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Finding a suitable benchmark for commercial bank branches using DEA

M.R.Mozaffari*, F.hosseinzadeh Lotfi [b], J.Gerami [a]

(a) *Department of Mathematics, Science and Research Branch,Islamic Azad University,Fars, Iran*

(b) *Department of Mathematics, Science and Research Branch,Islamic Azad University,Tehran, Iran*

Abstract

This paper proposes a suitable benchmark for inefficient commercial bank branches by using Data Envelopment Analysis (DEA). In order to render an inefficient bank branch efficient, it is necessary to decrease inputs and increase outputs. As there are priorities for decreasing some certain inputs and increasing some certain outputs over other inputs and outputs, respectively, it is necessary to consider and incorporate the managers' view regarding the priorities in the models which are applied. So, in this paper, using the enhanced Russell model for an inefficient bank branch, we propose the decrease in inputs and the increase in outputs, taking the managers' priorities into account. Finally, in a numerical example, we apply the proposed procedure to the authentic information from 30 commercial bank branches, to show the application of the procedure.

Keywords: Data Envelopment Analysis, Benchmark, Linear programming.

1 Introduction

Data Envelopment Analysis (DEA), by Charnes, Cooper and Rhodes [3], is a method for evaluating the relative efficiency of comparable entities referred to as Decision Making Units (DMUs). DEA forms a production possibility frontier, or an efficient surface. If a DMU lies on the surface, i.e., there is no other DMU that can either produce the same outputs by consuming less inputs (input-oriented DEA) or produce more outputs by consuming the same amount of inputs (output-oriented DEA), it is referred to as an efficient unit, otherwise inefficient. DEA also provides efficiency scores and reference units for inefficient DMUs. The efficiency score tells the percentage by which a DMU should decrease its inputs (input-oriented DEA) or increase its outputs (output oriented DEA) in order to become efficient. Reference units are hypothetical units on the efficient surface, which can be regarded as target units for inefficient units. In the traditional DEA, they are obtained by projecting an inefficient DMU radially onto the efficient surface. The production theoretical argument for this principle is that the DMU preserves its current input-output mix. However, from a managerial point of

* Corresponding author, Email address: mozaffari23@yahoo.com, Tel:(+98728)4692104

view, it is possible that some other solution on the efficient surface might be a more preferable target, i.e., there exists an input-output mix that is more suitable for the inefficient unit than the one obtained through radial projection. One line of research in DEA concentrates on finding these targets for inefficient DMUs (Golany [6], Thanassoulis and Dyson [11] and Zhu [12]). The DMU can use the targets as goals or benchmarking units when working its way toward efficiency.

In Golany [6], a set of hypothetical reference units is generated and presented to the DMU. The DMU may choose one of them as a target, or new reference units may be generated. In Thanassoulis and Dyson [11], the DMU may articulate its preferences as a set of preference weights over improvements for different input-output levels, or as an ideal target (neither necessarily optimal nor feasible). The target corresponding to the preference weights or the ideal target is then calculated. In the model by Zhu [12], the DMU articulates its preferences as weights reflecting the relative degree of desirability of the potential adjustments of current input or output levels. One major problem with a radial measure of technical efficiency is that it does not reflect all identifiable potential for increasing outputs and reducing inputs.

In economics, the concept of efficiency is intimately related to the idea of Pareto optimality. An input-output bundle is not Pareto optimal if there remains the possibility of any net increase in outputs or net reduction in inputs. When positive output and input slacks are present at the optimal solution of a CCR or BCC linear programming problem in DEA, the corresponding radial projection of an observed input-output combination does not meet the criterion of Pareto optimality and should not qualify as an efficient point. The nonradial Russell measure was proposed by Färe and Lovell [5]. Russell [9] pointed out that this measure fails to satisfy a number of desirable properties of an efficiency measure.For further explanation, notice the following example. Consider the evaluation of various branches of a commercial bank. We have the following inputs/outputs:

Inputs: number of employees in the branch, area of the branch building, operation costs and equipment. Outputs: resources, granted loans, total revenue, services and customer satisfaction.

In the classic models CCR, BCC, revised Russell and SBM, in order to make an inefficient branch efficient, we have to decrease all inputs and increase all outputs to project the DMU onto the efficient frontier. In evaluating these branches, the management declares that the priority for making inefficient branches efficient is to decrease costs and the number of employees, and to increase services and customer satisfaction. To make it clear, as the first priority, they should try to project the branch onto the strong efficiency frontier by decreasing costs and the number of employees, and increasing services and customer satisfaction. Otherwise, on the second priority, they should cover the weaknesses existing in the former stage by decreasing the equipment and the area of the branch building, and increasing resources, granted loans, and total revenue. The importance of the matter is that the equipment and the area of the branch building are of vital importance to the management, and decreasing them may damage the banking process. Considering the fact that there are some restrictions in collecting resources, granting loans, and earning revenues, the management has decided to put them in the second priority.

This paper is organized as follows. Section 2 provides a brief overview of DEA methodology and benchmarking. Section 3 proposes a modified model to incorporate the managers views in the enhanced Russell model. Section 4 contains an application of the proposed model by a numerical example for 30 bank branches. Section 5 provides the conclusion.

2 Preliminaries

Consider n $DMUs$ with m inputs and s outputs. The input and output vectors of DMU_j $(j=1,\dots,n)$ are $X_j=(x_{1j},\dots,x_{mj})^t, Y_j=(y_{1j},\dots,y_{sj})^t$ where $X_j\ge 0, X_j\ne 0, Y_j\ge 0, Y_j\ne 0.$

By using the variable returns to scale, convexity, and possibility postulates, the non-empty production possibility set (PPS) is defined as follows:

$$T_v=\left\{(X,Y): X\ge \sum_{j=1}^{n}\lambda_j X_j, Y\le \sum_{j=1}^{n}\lambda_j Y_j, \sum_{j=1}^{n}\lambda_j=1, \lambda_j\ge 0, j=1\dots,n\right\}$$

By the above definition, the BCC model proposed by Banker et al. [1] is as follows:

$$\begin{aligned}
\text{Min}\quad & \theta-\varepsilon\Big[\sum_{i=1}^{m}s_i^{-}+\sum_{r=1}^{s}s_r^{+}\Big]\\
S.t\quad & \sum_{j=1}^{n}\lambda_j x_{ij}+s_i^{-}=\theta x_{ip}, \qquad i=1,\dots,m\\
& \sum_{j=1}^{n}\lambda_j y_{rj}-s_r^{+}=y_{rp}, \qquad r=1,\dots,s\\
& \sum_{j=1}^{n}\lambda_j=1\\
& \lambda_j\ge 0, \qquad j=1,\dots,n\\
& s_i^{-}\ge 0, \qquad i=1,\dots,m\\
& s_r^{+}\ge 0, \qquad r=1,\dots,s.
\end{aligned}\tag{1}$$

Clearly, the evaluated DMU_p is efficient if and only if $\theta^*=1$ and all slack variables in the optimal solution are zero in problem (1). Koopmans [3] defined an input-output vector as technically efficient if and only if increasing any output or decreasing any input is possible only by decreasing some other output or increasing some other input, respectively. A nonradial Pareto-Koopmans measure of technical efficiency for DMU_p can be computed as follows:

$$
\begin{aligned}
\text{Min} \quad & \tau = \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i}{\frac{1}{s}\sum_{r=1}^{s}\phi_r} \\
S.t \quad & \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta_i x_{ip}, \qquad i = 1,\dots,m \\
& \sum_{j=1}^{n}\lambda_j y_{rj} \ge \phi_r y_{rp}, \qquad r = 1,\dots,s \\
& \sum_{j=1}^{n}\lambda_j = 1 \\
& \lambda_j \ge 0, \qquad j = 1,\dots,n \\
& \theta_i \le 1, \qquad i = 1,\dots,m \\
& \phi_r \ge 1, \qquad r = 1,\dots,s
\end{aligned} \tag{2}
$$

In model (2) we suppose that all inputs and outputs are of the same importance in determining the efficiency. That is to say, the priority in increasing or decreasing the outputs or the inputs, respectively, is the same for all inputs and outputs. If there are different importance for different inputs/outputs, we can use the following objective function instead of objective function (2).

$$
\frac{\frac{1}{\sum_{i=1}^{m}\alpha_i}\sum_{i=1}^{m}\alpha_i\theta_i}{\frac{1}{\sum_{r=1}^{s}\beta_r}\sum_{r=1}^{s}\beta_r\phi_r} \quad ,
$$

where α_i , β_r show the importance of θ_i , φ_r , respectively.

Definition1. A DMU is called the benchmark for an inefficient unit if the DMU is on the efficiency frontier and its inputs are not greater than (less than or equal to) and its outputs are not less than those of that unit.

DMUs can benefit from benchmarking for continuous development and creating appropriate conditions. Benchmarking cannot generally be used to solve the problems in an organization, but it can be employed to accept activities prior to introducing novel processes. Benchmarking is, therefore, beyond comparison, as awareness of conditions increases willingness for change; Measures should be taken only when an organization is ready for change, and in order for a unit to be a successful benchmark, the necessary preparations must be carried out and appropriate conditions should be brought about.

3 A modification to the enhanced Russell model

Suppose that the management wants to give absolute priority to group of inputs over other inputs to decrease, and also to a group of outputs over others to increase, and reach the frontier. I_1 and I_2 are indices of the input set that are of the first and second priority, respectively, for the manager to decrease the inputs and O_1 and O_2 are indices of the output set that are of the first and second priority, respectively, for the manager to increase the outputs. For this propose, suppose inputs and outputs vectors of DMU_j $(j=1,\dots,n)$ are as follows, respectively.

$$X_j=(X_{I_1 j},X_{I_2 j}),\quad \text{where}\quad X_j>0,\quad I_1\cup I_2=\{1,2,\dots,m\},\quad |I_1|=p,\quad |I_2|=q$$

$$Y_j=(Y_{O_1 j},Y_{O_2 j}),\quad \text{where}\quad Y_j>0,\quad O_1\cup O_2=\{1,2,\dots,s\},\quad |O_1|=k,\quad |O_2|=l.$$

The model used in this form will be as follows:

$$\begin{array}{llll}
\text{Min} & R_e(X_p,Y_p)=\dfrac{\frac{1}{Mp+q}\left(M\sum_{i\in I_1}\theta_i+\sum_{i\in I_2}\theta_i\right)}{\frac{1}{Mk+l}\left(M\sum_{r\in O_1}\phi_r+\sum_{r\in O_2}\phi_r\right)} & & \\
S.t & \sum_{j=1}^{n}\lambda_j x_{ij}\le\theta_i x_{ip}, & i\in I_1\cup I_2 & (3)\\
 & \sum_{j=1}^{n}\lambda_j y_{rj}\ge\phi_r y_{rp}, & r\in O_1\cup O_2 & \\
 & \sum_{j=1}^{n}\lambda_j=1 & & \\
 & \lambda_j\ge 0, & j=1,\dots,n & \\
 & \theta_i\le 1, & i\in I_1\cup I_2 & \\
 & \phi_r\ge 1, & r\in O_1\cup O_2. &
\end{array}$$

In model (3), M is a large number which is used to show the view of the manager in the objective function. We can define R_e as the ratio of input average efficiency to output average efficiency. Therefore, we have:

$$\text{input average efficiency}=\frac{1}{Mp+q}\left(M\sum_{i\in I_1}\theta_i^*+\sum_{i\in I_2}\theta_i^*\right)$$

$$\text{output average efficiency}=\frac{1}{Mk+l}\left(M\sum_{r\in O_1}\varphi_r^*+\sum_{r\in O_2}\varphi_r^*\right)$$

Theorem 1. In each optimal solution of model (3) all input and output constraints are binding.

Proof. Let $(\lambda^*,\theta_{I_1}^*,\theta_{I_2}^*,\varphi_{O_1}^*,\varphi_{O_2}^*)$ be an optimal solution of the model (3). By contradiction, suppose that there exists $t\in I_1\cup I_2$, such that $\sum_{j=1}^{n}\lambda_j^* x_{tj}<\theta_t^* x_{tp}$, hence, $\sum_{j=1}^{n}\lambda_j^* x_{tj}=\bar{\theta}_t x_{tp}$ and $\bar{\theta}_t<\theta_t^*$. We put $(\bar{\lambda},\bar{\theta}_{I_1},\bar{\theta}_{I_2},\bar{\varphi}_{O_1},\bar{\varphi}_{O_2})$ is a feasible solution for model (3) such that $\bar{\lambda}=\lambda^*$, $\bar{\theta}_{I_2}=\theta_{I_2}^*$, $\bar{\varphi}_{O_1}=\varphi_{O_1}^*$, $\bar{\varphi}_{O_2}=\varphi_{O_2}^*$, $\bar{\theta}_i=\theta_i^*$, $i=1,\cdots p$ $,i\ne t$ and

$\overline{\theta}_t = \dfrac{\sum_{j=1}^{n}\lambda_j^* x_{tj}}{x_{tp}}$. We have $\dfrac{\frac{1}{Mp+q}(M\sum_{i\in I_1}\overline{\theta}_i + \sum_{i\in I_2}\overline{\theta}_i)}{\frac{1}{Mk+l}(M\sum_{r\in O_1}\overline{\varphi}_r + \sum_{r\in O_2}\overline{\varphi}_r)} < \dfrac{\frac{1}{Mp+q}(M\sum_{i\in I_1}\theta_i^* + \sum_{i\in I_2}\theta_i^*)}{\frac{1}{Mk+l}(M\sum_{r\in O_1}\varphi_r^* + \sum_{r\in O_2}\varphi_r^*)}$,
which is a contradiction. The proof is, therefore, completed.

Theorem 2. $0 < R_e \le 1$.

Proof. First, to prove $R_e \le 1$, it is obvious that $(\lambda, \theta_{I_1}, \theta_{I_2}, \varphi_{O_1}, \varphi_{O_2})$ is a feasible solution of (3) and the objective function value for this solution is 1 , and regarding minimization we have $R_e \le 1$. Now, to prove $R_e > 0$, by $\theta_{I_1} \ge 0, \theta_{I_2} \ge 0$, $\varphi_{O_1} \ge 1$, $\varphi_{O_2} \ge 1$ we have $R_e \ge 0$.

We have to show that $R_e \ne 0$. By contradiction, suppose $R_e = 0$, therefore $\theta_{I_1} = 0$, $\theta_{I_2} = 0$ and in the input constraints we have $\sum_{j=1}^{n}\lambda_j x_{ij} \le \theta_i x_{ip} = 0$, $i \in I_1 \cup I_2$, so, for all j, $\lambda_j = 0$.

Also, we have $0 = \sum_{j=1}^{n}\lambda_j y_{rj} \ge \varphi_r y_{rp}$, $r \in O_1 \cup O_2$, in the output constraints. Regarding the fact that all φ_{O_1}, φ_{O_2} are strictly positive, we must have for all r, $y_{rp} = 0$, and this contradicts the assumption.

Theorem 3. $R_e = 1$ if and only if DMU_p being evaluated is Koopmans-efficient.

Proof. Suppose $R_e = 1$. To prove that DMU_p is Pareto efficient, i.e., it is not dominated by any member in T_v, by contradiction assume there is $(X,Y) \in T_v$ such that $(-X,Y) \underset{\neq}{\ge} (-X_p, Y_p)$. Therefore, there exists $\overline{\lambda} \ge 0$ such that $(-\sum_{j=1}^{n}\overline{\lambda}_j X_j, \sum_{j=1}^{n}\overline{\lambda}_j Y_j) \underset{\neq}{\ge} (-X_p, Y_p)$. Thus, there exists $t \in I_1 \cup I_2$, $\sum_{j=1}^{n}\overline{\lambda}_j x_{tj} < x_{tp}$ or there exists $q \in O_1 \cup O_2$, $\sum_{j=1}^{n}\overline{\lambda}_j y_{qj} > y_{qp}$.

Without loss of generality, suppose there exists $t \in I_1 \cup I_2$ $\sum_{j=1}^{n}\overline{\lambda}_j x_{tj} < x_{tp}$, so define $\overline{\theta}_t = \dfrac{\sum_{j=1}^{n}\overline{\lambda}_j x_{tj}}{x_{tp}} < 1$. Set $\overline{\theta}_t = \dfrac{\sum_{j=1}^{n}\overline{\lambda}_j x_{tj}}{x_{tp}}$, $\overline{\theta}_i = 1$, $i \in I_1 \cup I_2, i \ne t$ $\overline{\varphi}_r = 1, r \in O_1 \cup O_2$.

Then $(\overline{\lambda}, \overline{\theta}_{I_1}, \overline{\theta}_{I_2}, \overline{\varphi}_{O_1}, \overline{\varphi}_{O_2})$ is a feasible solution of the model and we have

$$\frac{\frac{1}{Mp+q}(M\sum_{i\in I_1, i\ne t}\overline{\theta}_i + \sum_{i\in I_2}\overline{\theta}_i)}{\frac{1}{Mk+l}(M\sum_{r\in O_1}\overline{\varphi}_r + \sum_{r\in O_2}\overline{\varphi}_r)} < 1,$$

which is contradiction.

Now, suppose that DMU_p is Pareto efficient. To prove $R_e = 1$, by contradiction assume $R_e \neq 1$. By Theorem 1, we must have $R_e < 1$. We must have for all i, $\theta_i \leq 1$, and for all r, $\varphi_r \geq 1$, there exists t, $\theta_t^* < 1$ or there exists q, $\varphi_q^* > 1$.

Without loss of generality, assume that there exists t, $\theta_t^* < 1$. Thus we have

$$\sum_{j=1}^{n} \lambda_j^* x_{ij} \leq \theta_i^* x_{ip}, i \in I_1 \cup I_2, \sum_{j=1}^{n} \lambda_j^* y_{rj} \geq \varphi_r^* y_{rp}, \quad r \in O_1 \cup O_2 \quad \theta_i^* \leq 1, \quad i \in I_1 \cup I_2, \varphi_r^* \geq 1, r \in O_1 \cup O_2.$$

Considering $\theta_t^* < 1$, we have

$$\sum_{j=1}^{n} \lambda_j^* x_{ij} \leq \theta_i^* x_{ip} \leq x_{ip}, \quad i \in I_1 \cup I_2, i \neq t, \sum_{j=1}^{n} \lambda_j^* x_{tj} \leq \theta_t^* x_{tp} \leq x_{tp}, \quad \sum_{j=1}^{n} \lambda_j^* y_{rj} \geq \varphi_r^* y_{rp} \geq y_{rp}, r \in O_1 \cup O_2.$$

That is to say, a point in T_v has been found such that $(-\sum_{j=1}^{n} \lambda_j^* X_j, \sum_{j=1}^{n} \lambda_j^* Y_j) \geq (-X_p, Y_p)$,

which implies that DMU_p is not Pareto efficient, and this is a contradiction.

Theorem 4. R_e is strongly monotonic in inputs and outputs.

Proof. Suppose that, in the evaluation of DMU_p, $(\lambda^*, \theta_{I_1}^*, \theta_{I_2}^*, \varphi_{O_1}^*, \varphi_{O_2}^*)$ is the optimal solution, then we have $\sum_{j=1}^{n} \lambda_j^* x_{ij} = \theta_i^* x_{ip}, \quad i \in I_1 \cup I_2, \quad \sum_{j=1}^{n} \lambda_j^* y_{rj} = \varphi_r^* y_{rp}, \quad r \in O_1 \cup O_2.$

Now assume that we increase at least one of the inputs by a constant ω, say $\bar{x}_{tp} = x_{tp} + \omega \quad (\omega > 0)$.

Consider the t^{th} input constrains $\sum_{j=1}^{n} \lambda_j^* x_{tj} = \theta_t^* (x_{tp} + \omega), \quad t \in I_1.$

Suppose $\bar{\theta}_t = \dfrac{\sum_{j=1, j\neq p}^{n} \lambda_j^* x_{tj} + \lambda_p^* (x_{tp} + \omega)}{x_{tp} + k}$. Regarding $\theta_t^* = \dfrac{\sum_{j=1}^{n} \lambda_j^* x_{tj}}{x_{tp}}$, note that we always have $\lambda_p^* \leq 1$. Therefore, we have

$$\bar{\theta}_t = \frac{\sum_{j=1, j\neq p}^{n} \lambda_j^* x_{tj} + \lambda_p^* (x_{tp} + \omega)}{x_{tp} + \omega} < \frac{\sum_{j=1}^{n} \lambda_j^* x_{tj}}{x_{tp}} = \theta_t^*.$$

Therefore, by defining $\bar{\theta}_i = \theta_i^*, \quad i \in I_1, i \neq t$, the solution $(\lambda^*, \bar{\theta}_{I_1}, \theta_{I_2}^*, \varphi_{O_1}^*, \varphi_{O_2}^*)$ is a feasible solution of DMU under evaluation , and we have

$$\frac{\frac{1}{Mp+q}(M\sum_{i\in I_1}\bar{\theta}_i+\sum_{i\in I_2}\theta_i^*)}{\frac{1}{Mk+l}(M\sum_{r\in O_1}\varphi_r^*+\sum_{r\in O_2}\varphi_r^*)}<\frac{\frac{1}{Mp+q}(M\sum_{i\in I_1}\theta_i^*+\sum_{i\in I_2}\theta_i^*)}{\frac{1}{Mk+l}(M\sum_{r\in O_1}\varphi_r^*+\sum_{r\in O_2}\varphi_r^*)}.$$

This show that R_e is strictly decreasing with respect to the inputs of the DMU under evaluation. It can be shown, in a similar manner, that R_e is strictly increasing with respect to the outputs of the DMU under evaluation.

Theorem 5. R_e satisfies the following conditions:

(i) If $\gamma > (<) 1$ then $R_e(\gamma X_p, Y_p) \le (\ge)\ \frac{1}{\gamma} R_e(X_p, Y_p)$.

(ii)If $\gamma < (>) 1$ then $R_e(X_p, \gamma Y_p) \le (\ge)\ \gamma R_e(X_p, Y_p)$.

Proof. (i) Suppose that $(\lambda^*, \theta_{I_1}^*, \theta_{I_2}^*, \varphi_{O_1}^*, \varphi_{O_2}^*)$ is the optimal solution in the evaluation of DMU_p. Then $(\lambda^*, \frac{1}{\gamma}\theta_{I_1}^*, \frac{1}{\gamma}\theta_{I_2}^*, \varphi_{O_1}^*, \varphi_{O_2}^*)$ is a feasible solution in the evaluation of $(\gamma X_p, Y_p)$, and we have

$$R_e(\gamma X_p, Y_p) \le \frac{\frac{1}{Mp+q}(M\sum_{i\in I_1}\frac{1}{\gamma}\theta_i^*+\sum_{i\in I_2}\frac{1}{\gamma}\theta_i^*)}{\frac{1}{Mk+l}(M\sum_{r\in O_1}\varphi_r^*+\sum_{r\in O_2}\varphi_r^*)}=\frac{1}{\gamma}R_e(X_p, Y_p).$$

(ii) Suppose that $(\lambda^*, \theta_{I_1}^*, \theta_{I_2}^*, \varphi_{O_1}^*, \varphi_{O_2}^*)$ is the optimal solution in the evaluation of (X_p, Y_p). The, by assuming $(\lambda^*, \theta_{I_1}^*, \theta_{I_2}^*, \frac{1}{\gamma}\varphi_{O_1}^*, \frac{1}{\gamma}\varphi_{O_2}^*)$ is a feasible solution in the model evaluating DMU_p, and we have $R_e(X_p, \gamma Y_p) \le \frac{\frac{1}{Mp+q}(M\sum_{i\in I_1}\theta_i^*+\sum_{i\in I_2}\theta_i^*)}{\frac{1}{Mk+l}(M\sum_{r\in O_1}\frac{1}{\gamma}\varphi_r^*+\sum_{r\in O_2}\frac{1}{\gamma}\varphi_r^*)}=\gamma R_e(X_p, Y_p).$

Therefore, considering Theorem 1 and the above variable alteration in model (3), the SBM model is modified to the following model :

$$
\begin{aligned}
\pi^{*}=Min \quad & \pi=\frac{1-\frac{1}{Mp+q}\left(M\sum_{i\in I_1}\frac{s_i^-}{x_{ip}}+\sum_{i\in I_2}\frac{s_i^-}{x_{ip}}\right)}{1+\frac{1}{Mk+l}\left(M\sum_{r\in O_1}\frac{s_r^+}{y_{rp}}+\sum_{r\in O_2}\frac{s_r^+}{y_{rp}}\right)} \\
S.t \quad & \sum_{j=1}^{n}\lambda_j x_{ij}+s_i^-=x_{ip}, \qquad i\in I_1\cup I_2 \\
& \sum_{j=1}^{n}\lambda_j y_{rj}-s_r^+=y_{rp}, \qquad r\in O_1\cup O_2 \\
& \sum_{j=1}^{n}\lambda_j=1 \\
& \lambda_j\geq 0, \qquad j=1,\ldots,n \\
& s_i^-\geq 0, \qquad i\in I_1\cup I_2 \\
& s_r^+\geq 0, \qquad r\in O_1\cup O_2 .
\end{aligned}
\tag{4}
$$

If Model (4) has multiple optimal solutions, there are several benchmarks for DMUp and more choices of suitable benchmarks for the management to choose from. Therefore, in order to find the multiple optimal solutions of Model (4), we solve it again after adding the constraint $\pi^{*}=\pi$. It is not, of course, possible to find all the alternative solutions; One can only determine the extreme DMUs and find their convex combination as the desirable benchmark.

To solve Model (4), we obtain Model (5) by a change of variable, and then we transform the fractional linear model to a linear programming model. Finally, we put the optimal solution of Model (5) in $\lambda_j^{*}=\frac{\mu_j^{*}}{\beta^{*}}, s_i^{-*}=\frac{t_i^{-*}}{\beta^{*}}, s_r^{+*}=\frac{t_r^{+*}}{\beta^{*}}$ to obtain the optimal solutions and the desirable benchmark for the DMUs.

$$
\begin{aligned}
\text{Min} \quad & \beta - \frac{1}{Mp+q}\Big(M \sum_{i \in I_1} \frac{t_i^-}{x_{ip}} + \sum_{i \in I_2} \frac{t_i^-}{x_{ip}}\Big) \\
S.t \quad & \beta + \frac{1}{Mk+l}\Big(M \sum_{r \in O_1} \frac{t_r^+}{y_{rp}} + \sum_{r \in O_2} \frac{t_r^+}{y_{rp}}\Big) = 1 \\
& \sum_{j=1}^{n} \mu_j x_{ij} + t_i^- = \beta x_{ip}, \qquad i \in I_1 \cup I_2 \\
& \sum_{j=1}^{n} \mu_j y_{rj} - t_r^+ = \beta y_{rp}, \qquad r \in O_1 \cup O_2 \qquad\qquad (5) \\
& \sum_{j=1}^{n} \mu_j = \beta \\
& \mu_j \geq 0, \qquad j = 1, \ldots, n \\
& t_i^- \geq 0, \qquad i \in I_1 \cup I_2 \\
& t_r^+ \geq 0, \qquad r \in O_1 \cup O_2 \\
& \beta \geq 0.
\end{aligned}
$$

To be precise, we know that from any optimal solution of model (5) with $\beta > 0$, we can obtain an optimal solution of model (3) through the change of variables (4). Considering (β, t_i^-, t_r^+) be the extreme points of the model (5) the value of M is given by the following equation.

$$
\frac{M}{Mp+q} = Min\left\{ \frac{\beta - \beta^*}{\sum_i \frac{t_i^-}{x_{ip}} - \sum_i \frac{t_i^{-*}}{x_{ip}}} \,\middle|\, \Big(\sum_i \frac{t_i^-}{x_{ip}} - \sum_i \frac{t_i^{-*}}{x_{ip}}\Big) > 0 \right\}.
$$

4 An application

The real data set, documented in Table 1, contains 30 banks with four inputs and four outputs, where the set of inputs/outputs are given as follows:

Inputs:
- Level of education of the staff.
- Number of the staff in the branch.
- Working experience of the staff.
- Interest paid to customers.

Outputs:
- Resources.
- Expenditure.
- Revenues.
- Volume of activity.

The data for the above inputs and outputs are shown in Table 1. In this section we solve linear model (5) with GAMS software. The parameter value 'M' in models (4) and (5) has been considered equal to 100.

Table 1

Data contains 30 banks with four inputs and four outputs

DMUs	i_1	i_2	i_3	i_4	o_1	o_2	o_3	o_4
1	73504.25	86	4000	61196520.56	57029.55	40.88	1269	175
2	36582.75	89.77	2565	66287094.89	36872	18.55	8543.33	313.11
3	25002.25	87	1343	47612874.67	38680	20.22	6594.77	274.88
4	36684.75	94	1500	349278138.6	35933	32.44	10516.55	248.55
5	36834.5	83.44	1680	68356757.22	54457.77	30.11	9684.66	221.66
6	62611.62	97	3750	75508661.33	72277.11	11.77	8022	329
7	41572.77	90.66	3313	114264317.2	36625.44	101.22	14513.33	264.66
8	55949.62	92	1500	74950922.22	46360.33	17.11	1622.55	226.11
9	95522.75	89.55	1600	106720450.6	86063.33	71.44	10645	297.88
10	59080.62	95.22	1725	66010355.44	47242.11	26.22	6824.33	215.77
11	40736.12	78.33	1920	86613046.78	38977.77	184.11	12226	178.33
12	27300.62	89.66	4433	69942333.33	38214.88	21.77	7561.77	157.44
13	63295	106.33	2500	78657408.78	58340.44	45.22	7584	269.55
14	94969.87	93.77	2800	78951533.89	88472.33	40	661.22	136.22
15	50062	93.44	1630	69126959.33	50499	14.33	10264.11	125.11
16	45926.25	85	1127	146206909	47907.22	25	7491.55	188.55
17	82202.5	104	3400	107289969.8	59579.77	19.88	4952.77	157.33
18	88782.5	92.33	1304	165532950.4	83075.11	23.44	4917	141.55
19	87247.25	96.11	4206	68355245.33	51026.55	17.55	1528.33	217.77
20	33196.5	100.44	1340	92342642.33	29658.11	77.33	14766.33	306.22
21	28402.75	89.33	1393	44055007.78	27735	19.22	940.66	165.11
22	122897.87	121.88	2191	94312914.11	102855.11	47.55	2510.44	238
23	32587.75	100	2140	89070348.78	34063.66	23.11	2110.88	349
24	60866.37	92	1231	69549538.56	53731.33	63.33	10219.55	163.55
25	86429.87	90	1960	164581079.9	75776.55	53.88	4480.33	253.44
26	253690	890	13430	694238180	1000	3	650	27
27	1002200	920	16000	1829518580	9000	17	164	29
28	66803.12	93.55	1603	143921177.8	72552.66	75.88	12091.22	184.77
29	40156.12	82.33	2300	65099487.67	38630.77	24.88	1460.55	75.33
30	986780	950	13040	3891855920	1800	7	491	14

Now in Table 1, the suitable index for inefficient branches can be suggested by considering the priority of decreasing i_1, i_2 and i_3 and also priority of increasing o_3 and o_4.

($I_1 = \{1,2,3\}, I_2 = \{4\}, \quad O_1 = \{3,4\}, \quad O_2 = \{1,2\}$). Therefore, we calculate optimal values in order to evaluate the bank branches by model (5). Then considering the change of variables, the optimal solution of model (4) will be obtained. We put the optimal value of model (4) in the Table

Table 2

Benchmarks for each branch by using the proposed approach.

DMUs	i_1	i_2	i_3	i_4	o_1	o_2	o_3	o_4
1	73504.25	86.00	4000.00	6.11965E+7	57029.56	40.89	1269.00	175.00
2	36582.75	89.78	2565.00	6.62871E+7	36872.00	18.56	8543.33	313.11
3	25002.25	87.00	1343.00	4.76129E+7	38680.00	20.22	6594.78	274.89
4	36684.75	94.00	1500.00	8.87670E+7	35933.00	90.82	13564.99	272.14
5	36834.50	83.44	1680.00	6.83568E+7	54457.78	30.11	9684.67	221.67
6	62611.63	97.00	3750.00	7.55087E+7	72277.11	11.78	8022.00	329.00
7	41572.77	90.67	3313.00	1.14264E+8	36625.44	101.22	14513.33	264.67
8	38851.85	89.92	1500.00	7.49509E+7	46360.33	48.76	11170.70	240.34
9	95522.75	89.56	1600.00	1.06720E+8	86063.33	71.44	10645.00	297.89
10	39697.81	91.10	1725.00	6.60104E+7	47242.11	34.57	8676.21	290.85
11	40736.13	78.33	1920.00	8.66130E+7	38977.78	184.11	12226.00	178.33
12	27300.63	89.35	1408.82	5.52739E+7	38214.89	28.79	7893.31	281.21
13	53669.46	93.60	2500.00	7.86574E+7	58340.44	45.22	9632.53	300.76
14	94969.88	93.78	2800.00	7.89515E+7	88472.33	40.00	661.22	136.22
15	50062.00	93.44	1630.00	6.91270E+7	50499.00	14.33	10264.11	125.11
16	45926.25	85.00	1127.00	1.46207E+8	47907.22	25.00	7491.56	188.56
17	66259.14	94.67	1477.92	0.99970E+8	59579.78	74.21	12580.06	301.80
18	88782.50	92.33	1304.00	1.65533E+8	83075.11	23.44	4917.00	141.56
19	35453.47	85.13	1621.83	6.83552E+7	51026.56	32.96	9804.68	233.31
20	33196.50	100.44	1340.00	9.23426E+7	29658.11	77.33	14766.33	306.22
21	8402.75	89.33	1393.00	4.40550E+7	27735.00	19.22	940.67	165.11
22	122897.88	121.89	2191.00	9.43129E+7	102855.11	47.56	2510.44	238.00
23	32587.75	100.00	140.00	8.90703E+7	34063.67	23.11	2110.89	349.00
24	60866.38	92.00	1231.00	6.95495E+7	53731.33	63.33	10219.56	163.56
25	84279.99	90.00	1907.35	1.07781E+8	75776.56	77.02	11443.43	292.00
26	33196.50	100.44	1340.00	9.23426E+7	29658.11	77.33	14766.33	306.22
27	33196.50	100.44	1340.00	9.23426E+7	29658.11	77.33	14766.33	306.22
28	66803.13	93.56	1603.00	1.43921E+8	72552.67	75.89	12091.22	184.78
29	40156.13	82.33	2300.00	6.50995E+7	38630.78	24.89	1460.56	75.33
30	33196.50	100.44	1340.00	9.23426E+7	29658.11	77.33	14766.33	306.22

The results of Model (4) are provided in Table 2. The set of inefficient units is denoted by IN; For the data in Table 1, this set is

$$IN = \{DMU_4, DMU_8, DMU_{12}, DMU_{13}, DMU_{17}, DMU_{19}, DMU_{25}, DMU_{26}, DMU_{27}, DMU_{30}\}.$$

By Model (4) and the results of Table 2, the desirable benchmark for DMU_4 is obtained from the convex combination of units 5, 7, 9, 11, and 20; and for DMU_8 it is obtained from the convex combination of units 3, 5, 20 and 24. Moreover, DMU_{20} is a good benchmark for units 26, 27, and 30. Looking more closely at the inputs and outputs of DMU_{30}, one can observe that the necessary amount of decrease for inputs 1-3, which have higher priority, is 0.03, 0.11, and 0.10, respectively, and the decrease required for input 4 to reach DMU_{20} is 0.2.

5 Conclusion

The main nature of benchmarking is an attempt for improvement and its purpose is to provide performance levels and a quantitative criterion for the results. In internal benchmarking, a DMU is compared with another DMU within the organization, which leads to easier interactions, more profound analyses, and clarification of problems. In this paper suggested a model using an modified Russell to find a suitable index for inefficient commercial banks. The suggest model can introduce a suitable efficient branch in order to give priority to decrease some inputs also to give priority to increase some outputs. For practical application, the discretionary and nondiscretionary data can be determined and also the suggest model can be studied in mentioned in order to eliminate the priorities which are on the discretionary and nondiscretionary introduce the suitable and practical index.

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