dots,n)$ is specified by a pair of non-negative input and output vectors $(x_j, y_j) \in R_+^{m+s}$, in which inputs $x_{ij} (i=1,\dots,m)$ are utilized to produce outputs, $y_{rj} (r=1,\dots,s)$. These models cannot be used for the case in which DMUs include both negative and positive inputs and/or outputs. Poltera et al. [15] consider a DEA model which can be applied in the cases where input/ output data take positive and negative values. Rang Directional Measure (RDM) model proposed by Poltera et al. [15] goes as follows:

$$\begin{aligned}
&\max\ \beta \\
&s.t.\ \sum_{j=1}^{n}\lambda_j x_{ij} \le x_{io} - \beta R_{io} \qquad i=1,\dots,m, \\
&\sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro} + \beta R_{ro} \qquad r=1,\dots,s, \\
&\sum_{j=1}^{n}\lambda_j = 1, \qquad (2.11) \\
&\lambda_j \ge 0 \qquad j=1,\dots,n.
\end{aligned}$$

Ideal point (I) in the presence of negative data, is $I = (\max_j\{y_{rj} : r=1,\dots,s\}, \min_j\{x_{ij} : i=1,\dots,m\}$ where

$R_{io} = x_{io} - \min_j\{x_{ij} : j=1,\dots,n\},\ \ i=1,\dots,m,$
$R_{ro} = \max_j\{y_{rj} : j=1,\dots,n\} - y_{ro},\ \ r=1,\dots,s.$

The other models solve negative data such as Modified slacks-based measure model (MSBM), semi-oriented radial measure (SORM) and etc.

In this section, we define cross-efficiency in the presence of negative data under variable returns to scale (VRS). In financial literature, mean – variance model and mean-variance-skewness are proposed based on Data Envelopment Analysis, which the variance of the portfolios is used as an input to the DEA and expected return and skewness are the output. Thus, we know that unlike return and skewness, variance is the only variable in the model that takes non-negative values. In this paper, new cross-efficiency is introduced based on RDM model.

Definition 2.1

$$\varphi_{jo} = 1 - \delta_{jo} = 1 - \frac{\sum_{i=1}^{m} v_{io}^{*} x_{ij} - \sum_{r=1}^{s} u_{ro}^{*} y_{rj} + u_0}{\sum_{i=1}^{m} v_{io}^{*} R_{ij} + \sum_{r=1}^{s} u_{ro}^{*} R_{rj}} \tag{2.13}$$

is referred to as the cross-efficiency of DMU_o to DMU_j by peer-evaluation, with negative data, where $j=1,\dots,n;\ j\ne o$.

In the above mentioned ratio, DMU_o is the DMU under evaluation and $u_{ro}^{*}(r=1,\dots,s)$ and $v_{io}^{*}(i=1,\dots,m)$ are the optimal solution to the below (2.12) model. Thus, $x_{ij}(1,\dots,m)$ and $y_{rj}(r=1,\dots,s)$ are the input and output values of $DMU_j\ (j=1,\dots,n)$

$$\begin{aligned}
\min\ & \sum_{i=1}^{m} v_i x_{io} - \sum_{r=1}^{s} u_r y_{ro} + u_0 \\
\text{s.t.}\ & \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} + u_0 \ge 0 \\
& \sum_{i=1}^{m} v_i R_{io} - \sum_{r=1}^{s} u_r R_{ro} = 1 \\
& v_i \ge 0\ \ u_r \ge 0\ \ i=1,\dots,m\ \ r=1,\dots,s
\end{aligned}$$

The above model is RDM model dual.
Model (2.14) can be stated as follows:

$$\begin{aligned}
\max\ & \sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io} - u_0 \\
\text{s.t.}\ & \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} + u_0 \ge 0 \\
& \sum_{i=1}^{m} v_i R_{io} - \sum_{r=1}^{s} u_r R_{ro} = 1 \\
& v_i \ge 0\ \ u_r \ge 0\ \ i=1,\dots,m\ \ r=1,\dots,s
\end{aligned}$$

In the above mentioned ratio (2.13) for DMU under evaluation, the nominator represents the efficiency (difference between virtual outputs and virtual inputs). In addition, the denominator designates normalize vector. It is clear that the value δ_{jo} is equal to 0 or less than 1 and is stated as inefficiency measure. Thus, $1-\delta_{jo}$ is equal to or less than 1 and is stated as efficiency measure. For another DMU, the below theorem should be approved.

Theorem1: 1: $0 \le \delta_{jo} \le 1$

Respecting to the first restriction of (2.14) model which is RDM model dual, we have: $0 \le \delta_{jo}$

We should approve $\delta_{jo} \le 1$:

Then, by above mentioned ratio (2.13), we have:

$$\sum_{i=1}^{m} v_{io}^{*} x_{ij} - \sum_{r=1}^{s} u_{ro}^{*} y_{rj} + u_0 \le \sum_{i=1}^{m} v_{io}^{*} R_{ij} + \sum_{r=1}^{s} u_{ro}^{*} y_{rj}$$

$$\sum_{i=1}^{m} v_{io}^{*} x_{ij} - \sum_{r=1}^{s} u_{ro}^{*} y_{rj} + u_0 \le$$
$$\sum_{i=1}^{m} v_{io}^{*} (x_{ij} - \min\{x_{ij}\}) + \sum_{r=1}^{s} u_{ro}^{*} (\max\{y_{rj}\} - y_{rj})$$

$$\sum_{i=1}^{m} v_{io}^{*} x_{ij} - \sum_{r=1}^{s} u_{ro}^{*} y_{rj} + u_0 \le$$
$$\sum_{i=1}^{m} v_{io}^{*} x_{ij} - \sum_{i=1}^{m} v_{io}^{*} \min\{x_{ij}\} + \sum_{r=1}^{s} u_{ro}^{*} \max\{y_{rj}\} - \sum_{r=1}^{s} u_{ro}^{*} y_{rj}$$

Then, we prove below equation:

$$u_0 \le \sum_{r=1}^{s} u_{ro}^{*} \max\{y_{rj}\} - \sum_{i=1}^{m} v_{io}^{*} \min\{x_{ij}\}$$

Also, respecting to restriction $\sum \lambda_j = 1$, there is $\lambda > 0$. Then, respecting to complementary slackness conditions, one constraint is binding. Then:

$$u_0 = \sum_{r=1}^{s} u_{ro}^{*} y_{ro} - \sum_{i=1}^{m} v_{io}^{*} x_{io}$$

We have:

$$\begin{cases} \sum_{r=1}^{s} u_{ro}^{*} y_{ro} \le \sum_{r=1}^{s} u_{ro}^{*} \max\{y_{rj}\} \\ \sum_{i=1}^{m} v_{io}^{*} x_{io} \ge \sum_{i=1}^{m} v_{io}^{*} \min\{x_{ij}\} \end{cases}$$

$$\begin{cases} \sum_{r=1}^{s} u_{ro}^{*} y_{ro} \le \sum_{r=1}^{s} u_{ro}^{*} \max\{y_{rj}\} \\ -\sum_{i=1}^{m} v_{io}^{*} x_{io} \le -\sum_{i=1}^{m} v_{io}^{*} \min\{x_{ij}\} \end{cases}$$

Then, we have:

$$u_0 = \sum_{r=1}^{s} u_{ro}^{*} y_{ro} - \sum_{i=1}^{m} v_{io}^{*} x_{io} \le$$
$$\sum_{r=1}^{s} u_{ro}^{*} \max\{y_{rj}\} - \sum_{i=1}^{m} v_{io}^{*} \min\{x_{ij}\}$$

So the prove is completed.

Figure 1 illustrates the concepts dealing with cross-efficiency and RDM efficiency, using the units A, B, C, D, E and F in the presence of negative data. In Figure 1, the RDM inefficiency of DMU_F can be evaluated by

$$\beta = \frac{d(P,F)}{d(I,F)}$$

Hence, the efficiency measure equals:

$$1-\beta = \frac{d(I,F) - d(P,F)}{d(I,F)} = \frac{d(I,P)}{d(I,F)}$$

Equation (2.17) represents the RDM efficiency of DMU_F which is between 0 and 1. $d(I,F)$ and $d(I,P)$ denote the distance from the ideal point I to F and the distance from the ideal point I to P, respectively, and $d(P,F)$ is the distance from P to F.

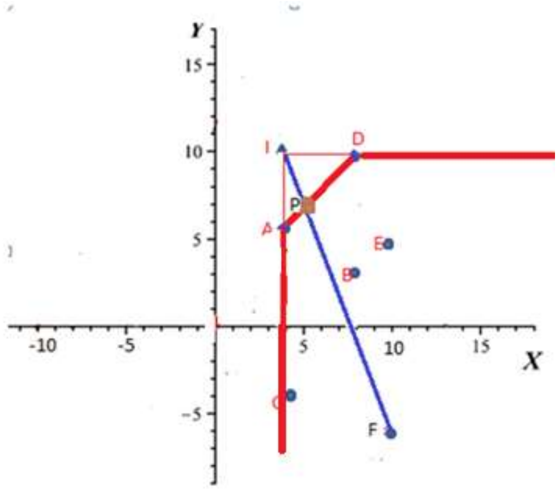

Figure1: RDM and BCC frontier in the presence of negative data

I is the ideal point. Using Figure 1, it is easy to see that the efficiency measure yielded by model RDM, $1-\beta$, is a distance measure between the observed and its target point P with reference to the ideal point.

2.4 OWA operators and their weight determination methods

An OWA operator of dimension n is a mapping $F:\Re^n \to \Re$ with an associated weight vector $W=(w_1,...,w_n)^T$ such that

$$w_1+...+w_n=1, \quad o\le w_i \le 1, \qquad i=1,...,n.$$

And

$$F(a_1,...,a_n)=\sum_{i=1}^{n} w_i b_i,$$

Where b_i is the i th largest of $a_1,...,a_n$.

OWA operators, introduced by yager [26], provide a unified framework for decision making under uncertainty, where different decision criteria such as maximax (optimistic), maximin (pessimistic), equally likely (Laplace) and Hurwicz criteria are characterized by different OWA operator weights.

For different weight selections, they are distinguished by the following orness degree [26]:

$$orness(W)=\frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i$$

The orness degree can be regarded as a measure of the optimism level of the DM.

To apply OWA operators for decision making, it is essential to determine the weights of OWA operators. The following models (2.20) and (2.21) are two important approaches for determining OWA operator weights under a given orness degree:

$$\max \; Disp(W)=\sum_{i=1}^{n} w_i \ln w_i,$$

$$\text{s.t.} \;\; orness(W)=\alpha=\frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i, \quad 0\le\alpha\le 1,$$

$$\sum_{i=1}^{n} w_i=1,$$

$$w_i\ge 0, \qquad\qquad i=1,...,n.$$

And

min δ

s.t.

$$
\begin{aligned}
&\text{orness}(\mathrm{W}) = \alpha = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)\,w_i, \quad 0 \le \alpha \le 1,\\
&\sum_{i=1}^{n} w_i = 1,\\
&w_i - w_{i+1} - \delta \le 0, \qquad i = 1,\dots,n-1, \qquad (2.21)\\
&w_i - w_{i+1} - \delta \ge 0, \qquad i = 1,\dots,n-1,\\
&w_i \ge 0, \qquad i = 1,\dots,n.
\end{aligned}
$$

Model (2.20), suggested by O'Hagan [14] maximizes the entropy of weight distribution and is thus referred to as the maximum entropy method, whereas model (2.21) that was proposed by Wang and Parkan [21] minimizes the maximum disparity between two adjacent weights and is thus called the minimax disparity approach.

The OWA operator weights determined by the above models have the following characteristics:

The weights are ordered. That is $w_1 \ge w_2 \ge \dots \ge w_n \ge 0$ if the orness degree $\alpha > 0.5$ and $0 \le w_1 \le w_2 \le \dots \le w_n$ if $\alpha \le 0.5$.

The weights have nothing to do with the magnitudes of the aggregates $b_1 \sim b_2$, but depend upon their ranking orders and the DM's optimism level (orness degree).

$w_1 = 1$ and $w_j = 0\ (j = 1)$ if $\alpha = 1$.Which means that the DM or investor is purely optimistic and considers only the biggest value $b_1 = \max_i(a_i)$ in decision analysis.

$w_n = 1$ and $w_j = 0\ (j \ne n)$ if $\alpha = 0$, which represents that the DM or investor is purely pessimistic and is only concerned with the most conservative value $b_n = \min_i(a_i)$ when making decision.

$w_1 = \dots = w_n = (1/n)$ if $\alpha = 0.5$, which stands for that the DM or investor is neutral and makes use of all the aggregates $b_1 \sim b_2$ equally in decision making.

$w_1,\dots,w_n$ determined by model (2.20) vary in the form of geometric progression, i.e. $w_{i+1}/w_i \equiv q$ for $i = 1,\dots,n-1$, where $q > 0$, while $w_1,\dots,w_n$ determined by model (2.21) vary in the form of arithmetical progression, namely, $w_i - w_{i+1} = d$ for $i = 1,\dots,K\ (K \le n)$ or $w_{i+1} - w_i = d$ for $i = K,\dots,n\ (K \ge 1)$ where $d > 0$.

3. Methodology

Return of assets; consist of money which we receive among period plus difference of buying and selling. Return is not definitely usually obvious. This uncertain in rate of expected return defined as deviation of return. Deviation of return is called risk. The investor's objective is to get the maximum possible return on an investment with the minimum possible risk. In this regard, mean-variance model Markowitz, expected return is treated as output and deviation as input. This model is due to the nature of the variance in quadratic form. Due to quadratic form investors prefer skewness which means that utility functions of investors are not quadratic. Thus, we know that unlike return and skewness, variance is the only variable in the model that takes non-negative values. The methodology in this paper starts with asset selection via cross-efficiency evaluation in presence of negative data using OWA operator weights. The data used for this methodology is from 20 Iranian stock companies. In many cases similar to this example there are a lot of assets. It is better that starts with asset selection. The choice of the asset can be random or discrete. The random choice of assets is usually biased and do not promise an optimum portfolio; hence it is more rational to have an objective choice while selecting the assets to be included in the portfolio. Among many evaluation methods, Data Envelopment Analysis (DEA) is one of the best ways for assessing the relative efficiency a group of homogenous decision making units (DMUs) that use multiple inputs to produce multiple outputs, originated from the work by charnes et al. [2]. Cross-efficiency evaluation is effective way of ranking decision making units (DMUs). It allows the overall efficiencies of the DMUs to be evaluated through self- and peer-evaluations. The self-evaluation allows the efficiencies of the DMUs to be evaluated with the most favorable weights so that each of them can achieve its best possible relative efficiency, whereas the peer-evaluation requires the efficiency of each DMU to be evaluated with the weights determined by the other DMUs. Table (3.1) shows cross-efficiency matrix for n DMUs with negative data. Traditional approaches for the cross-efficiency evaluation do not differentiate between self-evaluated and peer-evaluated efficiencies. A significant problem with these approaches is that the weight assigned to the self-evaluated efficiency of each DMU is fixed and has no way of incorporating the DM's or investor's subjective preferences in to the evaluation. For example, the investors may wish self-evaluated efficiencies to account for 20% or play a leading role in the final overall efficiency assessment. Obviously, equal evaluation has no method to obtain this purpose. To show the investor's subjective preferences on different efficiencies, the use of OWA operator weights is stated for cross-efficiency evaluation. This requires the re-ordering of the efficiencies, both self-evaluated and peer-evaluated, of each DMU, as shown in Table (3.2), where $w_1,\dots,w_n$ are OWA operator weights, $\vartheta_{ij}\ (i, j = 1,\dots,n)$ are re-ordered efficiencies of each DMU from the biggest to the smallest. Obviously, self-evaluated efficiencies are always ranked in the first place, i.e. $\vartheta_{i1} = \delta_{ii}^{*}\ i = 1,\dots,n$. In order to determine the weights of OWA operator, it is necessary for the investor to provide his/her preferences on different efficiencies or optimism level towards the best relative efficiencies. For example if the investor wants the self-evaluated efficiencies to account for 20% in the final overall efficiency assessment, then w_1 should take 0.2, whereas the other weights can be designated minimax disparity approach. The orness degree can be regarded as a measure of the optimism level of the investor. If the investor wants to self-evaluated to be more influenced, it should be used of orness> 0.5. And if investor wants to peer-evaluated to be more influenced, it should be used of orness< 0.5. Obviously, the best selection of stocks is not fixed. It varies with the investor's optimism level or subjective performance.

Table 3.1. cross-efficiency matrix for n DMUs with negative data

DMU	Target DMU				average crosses efficiency with negative data
	1	2	…	n	
1	φ_{11}	φ_{12}	…	φ_{1n}	$(\frac{1}{n})\sum_{k=1}^{n}\varphi_{1k}$
2	φ_{21}	φ_{22}	…	φ_{2n}	$(\frac{1}{n})\sum_{k=1}^{n}\varphi_{2k}$
⋮	⋮	⋮	…		⋮
n	φ_{n1}	φ_{n2}	…	φ_{nn}	$(\frac{1}{n})\sum_{k=1}^{n}\varphi_{nk}$

Table 3.2. Re-ordered cross-efficiency matrix of the n DMUs

DMU	Re-ordered efficiencies in descending order				Weighted average cross efficiency
	1st	2nd		nth	
	w_1	w_2	…	w_n	
1	ϑ_{11}	ϑ_{12}	…	ϑ_{1n}	$(\frac{1}{n})\sum_{k=1}^{n}\vartheta_{1k}$
2	ϑ_{21}	ϑ_{22}	…	ϑ_{2n}	$(\frac{1}{n})\sum_{k=1}^{n}\vartheta_{2k}$
⋮	⋮	⋮	…		⋮
n	ϑ_{n1}	ϑ_{n2}	…	ϑ_{nn}	$(\frac{1}{n})\sum_{k=1}^{n}\vartheta_{nk}$

4. Application in Stocks Company

We illustrate our approach in cross-efficiency evaluation in presence of negative data with OWA weight for a data set 20 Iranian stock companies. A list of stocks used is provided in Table (4.1). In this report, there is expected return and variance of stocks which expected return is considered as output and variance is as input. The example is received from Iranian stock companies and is about portfolio performance evaluation in a mean-variance framework. Three stocks are evaluated as efficient in model (2.11) as Mean-Variance RDM model (MV-RDM) which portfolio can be composed with them. But it is better to use cross-efficiency in presence of negative data to choose the best portfolio. Thus, we know that unlike return, variance is the only variable in the model that takes non-negative values.

Equation (2.13) is used for efficiency evaluation. In the analysis, the variance of the stocks is used as an input to the DEA and expected return is used as an output. Traditional approaches for the cross-efficiency evaluation do not differentiate between self-evaluated and peer-evaluated efficiencies. A main problem with these approaches is that the weight assigned to the self-evaluated efficiency of each DMU is fixed and has no way of incorporating the investor's subjective preferences in to the evaluation. Obviously, equal evaluation has no way to obtain this goal. To show the investor's subjective preferences on different efficiencies, the use of OWA operator weights is stated for cross-efficiency evaluation in Table (4.2). This requires the re-ordering of the efficiencies. The orness degree can be regarded as a measure of the optimism level of the investor. If the investor wants to self-evaluated to be more influenced, it should be used of orness> 0.5. And if investor wants to peer-evaluated to be more influenced, it should be used of orness< 0.5. In the traditional equal of cross-efficiencies, the weight assigned to the self-efficiencies, the weight assigned to the self-evaluated efficiencies is only $0.05\% = \left(\frac{1}{20}\right)$.

For an optimistic investor, he/she may wish the self-evaluated efficiencies to play a more role in the final overall efficiency assessment. For example, the investor may wish the weight for the self-evaluated efficiencies to account for 20% rather than 0.05% in the final overall efficiency assessment. As it seen in Tables (4.3), (4.4) ranks are not the same. We calculated these ranks for $\alpha = o.8$ and $\alpha = 0.2$. Some of the best ranks are designated according to investor. We consider six of the best ranks. Selecting of stocks to be included in portfolio is followed by six of the best ranks in Tables (4.5), (4.6) for $\alpha \geq 0.5$, $\alpha \leq 0.4$, respectively.

Table 4.1. Descriptive statistics of the Iranian stock companies

Stock companies	Expected return	Variance
VNVIN	7.285	6.534
VPARS	7.388	10.474
VBHMN	-2.193	3.720
VPASAR	10.853	4.256
DGABR	12.517	32.259
STRAN	9.052	70.764
FBAHNR	52.511	57.497
FMLI	-3.676	19.609
FVLAD	3.537	21.496
KCHINI	7.57	67.378
VTVSA	6.896	14.171
VLSAPA	1.888	29.002
VNFT	18.737	42.133
VTGART	1.302	12.419
VKHARZM	1.231	1.611
VSAKHT	14.741	11.429
KHSAPA	3.896	25.358
VSINA	2.967	4.856
RTKG	32.677	28.464
VBMLT	2.022	1.56

Table 4.2. OWA operator weights for cross efficiency evaluation

$\alpha=0.9$	$\alpha=0.8$	$\alpha=0.7$	$\alpha=0.6$	$\alpha=0.5$	$\alpha=0.4$	$\alpha=0.3$	$\alpha=0.2$	$\alpha=0.1$
0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.261	0.149	0.105	0.077	0.05	0.023	0	0	0
0.221	0.137	0.099	0.074	0.05	0.026	0	0	0
0.182	0.125	0.093	0.071	0.05	0.029	0.006	0	0
0.143	0.113	0.087	0.069	0.05	0.031	0.012	0	0
0.104	0.101	0.082	0.066	0.05	0.034	0.018	0	0
0.064	0.089	0.076	0.063	0.05	0.037	0.024	0	0
0.025	0.077	0.07	0.06	0.05	0.04	0.029	0	0
0	0.065	0.064	0.057	0.05	0.043	0.035	0.004	0
0	0.053	0.058	0.054	0.05	0.046	0.041	0.016	0
0	0.041	0.053	0.051	0.05	0.049	0.047	0.029	0
0	0.029	0.047	0.049	0.05	0.051	0.053	0.041	0
0	0.016	0.041	0.046	0.05	0.054	0.058	0.053	0
0	0.004	0.035	0.043	0.05	0.057	0.064	0.065	0
0	0	0.029	0.04	0.05	0.06	0.07	0.077	0.025
0	0	0.024	0.037	0.05	0.063	0.076	0.089	0.064
0	0	0.018	0.034	0.05	0.066	0.082	0.101	0.104
0	0	0.012	0.031	0.05	0.069	0.087	0.113	0.143
0	0	0.006	0.029	0.05	0.071	0.093	0.125	0.182
0	0	0	0.026	0.05	0.074	0.099	0.137	0.221
0	0	0	0.023	0.05	0.077	0.105	0.149	0.261

Table 4.3. Cross efficiency by optimism level of the investor for $\alpha \geq 0.5$

	Stock companies	MV RDM	Ranking RDM	$\alpha=0.9$	$\alpha=0.8$	$\alpha=0.7$	$\alpha=0.6$	$\alpha=0.5$	Ranking $\alpha=0.8$
1	VNVIN	0.89	7	0.812	0.837	0.850	0.856	0.860	5
2	VPARS	0.83	9	0.765	0.786	0.796	0.795	0.792	8
3	VBHMN	0.81	10	0.707	0.732	0.745	0.757	0.767	11
4	VPASAR	1	2	0.911	0.942	0.958	0.969	0.980	2
5	DGABR	0.65	14	0.622	0.630	0.632	0.615	0.597	13
6	STRAN	0.41	20	0.399	0.399	0.397	0.381	0.364	20
7	FBAHNR	1	3	0.916	0.892	0.879	0.778	0.728	4
8	FMLI	0.62	17	0.579	0.591	0.597	0.590	0.583	17
9	FVLAD	0.66	13	0.617	0.630	0.635	0.625	0.615	14
10	KCHINI	0.43	19	0.414	0.415	0.414	0.398	0.382	19
11	VTVSA	0.77	11	0.718	0.736	0.744	0.738	0.731	10
12	VLSAPA	0.59	18	0.552	0.562	0.565	0.553	0.541	18
13	VNFT	0.63	15	0.608	0.611	0.611	0.590	0.568	15
14	VTGART	0.72	12	0.667	0.686	0.694	0.692	0.689	12
15	VKHARZM	0.96	6	0.777	0.806	0.822	0.840	0.857	7
16	VSAKHT	0.96	5	0.876	0.898	0.908	0.902	0.895	3
17	KHSAPA	0.63	16	0.592	0.603	0.607	0.596	0.584	16
18	VSINA	0.83	8	0.756	0.783	0.797	0.806	0.815	9
19	RTKG	1	1	0.975	0.978	0.977	0.941	0.905	1
20	VBMLT	0.96	4	0.790	0.820	0.836	0.854	0.871	6

Table 4.4. Cross efficiency by OWA operator weights for $\alpha \le 0.4$

	Stock companies	MV RDM	Ranking RDM	$\alpha = 0.4$	$\alpha = 0.3$	$\alpha = 0.2$	$\alpha = 0.1$	Ranking $\alpha = 0.2$
1	VNVIN	0.89	7	0.864	0.868	0.863	0.852	4
2	VPARS	0.83	9	0.790	0.786	0.773	0.742	8
3	VBHMN	0.81	10	0.778	0.788	0.794	0.803	7
4	VPASAR	1	1	0.990	1.000	1.000	1.004	1
5	DGABR	0.65	14	0.579	0.561	0.530	0.462	16
6	STRAN	0.41	20	0.347	0.330	0.303	0.245	20
7	FBAHNR	1	3	0.728	0.676	0.606	0.453	12
8	FMLI	0.62	17	0.576	0.568	0.551	0.515	14
9	FVLAD	0.66	13	0.605	0.594	0.572	0.524	13
10	KCHINI	0.43	19	0.367	0.350	0.325	0.270	19
11	VTVSA	0.77	11	0.725	0.717	0.699	0.658	10
12	VLSAPA	0.59	18	0.529	0.516	0.493	0.442	17
13	VNFT	0.63	15	0.547	0.524	0.489	0.412	18
14	VTGART	0.72	12	0.686	0.682	0.669	0.640	11
15	VKHARZM	0.96	6	0.874	0.890	0.904	0.929	3
16	VSAKHT	0.96	5	0.888	0.880	0.858	0.811	5
17	KHSAPA	0.63	16	0.572	0.560	0.536	0.485	15
18	VSINA	0.83	8	0.824	0.832	0.834	0.835	6
19	RTKG	1	2	0.869	0.831	0.772	0.645	9
20	VBMLT	0.96	4	0.889	0.906	0.919	0.946	2

Table 4.5. Selecting the best assets for making portfolio for $\alpha \ge 0.5$

		Expected return	Variance	Ranking
19	RTKG	32.677	28.464	1
4	VPASAR	10.853	4.256	2
16	VSAKHT	14.741	11.429	3
7	FBAHNR	52.511	57.497	4
1	VNVIN	7.285	6.534	5
20	VBMLT	2.022	1.56	6

Table 4.6. Selecting the best assets for making portfolio for $\alpha \le 0.4$

		Expected return	Variance	Ranking
4	VNVIN	7.285	6.534	1
20	VBMLT	2.022	1.56	2
15	VKHARZM	1.231	1.611	3
1	VNVIN	7.285	6.534	4
16	VSAKHT	14.741	11.429	5
18	VSINA	2.967	4.856	6

5. Conclusion

In this paper, a new method is suggested for selecting the best of portfolio with one input (variance) and one output (expected return) in the DEA context in presence negative data. The cross-efficiency evaluation is an important method for ranking DMUs in DEA. Traditional approaches for the cross-efficiency evaluation do not differentiate between self-evaluated and peer-evaluated efficiencies. A main problem with these approaches is that the weight assigned to the self-evaluated efficiency of each DMU is fixed and has no way of incorporating the investor's subjective preferences in to the evaluation. To show the investor's subjective preferences on different efficiencies, the use of OWA operator weights is stated for cross-efficiency evaluation. In this case, if the investor wants to self-evaluated to be more influenced, it should be used of orness> 0.5. Thus, if investor wants to peer-evaluated to be more influenced, it should be used of orness< 0.5. In Tables 8,9 rankings have been designated for six of the best stocks via OWA operator weights in cross-efficiency evaluation in presence negative data. Since there are a large number of assets to invest in, this objective leads to two investment problems. First, the assets are selected for making portfolio and second, the proportion or weights are determined to be allocated to the selected assets. Selection of assets to be included in portfolio is followed by using cross-efficiency evaluation. Model (2.10) is used for this purpose. In this regard, this model is used to analyze the given 20 Iranian stock companies and six of the best stocks are obtained.

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