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Research Article

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Revise Approach to Measuring Congestion Based on the Comparison of Inputs

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Abstract

the method for measuring the congestion of Noura et al. [A.A. Noura, F. Hosseinzadeh Lotfi, G.R. Jahanshahloo, S. Fanati Rashidi, R.P. Barnett, A new method for measuring congestion in data envelopment analysis, Socio-Economic Planning Sciences, 44 (2010) 240-246] ,there is no problem for congestion detection In the case of one input and one output but in higher space is not able to detect congestion of some units. we offer modification of the method of measuring the congestion of Noura et al. The proposed method ability congestion units go up and this method detect all congestion units.

***Keywords*:** Data envelopment analysis; Congestion; Efficiency; Decision Making Unit.

1. Introduction

The focus of this paper is on the problem of congestion, which refers to a situation where the use of a particular input has increased by so much that output actually falls. Congestion can be viewed as an extreme form of technical inefficiency and, as such, can be regarded as a potentially serious practical problem. Fare and Svensson Proportional to the congestion by varying rule were defined and developed. [6]. Fare and Grosskopf to determine the functional role of input in the proposed

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congestion.[5,4]. Then Brocket et al [2] and cooper et al [3] Have developed a new method based on DEA to obtain input congestion. Other methods proposed by other scientists to evaluate congestion but In this paper, we pay to a method that is presented by Noura et al.
the method for measuring the congestion of Noura et al and his colleagues studied And we find that this approach ,there is no problem for congestion detection In the case of one input and one output but in higher space is not able to detect congestion of some units. we offer modification of the method of measuring the congestion of Noura et al. The proposed method ability congestion units go up and this method detect all congestion units.
Sections of this paper is as follows: In section 2 we expressed modification of the method of measuring the congestion of Noura et al. In section 3 By providing examples We describe the proposed method And compare the results with other methods of measuring congestion. And the end results are expressed.

2. Revise approach

Noura et al. [٧] have proposed a method for measuring the congestion at which they use. Comparing the measured densities of the inputs.
We found that this approach, there is no problem for congestion detection In the case of one input and one output but in higher space is not able to detect congestion of some units.[8]

We propose the following method for determining the congestion.
Suppose we have n DMUs with m inputs and s outputs, and that the vectors $x_j = (x_{1j}, \dots, x_{mj})^T$ and $y_j = (y_{1j}, \dots, y_{sj})^T$ denote the input and output values of $DMU_j (j = 1, \dots, n)$ respectively. First, we solve the output-oriented BCC (Banker, Charnes, Cooper) model (1), which assumes variable returns to scale (VRS), in order to obtain the efficiency of each DMU.

$$\emptyset_o^* = \text{Max}\ \emptyset_o + \varepsilon\left(\sum_{r=1}^{s} s_{ro}^+ + \sum_{i=1}^{m} s_{io}^-\right)$$

s.t.

$$\begin{aligned}
&\sum_{j=1}^{n} x_{ij}\lambda_j + s_{io}^- = x_{io}, \quad i = 1, \dots, m \\
&\sum_{j=1}^{n} y_{rj}\lambda_j + s_{ro}^+ = \emptyset_o y_{ro}, \quad r = 1, \dots, s \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&(\lambda_j, s_{ro}^+, s_{io}^-) \geq 0, \quad j = 1, \dots, n, \quad r = 1, \dots s, \quad i = 1, \dots, m
\end{aligned} \tag{1}$$

In (1), $\varepsilon > 0$ is a Non-Archimeadean element smaller than any positive real number. we solve Model (1), above, for each $DMU_j (j = 1, \dots, n)$ and obtain the optimal solution: $(\emptyset^*, \lambda^*, s^{+^*}, s^{-^*})$.
Denoting the $\emptyset^*$ corresponding to DMU_j by $\emptyset_j^*$ we define set E as follows:
E=$\{j | \emptyset_j^* = 1, \sum_{r=1}^{s} s_{rj}^* = 0\}$

As the congestion measurement method based on the comparison of inputs, Highest value in each entry for each component are calculated. We show Such input with $x_i^*(i = 1, \dots, m)$.

Step one: A) if $\emptyset_o^* > 1$ (or$\sum_{r=1}^{s} s_{ro}^* > 0$) and $x_{io} > x_i^*$, DMU_o is congestion. and the congestion of the input (i) against is $s_i^{ć} = x_{io} - x_i^*$.

B) if $\emptyset_o^* > 1$ (or $\sum_{r=1}^{s} s_{ro}^* > 0$) and $x_{io} = x_i^* = x_{io} - s_{io}^{-*}$, DMU_o on input i is not congestion.

C) if $\emptyset_o^* > 1$ (or $\sum_{r=1}^{s} s_{ro}^* > 0$) and $x_{io} \le x_i^* \neq x_{io} - s_{io}^{-*}$, go to the second step.

Step two: find:

$x_{ij} \le x_{it} < x_i^*,\ x_{io} - s_{io}^{-*} \le x_{it}\ \forall j(j, t \in E)$ (2)

If there was not in equations (2) applies, DMU_o on input i is not congestion.put: $x_{io} - s_{io}^{-*} = x_{it}$.

Put $x_{it} = x_i^*$ and go to the first step.

Total $s_i^{ć}$ is the congestion value of DMU_o.

The proposed method for every $DMU_o(o \notin E)$ One $DMU_o^* = (x_{1o}^*, \dots, x_{mo}^*, \emptyset^* y_{1o} + s_{1o}^{+*}, \dots, \emptyset^* y_{so} + s_{so}^{+*})$ Determined.

Theorem 1. If $DMU_o^* = (x_{1o}^*, \dots, x_{mo}^*, \emptyset^* y_{1o} + s_{1o}^{+*}, \dots, \emptyset^* y_{so} + s_{so}^{+*})$ Then $DMU_o^* \in PPS_{TV}$.

Proof. The method of determining x_{io}^* And that s_{io}^{-*} The maximum amount of assistance, we have:

$x_{io}^* \ge x_{io} - s_{io}^{-*}$

Further, since $(x_o - s_o^{-*}, \emptyset^* y_o + s_o^{+*}) \in PPS_{TV}$ and, given the production possibility principle [1], Output can be generated by input greater than $x_o - s_o^{-*}$, so

$DMU_o^* = (x_{1o}^*, \dots, x_{mo}^*, \emptyset^* y_{1o} + s_{1o}^{+*}, \dots, \emptyset^* y_{so} + s_{so}^{+*}) \in PPS_{TV}$.

Consider the following model:

$$\beta_o^* = \text{Max}\ \beta_o$$
s.t.
$$\sum_{j=1}^{n} x_{ij}\lambda_j = x_{io}, \quad i = 1, \dots, m$$
$$\sum_{j=1}^{n} y_{rj}\lambda_j - \beta_o y_{ro,} \ge 0 \quad r = 1, \dots, s \qquad (2)$$
$$\sum_{j=1}^{n} \lambda_j = 1$$
$$(\lambda_j, s_{ro}^{+}, s_{io}^{-}) \ge 0, \quad j = 1, \dots, n, \quad r = 1, \dots s, \quad i = 1, \dots, m.$$

Theorem 2: if $x_{io} > x_{io}^*$,then $\emptyset_o^* > \beta_o^*$ and FGL method based on the strong and weak access DMU_o is congestion.

Proof. Note that the first inequality constraint in model (1) with equality in the model (2) is replaced. Therefore, the auxiliary input is not possible. So we have: $\emptyset_o^* \ge \beta_o^*$ Suppose $x_{io} > x_{io}^*$ as $x_{io}^* \ge x_{io} - s_{io}^{-*}$ so $x_{io} > x_{io} - s_{io}^{-*}$so $\emptyset_o^* > \beta_o^*$ and FGL method based on the strong and weak access DMU_o is congestion.

3. Numerical Examples

Example 1

Six decisions With two inputs and one output is shown in Figure 1. The output units A,B,C,D,G Is equal to one And the output value of R is equal to 10.

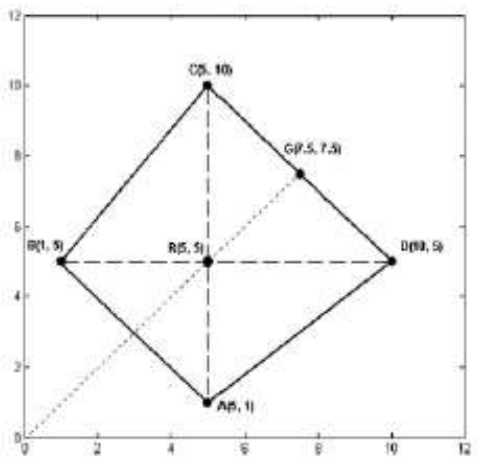

Figure 1

The measurement method is based on comparing the input congestion (Noura et al.) E={A,B,R} And congestion is calculated as follows:$\emptyset_C^*, \emptyset_D^*, \emptyset_G^* > 1$

$$\begin{cases} \boldsymbol{x_C = (5,10), x_C^* = (5,5)} \\ \boldsymbol{s_1^{\acute{c}} = 0, s_2^{\acute{c}} = 5 \rightarrow s_C^{\acute{c}} = 5} \\ \boldsymbol{x_D = (10,5), x_D^* = (5,5)} \\ \boldsymbol{s_1^{\acute{c}} = 5, s_2^{\acute{c}} = 0 \rightarrow s_D^{\acute{c}} = 5} \\ \boldsymbol{x_G = (7.5,7.5), x_G^* = (5,5)} \\ \boldsymbol{s_1^{\acute{c}} = 2.5, s_2^{\acute{c}} = 2.5 \rightarrow s_G^{\acute{c}} = 5} \end{cases}$$

So by Noura et al, the DMU G, D, C are subject to condensation.

In this example we solve the revise approach. (Proposed in Section 2.)

With this method E={A,B,R}. we have:

$\emptyset_C^*, \emptyset_D^*, \emptyset_G^* > 1$,

$$\begin{cases} \boldsymbol{x_C} = (5,10), \boldsymbol{x_C^*} = (5,5) \\ s_1^{\acute{c}} = 0, s_2^{\acute{c}} = 5 \rightarrow s_C^{\acute{c}} = 5 \\ \boldsymbol{x_D} = (10,5), \boldsymbol{x_D^*} = (5,5) \\ s_1^{\acute{c}} = 5, s_2^{\acute{c}} = 0 \rightarrow s_D^{\acute{c}} = 5 \\ \boldsymbol{x_G} = (7.5,7.5), \boldsymbol{x_G^*} = (5,5) \\ s_1^{\acute{c}} = 2.5, s_2^{\acute{c}} = 2.5 \rightarrow s_G^{\acute{c}} = 5 \end{cases}$$

The new method DMUs C,D,G are subject to condensation. As can be seen, for example, the results of both methods are equal.

Example 2

Consider the data in Table 1.

Table 1

DMU	Input 1	Input 2	Output 1	Output 2
A	1	1	1	1
B	2	2	2	2
C	2	3	2	1
D	3	3	1	1

According to conducted Noura and colleagues E={A,B,C} and $\boldsymbol{x}^* = (2,3) \geq \boldsymbol{x_j} \ \forall j (j \in E)$

We have: $\emptyset_D^* > 1, \boldsymbol{x_D} = (3,3) \rightarrow s_D^{\acute{c}} = 1$.

According to conducted Noura and colleagues Unit D is only subjected to compression.

The new method ,E={A,B} and we have:

$\emptyset_C^*, \emptyset_D^* > 1$,

$$\begin{cases} \boldsymbol{x_C} = (2,3), \boldsymbol{x_C^*} = (2,2) \\ s_1^{\acute{c}} = 0, s_2^{\acute{c}} = 1 \rightarrow s_C^{\acute{c}} = 1 \\ \boldsymbol{x_D} = (3,3), \boldsymbol{x_D^*} = (2,2) \\ s_1^{\acute{c}} = 1, s_2^{\acute{c}} = 1 \rightarrow s_D^{\acute{c}} = 2 \end{cases}$$

So with revise approach DMUs C,D are densities.

The results of the methods Tone and Sahoo (10) Sueyoshi (9) Noura et al. and revise approach for Example 2 is shown in Table 2.

Table 2

DMU	Noura et al	Tone and Sahoo	Sueyoshi	revise approach
A	Not congestion	Not congestion	Not congestion	Not congestion
B	Not congestion	Not congestion	Not congestion	Not congestion
C	Not congestion	congestion	Wide congestion	congestion
D	congestion	congestion	Wide congestion	congestion

As can be seen in the methods Tone and Sahoo, Sueyoshi and revise approach units C,D are densities. Method Noura et al. only Unit D has detect congestion and unit C does not detect the congestion.

Example 3

Consider the data in Table 3.

Table 3

DMU	Input 1	Input 2	Output 1	Output 2	Output 3	Output 4
A	2	2	2	2	2	2
B	2	2	2	3	2	2
C	2	2	2	2	3	2
D	2	2	2	2	2	3
H	1	1	2	2.5	3	2
F	1	3	2	2	2	4
G	2	1	2	2.5	2.25	3

based on the method of Noura et al E={A,B,C,D,H,F,G} and

$\boldsymbol{x}^* = (2,3) \geq \boldsymbol{x}_{\boldsymbol{j}} \ \forall j (j \in E)$

With this method, none of the units have condensation.

These are examples of revise approach to solve ,with this method E={B,H,F,G}. we have:

$$\sum_{r=1}^{2} s_{rA}^{+^*}, \sum_{r=1}^{2} s_{rC}^{+^*}, \sum_{r=1}^{2} s_{rD}^{+^*} > 0,$$

$$\begin{cases} \boldsymbol{x}_{\boldsymbol{A}} = (2,2), \boldsymbol{x}_{\boldsymbol{A}}^* = (1,1) s_1^{\acute{c}} = 1 \\ \quad , s_2^{\acute{c}} = 1 \rightarrow s_A^{\acute{c}} = 2 \\ \boldsymbol{x}_{\boldsymbol{C}} = (2,2), \boldsymbol{x}_{\boldsymbol{C}}^* = (1,1) s_1^{\acute{c}} = 1 \\ \quad , s_2^{\acute{c}} = 1 \rightarrow s_C^{\acute{c}} = 2 \\ \boldsymbol{x}_{\boldsymbol{D}} = (2,2), \boldsymbol{x}_{\boldsymbol{D}}^* = (2,1) s_1^{\acute{c}} = 0 \\ \quad , s_2^{\acute{c}} = 1 \rightarrow s_C^{\acute{c}} = 1 \end{cases}$$

so with the proposed method DMUs A,C,D are densities.

The results of the methods Cooper et al., Tone and sahoo, Sueyoshi, Noura et al. and revise approach for Example 3 is shown in Table 4.

Table 4

DMU	Cooper et al	Tone and sahoo	Sueyoshi	Noura et al	revise approach
A	Not congestion	Not congestion	Wide congestion	Not congestion	congestion
B	Not congestion	Not congestion	Not congestion	Not congestion	Not congestion
C	Not congestion	congestion	Wide congestion	Not congestion	congestion
D	Not congestion	congestion	Wide congestion	Not congestion	congestion
H	Not congestion	Not congestion	Not congestion	Not congestion	Not congestion
F	Not congestion	Not congestion	Not congestion	Not congestion	Not congestion
G	Not congestion	Not congestion	Not congestion	Not congestion	Not congestion

With Noura et al. and Cooper et al. method No one does compression. Tone and sahoo method has multiple optimal solutions.

Congestion determination in tone and sahoo method depend on which option to be choosen. with Sueyoshi method and revise approach units A,C,D are subject to condensation. Congestion determination in Sueyoshi method depend on real number of σ.

With revise approach units A,C,D are subject to condensation. Therefore, the proposed method can detect all congestion units.

5. Conclusions

In this paper, the method for measuring the congestion of Noura and his colleagues studied And we find that this approach ,there is no problem for congestion detection In the case of one input and one output but in higher space is not able to detect congestion of some units. Using the congestion of units that are modified to be able to identify. Revise approach don׳t have other methods difficulties and is a perfect method to determine congestion.

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