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Int. J. Data Envelopment Analysis (ISSN 2345-458X)

Vol.3, No.1, Year 2015 Article ID IJDEA-00311, 18 pages

Research Article

International Journal of Data Envelopment Analysis

Science and Research Branch (IAU)

The Most Revenue Efficiency with Price Uncertainty

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Received 26 July 2014, Revised 11 October 2014, Accepted 17 January 2015

Abstract

In this paper, a new revenue efficiency data envelopment analysis (RE-DEA) approach is considered for finding the most revenue efficient unit with price uncertainty in both optimistic and pessimistic perspectives. The optimistic and pessimistic perspectives use efficient frontier and inefficient frontier, respectively. An integrated model is introduced to find decision making units (DMUs) that can be a candidate for most revenue efficient unit, in both optimistic and pessimistic points. Consequently, the revenue efficiency of all DMUs is calculated with by solving one model. Then a mix integer programming (MIP) model is proposed for finding the most revenue efficient DMU with common set of weights. The proposed model ensures that just one unit has been revenue efficiency. To illustrate the applicability of the new approach, the model is utilized for data from 21 medical centers in Taiwan.

Keywords: Revenue Efficiency, Price Uncertainty, Decision Making Unit, Common Set of Weights.

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1. Introduction

Data envelopment analysis (DEA) is a non-parametric method that utilized linear programming (LP) techniques to empirically obtain the best production (efficient) frontier and evaluates the efficiencies of a set of similar organizations. In DEA models, efficiency is measured as the weighted sum of the outputs divided by the weighted sum of the inputs. Farrell [14] initially introduced a non-parametric approach to measure the efficiency of the firms, instead of estimating the conventional production functions. However, this study was limited to single input and single output. Charnes et al.[5] extended the Farrell view and presented a model that could measure efficiency of DMUs with multiple inputs and multiple outputs. This model was named CCR. In CCR model, if the DMU is inefficient, we use the image of the model on the efficient frontier. In this case the input size shrunk or output size expands that placed on the efficient frontier that is known in input oriented and output oriented, respectively. After Charnes et al. [5], in 1984, Banker et al. [3] were present the BCC model with changes in the CCR model. Whereas the CCR model, assumes constant returns to scale, the BCC model assumes variable returns to scale. In 1996, Cooper and Thompson [7] proposed new models for dealing technical inefficiency. In 1957, Farrell [14] introduced his concern about precise of prices to be used in cost efficiency. After Farrell assessment controversial and tables incorrect prices caused people like Charnes and Cooper [5] emphasized importance of technical efficiency measurement and necessary of uses them. But Farrell efforts were considered after controversy arose and content expressed in his article became the basic theory of measuring cost efficiency and constant prices for each decision of unit. In introduced model, the price of one unit to another unit could be distinguished. In his paper component of cost efficiency decomposed to technical efficiency and allocative efficiency and this useful and essential decomposition was known Farrell decomposition. Although Farrell model originally proposed to measure cost efficiency, it was a possibility with minor changes to become a revenue efficiency model and this was the benefit of his work. Since data envelopment analysis was proposed, this subject was widely used by agencies and organization to measure efficiency, determine the cost efficiency and revenue efficiency of the decision making unit. For the first time in 1985, Fare et al. [13] developed procedures for the empirical implementations of the CE and RE measures in DEA. Since then the aspect of measuring cost and revenue efficiencies have been explored in many studies. Sueyoshi [31] provided a theoretical framework related to DEA in which an analytical relationship among eight different efficiency concepts is defined and explored in terms of production and cost analysis. Puig-Junoy [25] studied and empirical analysis of the best production and cost frontiers of a sample of 94 acute care hospitals by DEA and regression model in two-stage approach. Tone[37], in 2002, pointed out the shortcomings of the cost and allocative efficiency as used in DEA literature and proposed a new approach to the cost efficiency evaluation. Ertay et al. [11] offered a min-max method consists of a parameter that should be selected for a trial and error method to reach the most efficient DMU. Jahanshahloo et al.[19] proposed a simplified version of the DEA cost efficiency model and decreased number of model's covariant as well as its variables. In 2010 Kuosmanen et al. [20] offered firm and industry level profit efficiency analysis using absolute and uniform shadow prices. Mozaffari et al. [24] proposed a new model for measuring cost and revenue efficiency. Fang and Li [12] developed a pair of two level mathematical programming models to calculate the upper and lower bounds of cost efficiency for each firm in the case of non-unique law of one price while keeping the industry's cost efficiency optimal. In the real-world economic instability, especially up and down world prices makes the detailed information about the input and output prices not be available. In the other words, determining the exact number of prices is not possible and so the prices are as uncertain. Uncertain prices can be fuzzy numbers, interval and so. Cooper et al. [8] considered new models for dealing with imprecise data in DEA. They transformed $IDEA$ models in two ordinary linear programming forms. They proposed the term of $IDEA$ in their studies for the first time. Kuasmanen and Posts [21] offered a new DEA model for computing upper and lower bounds for Farrell's CE measure. Despotis and Smirlis [9] developed an alternative approach for dealing with imprecise data in DEA. Zho [40] provided new constraints to CCR model corresponding to the imprecise data and then obtained a non-linear model to evaluated DMUs by uncertain data. Camanho and Dyson [4] considered pessimistic and optimistic approaches to dealing with CE with price uncertainty. Toloo et al. [33] proposed a new method for measuring overall profit efficiency with interval data. Mostafaee and Saljooghi [23] developed a method for the estimation of upper and lower bounds to the cost efficiency in situations of uncertain input and output. Emrouznejad et al. [10] proposed a new method for calculating an overall profit Malmquist productivity index with fuzzy and interval data. Rostamy-malkhalife and Aghayi [27] presented a new method for computing the efficiency of $DMUs$ with fuzzy data. Sahoo et al. [29] developed new models to evaluated cost and revenue efficiencies based on the directional measures of value-based which all satisfy several desirable properties of an ideal efficiency measure. The obtained model can be used for negative data.

Cook et al. [6] first utilized the idea of common set of weights to measure the relative efficiency of highway maintenance patrols and Roll et al. [26] extended. The common set of weights concepts help us to identify the efficient $DMUs$ in an identical condition. Hosseinzadeh Lotfi et al. [17] considered a new $MOLP$ model to determine the common set of weights for all DMUs. Saati et al. [28] proposed a common set of weight approach in two stages by the ideal DMU in DEA. Hosseinzadeh Lotfi et al. [15] offered an allocation mechanism that is based on a common dual weights approach. Hosseinzadeh Lotfi et al. [16] proposed a new model for centralized resource reduction and target setting by DEA approaches.

Many of the units are known to be efficient in the evaluation with DEA approaches. That's why it was considered necessary ranking $DMUs$ and researchers was trying to ranking $DMUs$ to better assess their performance. In 1986, Sexton et al. [30] proposed cross-efficient method for ranking $DMUs$. First, they calculated cross-efficiency score of all $DMUs$ and then ranked $DMUs$ with their scores. Anderson and Peterson [2] presented a method based on super efficiency scores. The super efficiency score is more than unity for the extremely efficient $DMUs$ and is equal to unity for the non-extremely efficient DMUs. Jahanshahloo et al. [18] proposed a new method for ranking $DMUs$ by L_1-norm with fuzzy data. Wang et al. [38] offered a new MRA method for comparing and ranking $DMUs$. Several studies identify the most efficient unit by ranking them have been proposed. In most DEA models when the aim is to calculate the most efficient unit, first calculate the score of the efficiency of the proposed method, and then rank the DMUs using existing methods. Therefore the best DMU selected. This will prolong the calculations. So if an issue, the aim is to calculate the most efficient unit, it is better to use from the ways that calculate most efficient model by solving just one model. Amin and Toloo [1] formulated a new integrated DEA model for finding the best CCR efficient DMUs. However Toloo and Nalchigar [35] extended it to variable returns to scale situation. Also, Toloo and Nalchigar [36] suggested a DEA approach for supplier selection in the presence of both cardinal and ordinal data. Toloo [32] expressed some drawbacks of previous studies and considered a new $MIP-DEA$ model to obtain the best BCC-efficient unit. Toloo and Ertay [34] formulated an integrated model for finding the most cost efficient automotive vendors with price uncertainty.

In this paper, a model is presented for measuring the most revenue efficiency, in both optimistic and pessimistic perspectives, when the output price vectors can be have the lower and upper bounds. The introduced model is an integrated model where finds DMU that could be a candidate for most revenue efficient DMU, in both optimistic and pessimistic cases. In this method, revenue efficiency of all $DMUs$ can be calculated by solving a model. It is also uses of a common set of weight for inputs and outputs. As well, according to that, it may exist more than one revenue efficient DMU, a mixed integer programming model is proposed using a common set of weight concept. So it introduces a DMU as most revenue efficient unit in both optimistic and pessimistic offered. Note that we use the efficient and inefficient frontier to the optimistic and pessimistic cases, respectively.

The paper proceeds as follows: In section 2, offered efficiency and inefficiency frontier. A model for measuring the revenue efficiency of DMUs in both optimistic and pessimistic cases is given in section 3. Finally, to illustrate the application of the proposed model, in section 4, the numerical example provided and then conclusions and recommendations for the future research are given.

2- Efficiency and Inefficiency Frontier

DEA is a method for estimating efficient frontier of the production possibility set (PPS). Since it is very difficult to obtain precise production function. So Farrell [16] contrast the PPS by using observations and rules, named its frontier; production function. The PPS in introduced as follows:

$T = \{(x, y) | x \text{ can be produce } y\}$

This section proposes two PPS in both optimistic and pessimistic cases. Then, the CCR model and its integrated model are given to estimate the DMUs in the output oriented in each optimistic and pessimistic cases.

2-1. Production Possibility Set in Optimistic Case using Efficient Frontier:

Consider the following properties for the PPS

1. The observed activities belong to PPS: For all $j \in \{1, 2, ...n\}$ we have $(x_j, y_j) \in PPS$
2. Constants returns to scale: For all $(x, y) \in PPS$ and for all $\lambda \geq 0$ we have: $(\lambda x, \lambda y) \in PPS$
3. Possibility principle: If $(x, y) \in PPS$, $\bar{x} \geq x$ and $\bar{y} \leq y$ then: $(\bar{x}, \bar{y}) \in PPS$
4. Convexity principle: If $(x, y) \in PPS$, $(\bar{x}, \bar{y}) \in PPS$ and $\lambda \in (0,1)$ then: $(\lambda x + (1-\lambda)\bar{x}, \lambda y + (1-\lambda)\bar{y}) \in PPS$
5. Minimum extrapolated principle: T is the intersection set of all sets satisfying postulates, 1,2,3,4.

According to the above principles, the non-empty PPS is defined as follows:

$$T_C = \left\{ (x,y) \middle| x \geq \sum_{j=1}^{n} \lambda_j x_j, y \leq \sum_{j=1}^{n} \lambda_j y_j, \lambda_j \geq 0, \forall j \right\}$$

Suppose that we have n $DMUs$ each consuming various amounts of m inputs to produce s outputs. Let X_j and Y_j be the input and output vectors, respectively. The following model can be utilized for calculating efficiency score of $DMUs$:

$$\begin{aligned} & Max \quad \varphi && (1) \\ & \text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io}, && \text{i=1,2,...,m,} \\ & \quad \sum_{j=1}^{n} \lambda_j y_{rj} \geq \varphi y_{ro}, && \text{r=1,2,...,s,} \\ & \quad \lambda_j \geq 0, && \text{j=1,2,...,n.} \end{aligned}$$

Suppose (φ^*, λ^*) is an optimal solution of model (1), DMU_o is efficient if and only if the optimal objective value is 1. Model (1) uses to image the unit under evaluation unit on the efficient frontier. In model (1), the aim is finding the virtual DMU that minimum inputs produce the maximum outputs.

The dual of model (1) is as follows:

$$\begin{aligned} & Min \quad \sum_{i=1}^{m} v_i x_{io} && (2) \\ & s.t. \quad \sum_{r=1}^{s} u_r y_{ro} = 1, \\ & \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, && \text{j=1,2,...,n,} \\ & \quad \mathrm{u}_r \geq 0, && \text{r=1,2,...,s,} \\ & \quad \mathrm{v}_i \geq 0, && \text{i=1,2,...,m.} \end{aligned}$$

Where y_{rj} is the amount of rth output for DMU_j ; x_{ij} the amount of ith input for DMU_j; u_r the weight of rth output; v_i the weight of ith input; n the number of $DMUs$; m the number of inputs; s the number of outputs; o the index of under evaluation DMU . If (v^*, u^*) is the optimal solution of model (2), then DMU_o is efficient if and only if the optimal object value is 1, i.e., $\varphi^* = \sum_{i=1}^{m} v_i^* x_{io} = 1$ When we consider constraints $\mathrm{u}_r \geq 0$ and $\mathrm{v}_i \geq 0$ so model (2) may not be the optimal solution. To resolve this problem, above constraints are amended as follows:

$$\begin{aligned} & Min \quad \sum_{i=1}^{m} v_i x_{io} && (3) \\ & s.t. \quad \sum_{r=1}^{s} u_r y_{ro} = 1, \\ & \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, && \text{j=1,2,...,n,} \\ & \quad \mathrm{u}_r \geq \varepsilon^*, && \text{r=1,2,...,s,} \\ & \quad \mathrm{v}_i \geq \varepsilon^*, && \text{i=1,2,...,m.} \end{aligned}$$

Where ε^* is the non-Archimedean infinitesimal.

Firstly, Charnes et al. [5] proposed ε^* to obtain the optimal value of weights (see [22]). Models (2) and (2) run n times in order to evaluate n DMUs. Therefore, we introduce the following integrated DEA model that minimizes the sum of deviation of all $DMUs$ from the efficiency frontier.

$$
\begin{aligned}
& Min \quad \sum_{j=1}^{n} d_j && (4) \\
& \text{s.t.} \quad \sum_{r=1}^{s} u_r y_{rj} \le 1, && j=1,2,...,n, \\
& \qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, && j=1,2,...,n, \\
& \qquad v_i \ge \varepsilon^*, && i=1,2,...,m, \\
& \qquad u_r \ge \varepsilon^*, && r=1,2,...,s, \\
& \qquad d_j \ge 0 && j=1,2,...,n.
\end{aligned}
$$

Where v and u are the, CSW for inputs and outputs, respectively.

Since, the constraint $\sum_{r=1}^{s} u_r y_{rj} = 1,$ increases the number of additional variables in the model. Then we can consider the constraint $\sum_{r=1}^{s} u_r y_{rj} \le 1$.

DMU_o is efficient if and only if we have $d_o^* = 0$ in model (4).

2-2- Production Possibility Set in Pessimistic Case using the Inefficient Frontier:

Consider the following properties for the PPS

1. The observed activities belong to PPS: for all $j \in \{1,2,...n\}$ we have $(x_j, y_j) \in PPS$

2. Constant returns to scale: for all $(x, y) \in PPS$ and for all $\lambda \ge 0$ we have: $(\lambda x, \lambda y) \in PPS$

3. Possibility principle: if $(x, y) \in PPS$, $\bar{x} \le x$ and $\bar{y} \ge y$ then: $(\bar{x}, \bar{y}) \in PPS$

4. Convexity principle: if $(x, y) \in PPS$, $(\bar{x}, \bar{y}) \in PPS$ and $\lambda \in (0,1)$ then: $(\lambda x + (1-\lambda)\bar{x}, \lambda y + (1-\lambda)\bar{y}) \in PPS$

5. Minimum extrapolation principle: T' is the intersection set of all sets satisfying principles, 1,2,3,4.

According to the above principles, the non-empty PPS is defined as follows:

$$
T'_C = \left\{ (x, y) \,\middle|\, x \le \sum_{j=1}^{n} \lambda_j x_j,\, y \ge \sum_{j=1}^{n} \lambda_j y_j,\, \lambda_j \ge 0, \forall j \right\}
$$

T'_C is the inefficient frontier.

Suppose that we have $n\ DMUs$ each consuming various amounts of m inputs to produce s outputs. Let X_j and Y_j be the input and output vectors, respectively. The following model can be utilized for calculating efficiency score of $DMUs$:

$$
\begin{aligned}
& Min \quad \varphi && (5) \\
& \text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \ge x_{io}, && i=1,2,...,m, \\
& \qquad \sum_{j=1}^{n} \lambda_j y_{rj} \le \varphi y_{ro}, && r=1,2,...,s, \\
& \qquad \lambda_j \ge 0, && j=1,2,...,n.
\end{aligned}
$$

Suppose $\left(\varphi^*, \lambda^*\right)$ is an optimal solution of model (5), DMU_o lies on the inefficient frontier, if and only if the optimal objective value is 1. Model (5) uses to image the under evaluation unit to the inefficient frontier. In model (5), the aim is finding the virtual DMU that maximum inputs produce the minimum outputs.

The dual of model (5) is as follows:

$$
\begin{aligned}
Max \quad & \sum_{i=1}^{m} v_i x_{io} && && (6)\\
s.t. \quad & \sum_{r=1}^{s} u_r y_{ro} = 1, \\
& \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \le 0, && j=1,2,...,n, \\
& u_r \ge 0, && r=1,2,...,s, \\
& v_i \ge 0, && i=1,2,...,m.
\end{aligned}
$$

Where y_{rj} is the amount of rth output of DMU_j ; x_{ij} the amount of ith input of DMU_j; u_r the weight of rth output; v_i the weight of ith input; n the number of $DMUs$; m the number of inputs; s the number of outputs; o the index of under evaluation DMU . Let $\left(v^*, u^*\right)$ is the optimal solution of model (6), then DMU_o lies on the inefficient frontier, if and only if the optimal objective value is 1. When we consider constraints $u_r \ge 0$ and $v_i \ge 0$ so model (6) may not be the optimal solution. To resolve this problem, above constraints are amended as follows:

$$
\begin{aligned}
Max \quad & \sum_{i=1}^{m} v_i x_{io} && && (7)\\
s.t. \quad & \sum_{r=1}^{s} u_r y_{ro} = 1, \\
& \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \le 0, && j=1,2,...,n, \\
& u_r \ge \varepsilon^*, && r=1,2,...,s, \\
& v_i \ge \varepsilon^*, && i=1,2,...,m.
\end{aligned}
$$

Where ε^* is the non-Archimedean infinitesimal.

As has been said, we consider the following integrated DEA model using T_C':

$$
\begin{aligned}
Min \quad & \sum_{j=1}^{n} d_j && && (8)\\
s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} \le 1, && j=1,2,...,n, \\
& \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} + d_j = 0, && j=1,2,...,n, \\
& u_r \ge \varepsilon^*, && r=1,2,...,s, \\
& v_i \ge \varepsilon^*, && i=1,2,...,m. \\
& d_j \ge 0 && j=1,2,...,n.
\end{aligned}
$$

Where y_{rj} is the amount of rth output of DMU_j ; x_{ij} the amount of ith input of DMU_j; u_r the weight of rth output; v_i the weight of ith input; d_j is the deviation of DMU_j; n the number of $DMUs$; m the number of inputs; s the number of outputs; o the index of under evaluation DMU .

DMU_o is inefficient if and only if we have $d_o^* = 0$ in model (8).

3. Revenue Efficiency

In the past years, due to fierce competition in the economic arena due to the increase in the number of economic enterprises, as well as the reliability and limitations of resources, efficient use of resources and information about earning, more than before can be felt. In circumstances where a serious lack of attention to this important matter, the survival of life organization, will doubt and challenge. If the aim, is to find a single input consumption equal to the input of under evaluation unit, most of the revenues from the sale of output greater than or equal to the output of the units acquired, under evaluation. The following model is known revenue efficiency, can be applied:

$$Max \quad \sum_{r=1}^{s} p_r y_r \qquad (9)$$

$$\begin{aligned} s.t. \quad & \mathrm{x}_o \geq \sum_{j=1}^{n} \lambda_j x_{ij}, && \text{i=1,2,...,m,} \\ & y_r \leq \sum_{j=1}^{n} \lambda_j y_{rj}, && \text{r=1,2,...,s,} \\ & \lambda_j \geq 0, && \text{j=1,2,...,n.} \end{aligned}$$

Where $p = (p_1, p_2, ..., p_s)$ is a known vector of price for outputs. If (y^*, λ^*) is an optimal solution of model (9), the revenue efficiency, given by the conventional DEA model of DMU_o is defined as the ratio of the optimal revenues to current revenues, i.e.:

$$RE_o = \frac{\sum_r p_r y^*}{\sum_r p_r y_{ro}}$$

DMU_o is revenue efficient, iff $RE_o = 1$.

This section contains two parts, which in each episode were originally a way, to calculate the amount of revenue efficiency, with uncertainty output prices that in the interval form, are considered. Then, we offer, LP model for finding units that can be a candidate to the most revenue efficient unit, with CSW and price uncertainty. In continuing a model of MIP that can calculate the unit of most revenue efficient, will offer.

3.1. Optimistic Case:

In this section, according to optimistic point of view, boarders efficient frontier for the calculation of the amount of the efficient revenue, delivers. Then integrated model for calculation of unit or units that can be a candidate to be most revenue efficient units, will be introduced. In continuing a MIP model that can calculate, the most revenue efficient with price uncertainty by common set of weight has to offer.

Suppose that we have n $DMUs$ each consuming various amounts of m inputs to produce s outputs. As the corresponding to each output, a price vector is introduced and for any price vector, an upper bound and a lower bound can be considered. Therefore, the proposed model is as follows:

$$\min \quad \sum_{i=1}^{m} v_i x_{io} \qquad (10)$$
$$s.t. \quad \sum_{r=1}^{s} u_r y_{ro} = 1,$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j=1,2,...,n,$$
$$\frac{p_{r^a}^{\min}}{p_{r^b}^{\max}} \le \frac{u_{r^a}}{u_{r^b}} \le \frac{p_{r^a}^{\max}}{p_{r^b}^{\min}} \qquad 1 \le r^a \prec r^b \le s,$$
$$v_i \ge \varepsilon^*, \qquad i=1,2,...,m.$$

Where u_{r^a} is the weight for output $r^a\left(r^b\right)$ of DMU_o; $p_{r^a}^{\min}\left(p_{r^b}^{\min}\right)$ is a minimum bound estimate of the price of output $r^a\left(r^b\right)$ of DMU_o and $p_{r^a}^{\max}\left(p_{r^b}^{\max}\right)$ is the maximum bound estimate of the price of output $r^a\left(r^b\right)$ of DMU_o and ε^* is the non-Archimedean infinitesimal. In DEA models, when not be limited to the weights, the weights can be very large values or include very small amounts of it. That makes that, for $DMUs$ very large or very small values of efficiency will be calculated. To fix this problem, the weight restrictions will be proposed. In this case, the weight of the issue will be restricted by the bounds. In this model the output weight ratio, by minimum and maximum of an output price vector, it has been limited to $\frac{p_{r^a}^{\min}}{p_{r^b}^{\max}} \le \frac{u_{r^a}}{u_{r^b}} \le \frac{p_{r^a}^{\max}}{p_{r^b}^{\min}}$. Since the existence of the constraint $\frac{p_{r^a}^{\min}}{p_{r^b}^{\max}} \le \frac{u_{r^a}}{u_{r^b}} \le \frac{p_{r^a}^{\max}}{p_{r^b}^{\min}}$, causes non-linear model, this constraint will be replaced with two constraints $u_{r^a} \times p_{r^b}^{\min} - u_{r^b} \times p_{r^a}^{\max} \le 0$ and $u_{r^a} \times p_{r^b}^{\max} - u_{r^b} \times p_{r^a}^{\min} \ge 0$. In fact, this model is concluded by adding $2 \times C_2^S$ limit on the output weights to model (3).

DMU_o is revenue efficient by model (10), if and only if , we had $\sum_{i=1}^{m} v^*_i x_{io} = 1$.

The following LP model can be utilized for attaining an assurance value of ε^* in the model (10):

$$\varepsilon^* = \max \ \varepsilon \qquad (11)$$
$$s.t. \quad \sum_{r=1}^{s} u_r y_{rj} \le 1, \qquad j=1,2,...,n,$$
$$\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \le 0, \qquad j=1,2,...,n,$$
$$u_{r^a} \times p_{r^b}^{\min} - u_{r^b} \times p_{r^a}^{\max} \le 0, \qquad 1 \le r^a \prec r^b \le s,$$
$$u_{r^a} \times p_{r^b}^{\max} - u_{r^b} \times p_{r^a}^{\min} \ge 0, \qquad 1 \le r^a \prec r^b \le s,$$
$$v_i - \varepsilon \ge 0, \qquad i=1,2,...,m.$$

It is easy to prove that the optimal objective value of model (11) is bounded and hence model (10), is feasible for $\varepsilon \in \left(0, \varepsilon^*\right]$.

The model (10) measures the optimistic RE with output price uncertainty. Now, to find a single revenue efficient DMU, there is need to solve one optimization problem for each DMU, rank all revenue efficient $DMUs$ (using one of the ranking approaches), and finally determine a DMU, with the highest rank score. To solve this problem, we propose the following integrated model:

$$
\begin{aligned}
& Min \ \sum_{j=1}^{n} d_j && (12) \\
& s.t. \ \sum_{r=1}^{s} u_r y_{rj} \le 1, && j=1,2,...,n, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, && j=1,2,...,n, \\
& u_{r^a} \times p_{r^b}^{\min} - u_{r^b} \times p_{r^a}^{\max} \le 0, && 1 \le r^a \prec r^b \le s, \\
& u_{r^a} \times p_{r^b}^{\max} - u_{r^b} \times p_{r^a}^{\min} \ge 0, && 1 \le r^a \prec r^b \le s, \\
& d_j \ge 0, && j=1,2,...,n, \\
& v_i \ge \varepsilon^*, && i=1,2,...,m.
\end{aligned}
$$

Where u_{r^a} is the common set of weights for output r^a; $p_{r^a}^{\max}$ and $p_{r^a}^{\min}$ are maximum and minimum bounds estimated for the price of output r^a; d_j is the deviation of DMU_j from efficiency and ε^* is the non-Archimedean infinitesimal. The common set of weights help us to identify the most revenue efficient DMU in an identical condition. Model (12) determines the most revenue efficient unit candidate(s) with CSW under an optimistic perspective. DMU_o is optimistic revenue efficient with CSW and price uncertainty, if and only if $d_o^* = 0$.

Let $E^{opt} = \{ j \,|d^*_j = 0 \}$. If E^{opt} is singleton and $O \in E^{opt}$, then model (12) can determine DMU_o as the most revenue efficient under an optimistic perspective. Otherwise, this model determines the most revenue efficient unit candidates. Now, we propose the following MIP model for finding the most revenue efficient DMU:

$$
\begin{aligned}
& Min \ \sum_{j=1}^{n} d_j && (13) \\
& s.t. \ \sum_{r=1}^{s} u_r y_{rj} \le 1, && j=1,2,...,n, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, && j=1,2,...,n, \\
& u_{r^a} \times p_{r^b}^{\min} - u_{r^b} \times p_{r^a}^{\max} \le 0, && 1 \le r^a \prec r^b \le s, \\
& u_{r^a} \times p_{r^b}^{\max} - u_{r^b} \times p_{r^a}^{\min} \ge 0, && 1 \le r^a \prec r^b \le s, \\
& \sum_{j=1}^{n} \theta_j = n-1, && \\
& d_j \le M\theta_j, && j=1,2,...,n, \\
& \theta_j \le N d_j, && j=1,2,...,n, \\
& \theta_j \in \{0,1\}, && j=1,2,...,n, \\
& v_i \ge \varepsilon^*, && i=1,2,...,m.
\end{aligned}
$$

Where M and N are large enough positive numbers and θ_j is auxiliary binary variable. If $\theta_j = 0$, then $d_j = 0$. As a result, according to the constraint $\sum_{j=1}^{n} \theta_j = n-1$, be obtained: $d_j = 0$. Therefore, the unit with the most revenue efficiency will be determined in optimistic perspective, if $\theta_j = 1$.

Categories, constraints $d_j \le M\,\theta_j$ and $\theta_j \le N d_j$, always are satisfy, and can be eliminated. Obviously, the constraint $\sum_{j=1}^{n} \theta_j = n-1$ ensures that above model will be obtain just one DMU with most revenue efficiency. The above model is a mixed integer linear programming problem. Note that, model

(13) can be applied independently from model (12).

2.3. Pessimistic Case:

In this section, according to the pessimistic point of view, boarders inefficient frontier for the calculation of the amount of the efficient revenue, delivers. Then integrated model for calculation of unit or units that can be a candidate to be most revenue efficient units will be introduced. In continuing a model of MIP that can calculate the most revenue efficient with price uncertainty with the use of common weight has to offer.

Suppose that we have n $DMUs$ each consuming various amounts of m inputs to produce s outputs. As the corresponding to each output, a price vector is introduced and for any price vector, an upper bound and a lower bound can be considered. Therefore, the proposed model is as follows:

$$
\begin{aligned}
\text{Min} \quad & \sum_{i=1}^{m} v_i x_{io} && (14)\\
s.t. \quad & \sum_{r=1}^{s} u_r y_{ro} = 1, &&\\
& \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \le 0, && j=1,2,...,n,\\
& \frac{p_{r^a}^{\min}}{p_{r^b}^{\max}} \le \frac{u_{r^a}}{u_{r^b}} \le \frac{p_{r^a}^{\max}}{p_{r^b}^{\min}} && 1 \le r^a \prec r^b \le s,\\
& v_i \ge \varepsilon^*, && i=1,2,...,m.
\end{aligned}
$$

Where $u_{r^a}\left(u_{r^b}\right)$ is the weight for output r^a $\left(r^b\right)$ of DMU_o; $p_{r^a}^{\min}\left(p_{r^b}^{\min}\right)$ is a minimum bound estimate of the price of output $r^a\left(r^b\right)$ of DMU_o and $p_{r^a}^{\max}\left(p_{r^b}^{\max}\right)$ is the maximum bound estimate of the price of output $r^a\left(r^b\right)$ of DMU_o and ε^* is the non-Archimedean infinitesimal. In this model the output weight ratio, by minimum and maximum of an output price vector, it has been limited to $\frac{p_{r^a}^{\min}}{p_{r^b}^{\max}} \le \frac{u_{r^a}}{u_{r^b}} \le \frac{p_{r^a}^{\max}}{p_{r^b}^{\min}}$. Since the existence of the constraint $\frac{p_{r^a}^{\min}}{p_{r^b}^{\max}} \le \frac{u_{r^a}}{u_{r^b}} \le \frac{p_{r^a}^{\max}}{p_{r^b}^{\min}}$, causes non-linear model, this constraint will be replaced with two constraints $u_{r^a} \times p_{r^b}^{\min} - u_{r^b} \times p_{r^a}^{\max} \le 0$ and $u_{r^a} \times p_{r^b}^{\max} - u_{r^b} \times p_{r^a}^{\min} \ge 0$. In fact, this model is obtained by adding $2 \times C_2^S$ limit on the output weights to model (7).

DMU_o is pessimistic revenue efficient if, $\sum_{i=1}^{m} v^*_i x_{io} = 1$.

Model (14) measures the pessimistic revenue efficiency. Now, to find a single revenue efficient, there is need to solve one optimization problem for each DMU , rank all revenue efficient $DMUs$ (using one of the ranking approaches), and finally determine a DMU , with the highest rank score. To solve this problem, we propose the following integrated model:

$$
\begin{aligned}
& Min \ \sum_{j=1}^{n} d_j && && (15) \\
& s.t. \ \sum_{r=1}^{s} u_r y_{rj} \le 1, && j=1,2,...,n, \\
& \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} + d_j = 0, && j=1,2,...,n, \\
& u_{r^a} \times p_{r^b}^{\min} - u_{r^b} \times p_{r^a}^{\max} \le 0, && 1 \le r^a \prec r^b \le s, \\
& u_{r^a} \times p_{r^b}^{\max} - u_{r^b} \times p_{r^a}^{\min} \ge 0, && 1 \le r^a \prec r^b \le s, \\
& d_j \ge 0, && j=1,2,...,n, \\
& v_i \ge \varepsilon^*, && i=1,2,...,m.
\end{aligned}
$$

Where u_{r^a} is the common set of weight for output r^a; $p_{r^a}^{\max}$ and $p_{r^a}^{\min}$ are maximum and minimum bounds estimated for the price of output r^a; d_j is the deviation of DMU_j from efficiency and ε^* is the non-Archimedean infinitesimal. Model (15) determines the most revenue efficient unit candidate(s) with CSW under a pessimistic perspective. The common set of weights help us to identify the most revenue efficient DMU in an identical condition. DMU_o is pessimistic revenue efficient with CSW and price uncertainty, if and only if, $d_o^* = 0$.

Let $E^{pes} = \left\{ j \,\middle|\, d^*_j = 0 \right\}$. If E^{pes} is singleton and $O \in E^{pes}$, then model (15) can determine DMU_o as the most revenue efficient under a pessimistic perspective. Otherwise, this model determines the most revenue efficient unit candidates. Now, we propose the following MIP model for finding the most revenue efficient DMU:

$$
\begin{aligned}
& Min \ \sum_{j=1}^{n} d_j && && (16) \\
& s.t. \ \sum_{r=1}^{s} u_r y_{rj} \le 1, && j=1,2,...,n, \\
& \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} + d_j = 0, && j=1,2,...,n, \\
& u_{r^a} \times p_{r^b}^{\min} - u_{r^b} \times p_{r^a}^{\max} \le 0, && 1 \le r^a \prec r^b \le s, \\
& u_{r^a} \times p_{r^b}^{\max} - u_{r^b} \times p_{r^a}^{\min} \ge 0, && 1 \le r^a \prec r^b \le s, \\
& \sum_{j=1}^{n} \theta_j = n-1, \\
& d_j \le M\theta_j, && j=1,2,...,n, \\
& \theta_j \le N d_j, && j=1,2,...,n, \\
& \theta_j \in \{0,1\}, && j=1,2,...,n, \\
& v_i \ge \varepsilon^*, && i=1,2,...,m.
\end{aligned}
$$

Where M and N are large enough positive numbers and θ_j is auxiliary binary variable. If $\theta_j = 0$, then $d_j = 0$. As a result, according to the constraint $\sum_{j=1}^{n} \theta_j = n-1$, be obtained: $d_j = 0$. Therefore, the unit with the most revenue efficiency will be determined in pessimistic perspective, if $\theta_j = 1$.

Categories constraints $d_j \le M\theta_j$ and $\theta_j \le N d_j$ ،always satisfy and can be eliminated. Obviously, the constraint $\sum_{j=1}^{n} \theta_j = n-1$ ensures that above model will be determine just one DMU with most revenue efficiency. Above model is a mixed integer linear programming problem. Note that, model

(16) can be applied independently from model (15).

4. Numerical Example

In this section, we utilize data of from 21 medical centers in Taiwan. (The data set is taken from Wei et al. [39]).

This data set contains two inputs (physicians, sickbeds) and three outputs (out-patients, in-patients, surgeries). In this section, for finding the most revenue efficient unit, the output prices are considered hypothetical. We utilized GAMS 24.2.1 package to solve this model.

Table1: The data set

DMUs	Sickbeds	Physicians	Out-patients	In-patients	Surgeries
1	2618	1106	2,029,864	680136	38714
2	1212	473	1003707	297719	18575
3	1721	531	1592960	408556	36658
4	2902	973	2596143	855467	75348
5	1389	447	1116161	337523	23803
6	1500	547	1476282	378658	22503
7	340	145	1300016	55003	5614
8	571	305	1052992	199780	26026
9	1168	369	1849711	326109	30967
10	921	372	1089975	209323	23847
11	920	316	334090	268723	15130
12	3236	1023	1954775	920215	56167
13	495	130	332741	136351	23423
14	1759	491	1465374	430407	35599
15	1357	390	1277752	368174	36006
16	2468	675	1825332	668467	32275
17	962	316	550700	247961	15618
18	745	272	1277899	217371	11671
19	1662	590	1916888	418205	21551
20	898	275	698945	209134	11748
21	1708	537	1702676	470437	32218
$p^{\max}$	-	-	۲۱۹۳	۳۳۷۰	۱۲۰۰
$p^{\min}$	-	-	۹۰	۳۷	۱۲۰

Firstly, by applying model (11) we have $\varepsilon^* = 0.000162$, that it is the assurance value of epsilon. Model (10) implies that $E^{opt} = \{7,8\}$ which means that DMU_7 and DMU_8 are the most revenue efficient DMU candidates under an optimistic perspective.

Finally by applying model (12), DMU_7, will be determined as the most revenue efficient DMU, whit optimistic point of view.

To find the most revenue efficient unit under a pessimistic perspective, we apply model (12), this model implies that, $E^{pes} = \{11\}$. Fortunately, in this case E^{pes} is singleton and the model (12) can individually find DMU_{11} as the most revenue efficient with pessimistic point of view. Therefore, it is unnecessary to utilize the MIP model (13). The results are shown in Table 2.

Table 2: The results

DMU	$d_j(MODEL\ 10)$	$d_j(MODEL\ 12)$	$d_j(MODEL\ 13)$
1	0.4403	0.4406	0.1301
2	0.1940	0.1941	0.0741
3	0.2360	0.2362	0.1810
4	0.4002	0.4005	0.3709
5	0.2070	0.2072	0.0989
6	0.2316	0.2316	0.201
7	0.0000	0.0000	0.2035
8	0.0000	0.0010	0.8578
9	0.1124	0.1125	0.3013
10	0.1244	0.1245	0.1352
11	0.1621	0.1622	0.0000
12	0.5107	0.5111	0.1613
13	0.0610	0.0610	0.0715
14	0.2429	0.2431	0.1623
15	0.1734	0.1736	0.1864
16	0.3626	0.3629	0.1648
17	0.1571	0.1572	0.0281
18	0.0779	0.0780	0.1878
19	0.2288	0.2290	0.2079
20	0.1358	0.1359	0.0493
21	0.2301	0.2302	0.2145

5. Conclusion and Further Research

This study investigated the most revenue efficient DMU with price uncertainty. For finding the most revenue efficient unit is two different views: the optimistic and pessimistic perspectives. For each perspective, is formulated a LP model for determining the most revenue efficient candidate(s) and MIP model for finding the most revenue efficient unit among this candidates. In this study optimistic state uses the efficient frontier and pessimistic state uses inefficient frontier. To illustrate the applicability of proposed approach is utilized for data from 21 medical centers in Taiwan. The idea in this paper can be extended for measuring profit efficiency with price uncertainty, negative data, imprecise data and stochastic data.

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