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Ranking all of efficient units using the proposed SBM model

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Abstract
One of the important issues discussed in DEA literature is ranking efficient units. This paper presents a new method based on slack based measure SBM with free variables in sign for ranking efficient units. The new method also can rank the efficient units non-extreme. It also does not have the problem of instability and infeasibility. The proposed model specifies the largest MPSS and after that other unit's portraits to this unit. Then, using examples will be compared with the approaches.

Keywords: Data Envelopment Analysis, Ranking, Efficiency.

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1. Introduction

The main idea of data envelopment analysis (DEA) to evaluate the performance of a set of homogeneous decision-making units in order to identifying efficient units. The value of efficiency an efficient unit will be showed with unit. Since in most of problems efficient units are more than unit, ranking methods have been proposed. Sexton et al [1] first presented an approach for ranking efficient units that are known as the cross-efficiency approach. Another important field in ranking is benchmarking methods such as Torgersen et al [2] suggested an approach method that based on an efficient unit is more efficient when it will be reference to more inefficient units. The most famous of ranking method that works based on supper efficiency first has been introduced by Anderson and Patterson [3]. In this way, the effect of removal efficient unit is a criterion for ranking that unit. Thrall [4], doula and Hickman [5] and Seiford and Zhou [6], Chen [7] showed that super efficiency models are infeasible in some conditions. For fixing this problem, Mehrabian et al [8], Tone [9,10], Jahanshahloo et al [11], have suggested their models. Furthermore, Adler et al [12], and Jahanshahloo et al [13], and HosseinzadehLotfi et al [14] have collected different ranking models and classified them.

The remainder of this paper is organized as follows. In Section 2, CCR and BCC model and SBM model and AP model are discussed. In Section 3, our SBM model is introduced. In what follow, we compare our approximation with the previous approximations. Numerical examples are used to illustrate the proposed approaches. Section 5 concludes the paper.

2- Background

Let we have n decision making units (DMUs) with the input vector of $x_j = (x_{1j}, \dots, x_{mj})$ and with the output vector of $y_j = (y_{1j}, \dots, y_{sj})$ so that $x_j \geq 0$ ، $x_j \neq 0$، $y_j \geq 0$، $y_j \neq 0$. The first DEA model for evaluating efficiency of DMUs was CCR model that introduced by Charnes, Cooper and Rhodes [4]. Production possibility set (T_C) has been defined as following:

$$T_C = \left\{ \begin{array}{l} (x, y):\ \sum_{j=1}^{n} x_j \lambda_j \leq x, \\ \qquad \sum_{j=1}^{n} y_j \lambda_j \geq y, \\ \qquad \lambda_j \geq 0\ j = 1, 2, \dots, n \end{array} \right\}$$

Input-oriented CCR model for assessment of relative Efficiency DMUo is as follows:

$$\begin{array}{lll} Min & \theta & (1) \\ s.t & \sum_{j=1}^{n} y_{rj}\lambda_j \geq y_{r\circ}, & r = 1, 2, \dots, s \\ & \sum_{j=1}^{n} x_{ij}\lambda_j \leq \theta x_{i\circ}, & i = 1, 2, \dots, m \\ & \lambda_j \geq 0 & \end{array}$$

With adding slack variables model (1) will be converted to the following model:

$$\begin{array}{lll} Min & \theta - \varepsilon(\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^-) & (2) \\ s.t & \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{r\circ}, & r = 1, 2, \dots, s \\ & \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta x_{i\circ}, & i = 1, 2, \dots, m \\ & \lambda_j, s_r^+, s_i^- \geq 0 & \end{array}$$

Definition 1 (CCR efficiency): DMU_o is efficient if and only if in the optimal solution of model (2) $\theta^* = 1$ and for each i, r have $s_r^{+*} = s_i^{-*} = 0$.

Banker, Charnes and Cooper [3] by adding the convexity constraint of $\sum_{j=1}^{n} \lambda_j = 1$ to CCR model introduced a new model that was called BCC model that is as follows:

$$\begin{aligned} Min \quad & \theta \qquad (3) \\ s.t \quad & \sum_{j=1}^{n} y_{rj}\lambda_j \geq y_{r\circ}, \qquad r=1,2,...,s \\ & \sum_{j=1}^{n} x_{ij}\lambda_j \leq \theta x_{i\circ}, \qquad i=1,2,...,m \\ & \sum_{j=1}^{n} \lambda_j = 1, \qquad \lambda_j \geq 0 \end{aligned}$$

Production possibility set in BCC model is shown with T_V and defined as following: $T_V = \{(x,y): x \geq \sum_{j=1}^{n} \lambda_j x_j , y \leq \sum_{j=1}^{n} \lambda_j y_j , \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \ (j = 1 \dots, n)\}$.

Definition 2 (BCC efficiency): DMU_o belonging to production possibility set T_V is technical efficiency if and only if $\theta^*_{BCC} = 1$. Otherwise DMU_o is inefficient and $(1 - \theta^*_{BCC})$ is the value of technical inefficiency in input-oriented model.

Slack based measure model

In this section, we will introduce SBM model which is stable to changes in measure of inputs and outputs. In SBM model, if replace $x_{ij}, \forall(i = 1, \dots, m)$ with $K_i x_{ij} = \hat{x}_{ij}$ and replace y_{rj} with $C_r y_{rj} = \hat{y}_{rj}$, there will be no changes in amount of obtained efficiency and this feature is known as independence of the unit. The fractional model of SBM has been introduced by Tone [9] as follows:

$$\begin{aligned} Min \quad & W = \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}}}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{ro}}} \qquad (4) \\ s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, \qquad i=1,2,....,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \qquad r=1,2,....,s \\ & s_i^- \geq 0, s_r^+ \geq 0, \lambda_j \geq 0. \ i=1,....,m, \ r=1,....,s, \ j=1,.....,n \end{aligned}$$

By introducing the positive variable of B, model (4) can be like following:

$$\begin{aligned} Min \quad & W = B - \frac{1}{m}\sum_{i=1}^{m}\frac{Bs_i^-}{x_{io}} \qquad (5) \\ s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, \qquad i=1,2,....,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \qquad r=1,2,....,s \\ & B + \frac{1}{s}\sum_{r=1}^{s}\frac{Bs_r^+}{y_{ro}} = 1 \\ & S_i^- \geq 0, s_r^+ \geq 0, \ \lambda_j \geq 0, \ B \geq \varepsilon . i=1,....,m, r=1,....,s \end{aligned}$$

Now, we will define:

$$S_i^- = BS_i^-, \quad S_r^+ = BS_r^+, \quad \mu_j = B\lambda_j$$

By such change of variables, model of (5) will be converted as follows:

$$\begin{aligned} Min \quad & W = B - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}} \qquad (6) \\ s.t \quad & \sum_{j=1}^{n} \mu_j x_{ij} + s_i^- = Bx_{io}, \qquad i=1,2,....,m \\ & \sum_{j=1}^{n} \mu_j y_{rj} - s_r^+ = By_{ro}, \qquad r=1,2,....,s \\ & B + \frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{ro}} = 1 \\ & s_i^- \geq 0, s_r^+ \geq 0, \ \lambda_j \geq 0, \ B \geq \varepsilon . i=1,....,m, r=1,....,s \end{aligned}$$

Suppose $(W^*, B^*, \mu_j^*, S_i^{-*}, S_r^{+*})$ is the optimal solution of model (6), then the optimal solution of model (4) will be:

$$W^* = w^*, \quad S_i^{-*} = \frac{s_i^{-*}}{B^*}, \quad S_r^{+*} = \frac{s_r^+}{B^*}, \quad \lambda_j^* = \frac{\mu_j^*}{B^*}$$

Definition 3 (SBM-efficient**):** A DMUo is SBM-efficient if and only if $W^* = 1 \left(S_r^{+*} = 0, S_i^{-*} = 0\right)$.

This condition is equivalent to $s^{-*} = 0$ and $s^{+*} = 0$, i.e., no input excess and no output shortfall in an optimal solution.

Theorem 1. $W^* \leq \theta^*_{CCR}$ **.**

Proof. Refer to Tone [9].

Theorem 2. A DMUo is CCR-efficient if and only if it is SBM-efficient.

Proof. Refer to Tone [9].

Anderson and Peterson model

In 1993, Andersen and Petersen introduced a new model for ranking efficient units. Their model is known as AP model in DEA literature. This model by removing the efficient unit of DMUo from the production possibility set (PPS) presents a new PPS as follows:

$$T_{AP}=\left\{\begin{pmatrix}x\\y\end{pmatrix}\;\middle|\;\begin{array}{l} x\geq\sum_{\substack{j=1\\ j\neq o}}^{n}\lambda_j x_j,\, y\leq\sum_{\substack{j=1\\ j\neq o}}^{n}\lambda_j y_j,\\ \lambda_j\geq 0,\ j=1,\dots,n,\ j\neq o\end{array}\right\}.$$

The proposed model is as follows:

$$\theta_o^*=Min\,\theta \qquad (7)$$

$$\begin{aligned} s.t\quad & \sum_{\substack{j=1\\ j\neq o}}^{n}\lambda_j x_{ij}\leq\theta x_{io},\ i=1,2,\dots,m\\ & \sum_{\substack{j=1\\ j\neq o}}^{n}\lambda_j y_{rj}\geq y_{ro},\ r=1,2,\dots,s\\ & \lambda_j\geq 0, \qquad j=1,2,\dots,n.\,j\neq o \end{aligned}$$

The methodology enables an extreme efficient unit o to achieve an efficiency score greater than one by removing the o-th constraint in the primal formulation; however, there are three problematic areas with this methodology

1. With this method it is not possible to rank non-extreme efficient units.
2. AP model may be infeasible for special data in input oriented case.
3. One of the AP problems is instability. In input-oriented case, if a DMU has, at least, an input near zero, its ranking score will be exceedingly large.

3- Proposed method

For ranking of efficient units, we will consider the following SBM model Subject to being unbound of slack variables, like below:

$$Min \quad W=\frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}}}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{ro}}} \qquad (8)$$

$$\begin{aligned} s.t\quad & \sum_{j=1}^{n}\lambda_j x_{ij}+s_i^-=x_{io}, \qquad i=1,2,\dots,m\\ & \sum_{j=1}^{n}\lambda_j y_{rj}-s_r^+=y_{ro}, \qquad r=1,2,\dots,s\\ & \sum_{j=1}^{n}\lambda_j=1, \qquad \lambda_j\geq 0.\ j=1,\dots,n \end{aligned}$$

With the change of variables, the above model will be converted to the following model:

$$Min \quad W=B-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}} \qquad (9)$$

$$\begin{aligned} s.t\quad & \sum_{j=1}^{n}\mu_j x_{ij}+s_i^-=Bx_{io}, \qquad i=1,2,\dots,m\\ & \sum_{j=1}^{n}\mu_j y_{rj}-s_r^+=By_{ro}, \qquad r=1,2,\dots,s\\ & \sum_{j=1}^{n}\mu_j=B\\ & B+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{ro}}=1\\ & \mu_j\geq 0,\ B\geq\varepsilon.\ j=1,\dots,n \end{aligned}$$

Theorem 3. Model (9) is feasible.

Proof. We put $\mu_\circ=1$ and $\mu_j=0,\ j\neq 0$, $j=1,2,3,\dots,n$, for $i=1,2,3,\dots,m,\ s_i^-=0$, and for $r=1,2,3,\dots,s,\quad s_r^+=0$. Then $\left(\mu_j=e_o, S^-=0, S^+=0\right)$ is a feasible solution for model (9).□

Theorem 4. The optimal value of the objective of model (2) is $0\leq W^*\leq 1$.

Proof: We put $\mu_\circ=1$ and $\mu_j=0,\ j\neq 0,\ j=1,2,3,\dots,n$, for $i=1,2,3,\dots,m,\ s_i^-=0$, and for $r=1,2,3,\dots,s,\ \ s_r^+=0$. Then

$\left(\mu_j = e_o, S^- = 0, S^+ = 0\right)$ is a feasible solution of model (9). The objective function value for this Solution is 1, and regarding minimization we have $W^* \le 1$. Improved activity corresponding to each decision making unit is on the efficient frontier; but for each frontier point it may not to be favored by the decision making unit. T_v Can see more on this issue; where the scale and size share a lot on determine efficiency. The unit that the amounts of its inputs and outputs have the most productivity, it is called MPSS. Concept of the most productivity scale, for the first time, has been presented by Banker and et al. (1984). For a unit that is not MPSS, by increasing input, the amount of the corresponding output is increased proportionally; or by decreasing input, maximum the amount of output is decreased proportionally. About MPSS, inputs and outputs have the best and the most economical size. MPSS recognition and imaging units on it, based on the market demand, causes if low needed, the units will be imaged on the smallest image of MPSS and if a lot needed, the units will be imaged on the largest image of MPSS.

Theorem 8: Suppose in evaluating of $\left(x_O, y_O\right) \in \delta T_V$, if in each optimal solution of model (7), all the slack variables are equal to zero and the amount of objective function is equal to 1, it means $W^* = 1 \quad, \left(\lambda^*, S^{-^*} = S^{+^*} = 0\right)$, then DMU$_O$ is MPSS.

Proof: Regarding $B \ge \varepsilon$ and $\mu_j \ge 0$ so sum of these non-negatives is a positive scalar, it means $\sum_{j=1}^{n} \mu_j = B$ so this restriction is redundant and the model will be converted like following:

$$
\begin{aligned}
Min \quad & W = B - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}} \qquad (10)\\
s.t \quad & \sum_{j=1}^{n}\mu_j x_{ij} + s_i^- = Bx_{io} \quad i = 1,2,....,m\\
& \sum_{j=1}^{n}\mu_j y_{rj} - s_r^+ = By_{ro} \quad r = 1,2,....,s\\
& B + \frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{ro}} = 1\\
& \mu_j \ge 0,\ B \ge \varepsilon.
\end{aligned}
$$

$\left(\mu_j = e_o, S^- = 0, S^+ = 0\right)$ is a feasible solution of model (10) and it applies by replacing in model (10). The objective function for this feasible solution is 1 and since it is minimization, so the objective function should be less than or equal to the feasible solution, it means $W^* \le 1$. According to theorem (2), the under evaluated DMU$_O$ also is efficient in CCR model and due to assume of the problem, so it is MPSS.☐

Theorem 9:
$W^* = B^* = 1 \quad, \left(\lambda^*, S^{-^*} = S^{+^*} = 0\right)$ are the largest images of MPSS corresponds to the unit of (X_o, Y_o).

Proof: 1) $(B^* X_o - S^{-*}, B^* Y_o + S^{+*}) \in T_v$

2) If $(B^* X_o - S^{-*}, B^* Y_o + S^{+*}) \in T_v$ is not MPSS then, there is $(\alpha(B^* X_o - S^{-*}), \beta(B^* Y_o + S^{+*})) \in T_v$ so that $\beta > \alpha$. Also there is $\{\lambda_j\}_{j=1}^{n}$ as:

$$
\begin{aligned}
& \sum_{j=1}^{n}\lambda_j X_j \le \alpha\left(B^* X_o - S^{-*}\right)\\
& \sum_{j=1}^{n}\lambda_j Y_j \ge \beta\left(B^* Y_o + S^{+*}\right)\\
& 1\lambda = 1,\ \lambda \ge 0 \qquad (11)
\end{aligned}
$$

By introducing slack variables and dividing restrictions on βB^*, it is resulted:

$$\sum_{j=1}^{n} \bar{\lambda}_j X_j + \bar{S}^- = \frac{\alpha}{\beta} X_o$$

$$\sum_{j=1}^{n} \bar{\lambda}_j X_j - \bar{S}^+ = Y_o$$

$$\bar{\lambda} \geq 0 \qquad (12)$$

That in which $\bar{\lambda}_j = \frac{1}{\beta B^*} \lambda_j, j = 1,\dots,n$, also we found a solution for CCR model in evaluating (X_o, Y_o) that we obtained $B^* = \theta^* = 1$ before but now, $\frac{\alpha}{\beta} < \theta^*$ that it means we have obtained a better solution and this is contradiction.

So $(B^* X_o - S^{-*}, B^* Y_o + S^{+*})$ is MPSS. $W^* = B^* = 1 \quad , (\lambda^*, S^{-^*} = S^{+^*} = 0)$ The maximum amount of W will result the largest image of MPSS. ☐

In most previous DEA model that are used to rank efficient units, by removing the efficient unit under evaluation among all units will be made a new PPS. Then, by finding the nearest competitor the distance will be ranking criteria. More distance means the unit has better rank. In our proposed method, any DMU will not be removed from the production possibility set. This approach by selecting best DMU other DMUs is image upon it. In this method, less distance than the Best DMU means the unit has better. The advantage of this method is that it is always feasible and stable. In addition, this method ranks non vertical DMUs while other methods can't do this. Now by evaluating the efficient DMUs using the proposed SBM model the DMU with optimal solution $W^* = 1 \quad , (\lambda^*, S^{-^*} = S^{+^*} = 0)$ has first rank and the DMU with optimal solution $W^* = 1 \quad , (\lambda^*, S^{-^*} \neq 0 \text{ or } S^{+^*} \neq 0)$ takes next rank. Similarly, the rest of DMUs are ranked.

4- A numerical example

Example 1: Table 1 presents five DMUs A, B, C, D and E with one input and one output. Ranking these units with models of AP, MAJ, MAJ modified and the proposed model shows that DMUD that is MPSS has first rank in the proposed model, second rank in AP model and third rank in MAJ and MAJ modified models. The unit of B that is MPSS has second rank in the proposed model, third rank in AP model second rank in MAJ model fourth rank in MAJ modified model. DMUE has third rank in the proposed model and first rank in MAJ modified model. This unit is infeasible in AP and MAJ models. DMUF that is non-vertical efficiency has fourth rank in the proposed model and other models can't rank this unit. DMUA has fifth rank in the proposed model, second rank in AP and MAJ models and finally this unit has second rank in MAJ modified model (Fig. 1):

Table 1: Inputs and Outputs and the results of ranking DMUs with ranking models

DMU	A	B	D	E	F
Input	2	4	8	10	12
Output	3	8	16	18	20
AP Value	1.5(1)	1.05(3)	1.10(2)	Infeasible	1
MAJ	2(1)	1.19(2)	1.18(3)	Infeasible	1
Modified MAJ	2(2)	1.13(4)	1.5(3)	3(1)	1
Proposed Model	0.75(5)	1.00(2) $s^-, s^+ \neq 0$	1.00(1)	0.90(3)	0.83(4)

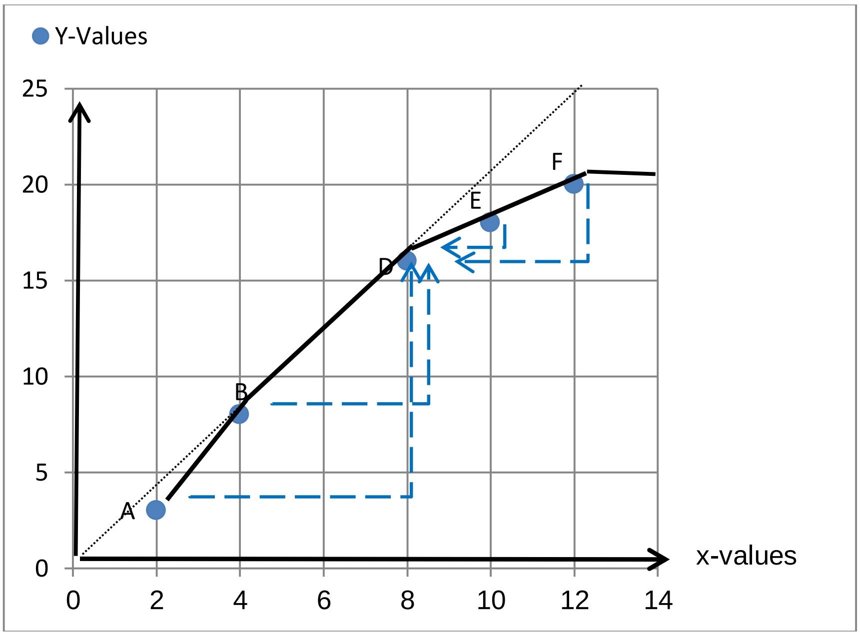

Fig. 1. Illustration of a simple 1 input, 1 output example

Example 2: This example has been taken from Chen (2004). We here use high-end housing price (1000US$), lower end housing monthly rental (US$), and number of Violent crimes as three DEA inputs and median household income (US$), number of population with bachelor's degree(million) and number of doctors (thousand) as three DEA outputs in evaluating fifteen cities of America. Data of these fifteen cities provided in Table 2. Among the fifteen city of America ten of them have the efficiency score equal one so they are efficient and the rest with the efficiency score less than one are in efficient. These efficient units are ranked with Chen's method and the proposed model. The results of ranking presented in Table 3. Based on the results the proposed model, DMU6, St Louis, DMU3, Philadelphia DMU8, Washington, and DMU4, Minneapolis, have ranks of 1, 2,4 and 5, respectively, which are similar to the ranking of Chen's method. DMU1, Seattle, DMU2, Denver, DMU5, Raleigh and DMU9, Pittsburgh, have ranks of 3, 10, 7 and 9, respectively, using Chen's method, but these DMUs or cities have ranks of 6, 9, 8 and 7, respectively, using the proposed model.

Table 2: Fifteen cities of America with three inputs and three outputs with their efficiency score in.

DMUs	Cities	House price	Rental	Violent	Income	B. Degree	Doctor	Efficiency score
DMU01	Seattle	586	581	1193.1	46,928	0.6534	9.878	1
DMU02	Denver	475	558	1131.6	42,879	0.5529	5.301	1
DMU03	Philadelphia	201	600	3468	43,576	1.135	18.2	1
DMU04	Minneapolis	299	609	1340.6	45,673	0.729	7.209	1
DMU05	Raleigh	318	613	634.7	40,990	0.319	4.94	1
DMU06	St Louis	265	558	657.5	39,079	0.515	8.5	1
DMU07	Cincinnati	467	580	882.4	38,455	0.3184	4.48	0.94968
DMU08	Washington	583	625	3286.7	54,291	1.7158	15.41	1
DMU09	Pittsburgh	347	535	917.04	34,534	0.4512	8.784	1
DMU10	Dallas	296	650	3714.3	41,984	1.2195	8.82	0.92652
DMU11	Atlanta	600	740	2963.1	43,249	0.9205	7.805	0.77243
DMU12	Baltimore	575	775	3240.8	43,291	0.5825	10.05	0.73827
DMU13	Boston	351	888	2197.1	46,444	1.04	18.208	1
DMU14	Milwaukee	283	727	778.35	41,841	0.321	4.665	1
DMU15	Nashville	431	695	1245.8	40,221	0.2365	3.575	0.80117

Table 3: Ranking of U.S cities using Chen's method and the proposed model

DMUs	γ	τ	S^a	$S^{a\wedge}$	Proposed Model
DMU01	1.44335	0.91456	1.26839 (3)	0.80370 (3)	0.791373(6)
DMU02	1.01593	1 (Infeasible)	1.00797 (10)	0.99216 (10)	0.620641(9)
DMU03	1 (Infeasible)	0.54672	1.41454 (2)	0.77336 (2)	1.00000 (2)
DMU04	1.22752	0.92080	1.15677 (6)	0.86773 (5)	0.836513 (5)
DMU05	1.16766	0.92032	1.12712 (7)	0.88837(7)	0.648143 (8)
DMU06	1.51628	0.66035	1.51531 (1)	0.65993(1)	1.00000 (1)
DMU07					
DMU08	1 (Infeasible)	0.65150	1.26746 (4)	0.82575 (4)	0.892164 (4)
DMU09	1.04529	1 (Infeasible)	1.02265 (9)	0.97834 (9)	0.778747 (7)
DMU10					
DMU11					
DMU12					
DMU13	1 (Infeasible)	0.75866	1.15906 (5)	0.87933 (6)	0.933001 (3)
DMU14	1.06559	0.97315	1.04659 (8)	0.95580 (8)	0.584501(10)
DMU15					

Example3: In Table 4 there are twelve DMUs with three inputs X_1, X_2 and X_3 and two outputs Y_1 and Y_2. This example was taken from Khodabakhshi et al (1012). Column Efficiency shows that DMUs 4, 5,6,8,9 and 12 are efficient. These DMUs have been ranked with AP and Khodabakhshi models and the proposed model. The results of ranking of these units are shown in Table 4. Based on these results, DMU9 has first rank in AP and

Khodabakhshi methods. Also, this unit has first rank in the proposed model. DMU4 has fourth rank in AP and Khodabakhshi methods but has second rank in the proposed model. DMU12 has third rank in Khodabakhshi method and the proposed model but this DMU is infeasible in AP model. DMU6 has third rank in AP model, sixth rank in Khodabakhshi method and fourth rank in the proposed model. DMU8 has fifth rank in three models. DMU5 has second rank in AP and Khodabakhshi models but this unit has sixth rank in the proposed model.

Table 4: Three Inputs and Three Outputs and the ranking DMUs with ranking models

DMU	X_1	X_2	X_3	Y_1	Y_2	Efficiency	AP	Khodabakhshi	Proposed Model
1	350	39	9	67	751	1.26			
2	298	26	8	73	611	1.03			
3	422	31	7	75	584	1.12			
4	281	16	9	70	665	1.00	1.20(4)	0.0937 (4)	0.811(2)
5	301	16	6	75	445	1.00	1.43(2)	0.0986(2)	0.677(6)
6	360	29	17	83	1070	1.00	1.21(3)	0.0810(6)	0.718(4)
7	540	18	10	72	457	1.06			
8	276	33	5	74	590	1.00	1.12(5)	0.0883(5)	0.689(5)
9	323	25	5	75	1074	1.00	1.87(1)	0.1313(1)	1.000(1)
10	444	64	6	74	1072	1.12			
11	323	25	5	25	350	3.00			
12	444	64	6	104	1199	1.00	Infeasible	0.0944(3)	0.805(3)

5- Conclusion

In this paper, a new method presented for ranking decision making units (DMUs). In the proposed method by finding the best decision-making unit other DMUs are imaged to this DMU. The distance between the best DMU and other DMUs will be taken as a criterion to rank efficient DMUs. The proposed model doesn't have the problems of unstable and infeasible that there are in some models of ranking. Finally, developing of the proposed model with fuzzy or stochastic data can be suggested for further research.

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