



Efficient Selection of Design Parameters in Multi-Objective Economic-Statistical Model of Attribute C Control Chart

S. Jafarian-Namin^a, A. Amiri^{b*}, E. Najafi^c

(a) *Department of Industrial Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran*

(b) *Industrial Engineering Department, Shahed University, Tehran, Iran*

(c) *Industrial Engineering Department, Islamic Azad University, Science and Research Branch, Tehran, Iran.*

Received 23 February 2014, Revised 10 June 2014, Accepted 29 June 2014

Abstract

Control chart is the most well-known chart to monitor the number of nonconformities per inspection unit where each sample consists of constant size. Generally, the design of a control chart requires determination of sample size, sampling interval, and control limits width. Optimally selecting these parameters depends on several process parameters, which have been considered from statistical and/or economic aspects in the literature. This study presents a multi-objective economic-statistical design (MOESD) of the C control chart. An algorithm using data envelopment analysis (DEA) is employed to solve this model. A numerical example is used to illustrate the algorithm procedure.

Keywords: C Control Chart, Multi-Objective Economic-Statistical Design, DEA.

*Corresponding author: amiri@shahed.ac.ir

1. Introduction

Statistical Process Control (SPC) is an industry-standard methodology for measuring, controlling and improving the quality and productivity of manufacturing and service enterprises. Control chart is one of the seven basic tools of SPC. A control chart is a graphical exhibition of plotted quality data obtained from a process in time order that can be used to maintain the process in the in-control status and improve it through reduction in the variability of the process through analyzing the process changes over time. Broadly, there are two classes of control charts: the variable and the attribute. Attribute control charts are used to monitor discrete and/or categorical data (for a comprehensive review see [1]). Among them, the classic Shewhart's C control chart is applied to monitor the number of nonconformities per inspection unit, distributed according to a Poisson distribution [2]. Despite the wider application in real practice for the relative simplicity in dealing with attributes than variables, the C control charts have been largely neglected unfortunately. Thus, in this study, the design of attribute C control charts is considered through corresponding parameters.

Generally, the design of a control chart requires the specification of three parameters, namely, sample size (n), sampling interval (h), and control limits width (k). Many researchers have considered optimally selecting these parameters. Traditionally, control charts are designed without taking into account economic aspects that experience substandard performances. Since the quality of a product and the cost of process control are affected by design parameters, it is of great importance to search for the best possible parameters. This in turn motivates many researchers to consider optimally selecting of the design parameters.

The first economic model to monitor the mean of a normal process under a single assignable cause was developed by Duncan [3]. This pure economic design was criticized for poor statistical properties. Thus, Saniga [4] introduced an economic-statistical model in which minimized the Duncan's cost model subject to statistical constraints. Since the importance of statistical properties is of the same as economic aspects, this approach seems ineffective and simultaneous optimization of both properties is required. As a multiple criteria decision-making, Chen and Liao [5] applied an approach to design X control chart in which the efficient solutions were selected using DEA. Recently, this approach has been considered for the design of the np control chart [6].

DEA is a powerful optimization approach to evaluate the relative efficiency of decision making units (DMUs) with multiple inputs and outputs. Charnes et al. [7] developed DEA via generalization of the Farrell's single input, single output efficiency measurement. In recent years, a large number of applications of DEA have been done by defining DMUs in various forms as needed. Two main reasons that make DEA more attractive are 1) the general and supple definition of a DMU, and 2) fairly few assumptions involved in the modeling. The first DEA approach gained a lot of attention is known as

the CCR model. In this mathematical programming model, the performance of a specific DMU is assessed with respect to the performance of the remaining DMUs.

By defining proper DMUs, in this paper, we present multi-objective economic-statistical design of the C control chart. The rest of the paper is organized as follows: In section 2, the C control chart is introduced. The multi-objective model is proposed in Section 3. In the next section, a brief description of the data envelopment analysis approach is given. In section 5, an algorithm using the DEA, proposed by Chen and Liao [5], is employed with some modifications to solve the proposed model. Section 6 includes a numerical example to illustrate the solution procedure and to perform some Comparisons. Finally, conclusions complete the paper.

2. The C Control Chart

It is often important to construct control charts for the number of defects, where each inspected unit of the same size may have several countable nonconformities. In such cases, the classic Shewhart C-chart is applied to monitor the number of nonconformities per inspection unit, distributed according to a Poisson distribution as follows:

$$P(x) = \frac{e^{-c} c^x}{x!} \quad x = 0, 1, \dots, \quad (1)$$

where $c > 0$ is the Poisson distribution parameter which represents its mean and variance in a preliminary inspection unit of constant size.

Commonly, a control chart has a central line (CL) to define the central tendency, and an upper control limit (UCL) and lower control limit (LCL) to exhibit the scope of expected variation of the plotted data. Hence, if the known value of c (obtained using historical information) is considered as the central line, the k -sigma upper and lower control limits are statistically computed for a C control chart as given in the following equations [2]:

$$UCL = c + k\sqrt{c}, \quad (2)$$

$$LCL = c - k\sqrt{c}. \quad (3)$$

In this way, the production process is permanently sampled in specific time intervals, inspected and the results are interpreted to see whether the outcomes are from one consistent and homogeneous process or not. Under the attitude of quality improvement, the assignable cause or causes of variation in the process must be discovered and eliminated so as to reach a stable and predictable process (i.e., 'in-control' state). In this study, a single assignable cause is assumed to occur.

If designing of a C control chart based on n inspection units (not necessarily integer) is of interest, new inspection unit is obtained with n multiplied by primary inspection unit. Therefore, the control limits are converted into (notice that the number of items inspected remains the same among all the new

subgroups):

$$UCL = nc + k\sqrt{nc}, \quad (4)$$

$$LCL = nc - k\sqrt{nc}. \quad (5)$$

In some cases, when the calculations yield zero or a negative value for LCL, researchers often set it to zero. However, this violates the definition of a lower control limit and Use of run rules. Because of the importance of points plotting below the LCL, indicating a possible reduction in the number of defects, we define a constraint to consider situations in which it is positive.

3. The Proposed Model

The proposed *C-MOESD* model is based on Duncan's economic model [3] by some adjustments to adapt with the C control chart and consider economic and statistical properties simultaneously. To simplify the mathematical manipulation and analysis, the following assumptions are considered to be hold:

1. The quality characteristic follows a Poisson distribution,
2. The process is either in-control or out-of-control state only and is initially in the 'in-control' state; that is, $c = c_0$,
3. When a random assignable cause of magnitude δ occurs, leads the process mean to shift from c_0 to $c_1 = c_0 + \delta\sqrt{c_0}$,
4. The occurrence of an assignable cause possesses an exponential distribution with mean time $1/\lambda$,
5. The process is allowed to continue during the search and repair.

A. Economic Cost Function

Duncan's [3] definition of the expected hourly cost is based on a single quality control cycle of monitoring the production process.

This cycle length consists of four components: 1) the period in which the process is in-control, 2) the time to signal during the out-of-control period, 3) the time to sample, inspect and interpret, and 4) the time to discover and repair the assignable cause.

Accordingly, an expected cycle length and an expected cost of the cycle can be formulated in economic model construction. As a ratio between the expected cost during a cycle and the expected cycle time length, the expected hourly cost in [3] is adapted to the C control chart:

$$E_L = \frac{f + vn}{h} + \frac{\lambda(W + TA + MB)}{1 + \lambda B}, \quad (6)$$

where:

- f is the fixed cost of sampling an inspection unit,
- v is the variable cost of sampling an inspection unit,
- W is the average cost to detect an assignable cause,
- T is the cost of verifying a false alarm,
- M is the hourly loss due to poor quality of units,
- A is the average number of false alarms per cycle:

$$A = \alpha / (e^{\lambda h} - 1) , \quad (7)$$

- α is probability of false alarm ([UCL] and <LCL> denote the integer round down and round up functions of control limits):

$$\alpha = 1 - \sum_{x=\langle LCL \rangle}^{[UCL]} (c_0)^x e^{-c_0} / x! , \quad (8)$$

- B is the average time of the process being in out-of-control state:

$$B = h / P - \tau + gn + D , \quad (9)$$

- P is detection power:

$$P = 1 - \sum_{x=\langle LCL \rangle}^{[UCL]} (c_1)^x e^{-c_1} / x! , \quad (10)$$

- τ is average time of occurrence of an assignable cause between samples:

$$\tau = \left(1 - (1 + \lambda h) e^{-\lambda h} \right) / (\lambda - \lambda e^{-\lambda h}) , \quad (11)$$

- g is time to sample and interpret the results,
- D is time required to find the assignable cause.

B. Multi-Objective Design of the C Control Chart

The goal of multi-objective decision making is to find a solution giving the best compromise between multiple objectives. In addition to statistical perspective, designing a control chart has several economic consequences as presented before. Thus, taking into account both properties, the *C-MOESD* model is formulated as follows:

$$\begin{aligned}
& \text{Min } E_L(S) \\
& \text{Max } ATS_0(S) \\
& \text{Min } ATS_1(S) \\
& \text{s.t. } E_L \leq E_L^U, \\
& \quad ATS_0 \geq ATS_0^L \\
& \quad ATS_1 \leq ATS_1^U \\
& \quad LCL > 0
\end{aligned} \tag{12}$$

where, E_L is the expected hourly cost, $S=(n, h, k)$ is a possible set of design parameters, $ATS_0=h/\alpha$ is the average time to signal when a false alarm occurs, and $ATS_1=h/P$ is the average time to signal when an assignable cause occurs. In addition, E_L^U , ATS_0^L and ATS_1^U are the desired bounds determined by DM, and LCL is the lower control limit.

The design of a control chart requires the specification of three decision variables, i.e. n , h and k . One of the most powerful methods to solve (12) is DEA method. Although DEA is applied for various control charts, it has not been used for design of C control chart yet. In this study, DEA method is used to search the optimal solution(s) in the model.

4. Data Envelopment Analysis

DEA is a powerful non-parametric approach to evaluate the relative efficiency of a group of decision making units (DMUs) with multiple inputs and outputs. The first DEA approach is known as the CCR model via generalization of the Farrell's single input, single output efficiency measurement [7]. This linear programming formulation can be either input-oriented or output-oriented. Assuming n DMUs, each with m inputs and s outputs, the efficiency of a specific DMU can be obtained by solving the input-oriented CCR model:

$$\begin{aligned}
& \text{Max } E_0(D) = \sum_{r=1}^s u_r Y_{r0} \\
& \text{s.t. } \sum_{i=1}^m v_i X_{i0} = 1 \\
& \sum_{r=1}^s u_r Y_{rj}(D) - \sum_{i=1}^m v_i X_{ij}(D) \leq 0, \quad j = 1, \dots, n, \\
& u_r \geq 0, \quad r = 1, \dots, s \\
& v_i \geq 0, \quad i = 1, \dots, m
\end{aligned} \tag{13}$$

where u_r is the weight of output r , v_i is the weight of input i , Y_{rj} is the value of output r for j th DMU, and X_{ij} is the value of input i for j th DMU. The performance of each DMU measured is relative to the remaining DMUs. A DMU is relatively inefficient if $E_0^* < 1$ and relatively efficient, strictly or weakly, if $E_0^* = 1$. In designing control charts, DMUs refer to feasible combinations of design parameters.

In the *C-MOESD* model, the objectives including E_L and ATS_1 are considered as inputs because of

their minimizing nature, and ATS_0 is probed as output. The model should be formulated for each DMU to find the set of weights, as decision variables, that maximize the relative efficiency of considered DMU. As a result, at least one of the DMUs will be efficient.

5. Solution Procedure

We intend to achieve a well-balanced trade-off between the economic and the statistical features. For this reason, Chen and Liao's [5] algorithm is employed by some adjustments for optimizing our proposed model. In fact, the modifications are as follows: 1) optimizing the model for the C control chart, 2) changing some of the objective functions and constraints by introducing h into them, 3) setting upper bound for expected hourly cost, 4) considering situations not to violate the definition of a LCL based on the nature of Poisson distribution. The solution algorithm for optimizing the proposed model is explained as follows:

Step 0. *Determining the possible combinations of design parameters.* Set various combinations of design parameters by putting bounds on them according to DM's needs. Due to the discrete optimization nature of the DEA method used in this algorithm, and to avoid additional computations related to dominated solutions, the analyzer can limit the solution space in advance. Then, the values of objective functions must be computed for each DMU.

Step 1. *Determination of feasible combinations.* By using the constraints in (12), gather the feasible combinations with the same sample size n into a set Q_n .

Step 2. *Partial solution selection.* Determine the non-dominated solution points for each set of Q_n .

Step 3. *Global Pareto solution selection.* Merge all determined solutions from step 2 into a set W and then select efficient design(s) among the scores calculated for all designs by *CCR* model (Although there are some DEA softwares, in this study, all calculations have been facilitated under a program coded in the MATLAB (version R2013b) environment).

6. A Numerical Example

In order to illustrate the results of the *C-MOESD* model, the number of nonconformities in a unit of size n is supposed to follow a Poisson distribution with mean $c_0=4$. Moreover, when an assignable cause with the rate of $\lambda=0.01$ occurs, it provides a shift of size $\delta=2$ in the process mean (and so $c_1=8$). The values of other parameters are listed in Table 1.

Table 1

Input values of parameters						
Cost factors					Time factors	
<i>M</i>	<i>T</i>	<i>W</i>	<i>f</i>	<i>V</i>	<i>g</i>	<i>D</i>
20	25	12.5	1	0.1	0.05	2

Firstly, the solution space is limited according to the bounds on design parameters: $1 \leq n \leq 30$ increases by 0.5, $0.1 \leq h \leq 4$ increases by 0.1, and $0.5 \leq k \leq 4$ increases by 0.5. Therefore, the number of possible combinations will be $59 \times 40 \times 8 = 18880$. For all of the possible combinations, the values of objective functions are computed. Then, the feasible combinations are specified by applying the following constraints: $E_L \leq 7$, $ATS_0 \geq 100$, $ATS_1 \leq 4$, and $LCL > 0$. Next, the *NDS* points for each set of Q_n are selected. Finally, the relative efficiency score of each *NDS* is calculated.

As shown in Table 2, two combinations of design parameters have received efficiency score 1 and thus are offered to the DM for final selection. In consequence, Table 3 shows the determined efficient units in addition to comparison with the pure economic design in which only the expected hourly cost in (12) is minimized (see Duncan's [3] model). Selected two efficient units by the proposed model have improved ATS_0 about 880.3% and 715.9%, and ATS_1 about 35.1% and 48.9%, respectively. However, E_L is increased about 109.9% and 162.2%, respectively. Despite the increase in cost, statistical performance is improved substantially using *C-MOESD* model. Moreover, in our designs, all the objectives are in the desired limits, while, the statistical objectives of pure economic design are not satisfactory.

In terms of efficiency values, our designs show significant difference against the pure economic design. This can totally confirm the improved performance of the Multi-Objective Economic-Statistical design and reveal the insufficiency of the pure economic design in such space.

Table 2.

Non-dominated solution points in the set W

<i>n</i>	<i>h</i>	<i>k</i>	ATS_0	ATS_1	E_L	<i>n</i>	<i>h</i>	<i>k</i>	ATS_0	ATS_1	E_L
2.5	0.4	3	101.17	1.89	4.22	7	0.7	3	159.34	3.31	3.70
2.5	0.5	3	126.46	2.37	3.62	7	0.8	3	182.11	3.79	3.46
2.5	0.6	3	151.76	2.84	3.25	7.5	0.4	3	118.66	2.09	5.52
2.5	0.7	3	177.05	3.32	3.01	7.5	0.5	3	148.32	2.62	4.69
2.5	0.8	3	202.34	3.79	2.84	7.5	0.6	3	177.99	3.14	4.16
3	0.4	3	112.06	1.98	4.34	7.5	0.7	3	207.65	3.66	3.81
3	0.5	3	140.07	2.47	3.73	8	0.4	3.5	481.54	3.67	5.78
3	0.6	3	168.08	2.97	3.35	8	0.5	3	111.04	2.35	4.80
3	0.7	3	196.10	3.46	3.10	8	0.6	3	133.25	2.82	4.24

Table 2. (continued)

<i>n</i>	<i>h</i>	<i>k</i>	<i>ATS</i> ₀	<i>ATS</i> ₁	<i>E_L</i>	<i>n</i>	<i>h</i>	<i>k</i>	<i>ATS</i> ₀	<i>ATS</i> ₁	<i>E_L</i>
3	0.8	3	224.11	3.96	2.93	8	0.7	3	155.45	3.29	3.86
3.5	0.3	3.5	463.25	2.98	5.62	8	0.8	3	177.66	3.77	3.59
3.5	0.4	3	129.79	2.10	4.46	8.5	0.4	3	116.11	2.08	5.78
*	3.5	0.4	617.66	3.98	4.65	8.5	0.5	3	145.14	2.60	4.90
3.5	0.5	3	162.24	2.63	3.84	8.5	0.6	3	174.17	3.13	4.34
3.5	0.6	3	194.69	3.15	3.45	8.5	0.7	3	203.20	3.65	3.96
3.5	0.7	3	227.14	3.68	3.19	9	0.4	3	151.32	2.31	5.90
4	0.3	3	115.87	1.69	5.72	9	0.5	3	189.15	2.89	5.01
*	4	0.3	514.07	3.13	5.82	9	0.6	3	226.98	3.46	4.45
4	0.4	3	154.50	2.25	4.59	9.5	0.4	3	117.40	2.10	6.04
4	0.5	3	193.12	2.82	3.95	9.5	0.4	3.5	570.27	3.98	6.21
4	0.6	3	231.75	3.38	3.56	9.5	0.5	3	146.75	2.62	5.11
4	0.7	3	270.37	3.94	3.30	9.5	0.6	3	176.10	3.15	4.51
4.5	0.3	3.5	287.38	2.44	5.90	9.5	0.7	3	205.46	3.67	4.11
4.5	0.4	3.5	383.17	3.25	4.81	10	0.5	3	116.37	2.41	5.21
4.5	0.5	3	114.30	2.34	4.06	10	0.6	3	139.65	2.89	4.59
4.5	0.6	3	137.17	2.81	3.61	10	0.7	3	162.92	3.37	4.17
4.5	0.7	3	160.03	3.27	3.32	10	0.8	3	186.20	3.85	3.87
4.5	0.8	3	182.89	3.74	3.11	10.5	0.4	3	121.53	2.13	6.29
5	0.3	3.5	342.63	2.64	6.09	10.5	0.4	3.5	551.20	3.95	6.47
5	0.4	3	115.37	2.04	4.86	10.5	0.5	3	151.92	2.67	5.32
5	0.4	3.5	456.85	3.51	4.97	10.5	0.6	3	182.30	3.20	4.69
5	0.5	3	144.22	2.55	4.16	10.5	0.7	3	212.69	3.73	4.27
5	0.6	3	173.06	3.06	3.71	11	0.5	3	122.86	2.46	5.42
5	0.7	3	201.90	3.57	3.42	11	0.6	3	147.43	2.96	4.77
5.5	0.3	3	109.53	1.68	6.24	11	0.7	3	172.00	3.45	4.32
5.5	0.3	3.5	413.37	2.87	6.29	11	0.8	3	196.57	3.94	4.01
5.5	0.4	3	146.04	2.24	4.99	11.5	0.4	3	128.02	2.18	6.55
5.5	0.4	3.5	551.16	3.82	5.15	11.5	0.4	3.5	547.34	3.95	6.73
5.5	0.5	3	182.55	2.80	4.27	11.5	0.5	3	160.03	2.73	5.53
5.5	0.6	3	219.06	3.35	3.82	11.5	0.6	3	192.04	3.27	4.88
5.5	0.7	3	255.57	3.91	3.53	11.5	0.7	3	224.04	3.82	4.43
6	0.3	3.5	503.03	3.14	6.50	12	0.4	3	105.18	2.03	6.69
6	0.5	3	122.99	2.42	4.37	12	0.5	3	131.47	2.53	5.63
6	0.6	3	147.58	2.91	3.88	12	0.6	3	157.77	3.04	4.95
6	0.7	3	172.18	3.39	3.56	12	0.7	3	184.06	3.55	4.48

Table 1. (continued)

n	h	k	ATS_0	ATS_1	E_L	n	h	k	ATS_0	ATS_1	E_L
6	0.8	3	196.78	3.88	3.33	12.5	0.4	3	136.65	2.25	6.81
6.5	0.3	3.5	328.88	2.62	6.61	12.5	0.4	3.5	555.07	4.00	6.99
6.5	0.4	3	127.10	2.14	5.25	12.5	0.5	3	170.82	2.81	5.74
6.5	0.4	3.5	438.50	3.49	5.36	12.5	0.6	3	204.98	3.37	5.06
6.5	0.5	3	158.87	2.67	4.48	12.5	0.7	3	239.14	3.93	4.60
6.5	0.6	3	190.64	3.20	3.99	13	0.4	3	113.73	2.09	6.95
6.5	0.7	3	222.42	3.74	3.66	13	0.5	3	142.17	2.62	5.84
7	0.3	3.5	411.07	2.89	6.81	13	0.6	3	170.60	3.14	5.13
7	0.4	3.5	548.10	3.85	5.54	13	0.7	3	199.03	3.66	4.65
7	0.5	3	113.82	2.37	4.58	13.5	0.5	3	184.21	2.90	5.96
7	0.6	3	136.58	2.84	4.06	13.5	0.6	3	221.05	3.48	5.25
14	0.5	3	154.98	2.71	6.05	18.5	0.5	3	138.67	2.61	6.99
14	0.6	3	185.97	3.25	5.32	18.5	0.6	3	166.41	3.14	6.10
14	0.7	3	216.97	3.80	4.81	18.5	0.7	3	194.14	3.66	5.48
14.5	0.5	3	131.78	2.55	6.16	19	0.6	3	212.17	3.46	6.21
14.5	0.6	3	158.13	3.06	5.39	19.5	0.6	3	187.48	3.29	6.29
14.5	0.7	3	184.49	3.57	4.87	19.5	0.7	3	218.72	3.84	5.65
15	0.5	3	170.03	2.82	6.27	20	0.6	3	166.59	3.14	6.36
15	0.6	3	204.03	3.38	5.50	20	0.7	3	194.36	3.67	5.71
15	0.7	3	238.04	3.94	4.98	20.5	0.6	3	211.49	3.46	6.48
15.5	0.5	3	145.73	2.65	6.37	21	0.6	3	188.53	3.31	6.55
15.5	0.6	3	174.87	3.18	5.57	21	0.7	3	219.95	3.86	5.88
15.5	0.7	3	204.02	3.72	5.03	21.5	0.6	3	168.88	3.17	6.63
16	0.5	3	187.49	2.93	6.48	21.5	0.7	3	197.03	3.69	5.94
16	0.6	3	224.99	3.52	5.69	22	0.6	3	213.51	3.48	6.74
16.5	0.5	3	161.80	2.77	6.58	22.5	0.6	3	191.74	3.33	6.82
16.5	0.6	3	194.16	3.32	5.76	22.5	0.7	3	223.70	3.89	6.11
16.5	0.7	3	226.52	3.88	5.20	23	0.6	3	172.94	3.20	6.89
17	0.5	3	140.72	2.62	6.68	23	0.7	3	201.76	3.73	6.17
17	0.6	3	168.86	3.15	5.84	24	0.7	3	229.62	3.94	6.35
17	0.7	3	197.01	3.67	5.26	24.5	0.7	3	208.31	3.79	6.41
17.5	0.5	3	180.23	2.90	6.80	25.5	0.7	3	237.49	4.00	6.58
17.5	0.6	3	216.28	3.48	5.95	26	0.7	3	216.53	3.85	6.64
18	0.5	3	157.57	2.75	6.89	26.5	0.7	3	198.06	3.72	6.70
18	0.6	3	189.09	3.30	6.02	27.5	0.7	3	226.36	3.93	6.88
18	0.7	3	220.60	3.85	5.43	28	0.7	3	207.89	3.80	6.94

* denotes the parameter combination with relative efficiency score of 1

Table 3.
Efficient design parameters for *C-MOESD* model
in comparison with pure economic model

Design	n	h	k	ATS_0	ATS_1	E_L
C-MOESD	3.5	0.4	3.5	617.66	3.98	4.66
	4	0.3	3.5	514.07	3.13	5.82
Pure Economic	1	2.5	2	63.01	6.13	2.22

7. Conclusion and Future Researches

In this study, we proposed a multi-objective economic-statistical model for the C control chart. The *DEA* approach was applied to specify efficient design parameters. Through a numerical example, the algorithm procedure was investigated in addition to comparison with pure economic design. According to the results, Multi-Objective Economic-Statistical design showed statistically improved performance compared to the pure economic design for the C control chart.

Using the other multi-objective decision making methods, and using the proposed model in occurrence of multiple assignable causes can be considered as future researches. In addition, models including assignable causes with random shifts and for variable sample sizes in addressing various control charts worth to be investigated.

References

- [1] Woodall, W. H. (1997). Control charts based on attribute data: bibliography and review. *Journal of Quality Technology*, 29(2), 172-183.
- [2] Montgomery, D. C. (2008). *Introduction to Statistical Quality Control* (6th ed). New York: Wiley.
- [3] Duncan, A. J. (1956). The economic design of \bar{x} charts used to maintain current control of a process. *Journal of the American Statistical Association*, 51(247), 228-242.
- [4] Saniga, E. M. (1989). Economic statistical control chart designs with an application to \bar{x} and R charts. *Technometrics*, 31(3), 313-320.
- [5] Chen, Y. K., Liao, H. C. (2004). Multi-criteria design of an \bar{x} control chart. *Computers & Industrial Engineering*, 46(4), 877-891.
- [6] Bashiri, M., Amiri, A., Asgari, A., Doroudyan, M. H. (2013). Multi-objective efficient design of np control chart using data envelopment analysis. *International Journal of Engineering*, 26(6), 621-630.
- [7] Charnes, A., Cooper, W. W., Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research*, 2(6), 429-444