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Data Envelopment Analysis Model and Benchmarking in a Hierarchical Structure with Dependent Parameters

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Abstract

Data envelopment analysis is one of the best methods to evaluate the performance of decision-making units. This method is also used for benchmarking. benchmarking is a tool to evaluate organizational performance with a learning approach from others, it is also one of the practical methods in continuous improvement of the benchmarking method. The importance of benchmarking in all industries is clear. This paper considers the after-sales service network of an automobile company in Iran to evaluate the model. According to the structure of this network, a hierarchical structure is considered for benchmarking. In this paper, the purpose is to provide a model for benchmarking decision-making units with hierarchical structure and dependent parameters. In the real world, most decision-making units have a hierarchical structure and this structure needs more attention by researchers also dependent parameters can have a high impact on benchmarking. The proposed model for the after-sales service network of an automobile company in Iran was implemented and the results show the high impact of dependent variables on benchmarking and has increased modeling accuracy. The accuracy of benchmarking is very important for the success of decision-making units and the results show that paying attention to the relationships between the parameters increases the accuracy of benchmarking and according to the proposed model, more accurate benchmarking can be achieved.

Keywords: Data Envelopment Analysis, Benchmarking, Dependent Parameters, Hierarchical Structure.

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1. Introduction

Performance evaluation is one of the most basic tasks of managers in every organization, for this purpose, various methods and models have been proposed by researchers [1]. One of the most widely used methods of performance evaluation is Data Envelopment Analysis (DEA) method. Based on the basic concepts [2], Chaarns et al. Conducted a measurement of the efficiency of decision-making units in 1978 using a linear programming model to estimate the production frontiers [3]. After that, many books and articles on DEA and DEA applications in various industries and topics were presented. The DEA method is a linear and non-parametric programming method. This method is used in cases where the purpose is to compare the outputs and inputs of DMUs. This method is a good tool for measuring and evaluating the relative efficiency of DMUs. Conventional and traditional statistical methods usually operate with strict approaches and in these methods, DMUs are evaluated by comparing according to the average of DMUs. While DEA is a maximalist approach, it compares and evaluates the parameters of each DMU with only the parameters of the best DMU. Lee et al. (2001) showed that data envelopment analysis has several advantages over the traditional performance evaluation approach [4]:

- In this method, the weight measurements are calculated according to the inputs and outputs values of each unit being compared, and there is no need to evaluate the weight of the inputs or outputs variables or prioritize them.
- DEA can be measured relative efficiency of DMUs with multiple inputs and outputs,

Therefore, data envelopment analysis is recommended to assist traditional benchmarking methods and provide guidance for managing DMUs [5]. Various experiences show that this method is a powerful tool for evaluating performance and benchmarking to improve efficiency of organizations. This method has been used in various studies [6-10]. Data envelopment analysis method has been widely used in performance evaluation and benchmarking studies since it was proposed by Changes et al.

First, DEA presented for evaluation performance of DMUs as a black box (regardless of their internal structure). Although many studies shows that this may not be a correct assumption, for example Cron and Sobol have shown that information technology (IT) unit (as an internal unit) can affect the performance of DMUs [11]. Therefore, there was a need to pay attention to the internal structure of DMUs and this has been one of the weaknesses of data envelopment analysis. To solve this problem, network data envelopment analysis is presented. The main purpose of network DEA studies is to open the black box of a system, in measuring performance of DMU component processes is considered. On the other hand, in the real world, some DMUs have a hierarchical structure that can be considered as a network structure that has received less attention from researchers. A hierarchical structure can have several levels, and as presented in the second section, most existing articles discuss only two levels. If there is only one level and there is no interaction between the headquarters and its affiliated units, the system can be considered as a parallel network.

Another issue that has received very little attention in data envelopment analysis models is the dependence of model's variables, especially the output variables of DMUs, because in the real world, model's parameters have a dependence and correlation coefficient with each other. For example, in evaluating the efficiency of banks is one of the output variables of the bank's income through receiving the installments of the facilities provided, which is directly related to the variable of the provided loan. Here, two basic questions are raised. first of all, why not consider both variables as one? Second, the values of the variables in each DMU are known, so what helps to consider this dependency? To answer the first question, accuracy of measuring the efficiency of DMUs is considered, because the variables are interdependent and not exactly equal (in other words, their correlation coefficient is not equal to 1 and other factors affect them). To answer second question, it should be noted that to measure performance it is true that this dependency may not be significant but it is very important for benchmarking, we cannot benchmark a virtual DMU for an inefficient DMU so that the dependence of the variables is logically observed have not been.

In the second section, the literature of the subject is presented and it is shows that the subject of benchmarking in hierarchical structures with dependent variables has been empty. The necessity of this issue can be seen in the case study. Iran khodro is considered as the main organization and further study shows that the after-sales service of this company (Isaco) has a hierarchical structure and at the lowest level (agencies) its two outputs that have a high correlation coefficient. Therefore, in this paper, a hierarchical data envelopment analysis model has been developed to benchmark the hierarchical structure (for all levels) for car after-sales service. For this purpose, after reviewing the literature on subject, in the third section, introductory concepts are presented. Then mathematical model for the hierarchical structure is presented in the fourth section, then in the fifth section the numerical results of the model are presented and finally in the sixth section the discussion and conclusion are presented.

2- Literature review

Data envelopment analysis is a mathematical programming method for evaluating the efficiency of decision-making units (DMUs) that have multiple inputs and multiple outputs. Measuring efficiency has always been of interest to researchers because of its importance in evaluating the performance of a company or organization. In 1957, Farrell began to measure efficiency for a production unit using a method similar to measuring efficiency in engineering. The case that Farrell considered to measure efficiency included an input and an output [2].

For the first time, Farrell defined a non-parametric production frontier by introducing a boundary method called the Farrell Production Frontiers and using mathematical equations, the distance of the decision-making unit from the mentioned boundary was measured as the efficiency of that unit. By expressing the most central structural ideas of models and methods of measuring productivity, in the form of increasing outputs and thus increasing efficiency without attracting more resources, he practically established the branches of optimization in mathematics, which was later published in 1987 by Changes, Cooper, and Rhodes developed the CCR model based on mathematical models and became known as data envelopment analysis [3]. Chanz et al. Describe data envelopment analysis as "a nonparametric method in operations research and economics for the estimation of production frontiers" [3]. Six years after the introduction of the CCR model, this model was fundamentally modified by Bunker, Charans, and Cooper, and a second model was developed in this field, called the BCC model [12].

Since DEA was introduced by Charles, Cooper, and Rhodes in 1978, many simple and powerful methods have been developed based on this method to evaluate the relative efficiency of decision-making units. DEA's popularity is due to its ability to measure the relative performance of DMUs with multiple inputs and outputs without prior weight. To date, research on DEA is still extensive and is being used in many fields. For example, the banking industry [13], information technology and information systems [14], education [15], airlines [16], Computer [17], Power Plant [18], Sports [19], Stock Market [20], Government [21], Supply chain [22] and etc.

DEA models were originally developed to measure the performance of a DMU regardless of its internal structure. In other words, in this method, DMUs are considered as a black box in which inputs are used to generate outputs and there is generally a positive correlation between the two. However, there are empirical studies that suggest this may not always be true and that the internal structure of DMUs should be considered. The first paper to possibly discuss this idea was by Charnes et al. [23], who found that recruiting the military has two processes: first, advertising, and then contract. Separating large operations into smaller processes helps identify the true impact of input factors. The simplest way is to divide the whole operation into two processes, such as [23, 24]. There are more complex cases in which the whole process is divided into more than two processes. In these structures, it may be a series structure, a parallel structure, or a mixture of these. These structures are generally called network structures, and the DEA method for measuring the performance of systems with a network structure is called network DEA [25].

The main purpose of DEA network studies is to open the black box of a system when measuring its performance. In recent years, many researchers have developed network structures such as [26-32].

Hierarchical structure is known as one of the network structures and according to the subject of this paper, in this section, the hierarchical structure literature has been presented. In 2004, Castley et al. proposed a model for measuring the performance of a one- and two-level hierarchical system. For single-level systems, system performance is the ratio of total output to total input. The same idea can be extended to two-level systems. In this study, two conditions were discussed, one was weight balance and the other was flow balancing [33]. Cook and Green (2005) discussed a hierarchical system consisting of several layers based on the idea presented in a study by Cook et al. (1998) [34]. The headquarters has several departments and each department in turn has several subdivisions. They used 10 Canadian power plant's data, consisting of 40 power plant units to explain their model [35]. In recent years, research on hierarchical structure has been conducted for further study [36-40]. As mentioned, many studies have been done on network data envelopment analysis (although due to the scope of the subject there are still areas for the development of network models) but there is much less research on hierarchical structures.

Some approaches have been developed in the DEA for hierarchical structure. For example, Fair and Primont are used a distance objective function for the performance of multi-factory companies used by Kao (1998) to measure the performance of eight forest areas in Taiwan [41]. Kao (2015) developed the relational model that he proposed in 2009, [42] to measure the efficiency of a hierarchical system. The efficiency of the system is the weighted average of the lower components of the hierarchy.

Zhang and Chen presented a hierarchical (two-level) performance evaluation model to evaluate the performance of China's high-tech industry in 2017 [38]. The structure considered by them can be considered one of the most complete hierarchical structures in the literature of this field.

In 2018, Chen et al. Presented a hierarchical data envelopment analysis model to build a multidimensional index and use it to reassess the Global Food Security Index of 2014, created by the Economist Intelligence Unit (EIU) in 110 countries. The results show that although the ranking is not significantly different from the EIU, the optimal scores and weights vary according to the level of the countries. In addition, they have done benchmarking by creating a performance center [37].

Boďa et al. proposed a solution to considering that one of the problems of hierarchical structures is the heterogeneity of DMUs [40].

Ghasemi et al. Have presented a data envelopment analysis model with the goal of measuring the relative efficiency of Farhangian University campuses that train school teachers in Iran. According to them, Farhangian University campuses have a hierarchical structure and need to provide appropriate models to evaluate their performance, and they proposed a hierarchical data envelopment analysis model to solve this problem [39].

Amini et al. Have presented a fuzzy hierarchical data envelopment analysis model based on a fuzzy network model that they have previously provided; they believe that ignoring the hierarchy causes error. With this model, they evaluated the efficiency of the budgeting system in 14 government organizations in Iran. They compared the results of the hierarchical model with the single layer model. The results show that the number of efficient decision-making units in the single layer model is eight efficient units, while in the hierarchical model it is only one efficient unit [43].

Also, the subject of dependent parameters in data envelopment analysis models has been less studied by researchers and most articles have examined external dependent parameters and their effect on performance [44-46], with search that was done, only one paper related to Ji et al. (Ji et al., 2015) was found in this area, which also turned all dependent variables into a variable and have solved the model that according to the purpose of this article, which is to find a model for decision-making units, this method cannot be used, so in this paper, an attempt has been made to cover this gap in the literature.

Table 1 summarizes some of the reviewed papers. In table 1, with emphasis on recent papers, papers from 2005 onwards in the subject of benchmarking in data envelopment analysis, papers on hierarchical structures in data envelopment analysis, data envelopment analysis and dependent variables are presented.

Table 1. Summary of reviewed papers

Row	Year	Ref	Title	Structure			Dependent parameters
				Black Box	Network	Hierarchical	
1	2005	[5]	Benchmarking marketing productivity using data envelopment analysis		×		

2	2005	[35]	Evaluating power plant efficiency: a hierarchical model			×	
3	2006	[6]	Benchmarking the operational efficiency of third-party logistics providers using data envelopment analysis	×			
4	2006	[7]	A Benchmarking Analysis of Spanish Commercial Airports. A Comparison Between SMOP and DEA Ranking Methods	×			
5	2006	[8]	Benchmarking with quality-adjusted DEA (Q-DEA) to seek lower-cost high-quality service: Evidence from a U.S.bank application	×			
6	2009	[47]	Elastic neural network method for multi-target tracking task allocation in wireless sensor network	×			
7	2009	[48]	Modeling the efficiency of top Arab banks: A DEA–neural network approach	×			
8	2009	[42]	Efficiency decomposition in network data envelopment analysis: A relational model		×		
9	2009	[49]	Modeling the competitive market efficiency of Egyptian companies: a probabilistic neural network	×			
10	2010	[50]	Evaluation of credit risk based on firm performance	×			
11	2011	[51]	The competitiveness of nations and implications for human development	×			
12	2011	[52]	An integrated data envelopment analysis-artificial neural network-rough set algorithm for assessment of personnel efficiency	×			
13	2012	[53]	A DEA approach for comparative analysis of service quality dimensions with a case study in hotel industry	×			
14	2013	[54]	A Comparison of Three-Stage DEA and Artificial Neural Network on the Operational Efficiency of Semi-Conductor Firms in Taiwan	×			
15	2013	[55]	Performance evaluation of HESA laboratory units: an integrated DEA-BSC approach	×			
16	2013	[56]	Efficiency evaluation of a private bank's branches with service quality approach by data envelopment analysis	×			
17	2014	[57]	Chinese company's distress prediction: an application of data envelopment analysis	×			
18	2015	[58]	Two-stage production modeling of large U.S. banks: A DEA-neural 4 network approach		×		
19	2015	[59]	Efficiency measurement for hierarchical network systems			×	
20	2015	[60]	Data envelopment analysis with interactive variables	×			×
21	2016	[10]	The impact of internationalization and diversification on construction industry performance	×			
22	2017	[61]	Dynamic Prediction of Financial Distress Using Malmquist DEA	×			
23	2017	[62]	Exploring the predictive potential of artificial neural networks in conjunction with DEA in railroad performance modeling	×			
24	2017	[63]	Three-stage performance modeling using DEA-BPNN for better practice benchmarking		×		
25	2018	[64]	Two-stage DEA-Truncated Regression: Application in banking efficiency and financial development		×		
26	2018	[65]	Neural network modeling for a two-stage production process with versatile variables: Predictive analysis for above-average performance		×		
27	2018	[36]	The geographical efficiency of education and research: The ranking of U.S. universities			×	
28	2019	[66]	A Comparative Analysis of Two-Stage Distress Prediction Models		×		
29	2019	[67]	A data-driven prediction approach for sports team performance and its application to National Basketball Association		×		
30	2019	[37]	A reassessment of the Global Food Security Index by using a hierarchical data envelopment analysis approach			×	
31	2019	[38]	Hierarchical network systems: An application to high-technology industry in China			×	

32	2020	[68]	Two-stage additive network DEA: Duality, frontier projection and divisional efficiency		×		
33	2020	[69]	Network data envelopment analysis in uncertain environment		×		
34	2020	[70]	Evaluation and prediction on total factor productivity of Chinese petroleum companies via three-stage DEA model and time series neural network model		×		
35	2020	[71]	A conic relaxation model for searching for the global optimum of network data envelopment analysis		×		
36	2020	[72]	A combined machine learning algorithms and DEA method for measuring and predicting the efficiency of Chinese manufacturing listed companies		×		
37	2020	[39]	Assessing the performance of organizations with the hierarchical structure using data envelopment analysis: An efficiency analysis of Farhangian University			×	
38	2020	[40]	Modeling a shared hierarchical structure in data envelopment analysis: An application to bank branches			×	
39	2021	[26]	Proportional trade-offs in network DEA		×		
40	2021	[27]	Production scale-based two-stage network data envelopment analysis		×		
41	2021	[28]	Performance evaluation of process industries resilience: Risk-based with a network approach		×		
42	2021	[29]	Efficiency measurement in multi-period network DEA model with feedback		×		
43	2021	[30]	An adjustable fuzzy chance-constrained network DEA approach with application to ranking investment firms		×		
44	2021	[31]	Efficiency stability region for two-stage production processes with intermediate products		×		
45	2021	[32]	Efficiency decomposition of the network DEA in variable returns to scale: An additive dissection in losses		×		
46	2021	[43]	A generalized fuzzy Multiple-Layer NDEA: An application to performance-based budgeting			×	
47	2021	[73]	Big data and portfolio optimization: A novel approach integrating DEA with multiple data sources		×		
48	2021	[74]	Benchmarking bank branches: A dynamic DEA approach		×		
This Paper						×	×

3- Introductory concepts

Efficiency means working well and is influenced by indicators within the organization such as profit per unit, sales per unit, and the like, which is expressed as the ratio of output to input. Data envelopment analysis is a linear programming method that uses the data of decision-making units to construct efficiency boundaries. The above boundary is made based on data of inputs and outputs, and in fact, the value of inefficiency of each decision-making unit is the distance of the unit to the efficiency boundary [23]. In 1986, Charnes and Cooper first proposed the first model of data envelopment analysis [23]. Data envelopment analysis calculates the deviation of each DMU from the efficiency boundary by plotting the efficiency boundary according to the production possibility set (PPS). The production possibility set is defined as follows [75].

$$PPS = \left\{ \begin{array}{l} (x, y) \mid x \geq \sum_{j=1}^{n} \lambda_j x_j , \\ 0 \leq y \leq \sum_{j=1}^{n} \lambda_j y_j , \lambda_j \geq 0 \end{array} \right\} \quad (1)$$

Data envelopment analysis models are generally divided into two main parts, CCR and BCC, the basic CCR model was proposed by Charnes and Cooper [23] and the BCC model in 1984 by Bunker et al. the CCR model and the addition of a new constraint were proposed [12]. Data envelopment analysis models are divided into two categories of envelopment and multiplication in terms of modeling. The first model of data envelopment analysis is called multiplication. The basis of this model is to define efficiency as the ratio of one output to one input. For example, in the CCR model, instead of using the ratio of one output to one input, the ratio of the weighted sum of outputs (virtual output) to the weighted sum of inputs (virtual input) is used to calculate technical efficiency [23].

Equation 2 presents the CCR multiplication form of the data envelopment analysis model.

$$E_o = Max \sum_{r=1}^{s} u_r . y_{r,o}$$

$$s.t. \sum_{i=1}^{m} v_i . x_{i,o} = 1 \qquad (2)$$

$$\sum_{r=1}^{s} u_r . y_{rj} - \sum_{i=1}^{m} v_i . x_{ij} \le 0, \; j = 1,...,n$$

$$u_r , v_i \ge \varepsilon , r = 1,....,s , \; i = 1,....,m$$

Equation 2 deals with the evaluation of n decision-making units, where each decision-making unit has m inputs and s outputs, in which y_{rj} are the outputs values and x_{ij} are the input values of the decision-making units.

By calculating the dual form of the multiplication model, the envelopment model is obtained. Equation 3 presents the CCR envelopment model.

$$E_o = Min\, \theta \qquad (3)$$

$$s.t. \; \sum_{j=1}^{n} \lambda_j . x_{ij} \le \theta x_{i,o} , i = 1,..., m$$

$$\sum_{j=1}^{n} \lambda_j . y_{rj} \ge y_{r,o} \;, r = 1,..., s$$

$$\lambda_j \ge \varepsilon \qquad , j = 1,...., n$$

4- Modeling

The envelopment model is used to obtain the desired benchmark for inefficient decision-making units, so that by calculating the lambda coefficients, the benchmark of each parameter is determined, according to the purpose of this paper, which is benchmarking. Figure 1 presents a hierarchical schematic model with corresponding parameters.

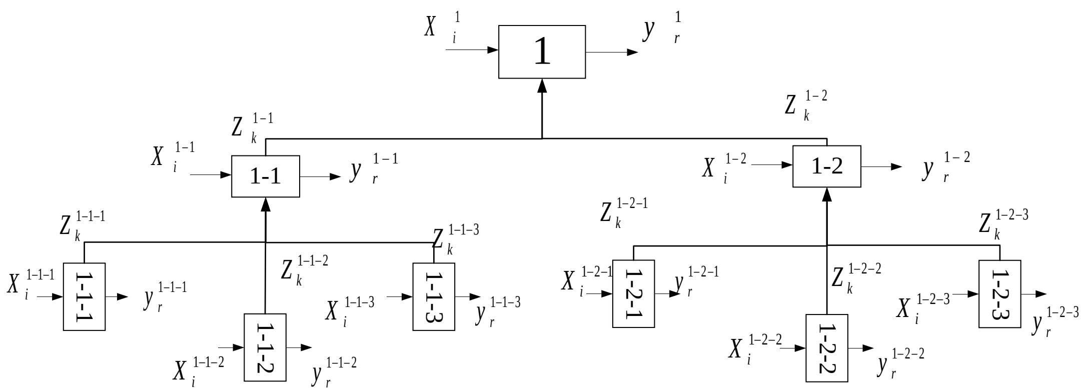

Figure 1. Schematic structure

The symbols used in the final model are introduced in Table 2.

Table 2. Define symbols

Symbol	Define
i	indices of inputs of each level
j	indices of DMUs
r	indices of outputs of each level
k	indices of outputs to the top level in each level
T	Number of DMUs below level 2
q	Number of DMUs below in level 3
m	Number of inputs of each level
s	Number of outputs of each level
d	Number of outputs to level up in each level
X_j^1	Vector of J^{th} DMU input parameters at level 1
Y_j^1	Vector of J^{th} DMU output parameters at level 1
X_j^{1-t}	Vector of J^{th} DMU input parameters in the Tth subunit of level 2
Y_j^{1-t}	Vector of J^{th} DMU output parameters in the Tth subunit of level 2
Z_j^{1-t}	Vector of J^{th} DMU intermediate parameters in the t^{th} subunit of level 2 to the higher level
X_j^{1-t-q}	Vector of J^{th} DMU input parameters in the q^{th} subunit of level 3
Y_j^{1-t-q}	Vector of J^{th} DMU output parameters in the q^{th} subunit of level 3
Z_j^{1-t-q}	Vector of J^{th} DMU intermediate parameters in the q^{th} subunit of level 3 to the higher level
V^1	Input weight variables at level 1
U^1	Output weight variables at level 1
V^{1-t}	Input weight variables in the t^{th} subunit of level 2
U^{1-t}	Output weight variables in the t^{th} subunit of level 2
W^{1-t}	Intermediate weight variables in the t^{th} subunit of level 2 to higher level
V^{1-t-q}	Input weight variables in the q^{th} subdivision of level 3
U^{1-t-q}	Output weight variables in the q^{th} subdivision of level 3
W^{1-t-q}	Intermediate weight variables in the q^{th} subunits of level 3 to higher level
θ_j^1	Efficiency of the j^{th} DMU at level 1
θ_j^A	Efficiency of the j^{th} DMU at throughout the whole structure
θ_j^{1-t}	Efficiency of the t^{th} subunits of the j^{th} DMU at level 2
θ_j^{1-t-q}	Efficiency of the q^{th} subunits of the j^{th} DMU at level 3
λ_j^1	Benchmarking coefficient of the j^{th} DMU at level 1
λ_j^A	Benchmarking coefficient of the j^{th} DMU at throughout the whole structure
λ_j^{2-t}	Benchmarking coefficient of the t^{th} subunits of the j^{th} DMU at level 2
λ_j^{3-t-q}	Benchmarking coefficient of the q^{th} subunits of the j^{th} DMU at level 3

In this section, first, the efficiency calculation fraction models are provided. It should be noted that there are 4 efficiency values, three values for each level and one for the whole structure (for the relationship between the levels and the feasibility of the model is inevitable), it is necessary to It should be noted that in all models, Sigma and input and output indices have been avoided to summarize the models.

The efficiency of the third level of DMUs (q is third level indices and t is second-level indices):

$$\theta_j^{1-t-q} = \frac{U^{1-t-q} Y_j^{1-t-q} + W^{1-t-q} Z_j^{1-t-q}}{V^{1-t-q} X_j^{1-t-q}} \quad (4)$$

The efficiency of the second level of DMUs (q is third level indices and t is second-level indices):

$$\theta_j^{1-t} = \frac{U^{1-t} Y_j^{1-t} + W^{1-t} Z_j^{1-t}}{V^{1-t} X_j^{1-t} + \sum_q W^{1-t-q} Z_j^{1-t-q}} \quad (5)$$

The efficiency of the first level of DMUs (t is second-level indices):

$$\theta_j^{1} = \frac{U^{1} Y_j^{1}}{V^{1} X_j^{1} + \sum_t W^{1-t} Z_j^{1-t}} \quad (6)$$

Whole structure efficiency (q is third level indices and t is second-level indices):

$$\theta_j^{A} = \quad (7)$$

$$\frac{U^{1} Y_j^{1} + \sum_t U^{1-t} Y_j^{1-t} + \sum_t \sum_q U^{1-t-q} Y_j^{1-t-q}}{V^{1} X_j^{1} + \sum_t V^{1-t} X_j^{1-t} + \sum_t \sum_q V^{1-t-q} X_j^{1-t-q}}$$

The general model is as follows (equation 8), in other words, the goal is to get the answer of this model.

$$\begin{aligned} & Max \quad \theta_p^{A} \qquad (8) \\ & ST. \\ & \qquad \theta_j^{1} \le 1 \quad \forall j \\ & \qquad \theta_j^{A} \le 1 \quad \forall j \\ & \qquad \theta_j^{1-t} \le 1 \quad \forall j, \forall t \\ & \qquad \theta_j^{1-t-q} \le 1 \quad \forall j, \forall t, \forall q \\ & \qquad \text{All Variables } \ge \varepsilon \end{aligned}$$

A multiplicative model is obtained by placing the fraction values in equation 8. By linearizing it, we reach the following model (equation 9):

$$Max \begin{cases} U^{1} Y_p^{1} + \sum_t U^{1-t} Y_p^{1-t} \\ + \sum_t \sum_p U^{1-t-q} Y_j^{1-t-q} \end{cases} \qquad (9)$$

$$ST.$$

$$\begin{cases} V^{1} X_p^{1} + \sum_t V^{1-t} X_p^{1-t} \\ + \sum_t \sum_q V^{1-t-q} X_p^{1-t-q} = 1 \end{cases}$$

$$U^{1} Y_j^{1} - V^{1} X_j^{1} - \sum_t W^{1-t} Z_j^{1-t} \le 0 \;, \forall j$$

$$\begin{cases} U^{1} Y_j^{1} + \sum_t U^{1-t} Y_j^{1-t} \\ + \sum_t \sum_q U^{1-t-q} Y_j^{1-t-q} \\ - V^{1} X_j^{1} - \sum_t V^{1-t} X_j^{1-t} \\ - \sum_t \sum_q V^{1-t-q} X_j^{1-t-q} \le 0 \quad \forall j \end{cases}$$

$$\begin{cases} U^{1-t} Y_j^{1-t} + W^{1-t} Z_j^{1-t} \\ - V^{1-t} X_j^{1-t} \\ - \sum_q W^{1-t-q} Z_j^{1-t-q} \le 0 \;, \forall j, \forall t \end{cases}$$

$$\begin{cases} U^{1-t-q} Y_j^{1-t-q} + W^{1-t-q} Z_j^{1-t-q} \\ - V^{1-t-q} X_j^{1-t-q} \le 0 \;, \forall j, \forall t, \forall q \end{cases}$$

$$\begin{cases} U^{1}, V^{1}, U^{1-t}, V^{1-t}, W^{1-t} \\ , U^{1-t-q}, V^{1-t-q}, W^{1-t-q} \ge \varepsilon \;, \forall t, \forall q \end{cases}$$

Constraints related to the weights of the parameters have also been added.

$$Max \quad \begin{cases} U^1 Y_p^1 + \sum_t U^{1-t} Y_p^{1-t} \\ + \sum_t \sum_p U^{1-t-q} Y_j^{1-t-q} \end{cases} \quad (10)$$

$ST.$

$$\begin{cases} V^1 X_p^1 + \sum_t V^{1-t} X_p^{1-t} \\ + \sum_t \sum_q V^{1-t-q} X_p^{1-t-q} = 1 \end{cases}$$

$$U^1 Y_j^1 - V^1 X_j^1 - \sum_t W^{1-t} Z_j^{1-t} \le 0 \ , \forall j$$

$$\begin{cases} U^1 Y_j^1 + \sum_t U^{1-t} Y_j^{1-t} \\ + \sum_t \sum_q U^{1-t-q} Y_j^{1-t-q} \\ - V^1 X_j^1 - \sum_t V^{1-t} X_j^{1-t} \\ - \sum_t \sum_q V^{1-t-q} X_j^{1-t-q} \le 0 \ , \forall j \end{cases}$$

$$\begin{cases} U^{1-t} Y_j^{1-t} + W^{1-t} Z_j^{1-t} \\ - V^{1-t} X_j^{1-t} \\ - \sum_q W^{1-t-q} Z_j^{1-t-q} \le 0 \ , \forall j , \forall t \end{cases}$$

$$\begin{cases} U^{1-t-q} Y_j^{1-t-q} + W^{1-t-q} Z_j^{1-t-q} \\ - V^{1-t-q} X_j^{1-t-q} \le 0 \ , \forall j , \forall t , \forall q \end{cases}$$

$$v_a^1 \le k.v_b^1 \quad ,(a,b) \in m_1$$
$$u_a^1 \le k.u_b^1 \quad ,(a,b) \in s_1$$
$$w_a^{1-t} \le k.w_b^{1-t} ,(a,b) \in d_2$$
$$v_a^{1-t} \le k.v_b^{1-t} \quad ,(a,b) \in i_2$$
$$u_a^{1-t} \le k.u_b^{1-t} \quad ,(a,b) \in s_2$$
$$v_a^{1-t-q} \le k.v_b^{1-t-q} \ ,(a,b) \in i_3$$
$$u_a^{1-t-q} \le k.u_b^{1-t-q} \ ,(a,b) \in s_3$$
$$w_a^{1-t-q} \le k.w_b^{1-t-q} \ ,(a,b) \in d_3$$

$$\begin{cases} U^1, V^1, U^{1-t}, V^{1-t} \\ , W^{1-t}, U^{1-t-q} \\ , V^{1-t-q}, W^{1-t-q} \ge \varepsilon \ , \forall t , \forall q \end{cases}$$

In the next step, to obtain the envelopment model, the dual form of the model equation 10 must be written. For this purpose, the corresponding variables of each constraint are also specified in the model equation 10. Finally, the model equation 11 is obtained.

$$Min \quad \begin{cases} \theta - \sum_i S_i^- - \sum_j S_j^+ \\ - \sum_t \sum_i S_{i,t}^- - \sum_t \sum_j S_{j,t}^+ \\ - \sum_t \sum_k S_{k,t}^* - \sum_t \sum_q \sum_i S_{i,t,q}^- \\ - \sum_t \sum_q \sum_j S_{j,t,q}^+ - \sum_t \sum_q \sum_k S_{k,t,q}^* \end{cases} \quad (11)$$

$ST.$

$$\begin{cases} \sum_j \lambda_j^1 X_j^1 + \sum_j \lambda_j^A X_j^1 \\ + \sum_{ci} \mu i 1_{ci} CX 1_{ci} \\ + S_i^- = \theta X_p^1 \ , \forall i \in m^1 \end{cases}$$

$$\begin{cases} \sum_j \lambda_j^1 Y_j^1 + \sum_j \lambda_j^A Y_j^1 \\ - \sum_{cr} \mu j 1_{cr} CY 1_{cr} \\ - S_j^+ = Y_p^1 \quad , \forall r \in s^1 \end{cases}$$

$$\begin{cases} \sum_j \lambda_j^{2-t} X_j^{1-t} + \sum_j \lambda_j^A X_j^{1-t} \\ + \sum_{ci} \mu i 2_{ci,t} CX 2_{ci,t} \\ + S_{i,t}^- = \theta X_p^{1-t} \ , \forall i \in m^{1-t} , \forall t \end{cases}$$

$$\begin{cases} \sum_j \lambda_j^{2-t} Y_j^{1-t} + \sum_j \lambda_j^A Y_j^{1-t} \\ - \sum_{cr} \mu j 2_{cr,t} CY 2_{cr,t} \\ - S_{j,t}^+ = Y_p^{1-t} \quad , \forall r \in s^{1-t} , \forall t \end{cases}$$

$$\begin{cases} \sum_j \lambda_j^{2-t} Z_j^{1-t} - \sum_j \lambda_j^1 Z_j^{1-t} \\ - \sum_{cd} \mu k 2_{cd,t} CZ 2_{cd,t} \\ - S_{k,t}^* = 0 \quad , \forall k \in d^{1-t} , \forall t \end{cases}$$

$$\begin{cases} \sum_j \lambda_j^{3-t-q} Z_j^{1-t-q} - \sum_j \lambda_j^{2-t} Z_j^{1-t-q} \\ -\sum_{cd} \mu k\, 3_{cd,t,q} CZ\, 3_{cd,t,q} \\ -S^{*}_{k,t,q} = 0 \quad , \forall k \in d^{1-t-q}, \forall t, \forall q \end{cases} \qquad (11)$$

$$\begin{cases} \sum_j \lambda_j^{3-t-q} X_j^{1-t-q} + \sum_j \lambda_j^{A} X_j^{1-t-q} \\ +\sum_{ci} \mu i\, 3_{ci,t,q} CX\, 3_{ci,t,q} + S^{-}_{i,t,q} \\ = \theta X_p^{1-t-q}, \forall i \in m^{1-t-q}, \forall t, \forall q \end{cases}$$

$$\begin{cases} \sum_j \lambda_j^{3-t-q} Y_j^{1-t-q} + \sum_j \lambda_j^{A} Y_j^{1-t-q} \\ -\sum_{cr} \mu j\, 3_{cr,t,q} CY\, 3_{cr,t,q} - S^{+}_{j,t,q} \\ = Y_p^{1-t-q}, \forall r \in s^{1-t-q}, \forall t, \forall q \end{cases}$$

$$\begin{cases} \lambda_j^{1}, \lambda_j^{A}, \lambda_j^{2-t}, \lambda_j^{3-t-q}, S_i^{-}, S_j^{+}, \\ S^{-}_{i,t}, S^{+}_{j,t}, S^{*}_{k,t}, S^{*}_{k,t,q}, S^{-}_{i,t,q}, \\ S^{+}_{j,t,q}, \mu j 1_{cr}, \mu j 1_{cr}, \mu i\, 2_{ci,t}, \\ \mu j\, 2_{cr,t}, \mu k\, 2_{cd,t} \geq 0 \end{cases}$$

$$\mu k\, 3_{cd,t,q}, \mu i\, 3_{ci,t,q}, \mu j\, 3_{cr,t,q} \geq 0$$

$$Min \quad \begin{cases} \theta - \sum_i S_i^{-} - \sum_j S_j^{+} \\ -\sum_t \sum_i S^{-}_{i,t} - \sum_t \sum_j S^{+}_{j,t} \\ -\sum_t \sum_k S^{*}_{k,t} - \sum_t \sum_q \sum_i S^{-}_{i,t,q} \\ -\sum_t \sum_q \sum_j S^{+}_{j,t,q} - \sum_t \sum_q \sum_k S^{*}_{k,t,q} \end{cases} \qquad (12)$$

$ST.$

$$\begin{cases} \sum_j \lambda_j^{1} X_j^{1} + \sum_j \lambda_j^{A} X_j^{1} \\ +\sum_{ci} \mu i\, 1_{ci} CX\, 1_{ci} + S_i^{-} \\ = \theta X_p^{1} \quad , \forall i \in m^{1} \end{cases}$$

$$\begin{cases} \sum_j \lambda_j^{1} Y_j^{1} + \sum_j \lambda_j^{A} Y_j^{1} \\ -\sum_{cr} \mu j\, 1_{cr} CY\, 1_{cr} - S_j^{+} \\ = Y_p^{1} \quad , \forall r \in s^{1} \end{cases}$$

$$\begin{cases} \sum_j \lambda_j^{2-t} X_j^{1-t} + \sum_j \lambda_j^{A} X_j^{1-t} \\ +\sum_{ci} \mu i\, 2_{ci,t} CX\, 2_{ci,t} + S^{-}_{i,t} \\ = \theta X_p^{1-t} \quad , \forall i \in m^{1-t}, \forall t \end{cases}$$

$$\begin{cases} \sum_j \lambda_j^{2-t} Y_j^{1-t} + \sum_j \lambda_j^{A} Y_j^{1-t} \\ -\sum_{cr} \mu j\, 2_{cr,t} CY\, 2_{cr,t} - S^{+}_{j,t} \\ = Y_p^{1-t} \quad , \forall r \in s^{1-t}, \forall t \end{cases}$$

$$\begin{cases} \sum_j \lambda_j^{2-t} Z_j^{1-t} - \sum_j \lambda_j^{1} Z_j^{1-t} \\ -\sum_{cd} \mu k\, 2_{cd,t} CZ\, 2_{cd,t} - S^{*}_{k,t} \\ = 0 \quad , \forall k \in d^{1-t}, \forall t \end{cases}$$

$$\begin{cases} \sum_j \lambda_j^{3-t-q} Z_j^{1-t-q} - \sum_j \lambda_j^{2-t} Z_j^{1-t-q} \\ -\sum_{cd} \mu k\, 3_{cd,t,q} CZ\, 3_{cd,t,q} - S^{*}_{k,t,q} \\ = 0 \quad , \forall k \in d^{1-t-q}, \forall t, \forall q \end{cases}$$

$$\begin{cases} \sum_j \lambda_j^{3-t-q} X_j^{1-t-q} + \sum_j \lambda_j^{A} X_j^{1-t-q} \\ +\sum_{ci} \mu i\, 3_{ci,t,q} CX\, 3_{ci,t,q} + S^{-}_{i,t,q} \\ = \theta X_p^{1-t-q}, \forall i \in m^{1-t-q}, \forall t, \forall q \end{cases}$$

$$\begin{cases} \sum_j \lambda_j^{3-t-q} Y_j^{1-t-q} + \sum_j \lambda_j^{A} Y_j^{1-t-q} \\ -\sum_{cr} \mu j\, 3_{cr,t,q} CY\, 3_{cr,t,q} - S^{+}_{j,t,q} \\ = Y_p^{1-t-q} \quad , \forall r \in s^{1-t-q}, \forall t, \forall q \end{cases}$$

$$\begin{cases} \sum_j \lambda_j^{1-t-q} y_{aj}^{1-t-q} \geq F(\sum_j \lambda_j^{1-t-q} y_{bj}^{1-t-q}) \\ , \forall\, Dependent\, (a,b) \end{cases}$$

$$\begin{cases} \lambda_j^{1}, \lambda_j^{A}, \lambda_j^{2-t}, \lambda_j^{3-t-q}, S_i^{-}, S_j^{+}, S^{-}_{i,t}, S^{+}_{j,t} \\ , S^{*}_{k,t}, S^{*}_{k,t,q}, S^{-}_{i,t,q}, S^{+}_{j,t,q}, \mu j 1_{cr} \\ , \mu j 1_{cr}, \mu i\, 2_{ci,t}, \mu j\, 2_{cr,t}, \mu k\, 2_{cd,t} \geq 0 \end{cases}$$

$$\mu k\, 3_{cd,t,q}, \mu i\, 3_{ci,t,q}, \mu j\, 3_{cr,t,q} \geq 0$$

The model equation 12 is proposed as the final model of this paper and in the next section, its numerical results are presented for the case study.

After solving the model, the total efficiency can be obtained through equation (7) and the efficiency of the components can be obtained through Equation (4), Equation (5), and Equation (6).

5- Numerical results

In this paper, the data of the after-sales service network of one of the largest after-sales service companies in Iran in 2020 has been used to implement the model. Figure 3 shows the structure of this network.

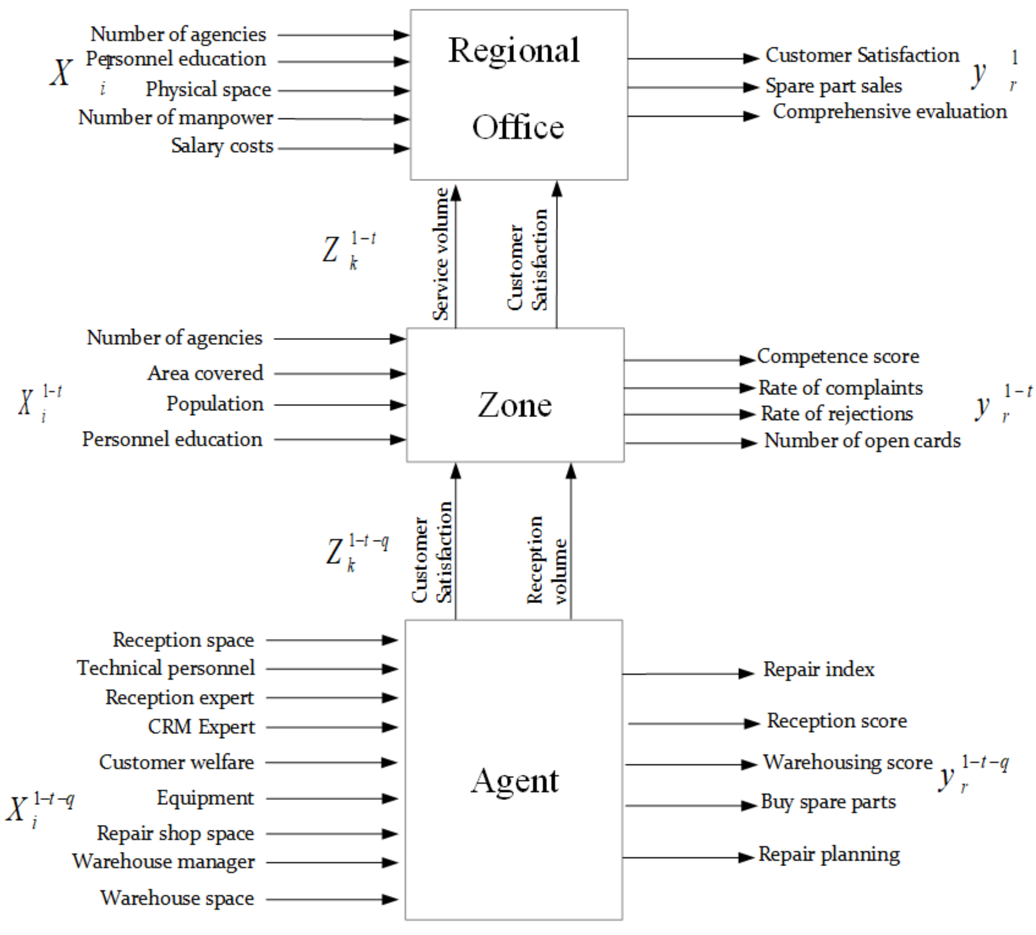

Figure 3 After-sales service network structure

It should be noted that due to the need for the number of DMUs to be homogeneous at different levels, by modifying the data in the first level (regional office) 11 DMU, in the second level (zone) for each DMU in the first level, 2 DMU and Finally for the third level (agent) for each DMU in the second level, there are 19 DMUs. Table 3 below defines the parameters.

Table 3. After-sales service network parameters

Parameters	Level	Define
Number of agencies	Regional Office	Number of agencies covered
Personnel education	Regional Office	Score index for the education of the staff on a scale of 100
Physical space	Regional Office	Physical space based on square meters
Number of manpower	Regional Office	Number of manpower
Salary costs	Regional Office	The cost of salaries paid to human resources
Customer Satisfaction	Regional Office	Customer satisfaction score on a scale of 1000
Spare part sales	Regional Office	The amount of piece sales based on Rials (Currency of Iran)
Comprehensive evaluation	Regional Office	An evaluation index based on 100 based on performance indicators
Number of agencies	Zone	Number of agencies covered
Area covered	Zone	Covered area based on square kilometers
population	Zone	The population of areas covered
Personnel education	Zone	Score index for the education of the staff on a scale of 100
Competence score	Zone	An evaluation index based on 100 based on the performance indicators of the covered agencies
Rate of complaints	Zone	The ratio of the number of complaints to the total reception of the covered agencies
Rate of rejections	Zone	Ratio of the number of unaccepted customers to the total reception of the covered agencies
Number of open cards	Zone	Number of open repair cards (customers whose car repair work has not been completed)
Service volume	Zone	Number of receptions
Customer Satisfaction	Zone	Customer satisfaction score on a scale of 1000
Reception space	Agent	Score on a scale of 100 for the reception area in accordance with the standard
Technical personnel	Agent	Score on a scale of 100 for technical personnel (skills, experience and knowledge) in accordance with the standard
Reception expert	Agent	Score on a scale of 100 for reception expert requirements (skills, experience and knowledge) in accordance with the standard
CRM Expert	Agent	Score on a scale of 100 for CRM expert requirements (skills, experience and knowledge) in accordance with the standard
Customer welfare	Agent	Scale of 100 on the status of customer welfare facilities in accordance with the standard
Equipment	Agent	Score 100 on standard equipment
Repair shop space	Agent	Score on a scale of 100 for the condition of the repair shop space in accordance with the standard
Warehouse manager	Agent	Score on a scale of 100 for warehouse manager (skills, experience and knowledge) in accordance with the standard
Warehouse space	Agent	Score 100 on a standard basis for warehouse space status
Repair index	Agent	Score for fixing all vehicle defects in a timely manner on a scale of 1000

Reception score	Agent	Score on a scale of 100 regarding the status of customer reception conditions
Warehousing score	Agent	Score 100 on the status of ordering and warehousing
Buy spare parts	Agent	The amount of the piece purchased from the parent company in Rials
Repair planning	Agent	Score 100 on the status of repair planning
Customer Satisfaction	Agent	Customer satisfaction score on a scale of 1000
Reception volume	Agent	Number of receptions

As can be seen, the parameters have their scale, from numbers less than 1 to the scale of population, area, and the number of receptions, which are numbers with high values, so at first, the data are normalized, to do this, each value is divided by the maximum value of that parameter so that all indicators are converted to numbers between 0 and 1. Appendix 1 presents the results of descriptive statistics of parameters data.

In the first step, a multiplicative model is used to obtain efficient and inefficient DMUs (it should be noted that the objective function of the models is based on the total performance θ^A and the goal is to obtain the level 1 performance θ^1) Table 4 presents the number of model parameters. According to that, the multiplicative model has 462 constraints and 636 variables.

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By solving this model, the results of Table 5 below are obtained. As the results show, only DMU No. 9 among all DMUs is efficient.

Table 4. Model parameters

Variable	Symbol	Number
Number of level 1 DMUs	n	11
Number of level 2 DMUs	t	2
Number of level 3 DMUs	q	19
Number of level 1 inputs	m^1	5
Number of level 1 outputs	s^1	3
Number of outputs of level 2 to level 1	d^{1-t}	2
Number of level 2 inputs	m^{1-t}	4
Number of level 2 outputs	s^{1-t}	4
Number of outputs of level 3 to level 2	d^{1-t-q}	2
Number of level 3 inputs	m^{1-t-q}	9
Number of level 3 outputs	s^{1-t-q}	5

Table 5. Efficiency results

DMU	θ^1	θ^A
1	0.721	1.000
2	0.762	1.000
3	0.824	1.000
4	0.678	1.000
5	0.820	1.000
6	0.917	1.000
7	0.734	1.000
8	0.850	1.000
9	1.000	1.000
10	0.778	1.000
11	0.568	1.000

Now to get the benchmarking we have to solve the envelopment model, before that we have to get the relationship of the dependent parameters. By further evaluation of the data, it is observed that in the agency level parameters, customer satisfaction score and repair index have a correlation coefficient of 0.714, so to apply this condition in the final model (to increase the accuracy of benchmarking) the relationship between the two must be Index to be specified. It should be noted that this shows that a dealership cannot have a low score in the repair index and on the other hand, can expect to be able to achieve a high score of customer satisfaction, one of the purposes of this article is that this condition in defining the benchmark for agencies to be considered.

It can be logically concluded that customer satisfaction is the result of the repair index, therefore, the repair index is an independent variable, and the customer satisfaction index is a dependent variable. Initially, different regression methods have been performed to determine the relationship between the two parameters, Table 6 presents the results of this analysis.

Table 6. Curve fitting output

Regression method	***R Square***	***Constant***	***b1***	***b2***	***b3***
Linear	0.519	0.327	0.511		
Logarithmic	0.5	0.822	0.384		
Inverse	0.488	1.093	-0.285		
Quadratic	0.518	0.816	-0.779	0.844	
Cubic	0.518	0.647	-0.122	0	0.359
Compound	0.503	0.415	2.041		
Power	0.495	0.829	0.537		
S	0.484	0.192	-0.399		
Growth	0.503	-0.879	0.714		
Exponential	0.503	0.415	0.714		

According to the results of Table 6, the constraint of the relationship between dependent variables is as follows.

Customer Satisfaction = (0.519× Repair Index) + 0.327

Due to the fact that customer satisfaction is of higher importance, then their relationship is modified as follows.

Customer Satisfaction ≥ (0.519× Repair Index) + 0.327

Finally, the last constraint of Equation 12 is modified as Equation 13

$$\sum_j Z_{j,1}^{1-t-q} \geq 0.511 \sum_j Y_{j,1}^{1-t-q} + 0.327 \qquad (13)$$

Table 7. Level 2 efficiency results

Zone	Efficiency	Zone	Efficiency
DMU1 - Zone 1	0.92	DMU1 - Zone 2	1
DMU2 - Zone 1	1	DMU2 - Zone 2	1
DMU3 - Zone 1	0.91	DMU3 - Zone 2	1
DMU4 - Zone 1	1	DMU4 - Zone 2	0.26
DMU5 - Zone 1	0.95	DMU5 - Zone 2	1
DMU6 - Zone 1	1	DMU6 - Zone 2	0.88
DMU7 - Zone 1	0.91	DMU7 - Zone 2	1
DMU8 - Zone 1	0.94	DMU8 - Zone 2	1
DMU9 - Zone 1	0.88	DMU9 - Zone 2	1
DMU10 - Zone 1	0.93	DMU10 - Zone 2	1
DMU11 - Zone 1	1	DMU11 - Zone 2	1

Level 1 efficiency results were previously presented in Table 7, below are Level 2 results.

The results of the envelopment model are used for benchmarking and according to the purpose of this paper, the envelopment model (Equation (12)) is the main model. In order to compare and identify the effect of the constraint of the dependent variables as well as the constraint applied to the model, the results of both models (with and without constraints) are presented. It should be noted that in the envelopment model at level 1 (regional office) we have two types of coefficients λ, λ corresponding to θ^1 and also corresponding to θ^A . Given that the goal was level 1 efficiency, λ's corresponding to this level is presented.

Table 8. Results of benchmarking coefficients without dependent parameters

λ^1	DMU 1	DMU 2	DMU 3	DMU 4	DMU 5	DMU 6	DMU 7	DMU 8	DMU 9	DMU 10	DMU 11
DMU1	0	0	0	0	0	0	0	0	0	0	0
DMU2	0	1	0	0	0	0	0	0	0	0	0
DMU3	0	0	0	0	0	0	0	0	0	0	0
DMU4	0	0	0	0	0	0	0	0	0	0	0
DMU5	0	0	0	0	0	0	0	0	0	0	0
DMU6	0	0	0	0	0	0	0	0	0	0	0
DMU7	0	0	0	0	0	0	0	0	0	0	0
DMU8	0	0	0	0	0	0	0	0	0	0	0
DMU9	0	0	0	0	0	0	0	0	0	0	0
DMU10	0	0	0	0	0	0.26	0.2	0.35	0.19	0	0
DMU11	0	0	0	0	0	0	0	0	0	0	0

Table 9. Results of benchmarking coefficients with dependent parameters

λ^1	DMU 1	DMU 2	DMU 3	DMU 4	DMU 5	DMU 6	DMU 7	DMU 8	DMU 9	DMU 10	DMU 11
DMU1	0.027	0	0	0	0	0	0	0	0	0	0
DMU2	0	0	0	0	0	0	0	0	0	0	0
DMU3	0	0	0.026	0	0	0	0	0	0	0	0
DMU4	0	0	0	0.026	0	0	0	0	0	0	0
DMU5	0	0	0	0	0.027	0	0	0	0	0	0
DMU6	0	0	0	0	0	1	0	0	0	0	0
DMU7	0	0	0	0	0	0	0.026	0	0	0	0
DMU8	0	0	0	0	0	0	0	0.025	0	0	0
DMU9	0	0	0	0	0	0	0	0	0.025	0	0
DMU10	0	0.001	0	0	0.005	0	0	0.019	0	0	0
DMU11	0	0	0	0	0	0	0	0	0	0	0.025

First, in Table 8, the results of the initial model without the constraint of dependent parameters are presented, and then in Table 9, the results with dependent parameters are presented.

6- Conclusion

The main purpose of this paper is to predictive benchmarking in a hierarchical structure with dependent parameters, but once again, we need to review the reason why dependent parameters need to be considered. For example, in our case study, the experts had a hypothesis that one of the most important indicators affecting customer satisfaction is the repair index. Therefore, these conditions should also be considered when defining a benchmark to an agency.

As seen in Table 8 and Table 9, the results of benchmarking coefficients for the two models were presented for the second level, by considering or ignoring the dependence of parameters, and its effect on changes in benchmarking coefficients has been determined. The results show that ignoring the dependence of the parameters reduces the application of the obtained coefficients, the reason for this is the following:

1 –The number of problem variables is high.

2 –The results of DMUs are close to each other and have little difference.

3- The number of DMUs at this level (level 1) is low.

Given that the outputs of the development model are usually very large tables (for example, in the third level of the table is 418 by 418) in this section, some examples of the results are described.

- The data envelopment analysis model at levels 2 and 3 has not provided practical performance when the dependent parameters have not been observed and most of the results have zero benchmarking coefficients, but, in the case of dependence of parameters and weight constraints, practical results have been obtained. However, it should be noted that this is due to the specific conditions of the case study and the proposed hierarchical model is not necessarily the reason for this.

- In the final output of the model, it is observed that the benchmarking coefficients are mostly current in the level 1 subset, for example, the benchmarking coefficients of the "Kerman" regional office zones are in the same Kerman regional office and in other regional offices have a value of zero or agent code 7-1-1 Which is the No. 1 agent of zone 1 of the "Azerbaijan" regional office, has benchmarked itself on the "Azerbaijan" regional office, although there is a case of violation of this case, for example, the agency code 5-1-12, which is the agency No. 12 in zone 1 of the "Hamedan" regional office, has benchmarked most of the offices. This is an advantage for the case study.
- The agency does its benchmarking from the offices and the agency-to-agency comparison is avoided.

Due to the limitations of this study, the following are suggested for future studies:

- Model development for dependent parameters with a negative correlation coefficient
- Model development for undesirable parameters
- Development of dependent parameters models for other structures

Model development for the dependence of parameters between different levels

Appendix 1

Results of descriptive statistics

Level	Parameters	Minimum	Maximum	Mean	Std. Deviation
Number of agencies	Regional Office	39	98	66	18
Personnel education	Regional Office	456	722	553	84
Physical space	Regional Office	150	400	256	74
Number of manpower	Regional Office	19	48	27	8
Salary costs	Regional Office	950000000	2400000000	1345454545	417405406
Customer Satisfaction	Regional Office	702	755	723	18
Spare part sales	Regional Office	6576020203	14712443550	8455663483	2259493851
Comprehensive evaluation	Regional Office	62	72	68	3
Number of agencies	Zone	19	41	30	6
Area covered	Zone	12981	434708	143043	116460
population	Zone	3801116	15399472	7711803	2848473
Personnel education	Zone	400	750	549	100
Competence score	Zone	54	70	61	4
Rate of complaints	Zone	0.26%	0.99%	0.45%	0.18%
Rate of rejections	Zone	0.02%	0.15%	0.05%	0.03%
Number of open cards	Zone	53	256	144	52
Service volume	Zone	5491	13741	8836	2107
Customer Satisfaction	Zone	694	756	725	19
Reception space	Agent	17	100	90	20
Technical personnel	Agent	20	100	87	10
Reception expert	Agent	0	100	80	21
CRM Expert	Agent	0	100	82	29
Customer welfare	Agent	43	100	89	11
Equipment	Agent	41	100	84	11
Repair shop space	Agent	39	100	94	8
Warehouse manager	Agent	0	100	92	17
Warehouse space	Agent	15	100	96	12
Repair index	Agent	603	914	779	57
Reception score	Agent	37	100	72	11
Warehousing score	Agent	8	100	77	17
Buy spare parts	Agent	14998394	78920136530	9654202552	9818043465
Repair planning	Agent	8	100	51	19
Customer Satisfaction	Agent	534	844	725	41
Reception volume	Agent	14	2741	465	348

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