Available online at http://ijdea.srbiau.ac.ir

Int. J. Data Envelopment Analysis (ISSN 2345-458X)

Vol. 10, No. 2, Year 2022 Article ID IJDEA-00422, pages 1-12
Research Article

International Journal of Data Envelopment Analysis

Science and Research Branch (IAU)

Resource allocation problems in data envelopment analysis with simultaneous shared costs and common revenue

Kaveh Khalili-Damghani,[*] Farzin Noorzadeh[2], Arash Apornak[3]

[(1,2)] Department of Industrial Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran
[(3)] Department of Industrial Engineering, University of Tehran, Tehran, Iran

Received 19 December 2021, Accepted 15 April 2022

Abstract
Allocating fixed shared cost and common revenue equitably to all decision making units (DMUs) are two interesting issues in resource allocation using data envelopment analysis (DEA). The existing methods have accomplished these distinctively. In this paper, we developed a single dada envelopment analysis (DEA) approach for equitable allocation of shared costs to inputs and common revenue to outputs, concurrently. The main contribution of this research in comparison with existing methods are: 1) both allocation of shared costs to inputs and common revenue to outputs are considered in a single model, simultaneously; 2) The computational efforts has been reduced and no LP required to be solved; 3) simultaneous changes of inputs and outputs have been considered in order to project a DMU towards efficient frontier. A numerical example, adopted form literature, is presented and discussed in order to illustrate the applicability and efficacy of proposed approach.

Keywords: Data envelopment analysis; Resource Allocation; Equitable allocation of shared cost; Equitable allocation of common revenue; Modified Russell model

[*] Corresponding author: Email: kaveh.khalili@gmail.com

1. Introduction

Data envelopment analysis (DEA) is used to evaluate the relative efficiency of homogenous decision making units (DMUs) with multiple inputs and outputs. Measuring relative efficiency has been at the center of the focus of researchers [1]. Technical efficiency analysis was first reported by [2], and it is the capacity of a decision-making unit (DMU) to achieve maximal output under a given set of inputs [3]. First introduced data envelopment analysis, he offered a model which could measure the efficiency with multiple inputs and outputs for constant return to scale. Then, developed the DEA models for variable return to scale [4,5].

One of the most important extensions of DEA is equitable allocation of shared costs to decision-maker units [5-8]. The aim of management is to allocate shared costs in a manner that the relative efficiency of DMUs are not changed, while the absolute efficiency might be affected by some changes. [9] proposed a model by which shared cost allocation was made by the two principles of invariance and Pareto minimality. Allocation of shared cost is called invariant when the relative efficiency of each DMU remains unchanged before and after the allocation. Allocation of shared cost is called input Pareto minimality (output pareto maximality) when no shared cost (resource) can be transferred from one DMU to another one without violating the principle of invariance. The equitable allocation is achieved through considering both above properties. Several mathematical programming should be run in order to accomplish a full analysis of resource allocation. Such a calculation might impose a high computational effort. Unfortunately, the proposed method by [9] was shown to have violation form Pareto minimality principle in method. [6] And [7] showed that an equitable allocation of shared cost and common revenues among DMUs can be achieved by using a set of formulas and without any need to solve complicated linear programming problems. Agasisti & Dal Bianco (2006), considered the problem of determining technical efficiency of the Italian universities, they illustrated that a core of universities that perform well for various input and output specifications [10]. Masiye (2007) used DEA model to meseaure the efficiency of health system performance: in Zambian hospitals, Results showed that inefficiency of resource use in hospitals is significant [11]. Akazili et al (2008), used data envelopment analysis to measure the extent of technical efficiency of public health centers in Ghana, the findings showed that 65% of health centers were technically inefficient and so were using resources that they did not actually need [12]. Marschall, P., & Flessa(2009), used DEA method in order to evaluate the relative efficiency of health centers in rural Burkina Faso [13]. Amirteimoori & Tabar (2010), presented a DEA-based method for allocating fixed resources or costs across a set of decision making units; they showed how output targets can be set at the same time as decisions are made about allocating input resources [14]. Khalili-Damghani&Abtahi (2011), Measured efficiency of just in time implementation using a fuzzy data envelopment analysis approach in a real case of Iranian dairy industries [15]. Lin (2011), focused on two main aspects: to obtain a new fixed costs or resources allocation approach by improving Cook and Zhu’s approach, and to set fixed targets according to the amount of fixed resources shared by individual DMUs. The results proved to be able to achieve a feasible costs or resources allocation compared to previous research [16]. Khalili-Damghani, et al (2012), proposed an integrated multi-objective framework for solving multi-period project selection problems, the proposed method is based on the Technique for Order Preference by Similarity to Ideal

Solution (TOPSIS) and an efficient version of the epsilon-constraint method. The proposed framework illustrated the efficacy of the procedures and algorithms [17]. Tavana et al (2013), proposed a fuzzy Multidimensional Multiple-choice Knapsack Problem (MMKP) formulation for project portfolio selection. The proposed model is composed of an Efficient Epsilon-Constraint (EEC) method and a customized multi-objective evolutionary algorithm. A Data Envelopment Analysis (DEA) model is used to prune the generated solutions into a limited and manageable set of implementable alternatives [18]. Khodabakhshi & Aryavash(2014), proposed the fair allocation of common fixed cost or revenue using DEA concept, this method was based on three principles: First, allocation must be directly proportional to the elements (inputs and outputs). Second, allocation must be inversely proportional to the elements that are inversely proportional to common fixed cost or revenue. Third, the elements that have no effect on common fixed cost or revenue must have no effect on allocation as well [19]. Khalili-Damghani, et al (2015), proposed a Hybrid Approach Based on Multi-Criteria Satisfaction Analysis (MUSA) and a NDEA to Evaluate Efficiency of Customer Services in Bank Branches, the proposed approach caused the total efficiency of main process and assigned the efficiency to customer expectations, customer satisfactions, and customer loyalties sub-processes in bank branches [20]. Yu, et al (2016), proposed an alternative approach to fixed cost allocation based on the two-stage network DEA (NDEA) and the concept of cross-efficiency. The study presented a numerical example to illustrate the applicability of the method. The results showed that if two DMUs have similar output profiles, the DMU with higher input values receives less fixed cost, whereas if two DMUs have similar input profiles [21]. Houshyar et al (2017), employed Dynamic DEA models in order to analyze the impacts of technological change on energy use efficiency and GHG mitigation of pomegranate [22]. Jin et al (2018), Determined the optimal carbon tax rate based on data envelopment analysis, in order to find an optimal carbon tax rate and to achieve the three objectives simultaneously, they considered this as a multiple criteria decision-making problem. Then, they proposed to use a centralized DEA approach to solve it [23]. Sarah & Khalili-Damghani (2019), proposed Fuzzy type-II De-Novo programming for resource allocation and target setting in network data envelopment analysis in a natural gas supply chain, the proposed method has two main modules. First, the most suitable system is designed using De-Novo programming. De-Novo programming is used to optimally determine the inputs and outputs of DMUs in network DEA rather than optimizing existing DMUs. Then, the optimal values of resources are allocated and optimal values of the targets are set in a complex network structure [24]. Li et al (2019), in their study proposed a new data envelopment analysis based approach for fixed cost allocation, in this paper; they addressed the fixed cost allocation problem in decentralized environment [25]. An et al (2020), proposed fixed cost allocation for two-stage systems with cooperative relationship using data envelopment analysis, taking this issue into account in the allocation process, they integrated cooperative game theory and the DEA methodology to generate a unique and fair allocation plan. The results confirm that each DMU can maximize its relative efficiency to one by a series of optimal variables after the fixed cost allocation [26]. Xie et al (2020), proposed

Fair allocation of wastewater discharge permits based on satisfaction criteria using data envelopment analysis, based on Max-min satisfaction, the data envelopment analysis (DEA) is applied for the achievement of tradable pollutant permits allocation [27]. Apornak et al (2021), Optimized human resource cost of an emergency hospital by using multi-objective algorithm [28], also in another research by Apornak et al (2021), used genetic algorithm approach in order to optimize Human resources in hospital emergency [29]. Apornak (2021), allocated Human resources in the hospital emergency department during COVID-19 pandemic [30], Tavana et al (2021) proposed A robust cross-efficiency data envelopment analysis model with undesirable outputs, they developed two DEA adaptations to rank DMUs characterized by uncertain data and undesirable outputs [31]. Kiaei & Kazemi matin (2022), used a new common set of weights approach to evaluate the units in both black-box and two-stage structures based on a unified criterion [32].

Allocating fixed shared cost and common revenue obtained from selling products equitably to all decision making units (DMUs) such that the relative efficiency of DMUs is not affected is an interesting problem in resource allocation field. Although there are several research work in the related areas but some pitfalls are existing which are to be resolved in this research. In this paper, we developed a single dada envelopment analysis (DEA) approach for equitable allocation of shared costs to inputs and common revenue to outputs, concurrently. The main contribution of this research in comparison with existing methods are:

1) both allocation of shared costs to inputs and common revenue to outputs are considered in a single model, simultaneously;

2) The computational efforts have been reduced and no LP required to be solved;

3) simultaneous changes of inputs and outputs have been considered in order to project a DMU towards efficient frontier.

The following sections of this paper are organized as below. In the second section, the basic concepts and preliminaries are defined and then the main problem of the study is presented. The Proposed approach with equitable allocation of shared cost and common revenue are presented in Section 3. In the Section 4, the simultaneous allocation of shared cost and common revenue through modified Russell model is developed. The theoretical properties of the proposed model are also discussed in Section 4. In Section 5, a numerical example is presented to illustrate the application of the proposed approach. In Section 6, the paper is summarized and concluded.

2. Basic concepts and Preliminaries

Suppose n homogenous DMU_j , $j=1,,\ldots,n$ use m inputs X_{ij}, $i=1,\ldots,m,\ j=1,\ldots,n$ to produce s output Y_{rj}, $r=1,\ldots,s;\ j=1,\ldots,n$. The multiplier form of output-oriented DEA-CCR model, which evaluates the DMU_p, is introduced as Model (1).

$$
\begin{aligned}
Min \quad & \sum_{i=1}^{m} v_i x_{ip} && (1)\\
& \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \ge 0, \quad j=1,\ldots,n\\
& \sum_{r=1}^{s} u_r y_{rp} = 1\\
& v_i \ge 0, \quad i=1,\ldots,m\\
& u_r \ge 0, \quad r=1,\ldots,s
\end{aligned}
$$

The envelopment form of output-oriented DEA-CCR model, which is dual of Model (1) is presented as Model (2).

$$
\begin{aligned}
Max \quad & \phi_p && (2)\\
& \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ip}, \quad i=1,\ldots,m\\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge \phi_p y_{rp}, \quad r=1,\ldots,s\\
& \lambda_j \ge 0, \quad j=1,\ldots,\mathrm{n}
\end{aligned}
$$

DMU_p is efficient, if and only if $\phi_p = 1$, and all the slack variables in optimum answer are equal to zero.

The multiplier form of input-oriented DEA-CCR model is introduced as Model (3).

$$\begin{aligned} Max \quad & \sum_{r=1}^{s} u_r y_{rp} \qquad (3) \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1,\dots,n \\ & \sum_{i=1}^{m} v_i x_{ip} = 1 \\ & v_i \ge 0, \quad i = 1,\dots,m \\ & u_r \ge 0, \quad r = 1,\dots,s \end{aligned}$$

The envelopment form of input-oriented DEA-CCR model is presented as Model (4).

$$\begin{aligned} Min \quad & \theta_p \qquad (4) \\ & \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ip}, \quad i = 1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \ge \theta_p y_{rp}, \quad r = 1,\dots,s \\ & \lambda_j \ge 0, \quad j = 1,\dots,\mathrm{n} \end{aligned}$$

DMU_p is efficient, if and only if $\theta_p = 1$, and all the slack variables in optimum answer are equal to zero.

2.1 Combined-Oriented Efficiency Evaluation

The projection toward efficient frontier can be even based on input-orientation or output-orientation as mentioned in previous section. Combined approaches consider simultaneous projection of a DMU based on both outputs and inputs criteria. In order to evaluate the DMU_p, based combined-orientations the following Model (5) is presented.

$$\begin{aligned} Min \quad & \frac{1}{m}\sum_{i=1}^{m} \theta_i + \frac{1}{s}\sum_{r=1}^{s} \frac{1}{\phi_r} \qquad (5) \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_i x_{ip}, \quad i = 1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \ge \varphi_r y_{rp}, \quad r = 1,\dots,s \\ & \theta_i \le 1, \quad i = 1,\dots m \\ & \phi_r \ge 1, \quad r = 1,\dots s \\ & \lambda_j \ge 0, \quad j = 1,\dots,\mathrm{n} \end{aligned}$$

Unfortunately, Model (5) is non-linear and the global optimal solution is hard to find. So, the following procedure, which is called modified Russell, is presented to resolve the non-linearity of Model (5). Russell proposed the following Model (6).

$$\begin{aligned} Min \quad & \mathrm{Re} = \frac{\frac{1}{m}\sum_{i=1}^{m} \theta_i}{\frac{1}{s}\sum_{r=1}^{s} \varphi_r} \qquad (6) \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_i x_{ip}, i = 1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \ge \varphi_r y_{rp}, r = 1,\dots,s \\ & \theta_i \le 1, i = 1,\dots m \\ & \phi_r \ge 1, r = 1,\dots s \\ & \lambda_j \ge 0, j = 1,\dots,\mathrm{n} \end{aligned}$$

The objective function of Model (6) is the mean of input-oriented efficiency scores divided by the mean of output-oriented efficiency scores. It is clear that Model (6) is a non-linear programming. The following variable exchange is done on Model (6).

$$\begin{cases} \theta_i = \dfrac{x_{ip} - s_i^-}{x_{ip}} = 1 - \dfrac{s_i^-}{x_{ip}}, \quad i = 1,\dots,m \\ \phi_r = \dfrac{y_{rp} + s_r^+}{y_{rp}} = 1 + \dfrac{s_r^+}{y_{rp}}, \quad r = 1,\dots,s \end{cases} \qquad (7)$$

The introduced variable in Equations (7) is replaced in Model (6) and the following Model (8) is developed.

$$Min\ \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{ip}}}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{rp}}} \qquad (8)$$

$$\begin{aligned} s.t.\quad & \sum_{j=1}^{n}\lambda_j x_{ij} = x_{ip} - s_i^-, i=1,\dots,m \\ & \sum_{j=1}^{n}\lambda_j y_{rj} = y_{rp} + s_r^+, r=1,\dots,s \\ & s_i^- \geq 0, i=1,\dots m \\ & s_r^+ \geq 0, r=1,\dots s \\ & \lambda_j \geq 0,\ j=1,\dots,\mathrm{n} \end{aligned}$$

The variable exchange (9) is done in order to make the model (8) linear.

$$\begin{cases} \beta = (1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{rp}})^{-1} \\ t_i^- = \beta s_i^- \quad i=1,\dots,m \\ t_r^+ = \beta s_r^+ \quad r=1,\dots,m \\ \mu_j = \beta\lambda_j \quad j=1,\dots,n \end{cases} \qquad (9)$$

Replacing Equations (9) in Model (8) will result in Model (10).

$$Min\ \psi = \beta - \frac{1}{m}\sum_{i=1}^{m}\frac{t_i^-}{x_{io}} \qquad (10)$$

$$\begin{aligned} s.t.\quad & \sum_{j=1}^{n}\mu_j x_{ij} = \beta x_{io} - t_i^-, i=1,\dots,m \\ & \sum_{j=1}^{n}\mu_j y_{rj} = \beta y_{ro} + t_r^+, r=1,\dots,s \\ & \beta + \frac{1}{s}\sum_{r=1}^{s}\frac{t_r^+}{y_{ro}} = 1 \\ & \beta \geq 0 \\ & \mu_j \geq 0,\ j=1,\dots,n \\ & t_i^- \geq 0, i=1,\dots,m \\ & t_r^+ \geq 0, r=1,\dots,s \end{aligned}$$

Model (10) is a linear programming (LP) and its global optimum value can easily be found using optimization software.

2.2 Equitable allocation of shared cost

Suppose we want to distribute a certain amount of costs (R) among n DMUs in a manner that the efficiency of each DMU before and after allocation does not change. The share of each DMU is assumed to be a variable that should be determined and called r_j, j=1… n. The r_j can be taken as the new input, So Model (1) can be re-written as Model (11).

$$Min\sum_{i=1}^{m} v_i x_{ip} + \mu r_p \qquad (11)$$

$$\begin{aligned} s.t.\ & -\sum_{r=1}^{s}u_r y_{rj} + \sum_{i=1}^{m}v_i x_{ij} + \mu r_j \geq 0,\ j=1,\dots,n \\ & \sum_{r=1}^{s}u_r y_{rp} = 1 \\ & v_i \geq 0, i=1,\dots,\mathrm{m} \\ & u_r \geq 0, r=1,\dots,\mathrm{s} \end{aligned}$$

As mentioned an equitable allocation should assure that the efficiency scores of DMUs remain fixed before and after allocation. This can be achieved when decision variable μ is not a basic variable in final optimal solution of Model (11). In other words, the relation (12) must be satisfied.

$$c_\mu - z_\mu \geq 0 \Rightarrow c_\mu - c_B B^{-1}a_\mu \geq 0 \qquad (12)$$

$$\Rightarrow r_p - \sum_{j=1}^{n}\lambda_j^* r_j \geq 0 \Rightarrow r_p \geq \sum_{j=1}^{n}\lambda_j^* r_j$$

Assume that $\lambda_j^*, \mathrm{j}=1,\dots,\mathrm{n}$ is the optimum answer of dual variable on Model (11).

The dual of Model (11) is written as Model (13).

$$\begin{aligned} Max \quad & \phi_p \qquad (13) \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ip}, i=1,\dots,m \\ & -\sum_{j=1}^{n} \lambda_j y_{rj} + \phi_p^{/} y_{rp} \le 0, r=1,\dots,s \\ & \sum_{j=1}^{n} \lambda_j r_j \le r_p \\ & \lambda_j \ge 0,\ j=1,\dots,\mathrm{n} \end{aligned}$$

If and only if the last constraint in Model (13) is redundant, by virtue of dual theorem in linear programming, the optimum value of Model (13) is Equal to that of Model (2). Based on Relation (12) it is clear that the last constraint in Model (13) is redundant. [6] showed that the equitable allocation of costs can be accomplished using (14).

$$r_j = R * \frac{\sum_{i=1}^{m} x_{ij}}{\sum_{q=1}^{n} \sum_{i=1}^{m} x_{iq}} \qquad (14)$$

2.3 Equitable allocation of common revenue

In order to achieve an equitable allocation of common revenue, the allocation should be made in a manner that the two principles of invariance and Pareto maximality are met.

Suppose p_j, $j=1,\dots,n$ is the revenue share of DMU_j from a whole revenue *P. The value of* p_j, $j=1,\dots,n$ is taken as $(r+1)$*-th* output of a DMU. [7] proposed Model (15) in order to equitable allocation of common revenue.

$$\begin{aligned} Min \quad & \theta_o^{'} \qquad (15) \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_o^{'} x_{io}, i=1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, r=1,\dots,s \\ & \sum_{j=1}^{n} \lambda_j p_j \ge p_o \\ & \lambda_j \ge 0,\ j=1,\dots,\mathrm{n} \end{aligned}$$

[7] Showed that the equitable allocation of common revenue can be accomplished using (16).

$$p_j = P \times \frac{\sum_{r=1}^{s} \alpha_r y_{rj}}{\sum_{q=1}^{n} \sum_{r=1}^{s} \alpha_r y_{rq}}, \quad j=1,\dots,n \qquad (16)$$

Where $\alpha=(\alpha_1,\dots,\alpha_r)$ is an optional vector (non-negative and non-zero). Moreover, the last constraint in Model (15) is redundant. So the optimum values of objective functions in Model (4) and Model (15) are Equal [7].

3. Proposed approach with equitable allocation of shared cost and common revenue

Considering the modified Russell Model (10), that incorporates both input and output orientations to project a DMU toward efficient frontier, simultaneously, and also using the models for allocation of shared cost (i.e., Equation 14) and allocation of revenue (i.e., Equation 16) to DMUs, we are going to develop an equitable allocation plan for both shared costs and revenues.

Consider the problem of simultaneous allocation of total shared cost $R = \sum_{j=1}^{n} r_j$ and total common revenue $P = \sum_{j=1}^{n} p_j$ to DMUs. The r_j, p_j $j=1,\dots,n$ are assumed as a new input and new output, respectively. So Model (10) can be developed as Model (17).

$$
\begin{aligned}
Min \quad & \psi^{/} = \beta - \frac{1}{m}\sum_{i=1}^{m}\frac{t_i^-}{x_{io}} \qquad (17) \\
s.t. \quad & \sum_{j=1}^{n}\mu_j x_{ij} = \beta x_{io} - t_i^-, i = 1,\dots,m \\
& \sum_{j=1}^{n}\mu_j y_{rj} = \beta y_{ro} + t_r^+, r = 1,\dots,s \\
& \beta + \frac{1}{s}\sum_{r=1}^{s}\frac{t_r^+}{y_{ro}} = 1, i = 1,\dots,m \\
& \sum_{j=1}^{n}\mu_j r_j = \beta r_o - t_{s+1}^+ \\
& \sum_{j=1}^{n}\mu_j p_j = \beta p_o + t_{m+1}^- \\
& \beta \geq 0 \\
& \mu_j \geq 0, j = 1,\dots,n \\
& t_i^- \geq 0, i = 1,\dots,m \\
& t_r^+ \geq 0, r = 1,\dots,s
\end{aligned}
$$

Following variable exchanges have been accomplished in order to linearize the Model (17).

$$
\begin{cases}
\beta = (1 + \frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{ro}})^{-1} \\
t_i^- = \beta s_i^- \quad i = 1,\dots,m \\
t_r^+ = \beta s_r^+ \quad r = 1,\dots,m \\
\mu_j = \beta\lambda_j \quad j = 1,\dots,n \\
\sum_{j=1}^{n} p_j = p \\
\sum_{j=1}^{n} r_j = R
\end{cases} \qquad (18)
$$

It is clear that according to Model (11) and Equation (12), the constraint $\sum_{j=1}^{n}\mu_j r_j = \beta r_o - t_{s+1}^+$ in Model (17) is redundant. So it has not impact on efficiency scores before and after allocation. In the same way, based on [7] the constraint $\sum_{j=1}^{n}\mu_j p_j = \beta p_o + t_{m+1}^-$ in Model (17) is also redundant. So, the optimum answers of Models (10) and (17) are equal. On the other hand, $\psi^* = \psi^{/*}$. Based on the preceding discussions, the efficiency has remained fixed before and after allocation, so an equitable allocation has been achieved considering both shared costs and shared common revenues. Model (17) is a generalization of simultaneous equitable allocation of share costs and common revenues. The equitable allocation of shared costs can be accomplished using (19).

4. Experimental Results and Numerical Example

In order to demonstrate the efficacy of proposed approach, a series of experiments have been accomplished. Three main strategies were considered in resource allocation (i.e., shared costs and common revenues). The results of all strategies are presented and discussed in this section. Let's define the strategies for resource allocation as follows:

Strategy I: Improvement of Weakness. In this strategy, decision makers are interested to allocate the shared costs and common revenues to DMUs which have less efficiency scores in order to improve the efficiency scores. In this strategy the priority of allocation is based on increasing order of efficiency scores. So, the less shared costs which is assumed as new inputs is allocated to the higher priority DMUs in this strategy and the more common revenue which is assumed as new output is allocated to higher priority DMUs. In this way, the weaker DMUs are improved. This strategy does not guaranty the similar efficiency scores of DMUs after allocation, but it guaranties the improvement of weak DMUs. This strategy is called effective strategy and is recommended for long term period of planning in which weak DMUs and consequently the total PPS are improved and the mean of efficiency scores of PPS is improved. It is notable that the efficiency scores of all DMUs have a chance to be improved in this strategy but the priority of improvement is based on weaker DMUs.

Strategy II: Enhancement of Strength. In this strategy, decision makers are interested to allocate the shared costs and common revenues to DMUs which have higher efficiency scores in order to enhance the efficiency scores. In this strategy the priority of allocation is based on decreasing order of efficiency scores. So, the more shared costs which is assumed as new inputs is allocated to the lower priority DMUs in this strategy and the less common revenue which is assumed as new output is allocated to low priority DMUs. In this way, the stronger DMUs are enhanced. This strategy does not guaranty the similar efficiency scores of DMUs after allocation, but it guaranties the improvement of strong DMUs. This strategy is called efficient strategy and is recommended for short term period of planning in which strong DMUs. It is notable that the efficiency scores of all DMUs have a chance to be improved in this strategy but the priority of improvement is based on stronger DMUs.

Strategy III: Simultaneous Equitable Resource Allocation of Shared Costs and Common Revenues. This strategy is implemented based on proposed approach of this study in which the simultaneous equitable allocation of shared costs and common revenue are done. Based on this strategy the new input and output are allocated to DMUs in a way that the efficiency scores before and after allocation are fixed. This strategy is a fair strategy in which new inputs and outputs cannot impact on efficiency scores of DMUs. The DMUs should improve using enhancement of their process and technology.

In real cases, the decision makers (DMs) may use each of the above strategies based on the situation of organization and period of planning.

An illustrative example is adopted from Charnes et al. (1989) to show the applicability and efficacy of proposed approach and other strategies [33]. Data for 28 DMUs are shown in Table 1. Each DMU in this example has three inputs and three outputs. The example is related to 28 cities in China in 1983. Outputs are included industrial gross, revenues, taxes, and retailing and inputs are included human force, petty cash, and capitals.

Suppose we want to distribute 2 million dollars as the new output and 28886717 as the new inputs among these cities. We allocated these new input and output based on all strategies. The strategy I and strategy II works based on efficiency scores of DMUs. So, first the efficiency scores of all DMUs are calculated. The efficiency scores have been calculated using Model (10).

In third strategy, the management seeks to allocate them in a manner that the relative efficiency of these cities are not changed. Suppose ψ^* is the optimum value of model (10) before allocation and ψ'^* is the optimum value of model (17) after allocation. Models (10) and (17) have been run and the results of all strategies are shown in Table 2.

Table 1: Numerical Example Adopted from Charnes et al. [7]

DMU	Input1	Input2	Input3	Output1	Output2	Output3
1	483.01	1397736	616961	6785798	1594957	1088699
2	371.95	855509	385453	2505984	545140	835745
3	268.23	685584	341941	2292025	406947	473600
4	202.02	452713	117429	1158016	135939	366165
5	197.93	471650	112634	1244124	204909	317709
6	187.96	423124	189743	1187130	190178	605037
7	148.04	367012	97004	658910	86514	239760
8	189.93	408311	111904	993238	1411954	353896
9	23.30	245542	91861	854188	135327	239360
10	119.91	305316	91710	606743	78357	208188
11	12992	295812	92409	736545	114365	298112
12	109.26	198703	53499	454684	67154	233733
13	89.7	210891	95642	494196	78992	118553
14	10926	282209	84202	842854	149186	243361
15	58.5	184992	49357	776285	116974	234875
16	72.17	222327	73907	490998	117854	118924
17	76.18	161159	47977	482448	67857	158250
18	73.21	144163	43312	515237	114883	101231
19	86.72	190043	55326	625514	173099	130423
20	69.09	158439	66640	382880	74126	123968
21	77.69	135046	46198	867467	65229	262876
22	97.42	206926	66120	830142	128279	242773
23	54.96	79563	43192	521684	37245	184055
24	67	144092	43350	869973	86859	194416
25	46.3	100431	31428	604715	55989	127586
26	65.12	96873	28112	601299	37088	224855
27	20.09	50717	54650	145792	11816	24442
28	69.81	117790	30976	319218	31816	169051

As seen, in Table2 shows that the efficiency score are not changed in strategy III after allocation. On the other hand, an efficient DMU is also efficient even after allocation and an inefficient DMU remains inefficient after allocation. The strategy I and strategy II allocate the shared costs and common revenues for improvement of weakness and enhancement of strength, respectively. It is obvious that the allocation plans are completely different in these strategies as they have different goals.

Figure 1 plots the allocation plans of all strategies.

As shown in Figure 1, the allocation plan for strategy III is so dense so more details are required. Figure 2 shows the percentage of allocated inputs and outputs

Table2: Allocated shared cost and common revenue to each DMU and efficiencies

DMUs	ψ^*	Strategy I		Strategy II		Strategy III		
		p_j	r_j	p_j	r_j	p_j	r_j	ψ'^*
1	1	1131476	0	0	121144.9	3201113.445	301623	1
2	0.4627411	523580.4	93510.82	873377.8	56058.71	1971858.995	134332	0.4627411
3	0.4736532	535927.2	91611.55	855638.9	57380.65	1632649.5458	11341142	0.4736532
4	0.2451478	277378.8	131383.3	1227101	29698.4	905991.4757	900805	0.2451478
5	0.3664369	414614.5	110272.7	1029932	44391.95	928449.5458	769699	0.3664369
6	0.5331285	603222	81259.77	758954.8	64585.78	973823.5074	548427	0.5331285
7	0.2293896	259548.8	134126	1252718	27789.37	737324.5775	439496	0.2293896
8	1	1131476	0	0	121144.9	826663.2313	57582	1
9	1	1131476	0	0	121144.9	536001.7252	785029	1
10	0.2299546	260188.1	134027.7	1251800	27857.82	630861.5125	681573	0.2299546
11	0.3161558	357722.7	119024.2	1111670	38300.65	616894.9598	1337486	0.3161558
12	0.2935951	332195.8	122951	1148345	35567.54	400791.6710	416543	0.2935951
13	0.2754376	311651	126111.3	1177862	33367.85	487070.1589	881576	0.2754376
14	0.4306443	459880.4	103309.6	964897	49238.48	582217.4330	765054	0.406443
15	0.65755156	744003.7	59603.73	556690.4	79658.99	372399.2032	407488	0.65755156
16	0.4355077	492766.5	98250.84	917648.9	52759.52	470682.3510	103009	0.4355077
17	0.3267077	369661.9	117187.7	1094516	39578.96	332333.5479	1039828	0.3267077
18	0.5019149	567904.6	86692.55	809696.2	60804.41	297920.3315	1186967	0.5019149
19	0.5603882	634065.7	76515.18	714640.9	67888.15	389906.41122	1338677	0.5603882
20	0.3333747	377205.4	116027.3	1083678	40386.63	357647.7409	847862	0.3333747
21	1	1131476	0	0	121144.9	288029.5045	840890	1
22	0.5465685	618429.1	78920.52	737106.5	66213.97	433888.3226	1191303	0.5465685
23	1	1131476	0	0	121144.9	195083.6214	286509	1
24	1	1131476	0	0	121144.9	297585.0465	200175	1
25	1	1131476	0	0	121144.9	209531.5690	444347	1
26	1	1131476	0	0	121144.9	198642.1156	877951	1
27	0.2056878	1131476	0	0	121144.9	167407.3924	363424	0.2056878
28	0.3150642	356487.5	119214.2	1113444	38168.41	236425.6841	397920	0.3150642

to efficient DMUs in strategy III.

It can be concluded from Figure 1 that 98% of all inputs has been allocated to DMU number 4, and 39% of revenues has been allocated to DMU number 3. It is notable that neither DMU_3 nor DMU_3 are not efficient DMUs. This allocation is called equitable as it guaranties the fixity of efficiency scores. In the general belief there are two classic strategies in resource allocation (i.e., Strategies I, and II). But, in many real cases the simultaneous equitable allocation of shared costs and common revenues are preferred. The proposed approach of this study simply provides this option as strategy III.

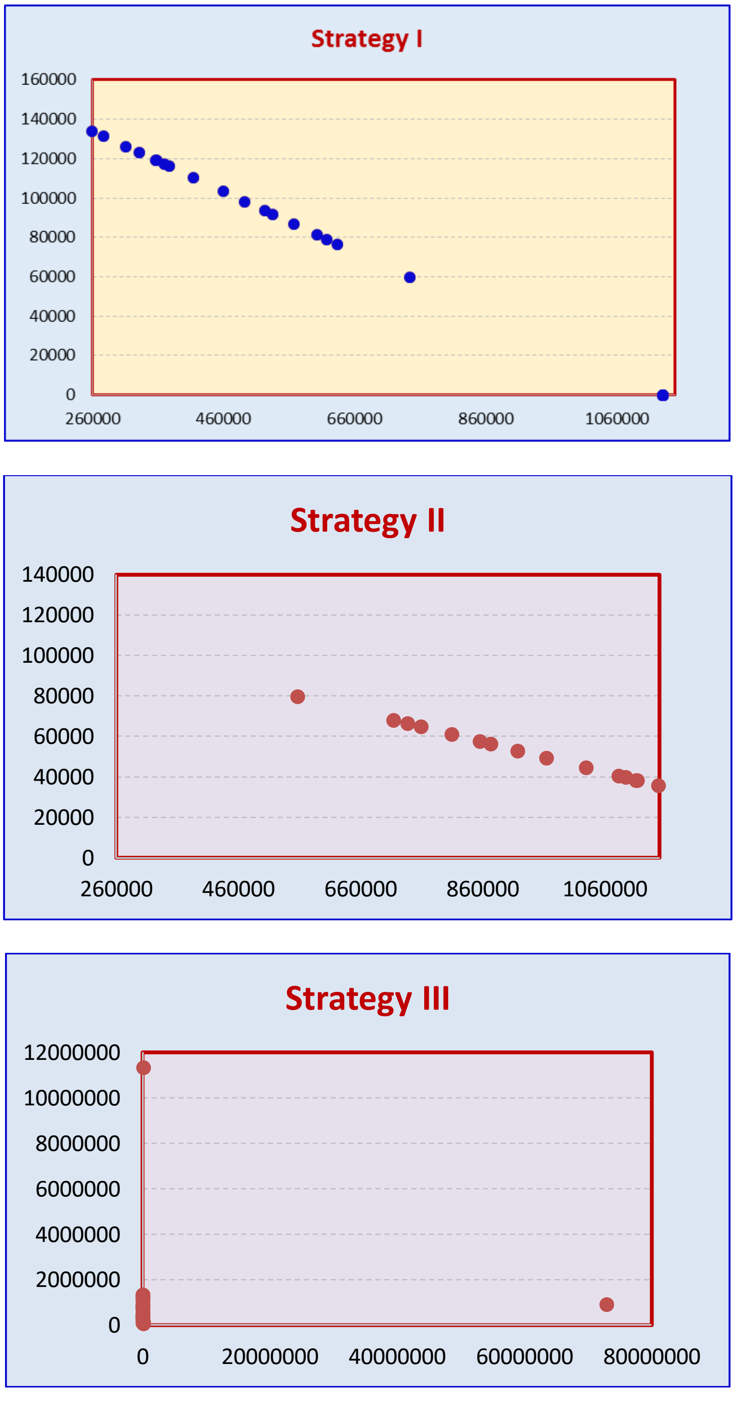

Figure1: Allocation plans for all strategies

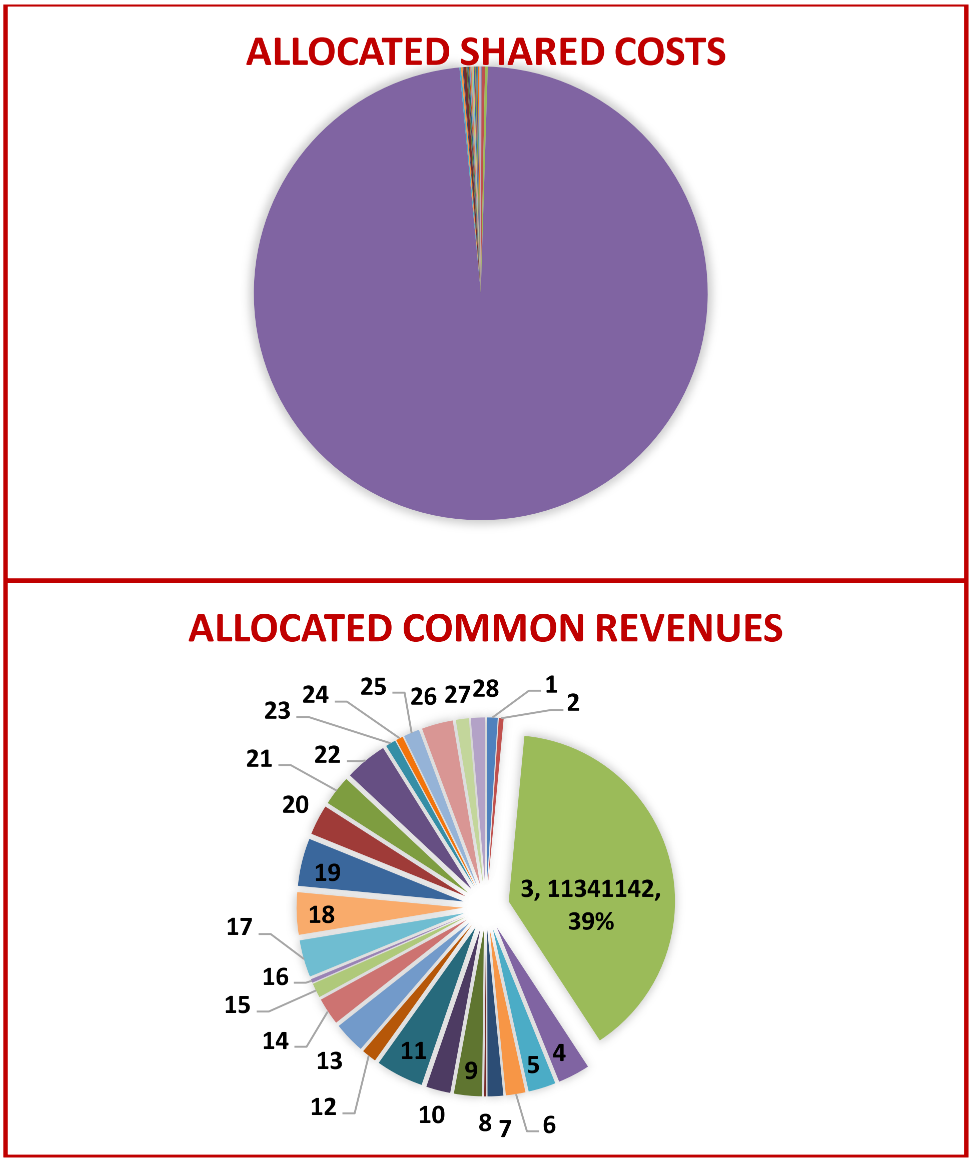

Figure 2: Allocated Shared Costs and Common Revenue to each DMU for Strategy III

7. Conclusion and Future Research Direction

Data envelopment analysis is a technique based on linear programming which is used to determine the relative efficiency of homogenous decision making units (DMUs) with multiple inputs and outputs. One of the main post-optimization and sensitivity analysis based on DEA approaches is resource allocation. There are several types of resource allocation models in DEA approach. In first type, called shared cost models, in spite of usual inputs of DMUs a certain amount of resources are available and should be allocated to DMUs such that the relative efficiency scores do not change. These types of problems are called equitable resource allocation through shared costs. In second type, called common revenue models, a certain amount of revenue should be distributed as a new output to all DMUs in a way that the efficiency scores do not change. The second types of problems are called equitable resource allocation through common revenue. In the previous researches the equitable resource allocation through shared costs approach and equitable resource allocation through common revenues approaches were proposed by distinctive models. On the other hand, there was no unique approach that can handle both shared costs and common revenues simultaneously. In this paper we have developed a unique approach in order to solve the resource allocation problems in DEA in presence of simultaneous shared costs and common revenue. The main contribution of the proposed approach is: 1) both allocation of shared costs to inputs and common revenue to outputs are considered in a single model, simultaneously; 2) The computational efforts has been reduced; 3) simultaneous changes of inputs and outputs have been considered in order to project a DMU towards efficient frontier; 4) concurrent allocation of inputs and outputs are accomplished in equitable manner. On the other hand, the shared costs and common revenues were added as new input and output, respectively, and the models were developed considering these new criteria. The final result of proposed approach revealed the amount of new input and amount of new outputs that should equitably be allocated to each DMU in order to sense no change in efficiency scores.

A numerical example, adopted form literature, is presented and discussed in order to illustrate the applicability and efficacy of proposed approach. The illustrative example presented the suitability of the proposed approach.

The proposed model in this paper can be applied on several real case study such as banks, insurance, service and production companies. The main idea of this research can be developed in uncertain situations in which the inputs, outputs, shared costs, and common revenue are not known exactly and are presented through linguistic terms parameterized by fuzzy sets. The common weight approach can be utilized in order to make the ranking process more adequate.

References

[1] Apornak A, Raissi S, Pourhassan MR. Solving flexible flow-shop problem using a hybrid multi criteria Taguchi based computer simulation model and DEA approach. Journal of Industrial and Systems Engineering. 2021 Feb 4;13(2):264-76.

[2] Farrell, M. J. (1957). The measurement of productive efficiency. Journal of the Royal Statistical Society: Series A (General), 120(3), 253-281.

[3] Zhao P, Zeng L, Li P, Lu H, Hu H, Li C, Zheng M, Li H, Yu Z, Yuan D, Xie J. China's transportation sector carbon dioxide emissions efficiency and its influencing factors based on the EBM DEA model with undesirable outputs and spatial Durbin model. Energy. 2022 Jan 1;238:121934.

[4] Charnes A, Cooper WW, Rhodes E. Measuring the efficiency of decision making units. European journal of operational research. 1978 Nov 1;2(6):429-44.

[5] Amirteimoori A, Kordrostami S. Allocating fixed costs and target setting: A DEA-based approach. Applied Mathematics and Computation. 2005 Dec 1;171(1):136-51.

[6] Jahanshahloo GR, Lotfi FH, Shoja N, Sanei M. An alternative approach for equitable allocation of shared costs by using DEA. Applied Mathematics and computation. 2004 May 25;153(1):267-74.

[7] Jahanshahloo GR, Lotfi FH, Moradi M. A DEA approach for fair allocation of common revenue. Applied Mathematics and Computation. 2005 Jan 27;160(3):719-24.

[8] Cook WD, Zhu J. Allocation of shared costs among decision making units: A DEA approach. Computers & Operations Research. 2005 Aug 1;32(8):2171-8.

[9] Cook WD, Kress M. Characterizing an equitable allocation of shared costs: A DEA approach. European Journal of Operational Research. 1999 Dec 16;119(3):652-61.

[10] Agasisti T, Dal Bianco A. Data envelopment analysis to the Italian university system: theoretical issues and policy implications. International Journal of Business Performance Management. 2006 Jan 1;8(4):344-67.

[11] Masiye F. Investigating health system performance: an application of data envelopment analysis to Zambian hospitals. BMC Health services research. 2007 Dec;7(1):1-1.

[12] Akazili J, Adjuik M, Jehu-Appiah C, Zere E. Using data envelopment analysis to measure the extent of technical efficiency of public health centres in Ghana. BMC international health and human rights. 2008 Dec;8(1):1-2.

[13] Marschall P, Flessa S. Assessing the efficiency of rural health centres in Burkina Faso: an application of Data Envelopment Analysis. Journal of Public Health. 2009 Apr;17(2):87-95.

[14] Amirteimoori A, Tabar MM. Resource allocation and target setting in data envelopment analysis. Expert

Systems with Applications. 2010 Apr 1;37(4):3036-9.

[15] Khalili-Damghani K, Abtahi AR. Measuring efficiency of just in time implementation using a fuzzy data envelopment analysis approach: real case of Iranian dairy industries. International Journal of Advanced Operations Management. 2011 Jan 1;3(3-4):337-54.

[16] Lin R. Allocating fixed costs or resources and setting targets via data envelopment analysis. Applied Mathematics and Computation. 2011 Mar 1;217(13):6349-58.

[17] Khalili-Damghani K, Tavana M, Sadi-Nezhad S. An integrated multi-objective framework for solving multi-period project selection problems. Applied Mathematics and Computation. 2012 Nov 25;219(6):3122-38.

[18] Tavana M, Khalili-Damghani K, Abtahi AR. A fuzzy multidimensional multiple-choice knapsack model for project portfolio selection using an evolutionary algorithm. Annals of Operations Research. 2013 Jul;206(1):449-83.

[19] Khodabakhshi M, Aryavash K. The fair allocation of common fixed cost or revenue using DEA concept. Annals of Operations Research. 2014 Mar;214(1):187-94.

[20] Khalili-Damghani K, Taghavi-Fard M, Karbaschi K. A hybrid approach based on multi-criteria satisfaction analysis (MUSA) and a network data envelopment analysis (NDEA) to evaluate efficiency of customer services in bank branches. Industrial Engineering and Management Systems. 2015;14(4):347-71.

[21] Yu MM, Chen LH, Hsiao B. A fixed cost allocation based on the two-stage network data envelopment approach. Journal of Business Research. 2016 May 1;69(5):1817-22.

[22] Houshyar E, Mahmoodi-Eshkaftaki M, Azadi H. Impacts of technological change on energy use efficiency and GHG mitigation of pomegranate: application of dynamic data envelopment analysis models. Journal of Cleaner Production. 2017 Sep 20;162:1180-91.

[23] Jin M, Shi X, Emrouznejad A, Yang F. Determining the optimal carbon tax rate based on data envelopment analysis. Journal of Cleaner Production. 2018 Jan 20;172:900-8.

[24] Sarah J, Khalili-Damghani K. Fuzzy type-II De-Novo programming for resource allocation and target setting in network data envelopment analysis: a natural gas supply chain. Expert Systems with Applications. 2019 Mar 1;117:312-29.

[25] Li F, Zhu Q, Liang L. A new data envelopment analysis based approach for fixed cost allocation. Annals of Operations Research. 2019 Mar;274(1):347-72.

[26] An Q, Wang P, Shi S. Fixed cost allocation for two-stage systems with cooperative relationship using data envelopment analysis. Computers & Industrial Engineering. 2020 Jul 1;145:106534.

[27] Xie Q, Xu Q, Zhu D, Rao K, Dai Q. Fair allocation of wastewater discharge permits based on satisfaction criteria using data envelopment analysis. Utilities Policy. 2020 Oct 1;66:101078.

[28] Apornak A, Raissi S, Keramati A, Khalili-Damghani K. Optimizing human resource cost of an emergency hospital using multi-objective Bat algorithm. International Journal of Healthcare Management. 2021 Jul 3;14(3):873-9.

[29] Apornak A, Raissi S, Keramati A, Khalili-Damghani K. Human resources optimization in hospital emergency using the genetic algorithm approach. International Journal of Healthcare Management. 2021 Oct 2;14(4):1441-8.

[30] Apornak A. Human resources allocation in the hospital emergency department during COVID-19 pandemic. International Journal of Healthcare Management. 2021 Jan 2;14(1):264-70.

[31] Tavana M, Toloo M, Aghayi N, Arabmaldar A. A robust cross-efficiency data envelopment analysis model with undesirable outputs. Expert Systems with Applications. 2021 Apr 1;167:114117.

[32] Kiaei H, Kazemi Matin R. New common set of weights method in black-box and two-stage data envelopment analysis. Annals of Operations Research. 2022 Feb;309(1):143-62.

[33] Charnes, A., Cooper, W. W., & Li, S. (1989). Using data envelopment analysis to evaluate efficiency in the economic performance of Chinese cities. Socio-Economic Planning Sciences, 23(6), 325-344.