Available online at http://ijdea.srbiau.ac.ir

Int. J. Data Envelopment Analysis (ISSN 2345-458X)

Vol.2, No.2, Year 2014 Article ID IJDEA-00223, 6 pages

Research Article

International Journal of Data Envelopment Analysis Science and Research Branch (IAU)

An Algorithm for Resource Allocation through the Classification of DMUs

M. Ahadzadeh Namin[a*], N. Ebrahimkhani Ghazi[b]

(a) Department of Mathematics, Shahr-e –Qods Branch, Islamic Azad University, Tehran, Iran.

(b) Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.

Received 10 January 2014, Revised 20 May 2014, Accepted 02 June 2014

Abstract

Data envelopment analysis (DEA) is a non-parametric method for assessing relative efficiency of decision-making units (DMUs). Every single decision-maker with the use of inputs produces outputs. These decision-making units will be defined by the production possibility set. Resource allocation to DMUs is one of the concerns of managers since managers can employ the results of this process to allocate resources in organizations appropriately and sufficiently and as a result achieve the lowest cost and highest level of profit. In this paper we will suggest an algorithm to select such efficient decision making units.

Keywords: DEA, classification, resource allocation, context-dependent.

1. Introduction

A mathematical programming method called Data envelopment analysis (DEA) measuring the relative efficiency of decision making units (DMUs) with multiple outputs and multiple inputs was introduced by Charnes, Cooper and Rhodes (CCR) [2]. In fact, a set of DMUs can be separated into different levels of efficient frontiers [4] which could be drawn on by managers to make efficient

* Corresponding author: Ahadzadehnamin@yahoo.com

decisions regarding resource allocation. In fact an important part of all businesses is *resource allocation* [7] which is typically set annually. The question is how to allocate resources? There are so many ways to answer this question. Basically clustering can solve the problem of grouping similar observations of a data set into the same cluster and dissimilar units into different clusters. The problem of [6] clustering high dimensional data is investigated by many research studies in many different fields. For identifying different classes of high dimensional data Amin G.R et al., 2011[1] suggested a data envelopment analysis (DEA) clustering approach. First, we use these clusters to allocate the infinite budget. With comprising the cost and revenue efficiency [5] for each cluster, allocation resource [3] has come under closer scrutiny. We have organized the rest of the paper as follows: the second section presents the new algorithm. Then the cost and revenue model is incorporated into the algorithm. We demonstrate our approach by applying it to a data set consisting of 15 DMUs with two inputs and one output.

2. An Algorithm for Resource Allocation

In this section we provide an algorithm to find a bunch of DMUs which have the lowest cost and most profit when source B allocated to DMUs.

Suppose that there are n decision making units, DMU_j, that use inputs X to produce outputs Y, that:

$$X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})$$
$$Y_j = (y_{1j}, y_{2j}, \ldots, y_{sj})$$

Step1: Find the efficiency value of DMUs with using the following model (CCR).

$$\begin{aligned} \text{Max} \quad & \sum_{r=1}^{s} u_r y_{rk} \qquad (1) \\ \text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j = 1, 2, \ldots, n \\ & \sum_{i=1}^{m} v_i x_{ik} = 1 \\ & u_r \ge \varepsilon, \quad v_i \ge \varepsilon, \quad r = 1, 2, \ldots, s; \quad i = 1, 2, \ldots, m \end{aligned}$$

That, $k \in \{1, 2, \ldots, n\}$. Suppose that:

$$E = \{ \text{ set of } DMU_j\text{s that have } \theta_j^* = 1, \ (U^*, V^*) \ne \bar{0} \ \}$$

$$\bar{E} = \{ \text{ set of } DMU_j\text{s that have } \theta_j^* \ne 1, \ (U^*, V^*) \ne \bar{0} \ \}$$

$$R = \{ \text{ set of } DMU_j\text{s that have zero weight at least one index} \}$$

Clearly; $n = | E \cup \bar{E} \cup R |$.

Step 2: The frontier of production possibility set is DMUs on efficient set E, that satisfied in:

$$PF(M) = f(x_{1M}, x_{2M}, ..., x_{mM}, y_{1M}, y_{2M}, ..., y_{sM}) = \sum_{r=1}^{s} u_r^* y_{rM} - \sum_{i=1}^{m} v_i^* x_{iM} = 0$$

Such (U^*, V^*) are the weights when DMU_M is evaluated. Frontiers of production possibility set are called $PF(1), PF(2), ..., PF(M), ..., PF(p)$.

If $DMU_k \in \bar{E}$ satisfies in $\sum_{r=1}^{s} u_r^* y_{rk} - \sum_{i=1}^{m} v_i^* \theta_k^* x_{ik} = 0$ constraints, where θ_k^* is the optimal solution and $(u_1^*, u_2^*, ..., u_s^*, v_1^*, v_2^*, ..., v_m^*)$ are the optimal weights when DMU_M is evaluated.

Then, put DMU_k in C(M) classification. Suppose that $C(1), C(2), ..., C(M), ..., C(P)$ are the existence classifications.

Step 3: If $DMU_k \in R$, then we can say that DMU_k needs revaluating for finding its classification. For achieving this aim, calculate the distances of DMU_k from all PPS frontiers (that means: $PF(1), PF(2), ..., PF(M), ..., PF(p)$). Suppose that the distances of DMU_k from frontiers, $PF(1), PF(2), ..., PF(M), ..., PF(p)$, are $d_{1k}, d_{2k}, ..., d_{Mk}, ..., d_{pk}$, respectively.

Step 4: Put $d_Z(k^*) = Max\{d_{1k}, d_{2k}, ..., d_{Mk}, ..., d_{pk}\}$, clearly $Z \in \{1, ..., M, ..., p\}$. Then we can put DMU_k in $C(Z)$ classification.

Step 5: Use the cost-profit model with **B** source for each bunch of DMUs in classifications $C(1), C(2), ..., C(M), ..., C(P)$.

Step 6: Each DMUs classification which has the lowest cost efficiency and upper profit efficiency introduces a bunch of DMUs that source B can be allocated for them.

Table 1. Data of DMUs

Units	I_1	I_2	O
DMU_1	872	3345	1
DMU_2	1162	2515	1
DMU_3	1924	1757	1
DMU_4	3004	1403	1
DMU_5	1596	5072	1
DMU_6	2756	4606	1
DMU_7	4354	2994	1

Table1.(continued)

DMU_8	4006	4550	1
DMU_9	1089	6359	1
DMU_{10}	5267	1892	1
DMU_{11}	7644	3183	1
DMU_{12}	1217	5774	1
DMU_{13}	1556	2123	1
DMU_{14}	2222	1659	1
DMU_{15}	2607	1546	1

3. Example

Suppose that we have 15 DMUs with two inputs and one output that shows in Fig 1 (see table1). By using model (1) we have:

$$E = \{DMU_1, DMU_2, DMU_3, DMU_4, DMU_{13}, DMU_{14}, DMU_{15}\}$$

$$\bar{E} = \{DMU_5, DMU_6, DMU_8, DMU_7\}$$

$$R = \{DMU_9, DMU_{10}, DMU_{11}, DMU_{12}\}$$

Clearly there are classifications for DMUs, that means:

Class 1: DMU_1, DMU_2

Class 2: DMU_2, DMU_3, DMU_{13}

Class 3: $DMU_3, DMU_4, DMU_{14}, DMU_{15}$

The PPS frontiers which are made by the above DMUs show that clearly DMU_5 is projected on the frontiers of class1, thus DMU_5 is put in class 1. Then we have

Class 1: DMU_1, DMU_2, DMU_5

Class 2: $DMU_2, DMU_3, DMU_{13}, DMU_6, DMU_8$

Class 3: $DMU_3, DMU_{14}, DMU_{15}, DMU_7$

To find classifications of $DMU_9, DMU_{10}, DMU_{11}, DMU_{12}$ it is enough to calculate distances from these DMUs to the frontiers of PPS with input oriented form and follow the step3, because these DMUs have at least one zero weight (See fig2).

If $DMU_{10} = (X_{A1}, X_{A2})$ and $C = (d_c.X_{A1}, d_c.X_{A2})$, $D = (d_D.X_{A1}, d_D.X_{A2}), E = (d_E.X_{A1}, d_E.X_{A2})$ then $d_c = Max\{d_C, d_D, d_E\}$ and DMU_{10} is projected on the blue frontier and the distance is d_3 and it is a member of class3. Thus, we have:

Class 1: $DMU_1, DMU_2, DMU_5, DMU_9, DMU_{12}$

Class 2: $DMU_2, DMU_3, DMU_{13}, DMU_6, DMU_8$

Class 3: $DMU_3, DMU_{14}, DMU_{15}, DMU_7, DMU_{10}, DMU_{11}$

Then, use model 2 for each classification groups until finding a group of DMUs classification that have lowest cost and upper profit efficiency when allocated **B** source.

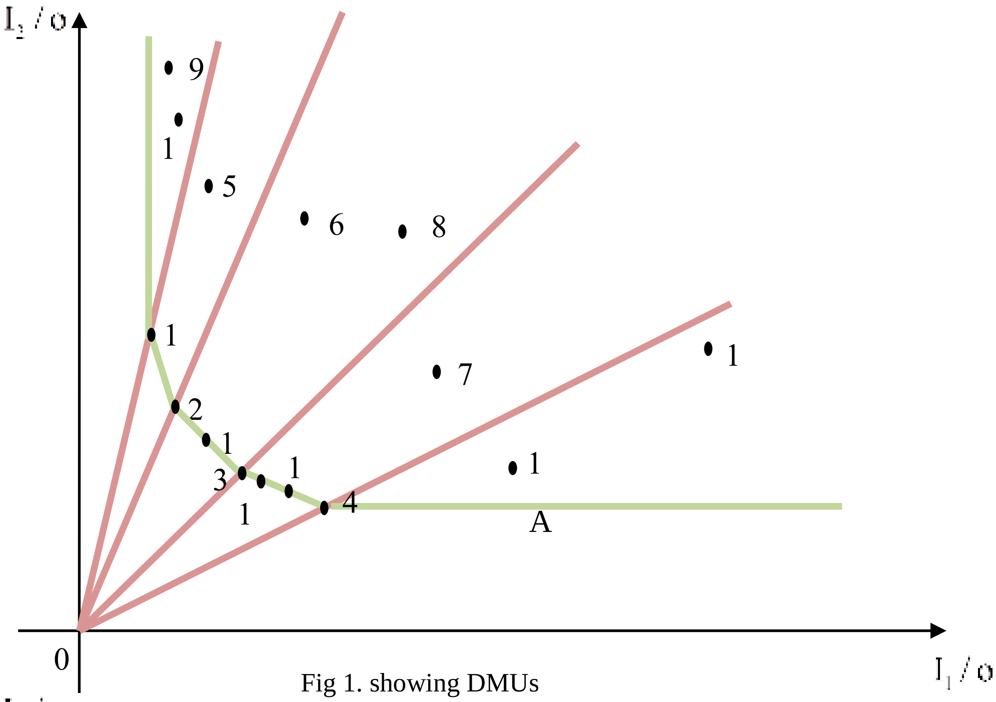

Fig 1. showing DMUs

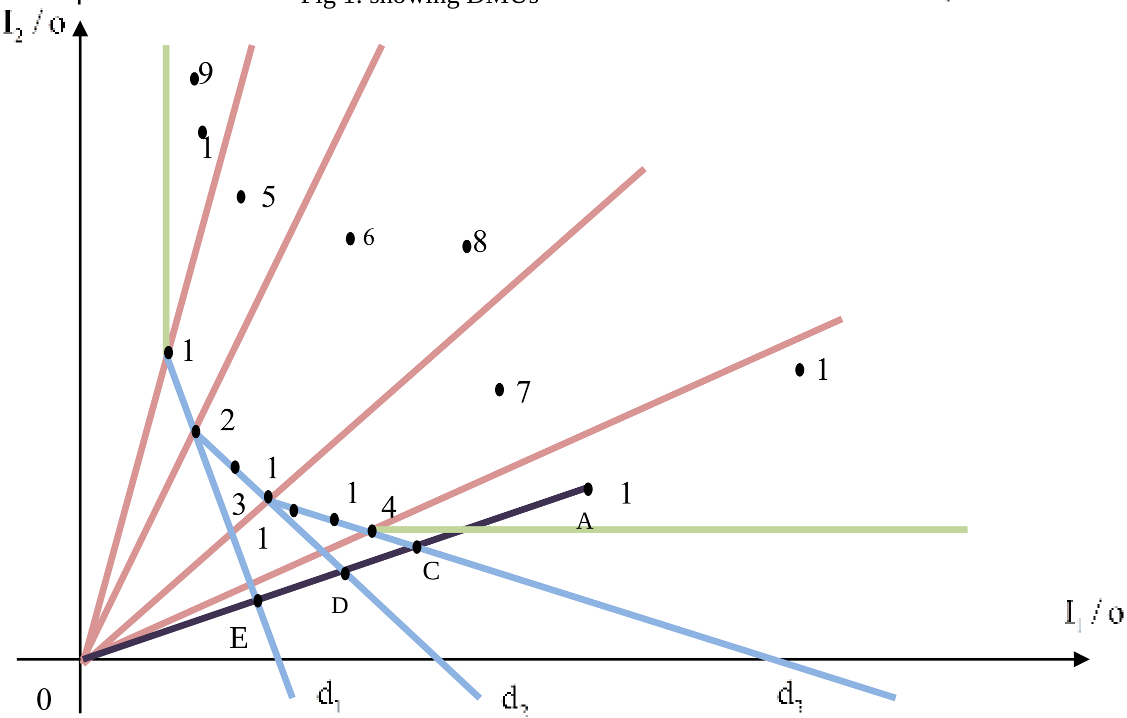

Fig 2. Image of DMU 10 in all frontiers of PPS

4. Conclusion

Sometimes in some organizations, there are fixed and limited sources and the need of allocating resources to some DMUs is important. In this paper we suggested an algorithm that helps to find a bunch of decision making units and logically allocate resource B for them while the lowest cost and upper profit efficiency in comparison with other classifications is evaluated. Then we gave an example to demonstrate the suggested algorithm. With the help of the algorithm figures we reached our goal and our approach proved to be successful.

References

[1] Amin G.R, Emrouznejad A. Rezaei S., (2011). Some clarifications on the DEA clustering approach, *European Journal of Operational Research,* 215, 498-501.

[2] Charnes A, Cooper WW, Rhodes E, (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research*; 2:429–44.

[3] Leonard, S.T., Droege, M., 2008. The uses and benefits of cluster analysis in pharmacy research. *Research in Social and Administrative Pharmacy* 4, 1–11.

[4] Seiford LM, Zhu J., (2003). Context- dependent data envelopment Analysis: measuring attractiveness and progress, *QMEGA*, 3(5), 397-408.

[5] Quanling Wei., Tsung-Sheng Chang., (2011). Optimal profit-maximizing system design data envelopment analysis models. Computer & Industrial Engineering, 1275-1284.

[6] Tversky A., Simonson I., (1993). Context-dependent preferences. *Management sciences*. 39, 1179-89.

[7] Yan, H., Wei, Q. L., & Hao, G. (2002). DEA models for resource reallocation and production input/output estimation. *European Journal of Operational Research*, 136, 19–31.