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Computation of output losses due to congestion in data envelopment analysis

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Abstract

Data Envelopment Analysis (DEA) is an approach for evaluating performances of Decision Making Units (DMUs). The performances of DMUs are affected by the amount of sources that DMUs used. Usually increases in inputs cause increases in outputs. However, there are situations where increases in one or more inputs generate a reduction in one or more outputs. In such situations there is congestion in inputs or production process. In this study, we review two approaches that are available in the DEA literature for evaluating congestion. Afterwards, we focus on output losses due to congestion, and a model is introduced to compute output reduction. Then, the mentioned models are applied on an empirical example and the results are presented and interpreted.

Keywords: Data Envelopment Analysis, Decision making unit, Inefficiency, Congestion.

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1. Introduction

Charnes, Cooper and Rhodes (CCR) developed Data envelopment analysis (DEA) in 1978 by their famous article [3]. Later in 1984, Banker, Charnes and Cooper (BCC) [1] presented a variable returns to scale type of the CCR model that was called BCC model. Since 1978 there has been a spurt of extensive investigations on DEA. Today, many scholars all over the world are working in this domain [4, 7, 8, 10, 13, 17, 18]. The objective of DEA models is assessing performances of decision making units (DMUs). The performances of DMUs are affected by the amount of sources that DMUs used. Usually increases in inputs cause increases in outputs. However, there are situations where increases in

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one or more inputs generate a reduction in one or more outputs. For example, in an underground coal mine, too many men decrease the output of coal. In such situations there is congestion in inputs or production process [5]. The definition we use is as follows:

Definition 1. Congestion is said to occur when the output that is maximally possible can be increased by reducing one or more inputs without improving any other inputs or outputs. Conversely congestion is said to occur when some of the outputs that are maximally possible are reduced by increasing one or more inputs without improving any other inputs or outputs [6].

The first paper that studied congestion -long disregarded in the economics- was the one by Fare and Svensson [15] in 1980. In that paper three forms of congestion were defined and described for a production function of single output. Later, Fare and Grosskopf [12] and Fare et al. [14] expanded a data envelopment analysis (DEA) model to compute the impact of congestion. Their model is a radial approach that calculates the congestion impact as ratio of the observed amounts to the expected amounts. It shows only existence or non-existence of congestion but it cannot identify congestion correctly in all cases. It is because their approach focuses attention on efficiency computation while congestion is a kind of inefficiency.

Another approach originally studied by Cooper et al. [11] is a slacks-based approach that calculates the congestion impact as the difference between the observed amounts and the expected amounts. This approach has some strong points compared to the previous method. It determines the congested inputs and provides a measure for the amount of congestion in each input. Later, Cooper et al. [9] expanded a unified additive model for determining congestion using additive models.

However, in both economics and OR studies the speed of progress of investigations into congestion has accelerated after the Fare et al. works [13, 14].

Jahanshahloo and Khodabakhshi [16, 18] introduced an input relaxation model for improving outputs and calculated the input congestion based on the proposed model.

In addition to the above publications, other studies have done separately by Wei and Yan [23] and Tone and Sahoo [22]. These two studies declare the congestion impact in terms of immoderate inputs. According to the definition, congestion occurs when increases in some inputs results in decreases in some outputs. Hence, congestion can also be determined as shortfalls in outputs. In this way, it is easier to declare the congestion in terms of outputs. The models introduced by Wei and Yan and Tone and Sahoo are expanded from the output viewpoint. Wei and Yan in an another work [24] have simultaneously studied the problems of congestion and different kinds of returns to scale by output oriented DEA models and recognize the necessary and sufficient conditions for the evidence of congestion and different kinds of returns to scale.

Sueyoshi and Sekitani presented an approach [20] which is able to assess congestion under the occurence of multiple solutions.

Noura et al. [19] in 2010 proposed a new method that considerably reduces the computational effort required for calculating congestion. According to the definition 1, it is discovered that congestion occurs in large sizes. The idea of this approach is to select the maximum amounts of each input between efficient DMUs. Then, it compares these amounts with inputs of other DMUs. It is because efficient DMUs are not congested.

The rest of this paper is organized as follows. Section 2 reviews two approaches for evaluating congestion. In section 3 we discuss calculating output losses due to congestion. Finally, in section 4 we present the results of the mentioned methods on an example adopted from Tone and Sahoo [22]. Section 5 provides conclusions and a summary of the article.

2. Two congestion models

Consider n DMUs. The jth DMU $(j = 1,\dots,n)$ consumes an input vector (X_j) and produces an output vector (Y_j). Also, it is assumed that $X_j \in R^m$, $Y_j \in R^s$ and $X_j \gneqq 0$, $Y_j \gneqq 0$ for all $j = 1,\dots,n$. As given in Charnes et al. [3] the efficiency of a specific DMU (o) can be evaluated by either of the following two DEA models:

Input orientation model:

$$
\begin{aligned}
\theta^* \quad &= \quad \text{Min} \quad \theta \\
s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io} ; \qquad i = 1,2,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} ; \qquad r = 1,2,\dots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 ; \\
& \lambda_j \geq 0 ; \qquad j = 1,\dots,n
\end{aligned}
\tag{1}
$$

Output orientation model:

$$
\begin{aligned}
\varphi^* \quad &= \quad \text{Max} \quad \varphi \\
s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} ; \qquad i = 1,2,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq \varphi y_{ro} ; \qquad r = 1,2,\dots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 ; \\
& \lambda_j \geq 0 ; \qquad j = 1,\dots,n
\end{aligned}
\tag{2}
$$

Here, x_{io} and y_{ro} are respectively the ith input and the rth output for the DMU under evaluation. Corresponding to the $m+s$ input and output constraints in (1) or (2), some none-zero input and output slacks, s_i^- and s_r^+, may exist in some multiple optimal solutions. After which, the following models are applied:

$$\begin{aligned}
\text{Max} \quad & \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\
s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta^* x_{io}\,; \quad i = 1,2,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}\,; \quad r = 1,2,\dots,s \qquad (3) \\
& \sum_{j=1}^{n} \lambda_j = 1\,; \\
& (\lambda_j, s_i^-, s_r^+) \geq 0\,; \quad j = 1,\dots,n \quad i = 1,\dots,m \quad r = 1,\dots,s
\end{aligned}$$

$$\begin{aligned}
\text{Max} \quad & \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\
s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}\,; \quad i = 1,2,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \varphi^* y_{ro}\,; \quad r = 1,2,\dots,s \qquad (4) \\
& \sum_{j=1}^{n} \lambda_j = 1\,; \\
& (\lambda_j, s_i^-, s_r^+) \geq 0\,; \quad j = 1,\dots,n \quad i = 1,\dots,m \quad r = 1,\dots,s
\end{aligned}$$

Definition 2. An optimal solution of s_i^- and s_r^+ in (3) and (4) are respectively called DEA input and output slack values.

Definition 3. A DMU_o evaluated in the above manner will be found to be DEA efficient if and only if the following conditions are satisfied:
(i) $\theta^* = 1$ (or $\varphi^* = 1$) *(ii)* $s_i^{-*} = s_r^{+*} = 0$ $(\forall i, r)$

2.1. The BCSW approach

This approach was first published by Cooper et al. [11] in 1996. Then, Brockett et al. [2] in 1998 examined it on real data and expanded it to check tradeoffs between employment and output which could be used to increase employment or increase output (or both) in Chinese production.
This approach also progresses in a two-stage way. It's idea is, initially, to define the projection point on the efficiency frontier via (1) and (3) (or (2) and (4)), then fixes the outputs to those of the projection

point and computes the maximum amount of inputs that can be augmented to the projection's inputs by the following models:

Input orientation:

$$
\begin{aligned}
\text{Max} \quad & \sum_{i=1}^{m} \delta_i^+ \\
s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} - \delta_i^+ = \hat{x}_{io} = \theta^* x_{io} - s_i^{-*}; \quad i = 1,2,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} = \hat{y}_{ro} = y_{ro} + s_r^{+*}; \quad r = 1,2,\dots,s \qquad (5) \\
& \sum_{j=1}^{n} \lambda_j = 1; \quad \delta_i^+ \le s_i^{-*}; \quad i = 1,\dots,m \\
& (\lambda_j, \delta_i^+) \ge 0; \quad j = 1,\dots,n \quad i = 1,\dots,m
\end{aligned}
$$

Output orientation:

$$
\begin{aligned}
\text{Max} \quad & \sum_{i=1}^{m} \delta_i^+ \\
s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} - \delta_i^+ = \hat{x}_{io} = x_{io} - s_i^{-*}; \quad i = 1,2,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} = \hat{y}_{ro} = \varphi^* y_{ro} + s_r^{+*}; \quad r = 1,2,\dots,s \qquad (6) \\
& \sum_{j=1}^{n} \lambda_j = 1; \quad \delta_i^+ \le s_i^{-*}; \quad i = 1,\dots,m \\
& (\lambda_j, \delta_i^+) \ge 0; \quad j = 1,\dots,n \quad i = 1,\dots,m
\end{aligned}
$$

Afterwards, the amount of congestion in each input (s_i^c) can be determined by the difference between each pair of s_i^{-*} and δ_i^{+*}, where δ_i^{+*} are optimal values in (5) or (6). That is:

$$s_i^c = s_i^{-*} - \delta_i^{+*}; \qquad i = 1,\dots,m \qquad (7)$$

We substitute the previous equality by the following equation, $s_i^{-*} = s_i^c + \delta_i^{+*}$ $(i = 1,\dots,m)$. The "total slack" obtained in stage 1, represented by s_i^{-*}, is separated into a value δ_i^{+*}, indicating a "technical inefficiency" component, and a value s_i^c, indicating a "congesting" component in input i.

It should be nooted that using an input oriented BCSW approach usually is not fruitful. It is because moving in a surface with fixed outputs causes the input slacks reach to its maximum value, and therefore, the output value of the projection point is equal to the output of DMU_o , and hence, output slacks become zero. This indicates pure technical inefficiency and no congestion, because input reduction does not alter the output.

2.2. The new method

Noura, Hosseinzade Lotfi, Jahanshahloo, Rashidi and Parker [19] offered a new method that requires fewer calculations and compared its performance with those of existing methodologies. In this method, first models (2) and (4) are solved for each DMU_j $(j = 1, \dots, n)$ and the optimal solution $(\varphi^*, \lambda^*, s^{-*}, s^{+*})$ is obtained. Denoting the φ^* corresponding to DMU_j by φ_j^*, set E is defined as follows:

$$E = \left\{ j \mid \varphi_j^* = 1 \right\}$$

Among the DMUs in set *E*, there exists at least one that has the highest consumption in its first input component compared with the first input component of the remaining DMUs of set E. That is to say,

$$\exists\, l\ (l \in E)\ ;\ \forall\, j\ (j \in E)\ \Longrightarrow\ x_{1l}\ \geq\ x_{1j}$$

x_{1l} is denoted by x_1^*. Then, among the DMUs in *E*, a DMU is found that has the highest consumption in its second input component compared to the remaining DMUs in *E*. In other words,

$$\exists\, t\ (t \in E)\ ;\ \forall\, j\ (j \in E)\ \Longrightarrow\ x_{2t}\ \geq\ x_{2j}$$

x_{2t} is denoted by x_2^*. In a similar manner, for all input components, *i=1,...,m*, a DMU can be identified in *E* whose input consumption is higher than that of all other DMUs in the set E. Such an input is denoted by x_i^* $(i = 1, \dots, m)$. Afterwards, congestion is defined as follows:

Definition 4. Congestion is present if and only if, in an optimal solution $(\varphi^*, \lambda^*, s^{-*}, s^{+*})$ of (4) for evaluating DMU_o, at least one of the following two conditions is satisfied:

(i) $\varphi^* > 1$, and there is at least one $x_{io} > x_i^*$ $(i = 1, \dots, m)$.

(ii) There exists at least one $s_r^{+*} > 0, (r = 1, \dots, s)$ and at least one $x_{io} > x_i^*$ $(i = 1, \dots, m)$.

Then, the amount of congestion in the *i*th input of DMU_o is denoted by $s_i^{c'}$, where $x_{io} > x_i^*$ and is defined as:

$$s_i^{c'} = x_{io} - x_i^*\ ; \qquad\qquad i = 1, \dots, m$$

Also, congestion is considered not present when $x_{io} \leq x_i^*$. Then sum of all $s_i^{c'}$ is the amount of congestion in DMU_o. Then they demonstrated three theorems. In the first theorem, they defined DMU_o^* as:

$$DMU_o^* = (x_1^*, x_2^*, \dots, x_m^*, \varphi^* y_{10} + s_1^{+*}, \varphi^* y_{20} + s_2^{+*}, \dots, \varphi^* y_{s0} + s_s^{+*})$$

and proved that this virtual DMU is in the production possibility set. In the second theorem they proved that the congestion calculated by Cooper (s_i^c) is equal to that calculated by this new method $(s_i^{c'})$, where $x_{io} > x_i^*$. In the third theorem they proved that congestion is not present in DMU_o, when $x_{io} \leq x_i^*$ $(\forall i)$.

3. Output reduction due to input congestion

Identifying and eliminating congestion have two important advantages:

1. Congestion there exists in inputs and inputs have costs, hence, eliminating congestion minimizes the cost of production.
2. According to the definition 1, congestion causes reduction in outputs, therefore, eliminating congestion increases outputs.

Suppose that using the approaches declared in previous section, we identify congested inputs and the amount of congestion in each input. Now we want to calculate the outputs losses due to input congestion. That is, we are going to see that eliminating congestion causes how much increase in outputs.

The congested DMU may have some output losses due to inefficiency. For calculating output losses due to congestion, DMU_o is first projected on the boundary of T_{NEW} with output oriented NEW model:

$$\begin{aligned} \text{Max} \quad & \varphi \\ s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} = x_{io}\,; && i = 1,2,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq \varphi\, y_{ro}\,; && r = 1,2,\dots,s \\ & \sum_{j=1}^{n} \lambda_j = 1\,; \\ & \lambda_j \geq 0\,; && j = 1,\dots,n \end{aligned} \tag{8}$$

The projection point $(\hat{x}_o, \hat{y}_0) = (x_o, \varphi^* y_0)$ is on the boundary of T_{NEW} and all of its output losses are due to congestion. The amount of congestion in *i*th input (s_i^c) is definite, thus, subtracting this value from *i*th input; we find the maximum amount that augmented to *r*th output. Therefore, we use following model:

$$\begin{aligned} \text{Max} \quad & \sum_{r=1}^{s} \xi_r \\ s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} = x_{io} - s_i^c\,; && i = 1,2,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} = \varphi^*\, y_{ro} + \xi_r\,; && r = 1,2,\dots,s \\ & \sum_{j=1}^{n} \lambda_j = 1\,; \\ & (\lambda_j, \xi_r) \geq 0\,; && j = 1,\dots,n \quad \text{r} = 1,\dots,\text{s} \end{aligned} \tag{9}$$

After solving (9), the amount of ξ_r^* indicates the output losses due to congestion in the *r*th output, and the optimum value of the objective function$\sum_{r=1}^{s} \xi_r^*$, indicates the total amount of output losses due to congestion.

Example 1

To more clear the discussion, we apply model (9) to an example in the case of one input and one output. Consider following DMUs:

A(1,1) , B(2,3) , C(4,3) , D(5,$\frac{3}{2}$) , E($\frac{13}{3}$,2)

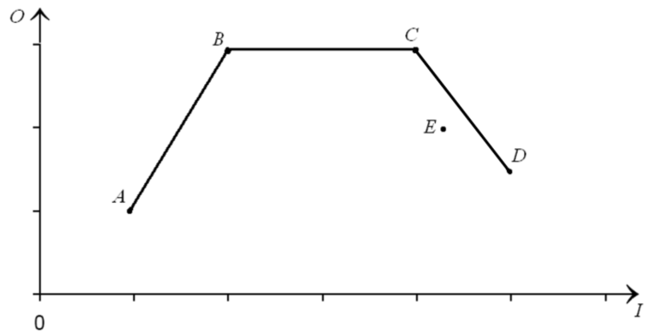

Figure 1: The PPS of example 1

See figure 1. Using the new method we find that D and E are congested. The amount of congestion in D's input is 1 and in E's input is $\frac{1}{3}$. Thus, $s_D^c = 1$ and $s_E^c = \frac{1}{3}$. D is on the frontier of T_{NEW}, thus, all output losses of D is due to congestion. Applying model (9) for D we have:

$$\lambda_A^* = \lambda_B^* = \lambda_D^* = \lambda_E^* = 0 \,, \lambda_C^* = 1 \,, \xi^* = \frac{3}{2} \ ,$$

that $\xi^* = \frac{3}{2}$ shows correctly output losses of D due to congestion. However E is not on the frontier of T_{NEW}, hence, it has some output losses due to inefficiency. First using model (8) we project E on the frontier of T_{NEW}. Then we have $\varphi^* = \frac{5}{4}$ and the projection point is $(\hat{x}_E, \hat{y}_E) = (\frac{13}{3}, \frac{5}{2})$. Now we evaluate $(\hat{x}_E, \hat{y}_E)$ with model (9), the optimal solution is:

$$\lambda_A^* = \lambda_B^* = \lambda_D^* = \lambda_E^* = 0 \,, \lambda_C^* = 1 \,, \xi^* = \frac{1}{2} \ ,$$

that $\xi^* = \frac{1}{2}$ shows correctly output losses of E due to congestion. s_2^c

Table 1:
Chain stores data set

Year	DMU	Number	Area	Sales
1975	1	2412	5480	41,091
1976	2	3163	6233	48,367
1977	3	3350	6798	56,000
1978	4	3371	7274	60,940
1979	5	3778	7992	69,046
1980	6	4020	8500	77,347
1981	7	5029	9246	85,805
1982	8	5164	9639	90,433
1983	9	5285	9981	95,640
1984	10	5618	10,276	100,257
1985	11	5981	10,521	105,944
1986	12	6217	10,766	109,857
1987	13	6455	11,144	116,114
1988	14	6674	11,418	125,404
1989	15	6829	11,717	131,862
1990	16	6995	11,987	140,817
1991	17	7338	12,463	150,583
1992	18	7946	13,426	152,943
1993	19	8236	14,147	155,128
1994	20	7722	15,014	158,714
1995	21	7727	15,022	161,739
1996	22	7822	16,191	169,786
1997	23	7531	16,969	167,195
1998	24	7201	17,627	167,187
1999	25	7281	18,364	165,480
2000	26	7053	19,698	162,847
2001	27	6067	16,176	154,671

4. An empirical study

Here we evaluate congestion over time of the operations of a set of chain stores in Japan for a period of 27 years from 1975 through 2001. This data that adopted from [22] have one output: annual sales (unit: hundred million yen), and two inputs: the number of stores and the total area of stores (unit: 1000 m^2). Table 1 reports such data.

Table 2:

The amounts of congestion

DMU	Year	BCSW_O		New method	
		s_1^c	s_2^c	s_1^c	s_2^c
1975	1				
1976	2	219.81			
1977	3	8.25			
1978	4				
1979	5				
1980	6				
1981	7				
1982	8				
1983	9				
1984	10				
1985	11	12.94			
1986	12	76.11			
1987	13	47.46			
1988	14	73.17			
1989	15	17.25			
1990	16				
1991	17				
1992	18	482.98		124	
1993	19	679.37		414	
1994	20	52.81			
1995	21	56.77			
1996	22				
1997	23		105.09		
1998	24				
1999	25		921.99		737
2000	26		2,260.37		2071
2001	27				

As is seen in table 1, there is a slow but steady rise in the number of chain stores till 1993 after which the trend continues declining consistently. Except the last year, the total area is consistently rising throughout. The output, annual sales has an increasing trend until 1996 after which the trend is consistently declining.

Table 2 shows the congested inputs and the amounts of congestion in mentioned models. In this example the results of the new method is different from the results of the Cooper one.

According to definition 1, when a DMU is congested it cannot obtain more output by increasing the input. Now, suppose the years 1976 and 1991. If we compare the inputs and the output of these two years, we have:

inputs of DMU_{22} = (7822,16191) > (3163,6233) = inputs of DMU_2, output of DMU_{22} = 169786 > 48367 = output of DMU_2.

Therefore, DMU_{22} has been able to obtain more output by consuming more inputs than DMU_2. Thus, decrease in the output of DMU_2 can be associated with inefficiency and no congestion. From another viewpoint, according to definition 1, congestion occurs in large sizes; hence, we should verify congestion in large sizes. The new method acts in this way. Consequently, it seems that the new method is more valid.

Table 3 shows the results of output reduction due to congestion. Here we use the amounts of input congestion calculated by Cooper et al. method and the new method and then apply equations (8) and (9).

Table 3:

Results of output reduction

Year	DMU	Cooper et al. method	New method
1975	1		
1976	2	4530.82	
1977	3	169.94	
1978	4		
1979	5		
1980	6		
1981	7		
1982	8		
1983	9		
1984	10		
1985	11	266.72	
1986	12	4827.12	
1987	13	1162.97	
1988	14	8793.64	
1989	15	355.53	
1990	16		
1991	17		
1992	18	2600.39	667.64
1993	19	4129.31	2700.52
1994	20	284.31	
1995	21	305.65	
1996	22		
1997	23	226.17	
1998	24		
1999	25	1984.28	1586.15
2000	26	2706.52	2479.77
2001	27		

5. Summary and conclusion

In this paper, we reviewed two approaches that are available in the DEA literature for evaluating congestion. The congestion model introduced by Cooper et al. [9] determines the congested inputs and the amounts of congestion. This model, first determines the projection point of the DMU under evaluation and then by assessing the projection point, it finds the maximum value that can augmented to the projection's inputs and remain in T_{NEW}. This model suffers from an occurrence of multiple solutions.

The new method introduced by Noura et al. [19] identifies the congested input and the amount of congestion with fewer computations compared with other models. It seems that the results of the new method are more valid than other ones. Thus, we suggest more serious researches on the new method as a future work. Also, this model does not have the problem of multiple solutions.

Then, we introduced a model to compute output losses due to congestion and afterwards we applied the mentioned method on an empirical example and presented the results.

References

[1] Banker, R.D., Charnes, A., Cooper, W.W., Some models for estimating technical and scale inefficiencies in data envelopment analysis. Management Science, 30 (1984) 1078-1092.

[2] Brockett, P.L, Cooper, W.W., Shin, H.C., Wang, Y., Inefficiency and congestion in Chinese production before and after the 1978 economic reforms. Socio-Economic Planning Sciences, 32 (1998) 1-20.

[3] Charnes, A., Cooper, W.W., Rhodes, E., Measuring the efficiency of DMUs. European Journal of Operational Research, 2 (1978) 429-444.

[4] Cooper, W.W., Deng, H., Gu, B., Li, S., Thrall, R.M., Using DEA to improve the management of congestion in Chinese industries. Socio-Economic Planning Sciences, 35 (2001) 227-242.

[5] Cooper, W.W., Deng, H., Huang, Z.M., Li, S.X., A one-model approach to congestion in data envelopment analysis. Socio-Economic Planning Sciences, 36 (2002) 231-238.

[6] Cooper, W.W., Deng, H., Seiford, L.M, Zhu, J., Congestion: its identifications and management with DEA. In: Cooper, w.w, Seiford, L.M., Zhu, J., editors. Handbook of data envelopment analysis. Boston: Kluwer (2004) 177-201.

[7] Cooper, W.W., Gu, B., Li, S., Comparisons and evaluations of alternative approaches to the treatment of congestion in DEA. European Journal of Operational Research, 132 (2001) 62-74.

[8] Cooper, W.W., Park, K.S., Pastor, J.T., RAM: A Range Adjusted Measure of inefficiency for use with additive models and relations to other models and measures in DEA. Journal of Productivity Analysis, 11 (1999) 5-42.

[9] Cooper, W.W., Seiford, L.M., Zhu, J., A unified additive model approach for evaluating inefficiency and congestion with associated measures in DEA. Socio-Economic Planning Sciences, 34 (2000) 1-25.

[10] Cooper, W.W., Seiford, L.M., Zhu, J., Slacks and congestion: response to a comment by R.Fare and S.Grosskopf. Socio-Economic Planning Sciences, 35 (2002) 205-215.

[11] Cooper, W.W., Thompson, R.G., Thrall, R.M., Introduction: extensions and new developments in DEA. Annals of operations Research, 66 (1996) 3-45.

[12] Fare, R., Grosskopf, S., Measuring congestion in production. Zeitschrift fur Nationalokonomie, 43 (1983) 257-271.

[13] Fare, R., Grosskopf, S., When can slacks be used to identify congestion? An answer to W.W. Cooper, L. Seiford and J. Zhu. Socio-Economic Planning Sciences, 35 (2001) 217-221.

[14] Fare, R., Grosskopf, S., Lovell, C.A.C., The Measurement of Efficiency of Production. Kluwer-Nijhoff Publishing, Boston, MA (1985).

[15] Fare, R., Svensson, L., Congestion of production factors. Econometrica, 48 (1980) 1745-1753.

[16] Jahanshahloo, G.R., Khodabakhshi, M., Suitable combination of inputs for improving outputs in DEA with determining input congestion considering textile industry of China. Applied Mathematics and Computation, 151 (2004) 263-273.

[17] Kao, C., Congestion measurement and elimination under the framework of data envelopment analysis. International Journal of Production Economics, 123 (2010) 257-265.

[18] Khodabakhshi, M., A one-model approach based on relaxed combinations of inputs for evaluating input congestion in DEA. Journal of Computational and Applied Mathematics, 230 (2009) 443-450.

[19] Noura, A. A., Hosseinzadeh Lotfi, F., Jahanshahloo, G.R., Fanati Rashidi, S., Parker, B.R., A new method for measuring congestion in data envelopment analysis. Socio-Economic Planning Sciences, 44 (2010) 240-246.

[20] Sueyoshi, T., Sekitani, K., DEA congestion and returns to scale under an occurrence of multiple optimal projections. European Journal of Operational Research, 194 (2009) 592-607.

[21] Tone, K., Cooper, W.W., Seiford, L.M., Data Envelopment Analysis: a comprehensive text with models, applications, references and DEA-solver software. Second Edition, Springer, New York (2007).

[22] Tone, K., Sahoo, B.K., Degree of scale economies and congestion: A unified DEA approach. European Journal of Operational Research, 158 (2004) 755-772.

[23] Wei, Q.L., Yan, H., Congestion and returns to scale in data envelopment Analysis. European Journal of Operational Research, 153 (2004) 641-660.

[24] Wei, Q.L., Yan, H., Weak congestion in output additive data envelopment analysis. Socio-Economic Planning Sciences, 43 (2009) 40-54.