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# Profit efficiency in non-competitive markets

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**Abstract**
This paper develops a non-radial linear programming model based on the concept of directional distance function to examine the profit efficiency (inefficiency) of decision making units (DMUs) in the case where the market prices are available but can be controlled by units, and DMUs can influence simultaneously the prices and quantities of inputs and outputs to maximize the total profit. The optimal solution of the proposed model can help decision makers to evaluate and improve the profit efficiency of DMUs by changing the prices and quantities of inputs and outputs. The proposed model satisfies the important properties, units invariance and translation invariance. We present a decomposition of the profit inefficiency obtained by the optimal solution of the proposed model which can individualize quantity and price contributions to the profit inefficiency of under evaluation DMU. Finally, an empirical application to a set of 50 bank branches will be presented to illustrate the proposed approach.

**Keywords:** Data envelopment analysis; linear programming; directional distance function; profit efficiency

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## 1. Introduction

Data envelopment analysis (DEA) is one of the best approaches to evaluate the performance of DMUs. DEA studies about the efficiency of DMUs help decision maker to identify and remove the sources of inefficiency. There are various DEA models that can be used for measuring the efficiency of DMUs in situations where unit cost or price is not known. For instance, radial models such as CCR and BCC [1,2] as well as non-radial models such as additive [3] and SBM models [4].

Other DEA models also have been developed to examine the efficiency of DMUs for the case where input/output prices are available. For example, Farrell [5], Ray and Kim [6], Tone [7], Tone and Sahoo [8], Maniadakis and Thanassoulis [9] and Sahoo and Tone [17] developed models for measuring the cost efficiency (CE), revenue efficiency (RE) and profit efficiency (PE) when the market prices are exactly known. In such markets that are named competitive markets, the input/output prices are exogenously fixed and units are named price taker. Hence, for improving the efficiency, DMUs are only allowed to change input or output quantities.

In non-competitive markets, DMUs have the ability to influence somewhat the market prices. In this situation, DMUs can maximize the CE, RE and PE measures by changing the input/output quantities as well as input/output prices. In this connection, Portela and Thanassoulis [11] proposed a model to evaluate the cost efficiency of DMUs. In their approach quantities and costs of inputs can vary simultaneously, and quantity and price contributions to the cost inefficiency can be individualized [11-15].

In this paper, a directional distance function approach is developed to measure a degree of profit inefficiency in the case where DMUs can influence the prices as well as quantities of inputs and outputs to maximize the total profit. By the proposed model we can have simultaneously non-radial improvements in the input/output prices and also input/output quantities along any arbitrary selected direction vector. In addition, we decompose the obtained profit inefficiency and study the effects of changing the prices and quantities to it.

The rest of paper proceeds as follows. Section 2 outlines the DEA methodology. Section 3 develops a model to study about profit efficiency. In section 4 we decompose the profit inefficiency. Section 5 shows the application of the proposed approach. Section 6 concludes.

## 2. Methodology

Assume that there are $n$ DMUs ; each $\text{DMU}_j$, $j=1,\ldots,n$, consumes $m$ inputs, $x_{ij}$, $i=1,\ldots,m,$ and produces $s$ outputs, $y_{rj}$, $r=1,\ldots,s.$ In addition, assume that $c_j=(c_{1j},\ldots,c_{mj})$ and $p_j=(p_{1j},\ldots,p_{sj})$ be respectively observed input and output price vectors corresponding to $\text{DMU}_j$.

The profit efficiency of $\text{DMU}_o$ (under evaluation DMU) can be defined in ratio form by $(p_o y_o - c_o x_o)/(p_o y^* - c_o x^*)$ . The term $p_o y^* - c_o x^*$ is the maximum profit that can be obtained by correcting the input/output combination at the market prices. Maximum profit can be computed by solving the following profit maximization problem [16]:

$$p_o y^* - c_o x^* = \max \left\{ \sum_{r=1}^{s} p_{ro} y_r - \sum_{i=1}^{m} c_{io} x_i \middle| x_i = \sum_{j=1}^{J} \lambda_j x_{ij} \le x_o,\ i=1,\ldots,m, \right. \quad (1)$$
$$\left. y_r = \sum_{j=1}^{J} \lambda_j y_{rj} \ge y_o,\ r=1,\ldots,s, \lambda_j \ge 0,\ j=1,\ldots,J \right\}$$

Model (1) can be applied in situation where the market prices are exogenously fixed and DMUs have no control over the prices.

The concept of the directional distance function was presented by Chambers et al. [17] to examine the efficiency of DMUs . They introduced a non-zero directional vector $g=(g_x, g_y)$ and solved the following optimization problem:

$$\max\left\{\beta \middle| \begin{array}{l} \sum_{j=1}^{n}\lambda_j x_{ij} \le x_{io}-\beta g_i,\ i=1,\dots,m, \\ \sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro}+\beta g_r,\ r=1,\dots,s \\ \lambda_j \ge o,\ j=1,\dots,n,\ \beta \ge o \end{array}\right\}. \quad (2)$$

This model provide a radial measure of inefficiency of $\mathrm{DMU}_o$. If $\beta^*=0$, $\mathrm{DMU}_o$ will be efficient and it will be inefficient if $\beta^*>0$. The directional distance function can also be extended to the non-radial form (see e.g. [15]).

In this paper, we extend the non-radial form of the directional distance function for the case where the market prices can be controlled by DMUs.

## 3. A directional profit efficiency when prices are not fixed

In order to measure a degree of profit inefficiency of $\mathrm{DMU}_o$ in a non-competitive market, we consider an arbitrary directional vector $g_o=(g_{x_o}, g_{y_o}, g_{c_o}, g_{p_o})\ne 0$ where, $g_{x_o}, g_{c_o}\in R^m$ and $g_{y_o}, g_{p_o}\in R^s$, and propose the non-radial linear programming model as follows:

$$\vec{D}_o^* = \vec{D}_o(x_o, y_o, c_o, p_o, g_o) =$$

$$\max\left\{ \frac{1}{4}\left(\frac{1}{m}\sum_{i=1}^{m}(\alpha_i+\sigma_i)+\frac{1}{s}\sum_{r=1}^{s}(\beta_r+\gamma_r)\right) \middle| \begin{array}{ll} \sum_{j=1}^{n}\lambda_j x_{ij} \le x_{io}-\alpha_i g_{x_{io}}, & i=1,\dots,m, \\ \sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro}+\beta_r g_{y_{ro}}, & r=1,\dots,s, \\ \sum_{j=1}^{n} z_{ij} c_{ij} \le c_{io}-\sigma_i g_{c_{io}}, & i=1,\dots,m, \\ \sum_{j=1}^{n} k_{rj} p_{rj} \ge p_{ro}+\gamma_r g_{p_{ro}}, & r=1,\dots,s, \\ \sum_{j=1}^{n} z_{ij}=1,\ i=1,\dots,m,\ \sum_{j=1}^{n} k_{rj}=1, & \\ r=1,\dots,s,\ \alpha_i,\sigma_i\ge 0,\ \theta_{i1}\le\sigma_i\le\theta_{i2}(\forall i), & \\ \beta_r,\gamma_r\ge 0,\ \eta_{r1}\le\gamma_r\le\eta_{r2}(\forall r) & \end{array}\right\}. \quad (3)$$

The parameters $\theta_{i1}$ and $\theta_{i2}$ are determined by decision maker and are respectively the lower and upper bound for the level of the input prices. These parameters have the values between 0 and 1. The parameters $\eta_{r1}$ and $\eta_{r2}$ have the same interpretation for the output prices of $\mathrm{DMU}_o$.

The feasible activities $(x, y)$ that define the production possibility set in model (1) are considered as the feasible activities in model (3). Furthermore, based on the constraints, $\sum_{j=1}^{n} z_{ij}c_{ij}\le c_{io}-\sigma_i g_{c_{io}}$ and $\sum_{j=1}^{n} z_{ij}=1$, the feasible price for an input is defined as a convex combination of observed prices for it across the DMUs. In fact, the observed input prices help decision maker to make the best prices for each input. The output prices constraints, $\sum_{j=1}^{n} k_{rj}p_{rj}\ge p_{ro}+\gamma_r g_{p_{ro}}$ and

$\sum_{j=1}^{n} k_{rj} = 1$, have also the similar interpretation.

In model (3), we assume that there is no causal correspondence between input and output prices and also between input/output prices and input/output quantities. Thus, the different intensity variables $\lambda_j$, $z_{ij}$ and $k_{rj}$ have been considered for input/output quantities, input prices and output prices, respectively. For example, if we assume that there is a causal correspondence between input and output prices, then we can use the intensity variables $z_{ij}$ instead of $k_{rj}$ in the output price constraints. The model (3) is developed under CRS assumption. The constraint $\sum_{j=1}^{n} \lambda_j = 1$ can be added in the case of VRS technology.

In this study, we select the directional vector $g_o$ as: $g_{x_{io}} = x_{io}; i = 1, \ldots, m,$ $g_{y_{ro}} = y_{ro}; r = 1, \ldots, s,$ $g_{c_{io}} = c_{io}; i = 1, \ldots, m,$ and $g_{p_{ro}} = p_{ro}; r = 1, \ldots, s$.

Model (3) has a feasible solution $\lambda_o = 1,$ $\lambda_j = 0\ (j \neq o),$ $\alpha_i = \sigma_i = o\ (\forall i),$ $\beta_r = \gamma_r = o\ (\forall r),$ $z_{io} = 1,$ $z_{ij} = 0\ (\forall i, \forall j \neq o),$ $k_{ro} = 1,$ $k_{rj} = 0\ (\forall r, \forall j \neq o).$ Therefore, the proposed model is always feasible.

**Definition 3.1 (Directional profit efficiency)**

A DMU is directional profit efficient if and only if $\overrightarrow{D}_o^* = 0.$

We consider the value of $\overrightarrow{D}_o^*$ as a degree of profit inefficiency of $\text{DMU}_o$ and it follows that, if $\overrightarrow{D}_o^* > 0$, $\text{DMU}_o$ is directional profit inefficient.

The value of $\alpha_i$ estimates the possible improvement in the quantity of *i*th input of $\text{DMU}_o$. Thus, $\frac{1}{m}\sum_{i=1}^{m} \alpha_i$ presents the average inefficiency score for its input quantities. Interpretations of the values of $\sigma_i$, $\beta_r$ and $\gamma_r$ are similar to that of $\alpha_i$.

Based on the optimal solution to model (3), directional profit efficiency measure for $\text{DMU}_o$ will be as

$$PE_o = \frac{PR_o}{PR_o^*} = \frac{\sum_{r=1}^{s} p_{ro} y_{ro} - \sum_{i=1}^{m} c_{io} x_{io}}{\sum_{r=1}^{s} p_{ro}^* y_{ro}^* - \sum_{i=1}^{m} c_{io}^* x_{io}^*}, \quad (4)$$

where,

$$\begin{aligned} x_{io}^* &= x_{io} - \alpha_i^* g_{x_{io}}, \quad i = 1, \ldots, m, \\ y_{ro}^* &= y_{ro} + \beta_r^* g_{y_{ro}}, \quad r = 1, \ldots, s, \end{aligned} \quad (5)$$

$$\begin{aligned} c_{io}^* &= c_{io} - \sigma_i^* g_{c_{io}}, \quad i = 1, \ldots, m, \\ p_{ro}^* &= p_{ro} + \gamma_r^* g_{p_{ro}}, \quad r = 1, \ldots, s. \end{aligned} \quad (6)$$

(5) and (6) are respectively the quantity and price targets of $\text{DMU}_o$. $\text{DMU}_o$ can maximize its profit by choosing the targets.

It is clear that, the new measure of the degree of profit inefficiency is monotone increasing in each $\alpha_i$, $\sigma_i\ (\forall i)$ and $\beta_r$, $\gamma_r\ (\forall r)$. In addition, the proposed measure satisfies the following important properties:

**a) Units invariance:** By selecting a direction vector $g_o$ so that the units of measurement of its components, $g_{x_{io}}, g_{c_{io}}\ (\forall i)$ and $g_{y_{ro}}, g_{p_{ro}}\ (\forall r)$, are respectively the same as those of $x_{io}$, $c_{io}$, $y_{ro}$ and $p_{ro}$, the proposed measure, $\overrightarrow{D}_o^*$, will be units invariant. To show this property, consider the constraint of the $i$th input quantity. We rescale the $i$th input quantity $x_{ij}$ into $q_i x_{ij}$. The $i$th component of the direction vector, $g_{x_{io}}$, will also change to $q_i g_{x_{io}}$. Thus, we have

$$\sum_{j=1}^{n} \lambda_j (q_i x_{ij}) \le (q_i x_{io}) - \alpha_i (q_i g_{x_{io}}), \quad i = 1, \ldots, m, \tag{7}$$

After eliminating $q_i$ on both sides, the above constraint is translated into

$$\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} - \alpha_i g_{x_{io}}, \ i = 1, \ldots, m, \tag{8}$$

that is the $i$th input quantity constraint in (3). Similarly, we can show this for the output quantity, input cost and output price constraints.

**b) Translation invariance:** In order to satisfy this property, two conditions must be met. The first condition is $\sum_{j=1}^{n} \lambda_j = 1$, and the second is the direction vector $g_o$ should be selected in a way that it does not change after translating the input/output quantities and input/output prices [1]. For example, the components of $g_o$ can be selected as $g_{x_{io}} = x_{io} - \min_j \{x_{ij}\}$, $g_{c_{io}} = c_{io} - \min_j \{c_{ij}\} (\forall i)$, $g_{y_{ro}} = \max_j \{y_{rj}\} - y_{ro}$ and $g_{p_{ro}} = \max_j \{p_{rj}\} - p_{ro} \ (\forall r)$. We show this property by translating $x_{ij}$ into $\tilde{x}_{ij} = x_{ij} + w_i$. In this case, we have:

$$\sum_{j=1}^{n} \lambda_j (x_{ij} + w_i) \le x_{io} + w_i - \alpha_i g_{x_{io}}, \ i = 1, \ldots, m, \tag{9}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} + w_i \sum_{j=1}^{n} \lambda_j \le x_{io} + w_i - \alpha_i g_{x_{io}}, \ i = 1, \ldots, m, \tag{10}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} + w_i \le x_{io} + w_i - \alpha_i g_{x_{io}}, \ i = 1, \ldots, m, \tag{11}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} - \alpha_i g_{x_{io}}, \ i = 1, \ldots, m. \tag{12}$$

It can be seen that the obtained constraint in (12) is the same as the original constraint in (3). It can be proved for all other constraints of model (3). It means that under the conditions mentioned above the proposed measure satisfy translation invariance.

In the special cases, if the input price vector be only known, model (3) is transformed to a cost model and if the output price vector be only at hand, the model is converted to a revenue model.

**Theorem 2.1** The projection of $\mathrm{DMU}_o$ defined by (5), is profit efficient under the prices defined by (6).

**Proof.** The degree of profit inefficiency of $\left(x_{io}^*, y_{ro}^*\right)$ under the prices $c_{io}^*$ and $p_{ro}^*$ can be evaluated by solving model (3). Let an optimal solution and the optimal value of the model (3) for $\left(x_{io}^*, y_{ro}^*, c_{io}^*, p_{ro}^*\right)$ are respectively, $\hat{\alpha}_i, \hat{\sigma}_i \ (\forall i)$, $\hat{\beta}_r, \hat{\gamma}_r \ (\forall r)$, $\hat{\lambda}_j (\forall j)$, $\hat{z}_{ij} (\forall i, \forall j)$, $\hat{k}_{rj} (\forall r, \forall j)$, and $\hat{\vec{D}}_o$. Hence, it holds that

$$\sum_{j=1}^{n} \hat{\lambda}_j x_{ij} \le x_{io}^* - \hat{\alpha}_i g_{x_{io}} = x_{io} - \alpha_i^* g_{x_{io}} - \hat{\alpha}_i g_{x_{io}} = x_{io} - (\alpha_i^* + \hat{\alpha}_i) g_{x_{io}}, \ i = 1, \ldots, m, \tag{13}$$

$$\sum_{j=1}^{n} \hat{\lambda}_j y_{rj} \ge y_{ro}^* + \hat{\beta}_r g_{y_{ro}} = y_{ro} + \beta_r^* g_{y_{ro}} + \hat{\beta}_r g_{y_{ro}} = y_{ro} + (\beta_r^* + \hat{\beta}_r) g_{y_{ro}}, \ r = 1, \ldots, s, \tag{14}$$

$$\sum_{j=1}^{n}\hat{z}_{ij}c_{ij} \le c_{io}^{*} - \hat{\sigma}_i g_{c_{io}} = c_{io} - \sigma_i^{*} g_{c_{io}} - \hat{\sigma}_i g_{c_{io}} = c_{io}^{t} - (\sigma_i^{*} + \hat{\sigma}_i) g_{c_{io}}, \quad i = 1,\ldots,m, \tag{15}$$

$$\sum_{j=1}^{n}\hat{k}_{rj}p_{rj} \ge p_{ro}^{*} + \hat{\gamma}_i g_{p_{io}} = p_{ro} + \gamma_r^{*} g_{c_{ro}} + \hat{\gamma}_r g_{p_{ro}} = p_{ro} + (\gamma_r^{*} + \hat{\gamma}_r) g_{p_{ro}}, \; r = 1,\ldots,s, \tag{16}$$

$$\sum_{j=1}^{n}\hat{z}_{ij} = 1, \quad i = 1,\ldots,m, \tag{17}$$

$$\sum_{j=1}^{n}\hat{k}_{rj} = 1, \; r = 1,\ldots,s, \tag{18}$$

We can rewrite the above constraints as,

$$\sum_{j=1}^{n}\hat{\lambda}_j x_{ij} \le x_{io} - \tilde{\alpha}_i\, g_{x_{io}}, \, i = 1,\ldots,m, \qquad \sum_{j=1}^{n}\hat{\lambda}_j y_{rj} \ge y_{ro} + \tilde{\beta}_r\, g_{y_{ro}}, \; r = 1,\ldots,s, \tag{19}$$

$$\sum_{j=1}^{n}\hat{z}_{ij}c_{ij} \le c_{io} - \tilde{\sigma}_i\, g_{c_{io}}, \, i = 1,\ldots,m, \qquad \sum_{j=1}^{n}\hat{k}_{rj}p_{rj} \ge p_{ro} + \tilde{\gamma}_r\, g_{p_{ro}}, \; r = 1,\ldots,s, \tag{20}$$

$$\sum_{j=1}^{n}\hat{z}_{ij} = 1, \quad i = 1,\ldots,m, \qquad \sum_{j=1}^{n}\hat{k}_{rj} = 1, \, r = 1,\ldots,s. \tag{21}$$

It means that $\tilde{\alpha}_i, \tilde{\sigma}_i \; (\forall i)$, $\tilde{\beta}_r, \tilde{\gamma}_r \; (\forall r)$, $\hat{\lambda}_j (\forall j)$, $\hat{z}_{ij} (\forall i, \forall j)$, $\hat{k}_{rj} (\forall r, \forall j)$, is a feasible solution of model (3) for $\left(x_{io}, y_{ro}, c_{io}, p_{ro}\right)$. For this feasible solution, the value of the objective function will be $\tilde{\vec{D}}_o = \frac{1}{4}\left(\frac{1}{m}\sum_{i=1}^{m}(\tilde{\alpha}_i + \tilde{\sigma}_i) + \frac{1}{s}\sum_{r=1}^{s}(\tilde{\beta}_r + \tilde{\gamma}_r)\right) = \hat{\vec{D}}_o + \vec{D}_o^{*}$. Since in the evaluation of $\left(x_{io}, y_{ro}, c_{io}, p_{ro}\right)$ by model (3), $\vec{D}_o^{*} = \frac{1}{4}\left(\frac{1}{m}\sum_{i=1}^{m}(\alpha_i^{*} + \sigma_i^{*}) + \frac{1}{s}\sum_{r=1}^{s}(\beta_r^{*} + \gamma_r^{*})\right)$ is maximal, we have $\tilde{\vec{D}}_o \le \vec{D}_o^{*}$ and hence, $\hat{\vec{D}}_o = 0$. Since $\hat{\alpha}_i, \hat{\sigma}_i \; (\forall i)$, $\hat{\beta}_r, \hat{\gamma}_r \; (\forall r)$, is considered as the optimal solution of the model (3) in the evaluation of $\left(x_{io}^{*}, y_{ro}^{*}, c_{io}^{*}, p_{ro}^{*}\right)$, we have $\hat{\vec{D}}_o = \frac{1}{4}\left(\frac{1}{m}\sum_{i=1}^{m}(\hat{\alpha}_i + \hat{\sigma}_i) + \frac{1}{s}\sum_{r=1}^{s}(\hat{\beta}_r + \hat{\gamma}_r)\right) = 0$. On the other hand, according to model (3), $\hat{\alpha}_i, \hat{\sigma}_i, \hat{\beta}_r, \hat{\gamma}_r \ge 0 \; (\forall i, r)$. Therefore, it can be conclude that $\hat{\alpha}_i = \hat{\sigma}_i = \hat{\beta}_r = \hat{\gamma}_r = 0 \; (\forall i, r)$, and hence, according to the definition 3.1, the projection of $\text{DMU}_o$ obtained by model (3) is profit efficient. □

## 4. Decomposition of the proposed directional profit efficiency measure

Directional profit efficiency measure presented in the former section cannot be decomposed. Therefore, one cannot individualize quantity and price contributions to the profit inefficiency of under evaluation DMU. Following Portela and Tanassoulis [14], we can compute a directional profit inefficiency measure to express the proportion of optimal profit which can be obtained after applying the quantities $(x_{io}^{*}, y_{ro}^{*})$ and prices $(c_{io}^{*}, p_{ro}^{*})$ for $\text{DMU}_o$. We can compute and decompose this inefficiency into two components respectively, as

$$1 - PE_o = 1 - \frac{PR_o}{PR_o^{*}} = \frac{PR_o^{*} - PR_o}{PR_o^{*}}, \tag{22}$$

$$1-PE_o = \frac{\sum_{r=1}^{s} p_{ro}^{*}(y_{ro}^{*}-y_{ro})-\sum_{i=1}^{m} c_{io}^{*}(x_{io}^{*}-x_{io})}{PR_o^{*}} + \frac{\sum_{r=1}^{s} y_{ro}(p_{ro}^{*}-p_{ro})-\sum_{i=1}^{m} x_{io}(c_{io}^{*}-c_{io})}{PR_o^{*}} \quad (23)$$

$= \text{quantity effect} + \text{price effect}.$

The first ratio in the right hand side of (23) represents the proportion of the increase in profit that is obtained after replacing the observed quantities of inputs and outputs with the optimal quantities (input/output inefficiency) and the second ratio represents the proportion that is obtained after replacing the observed prices of inputs and outputs with the optimal prices (market price inefficiency).

Decomposition of directional profit inefficiency measure can also be performed in a way that changes in quantities are weighted by observed prices instead of optimal prices and changes in prices are weighted by optimal quantities instead of observed quantities. This type of decomposition is shown in (24).

$$1-PE_o = \frac{\sum_{r=1}^{s} p_{ro}(y_{ro}^{*}-y_{ro})-\sum_{i=1}^{m} c_{io}(x_{io}^{*}-x_{io})}{PR_o^{*}} + \frac{\sum_{r=1}^{s} y_{ro}^{*}(p_{ro}^{*}-p_{ro})-\sum_{i=1}^{m} x_{io}^{*}(c_{io}^{*}-c_{io})}{PR_o^{*}} \quad (24)$$

$= \text{quantity effect} + \text{price effect}.$

Furthermore, since the set of weights is arbitrary [11], decomposition can be expressed as follows:

$$1-PE_o = \frac{\sum_{r=1}^{s}\left(\frac{p_{ro}^{*}+p_{ro}}{2}\right)(y_{ro}^{*}-y_{ro})-}{PR_o^{*}} \frac{\sum_{i=1}^{m}\left(\frac{c_{io}^{*}+c_{io}}{2}\right)(x_{io}^{*}-x_{io})}{PR_o^{*}} + \frac{\sum_{r=1}^{s}\left(\frac{y_{ro}^{*}+y_{ro}}{2}\right)(p_{ro}^{*}-p_{ro})-}{PR_o^{*}} \frac{\sum_{i=1}^{m}\left(\frac{x_{io}^{*}+x_{io}}{2}\right)(c_{io}^{*}-c_{io})}{PR_o^{*}} \quad (25)$$

$= \text{quantity effect} + \text{price effect}.$

Each component in (25) can be decomposed into two components, radial and mix components, which can consider instead of technical and allocative efficiency in classical cost decomposition. The decompositions for quantity and price effects are respectively exhibited in (26) and (27),

$$\text{quantity effect} = \frac{\sum_{r=1}^{s}\left(\frac{p_{ro}^{*}+p_{ro}}{2}\right)(y_{ro}^{*R}-y_{ro})-}{PR_o^{*}} \frac{\sum_{i=1}^{m}\left(\frac{c_{io}^{*}+c_{io}}{2}\right)(x_{io}^{*R}-x_{io})}{PR_o^{*}} + \frac{\sum_{r=1}^{s}\left(\frac{p_{ro}^{*}+p_{ro}}{2}\right)(y_{ro}^{*}-y_{ro}^{*R})-}{PR_o^{*}} \frac{\sum_{i=1}^{m}\left(\frac{c_{io}^{*}+c_{io}}{2}\right)(x_{io}^{*}-x_{io}^{*R})}{PR_o^{*}}, \quad (26)$$

price effect =

$$\frac{\sum_{r=1}^{s}\left(\frac{y_{ro}^{*}+y_{ro}}{2}\right)(p_{ro}^{*R}-p_{ro})-}{PR_o^{t*}}$$
$$\frac{\sum_{i=1}^{m}\left(\frac{x_{io}^{*}+x_{io}}{2}\right)(c_{io}^{*R}-c_{io})}{PR_o^{t*}}+ \tag{27}$$
$$\frac{\sum_{r=1}^{s}\left(\frac{y_{ro}^{*}+y_{ro}}{2}\right)(p_{ro}^{*}-p_{ro}^{*R})-}{PR_o^{*}}$$
$$\frac{\sum_{i=1}^{m}\left(\frac{x_{io}^{*}+x_{io}}{2}\right)(c_{io}^{*}-c_{io}^{*R})}{PR_o^{*}},$$

where

$$\begin{aligned} x_{io}^{*R} &= x_{io}-(\min_i \alpha_i^{*})g_{ix},\\ c_{io}^{*R} &= c_{io}-(\min_i \sigma_i^{*})g_{ic}, \quad i=1,\ldots,m,\\ y_{ro}^{*R} &= y_{ro}+(\min_r \beta_r^{*})g_{ry},\\ p_{ro}^{*R} &= p_{ro}+(\min_r \gamma_r^{*})g_{rp}, \quad r=1,\ldots,s. \end{aligned} \tag{28}$$

In fact, $(\min_i \alpha_i^{*})$ and $(\min_i \sigma_i^{*})$ estimate respectively the maximal feasible proportionate reduction in input quantities and prices. Similarly, $(\min_r \beta_r^{*})$ and $(\min_r \gamma_r^{*})$ estimate the maximal feasible proportionate expansion in output quantities and prices respectively, without altering the mix. Further reduction and expansion can be accomplished by changing the input/output quantities and prices proportions that are associated with the second components in (26) and (27).

The present decomposition is an extension of the approach expressed by Portela and Thanassoulis [11]. They used the Bennet indicators [18-20] to decompose the total cost savings and could separate the contribution of total cost which can be saved by changing the prices of inputs from that can be saved by changing the quantities.

## 5. Numerical example

### 5.1. Discussion on profit inefficiency and its decomposition

To demonstrate the ability of the proposed approach for evaluating the profit efficiency of DMUs, we design an illustrative example including five DMUs with two inputs and two outputs. We assume that the input and output price vectors are known, but units can somewhat influence them and therefore, improve their profit efficiency by selecting better price vectors and/or changing the input/output quantities. The input/output data are shown in Table 1.

Table 1. Quantity and price of inputs and outputs

| DMU | $x_1$ | $x_2$ | $c_1$ | $c_2$ | $y_1$ | $y_2$ | $p_1$ | $p_2$ |
|---|---|---|---|---|---|---|---|---|
| DMU1 | 2 | 4 | 3 | 6 | 4 | 8 | 6 | 12 |
| DMU2 | 4 | 8 | 3 | 6 | 2 | 4 | 6 | 12 |
| DMU3 | 4 | 8 | 1 | 2 | 2 | 4 | 18 | 36 |
| DMU4 | 4 | 8 | 4 | 2 | 2 | 4 | 30 | 50 |
| DMU5 | 10 | 16 | 2 | 1 | 3 | 6 | 2 | 5 |

DMU1 and DMU2 use the same input/output price vectors. Input levels consumed by DMU1 are half of those of DMU2, but its output quantities are the double of the output quantities of DMU2. Thus, we expect the degree of inefficiency of DMU2 be more than the degree of inefficiency of DMU1. This can be obtained by solving model (3) that its results are shown in Table 2. From the results in Table 4 it can be seen that two DMUs have the positive price effects, but quantity effect of DMU2 is only positive. It means that for reducing the profit inefficiency, DMU2 must change either quantity or price of its inputs and outputs, but DMU1 only need to change its price vectors.

Table 2. Results of model 1

| DMU | $\vec{D}_o$ | $\alpha_1$ | $\alpha_2$ | $\beta_1$ | $\beta_2$ | $\sigma_1$ | $\sigma_2$ | $\gamma_1$ | $\gamma_2$ |
|---|---|---|---|---|---|---|---|---|---|
| DMU1 | 0.25 | 0 | 0 | 0 | 0 | 0.5 | 0.5 | 0.5 | 0.5 |
| DMU2 | 1 | 0 | 0 | 3 | 3 | 0.5 | 0.5 | 0.5 | 0.5 |
| DMU3 | 0.924 | 0 | 0 | 3 | 3 | 0.0 | 0.5 | 0.5 | 0.39 |
| DMU4 | 0.875 | 0 | 0 | 3 | 3 | 0.5 | 0.5 | 0 | 0 |
| DMU5 | 1.296 | 0.2 | 0 | 4.33 | 4.33 | 0.5 | 0 | 0.5 | 0.5 |

The results in Table 2 show that the degree of profit inefficiency of DMU3 is less than that of DMU2. It is because of the price vectors used by DMU3 is better than that of DMU2. From Table 4 we can also see that these two DMUs have almost the same quantity effects but different price effects. In addition, based on the obtained results, DMU5 is the most inefficient among other DMUs. From the value of its price and quantity effects, we can conclude to improve the profit efficiency, this DMU must improve its input and/or output levels and also their prices. These results are also clear from data in Table 1 and 3.

It can be seen that from Table 3, how DMUs change the quantities and prices of their inputs and outputs to be profit efficient. For example, DMU5 should change its input/outputs quantities and their prices from (10,16), (3,6), (2,1) and (2,5) to (8,16), (16,32), (1,1), and (3,7.5) respectively, to be profit efficient.

Table 5 shows the measure of radial and non- radial changes in quantity and price vectors that are needed to improve the profit efficiency of under evaluation DMU.

Table 3. Projection points and profit efficiencies

| DMU | $i_1$ | $i_2$ | $o_1$ | $o_2$ | $c_1$ | $c_2$ | $p_1$ | $p_2$ | $PE_o$ |
|---|---|---|---|---|---|---|---|---|---|
| DMU1 | 2 | 4 | 4 | 8 | 1.5 | 3 | 9 | 18 | 0.55 |
| DMU2 | 4 | 8 | 8 | 16 | 1.5 | 3 | 9 | 18 | 0 |
| DMU3 | 4 | 8 | 8 | 16 | 1 | 1 | 27 | 50 | 0.16 |
| DMU4 | 4 | 8 | 8 | 16 | 2 | 1 | 30 | 50 | 0.22 |
| DMU5 | 8 | 16 | 16 | 32 | 1 | 1 | 3 | 7.5 | 0 |

Table 4. The results of stage 1 of decomposition

| DMU | $1-PE_o$ | quantity effect | price effect |
|---|---|---|---|
| DMU1 | 0.45 | 0 | 0.45 |
| DMU2 | 1 | 0.68 | 0.32 |
| DMU3 | 0.84 | 0.65 | 0.19 |
| DMU4 | 0.78 | 0.76 | 0.02 |
| DMU5 | 1 | 0.75 | 0.25 |

Table 5. The results of stage 2 of decomposition

| DMU | Non-radial part of quantity effect | Radial part of quantity effect | Non-radial part of price effect | radial part of price effect |
|---|---|---|---|---|
| DMU1 | 0 | 0 | 0 | 0.45 |
| DMU2 | 0 | 0.68 | 0 | 0.32 |
| DMU3 | 0 | 0.65 | 0.02 | 0.17 |
| DMU4 | 0 | 0.76 | 0 | 0.02 |
| DMU5 | 0.01 | 0.74 | 0.03 | 0.22 |

### 5.2. An empirical application: profit efficiency in U.S. banking

In this section, we apply the proposed approach for evaluating the profit efficiency of 50 U.S. bank branches during the year 1996 (Tables 6 and 7). The data was taken from Ray [21]. In this example units are price taker, but we assume that they are interested to examine the contribution of market prices in their profit inefficiency and also to know which price vector is the best for earning the maximum profit. By using the proposed decomposition, we can identify the reasons of their profit inefficiencies (Tables 8) and present the target levels of input/output quantities and prices (Table 9 and 10). Each unit consumes four inputs consist of, transaction deposits $(i_1)$, non-transaction deposits $(i_2)$, labor $(i_3)$, and capital $(i_4)$ for producing five outputs consist of, industrial loans $(o_1)$, consumer loans $(o_2)$, real estate loans $(o_3)$, investments $(o_4)$, and other income $(o_5)$. We evaluate the profit efficiency of units by using the optimal solution of model (4) for an special direction vector as $g_{x_{io}} = x_{io}; i = 1,\dots,m,$ $g_{y_{ro}} = y_{ro}; r = 1,\dots,s,$ $g_{c_{io}} = c_{io}; i = 1,\dots,m,$ and $g_{p_{ro}} = p_{ro}; r = 1,\dots,s$. In Table 8, the column 4 shows the obtained results of evaluating the profit inefficiency of bank branches by the proposed approach.

Table 6. Bank input quantity and price data

| B | $i_1$ | $i_2$ | $i_3$ | $i_4$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 111.805 | 434.194 | 0.411 | 19.356 | 0.006905 | 0.054842 | 34.8856 | 0.22928 |
| 2 | 154.721 | 311.423 | 0.203 | 8.266 | 0.010044 | 0.029718 | 32.3448 | 0.46443 |
| 3 | 76.975 | 396.428 | 0.083 | 5.795 | 0.008522 | 0.049931 | 55.807 | 0.12045 |
| 4 | 77.369 | 361.009 | 0.205 | 7.576 | 0.013326 | 0.052387 | 29.3659 | 0.18598 |
| 5 | 33.051 | 424.549 | 0.189 | 9.207 | 0.010741 | 0.04696 | 32.312 | 0.23297 |
| 6 | 130.316 | 363.854 | 0.178 | 5.67 | 0.001727 | 0.046073 | 28.3483 | 0.21746 |
| 7 | 95.421 | 369.313 | 0.185 | 11.238 | 0.009547 | 0.058695 | 30.227 | 0.11799 |
| 8 | 141.98 | 284.723 | 0.248 | 8.822 | 0.008776 | 0.052089 | 37.4435 | 0.3854 |
| 9 | 84.012 | 422.808 | 0.192 | 7.861 | 0.008606 | 0.043124 | 38.1719 | 0.24539 |
| 10 | 79.081 | 354.272 | 0.256 | 6.988 | 0.013315 | 0.04072 | 31.3477 | 0.32055 |
| 11 | 36.78 | 382.783 | 0.142 | 10.189 | 0.023355 | 0.045605 | 37.9507 | 0.14516 |
| 12 | 94.138 | 284.341 | 0.218 | 10.237 | 0.007383 | 0.048108 | 28.8119 | 0.2152 |
| 13 | 64.621 | 316.446 | 0.144 | 3.07 | 0.005184 | 0.044077 | 28.6736 | 0.20651 |
| 14 | 101.855 | 338.586 | 0.21 | 11.547 | 0.002278 | 0.034839 | 30.4857 | 0.22517 |
| 15 | 99.539 | 316.927 | 0.27 | 20.199 | 0.006148 | 0.041928 | 31.5185 | 0.15149 |
| 16 | 181.594 | 304.163 | 0.205 | 8.888 | 0.010061 | 0.032657 | 50.4537 | 0.28904 |
| 17 | 79.715 | 382.693 | 0.255 | 7.698 | 0.010299 | 0.035185 | 27.9412 | 0.20512 |
| 18 | 171.637 | 297.141 | 0.191 | 8.668 | 0.015632 | 0.046608 | 40.7853 | 0.20558 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 19 | 108.916 | 287.656 | 0.184 | 6.237 | 0.024422 | 0.051249 | 29.9565 | 0.24964 |
| 20 | 215.757 | 279.379 | 0.195 | 8.01 | 0.013436 | 0.052527 | 32.851 | 0.26841 |
| 21 | 116.651 | 340.618 | 0.214 | 5.253 | 0.012207 | 0.049539 | 31.028 | 0.45764 |
| 22 | 78.89 | 351.791 | 0.212 | 9.458 | 0.006515 | 0.046061 | 34.9434 | 0.2639 |
| 23 | 171.298 | 285.875 | 0.251 | 5.186 | 0.007875 | 0.042718 | 35.6892 | 0.61955 |
| 24 | 131.046 | 282 | 0.229 | 5.471 | 0.005555 | 0.039862 | 35.3974 | 0.26595 |
| 25 | 129.676 | 316.831 | 0.226 | 10.43 | 0.017027 | 0.04534 | 29.208 | 0.21055 |
| 26 | 136.549 | 310.071 | 0.275 | 9.483 | 0.008297 | 0.041249 | 34.32 | 0.24096 |
| 27 | 168.394 | 301.344 | 0.261 | 18.676 | 0.006633 | 0.049667 | 43.5402 | 0.18082 |
| 28 | 174.401 | 274.875 | 0.207 | 9.586 | 0.000872 | 0.038396 | 42.7633 | 0.26966 |
| 29 | 174.94 | 302.552 | 0.247 | 5.857 | 0.009243 | 0.046518 | 34.081 | 0.45433 |
| 30 | 231.463 | 330.746 | 0.209 | 12.092 | 0.006558 | 0.039988 | 43.5789 | 0.27464 |
| 31 | 108.419 | 327.439 | 0.251 | 11.223 | 0.013881 | 0.04781 | 27.8486 | 0.18337 |
| 32 | 144.217 | 336.406 | 0.273 | 15.439 | 0.008515 | 0.046685 | 29.3956 | 0.15804 |
| 33 | 221.628 | 294.729 | 0.259 | 10.933 | 0.019831 | 0.04726 | 37.938 | 0.16811 |
| 34 | 85.677 | 354.134 | 0.18 | 7.776 | 0.010003 | 0.052525 | 30.1222 | 0.19792 |
| 35 | 139.87 | 337.857 | 0.28 | 3.926 | 0.009652 | 0.048334 | 29.0357 | 0.68161 |
| 36 | 187.583 | 294.983 | 0.241 | 8.219 | 0.012618 | 0.043379 | 48.0747 | 0.31099 |
| 37 | 118.168 | 369.407 | 0.273 | 9.955 | 0.007904 | 0.043545 | 35.8901 | 0.25364 |
| 38 | 155.287 | 430.204 | 0.299 | 8.993 | 0.012158 | 0.048689 | 31.097 | 0.35316 |
| 39 | 223.944 | 283.096 | 0.186 | 8.244 | 0.014352 | 0.046807 | 50.629 | 0.29415 |
| 40 | 154.83 | 280.436 | 0.263 | 9.201 | 0.004741 | 0.043525 | 35.4791 | 0.35811 |
| 41 | 131.127 | 365.442 | 0.32 | 16.014 | 0.006627 | 0.048002 | 29.9063 | 0.18446 |
| 42 | 94.432 | 368.091 | 0.229 | 8.505 | 0.0097 | 0.052302 | 34.5109 | 0.31193 |
| 43 | 222.651 | 282.545 | 0.299 | 15.718 | 0.004905 | 0.034143 | 38.759 | 0.29648 |
| 44 | 116.617 | 326.074 | 0.231 | 8.274 | 0.009741 | 0.046244 | 29.1515 | 0.29236 |
| 45 | 193.806 | 236.212 | 0.175 | 5.151 | 0.018446 | 0.044308 | 43.6743 | 0.56688 |
| 46 | 73.233 | 486.438 | 0.22 | 3.46 | 0.007032 | 0.04908 | 49.705 | 0.82601 |
| 47 | 151.344 | 349.154 | 0.359 | 8.551 | 0.015567 | 0.024725 | 34.1309 | 0.42042 |
| 48 | 161.773 | 549.27 | 0.257 | 6.58 | 0.004179 | 0.04266 | 35.5681 | 0.49635 |
| 49 | 179.098 | 354.372 | 1.313 | 12.878 | 0.010257 | 0.047176 | 36.8104 | 0.8176 |
| 50 | 95.447 | 321.75 | 0.264 | 11.692 | 0.008832 | 0.045887 | 36.4924 | 0.17918 |

Table 7. Bank output quantity and price data

| B | $o_1$ | $o_2$ | $o_3$ | $o_4$ | $o_5$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 42.654 | 281.66 | 141.454 | 75.657 | 14.688 | 0.21967 | 0.1325 | 0.05154 | 0.06377 | 1 |
| 2 | 32.985 | 70.183 | 109.357 | 191.057 | 4.318 | 0.07849 | 0.10477 | 0.06728 | 0.024 | 1 |
| 3 | 75.474 | 8.832 | 290.18 | 155.438 | 0.944 | 0.0996 | 0.07892 | 0.07404 | 0.06026 | 1 |
| 4 | 57.935 | 74.259 | 196.96 | 98.871 | 2.433 | 0.09431 | 0.09999 | 0.07976 | 0.0555 | 1 |
| 5 | 39.382 | 49.084 | 316.682 | 48.674 | 3.138 | 0.12155 | 0.12601 | 0.06853 | 0.06811 | 1 |
| 6 | 41.054 | 33.29 | 247.589 | 148.686 | 3.751 | 0.08245 | 0.08567 | 0.08244 | 0.0547 | 1 |
| 7 | 50.278 | 75.52 | 286.727 | 53.148 | 3.015 | 0.09453 | 0.07766 | 0.09412 | 0.06933 | 1 |
| 8 | 87.693 | 52.779 | 165.261 | 56.463 | 9.432 | 0.09712 | 0.1374 | 0.05984 | 0.063564 | 1 |
| 9 | 28.026 | 55.779 | 239.118 | 208.537 | 6.249 | 0.09591 | 0.094 | 0.08016 | 0.057088 | 1 |
| 10 | 58.602 | 31.585 | 278.365 | 128.449 | 4.912 | 0.2933 | 0.15533 | 0.03119 | 0.054917 | 1 |
| 11 | 35.884 | 44.263 | 174.7 | 256.871 | 4.111 | 0.0938 | 0.09191 | 0.08498 | 0.05187 | 1 |
| 12 | 44.125 | 48.241 | 210.124 | 158.738 | 3.225 | 0.10701 | 0.092 | 0.08069 | 0.0529 | 1 |
| 13 | 55.637 | 64.486 | 150.87 | 185.25 | 4.47 | 0.07427 | 0.14135 | 0.07607 | 0.064092 | 1 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 14 | 31.702 | 105.386 | 200.102 | 85.255 | 6.652 | 0.0917 | 0.09085 | 0.08456 | 0.062401 | 1 |
| 15 | 34.788 | 50.011 | 246.324 | 159.393 | 3.236 | 0.10423 | 0.0797 | 0.08195 | 0.055 | 1 |
| 16 | 56.553 | 6.625 | 222.897 | 157.066 | 6.156 | 0.10938 | 0.19668 | 0.07467 | 0.0527 | 1 |
| 17 | 18.52 | 222.234 | 165.645 | 66.92 | 3.985 | 0.11134 | 0.08149 | 0.08404 | 0.0761 | 1 |
| 18 | 44.031 | 29.02 | 243.223 | 171.917 | 5.783 | 0.12314 | 0.08218 | 0.06223 | 0.06659 | 1 |
| 19 | 52.169 | 36.165 | 119.37 | 205.256 | 1.862 | 0.08449 | 0.08199 | 0.06468 | 0.05557 | 1 |
| 20 | 120.032 | 87.585 | 208.67 | 87.041 | 6.371 | 0.08048 | 0.07669 | 0.08122 | 0.07804 | 1 |
| 21 | 19.113 | 28.154 | 262.832 | 162.963 | 6.074 | 0.08743 | 0.12531 | 0.08745 | 0.06515 | 1 |
| 22 | 45.141 | 14.585 | 225.703 | 169.499 | 4.402 | 0.10492 | 0.0964 | 0.07889 | 0.063493 | 1 |
| 23 | 61.691 | 101.368 | 180.709 | 90.164 | 6.773 | 0.25077 | 0.07519 | 0.03253 | 0.056985 | 1 |
| 24 | 65.723 | 86.496 | 249.611 | 52.84 | 11.689 | 0.0881 | 0.09345 | 0.07759 | 0.066976 | 1 |
| 25 | 44.266 | 88.868 | 235.361 | 116.791 | 4.256 | 0.09987 | 0.10655 | 0.07983 | 0.069517 | 1 |
| 26 | 38.908 | 75.033 | 229.876 | 111.597 | 2.511 | 0.12327 | 0.08522 | 0.0766 | 0.068174 | 1 |
| 27 | 109.58 | 33.155 | 184.179 | 176.744 | 5.741 | 0.0689 | 0.11045 | 0.08624 | 0.068421 | 1 |
| 28 | 159.743 | 35.745 | 156.233 | 107.137 | 3.785 | 0.08646 | 0.08351 | 0.08051 | 0.0584 | 1 |
| 29 | 72.329 | 53.262 | 137.252 | 140.817 | 7.591 | 0.09664 | 0.11355 | 0.10683 | 0.07362 | 1 |
| 30 | 106.34 | 23.693 | 226.54 | 161.803 | 5.431 | 0.10021 | 0.10328 | 0.08419 | 0.053299 | 1 |
| 31 | 54.868 | 69.261 | 168.534 | 166.432 | 3.497 | 0.11752 | 0.09523 | 0.0743 | 0.060139 | 1 |
| 32 | 32.195 | 35.251 | 209.341 | 143.877 | 4.257 | 0.07625 | 0.1059 | 0.08361 | 0.062129 | 1 |
| 33 | 78.17 | 118.097 | 209.424 | 103.907 | 11.349 | 0.09687 | 0.11053 | 0.08966 | 0.05992 | 1 |
| 34 | 84.317 | 54.948 | 229.375 | 99.756 | 6.116 | 0.08989 | 0.09938 | 0.07628 | 0.067625 | 1 |
| 35 | 81.401 | 55.116 | 180.483 | 149.994 | 5.789 | 0.08437 | 0.09975 | 0.07544 | 0.0548 | 1 |
| 36 | 40.884 | 10.652 | 233.734 | 186.361 | 4.739 | 0.08568 | 0.08271 | 0.09283 | 0.061853 | 1 |
| 37 | 61.556 | 73.014 | 263.974 | 103.391 | 8.075 | 0.10053 | 0.10191 | 0.08727 | 0.0463 | 1 |
| 38 | 112.47 | 105.948 | 239.786 | 139.941 | 3.848 | 0.09438 | 0.06856 | 0.08076 | 0.057238 | 1 |
| 39 | 14.875 | 109.965 | 62.685 | 131.78 | 6.642 | 0.0876 | 0.13264 | 0.07739 | 0.060426 | 1 |
| 40 | 59.532 | 78.519 | 187.906 | 59.538 | 9.14 | 0.1007 | 0.08664 | 0.07836 | 0.069367 | 1 |
| 41 | 85.824 | 73.366 | 191.824 | 207.116 | 5.657 | 0.20274 | 0.08764 | 0.03422 | 0.061444 | 1 |
| 42 | 79.859 | 100.083 | 230.688 | 88.693 | 4.363 | 0.09003 | 0.09947 | 0.07976 | 0.05592 | 1 |
| 43 | 48.902 | 4.89 | 333.867 | 56.814 | 7.527 | 0.09431 | 0.20716 | 0.08438 | 0.07774 | 1 |
| 44 | 30.466 | 42.9 | 289.771 | 156.866 | 3.087 | 0.09607 | 0.10193 | 0.08328 | 0.062289 | 1 |
| 45 | 40.999 | 5.203 | 304.792 | 114.665 | 4.191 | 0.08456 | 0.12839 | 0.08187 | 0.066167 | 1 |
| 46 | 279.037 | 0.428 | 28.666 | 27.217 | 9.76 | 0.10653 | 0.09346 | 0.03436 | 0.05515 | 1 |
| 47 | 40.818 | 30.847 | 191.266 | 206.572 | 9.231 | 0.16385 | 0.184 | 0.05278 | 0.057956 | 1 |
| 48 | 63.333 | 86.147 | 167.996 | 280.677 | 16.237 | 0.09663 | 0.1114 | 0.0765 | 0.0695 | 1 |
| 49 | 51.656 | 107.739 | 228.967 | 57.192 | 77.482 | 0.07426 | 0.09884 | 0.0754 | 0.067107 | 1 |
| 50 | 17.836 | 6.684 | 204.33 | 321.243 | 4.704 | 0.07053 | 0.07346 | 0.08183 | 0.064001 | 1 |

Table 8. Profit inefficiency and its decomposition in U.S. banking

| B | $\vec{D}_o$ | $PE_o$ | $1-PE_o$ | quantity effect | | price effect | |
|---|---|---|---|---|---|---|---|
| | | | | radial part | Non-radial part | radial part | Non-radial part |
| 1 | 0.183 | 0.41 | 0.59 | 0 | 0 | 0.12 | 0.47 |
| 2 | 0.287 | 0.17 | 0.83 | 0 | 0.27 | 0.1 | 0.47 |
| 3 | 0.182 | 0.33 | 0.67 | 0 | 0 | 0.01 | 0.66 |
| 4 | 0.343 | 0.2 | 0.8 | 0 | 0.24 | 0.03 | 0.53 |
| 5 | 0.183 | 0.27 | 0.73 | 0 | 0 | 0.1 | 0.63 |
| 6 | 0.32 | 0.32 | 0.68 | 0 | 0.21 | 0.01 | 0.46 |
| 7 | 0.13 | 0.41 | 0.59 | 0 | 0 | 0 | 0.59 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 8 | 0.298 | 0.22 | 0.78 | 0 | 0.19 | 0.16 | 0.43 |
| 9 | 0.299 | 0.34 | 0.66 | 0 | 0.13 | 0.15 | 0.38 |
| 10 | 0.157 | 0.5 | 0.5 | 0 | 0 | 0.08 | 0.42 |
| 11 | 0.176 | 0.37 | 0.63 | 0 | 0 | 0.12 | 0.5 |
| 12 | 0.182 | 0.4 | 0.6 | 0 | 0 | 0.02 | 0.58 |
| 13 | 0.167 | 0.52 | 0.48 | 0 | 0 | 0.01 | 0.47 |
| 14 | 0.273 | 0.43 | 0.57 | 0 | 0.18 | 0.04 | 0.35 |
| 15 | 0.231 | 0.36 | 0.64 | 0 | 0.11 | 0.07 | 0.46 |
| 16 | 0.658 | 0.27 | 0.73 | 0 | 0.27 | 0.11 | 0.35 |
| 17 | 0.141 | 0.51 | 0.49 | 0 | 0 | 0 | 0.48 |
| 18 | 0.219 | 0.36 | 0.64 | 0 | 0.02 | 0.21 | 0.41 |
| 19 | 0.193 | 0.15 | 0.85 | 0 | 0 | 0.07 | 0.78 |
| 20 | 0.169 | 0.46 | 0.54 | 0 | 0 | 0.09 | 0.46 |
| 21 | 0.171 | 0.46 | 0.54 | 0 | 0 | 0.07 | 0.47 |
| 22 | 0.375 | 0.28 | 0.72 | 0 | 0.19 | 0.12 | 0.41 |
| 23 | 0.18 | 0.42 | 0.58 | 0 | 0 | 0.16 | 0.42 |
| 24 | 0.177 | 0.55 | 0.45 | 0 | 0 | 0.09 | 0.35 |
| 25 | 0.252 | 0.4 | 0.6 | 0 | 0.13 | 0.02 | 0.45 |
| 26 | 0.453 | 0.26 | 0.74 | 0 | 0.3 | 0.09 | 0.35 |
| 27 | 0.219 | 0.35 | 0.65 | 0 | 0.06 | 0.26 | 0.34 |
| 28 | 0.158 | 0.43 | 0.57 | 0 | 0 | 0 | 0.57 |
| 29 | 0.238 | 0.46 | 0.54 | 0 | 0.12 | 0.11 | 0.3 |
| 30 | 0.196 | 0.42 | 0.58 | 0 | 0 | 0.22 | 0.37 |
| 31 | 0.342 | 0.27 | 0.73 | 0 | 0.22 | 0 | 0.51 |
| 32 | 0.436 | 0.18 | 0.82 | 0 | 0.4 | 0.03 | 0.39 |
| 33 | 0.171 | 0.49 | 0.51 | 0 | 0 | 0.15 | 0.36 |
| 34 | 0.17 | 0.4 | 0.6 | 0 | 0 | 0.05 | 0.55 |
| 35 | 0.242 | 0.28 | 0.72 | 0 | 0.11 | 0.03 | 0.59 |
| 36 | 0.381 | 0.31 | 0.69 | 0 | 0.16 | 0.29 | 0.24 |
| 37 | 0.267 | 0.38 | 0.62 | 0 | 0.15 | 0.12 | 0.36 |
| 38 | 0.326 | 0.24 | 0.76 | 0 | 0.21 | 0.06 | 0.49 |
| 39 | 0.204 | 0.22 | 0.78 | 0 | 0 | 0.39 | 0.39 |
| 40 | 0.277 | 0.36 | 0.64 | 0 | 0.15 | 0.12 | 0.38 |
| 41 | 0.24 | 0.35 | 0.65 | 0 | 0.11 | 0.04 | 0.51 |
| 42 | 0.193 | 0.32 | 0.68 | 0 | 0 | 0.14 | 0.54 |
| 43 | 0.476 | 0.37 | 0.63 | 0 | 0.28 | 0.13 | 0.22 |
| 44 | 0.171 | 0.46 | 0.54 | 0 | 0 | 0.03 | 0.51 |
| 45 | 0.187 | 0.43 | 0.57 | 0 | 0 | 0.23 | 0.34 |
| 46 | 0.217 | 0.1 | 0.9 | 0 | 0 | 0.44 | 0.46 |
| 47 | 0.291 | 0.32 | 0.68 | 0 | 0.32 | 0 | 0.36 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 48 | 0.178 | 0.49 | 0.51 | 0 | 0 | 0.14 | 0.37 |
| 49 | 0.186 | 0.47 | 0.53 | 0 | 0 | 0.25 | 0.29 |
| 50 | 0.172 | 0.44 | 0.56 | 0 | 0 | 0.17 | 0.38 |

Table 9. Target levels of input/output quantities

| B | $i_1$ | $i_2$ | $i_3$ | $i_4$ | $o_1$ | $o_2$ | $o_3$ | $o_4$ | $o_5$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 111.805 | 434.194 | 0.411 | 19.356 | 42.654 | 281.66 | 141.454 | 75.657 | 14.688 |
| 2 | 73.579 | 311.423 | 0.203 | 6.424 | 43.172 | 70.183 | 153.721 | 191.057 | 6.255 |
| 3 | 76.975 | 396.428 | 0.083 | 5.795 | 75.474 | 8.832 | 290.18 | 155.438 | 0.944 |
| 4 | 77.369 | 361.009 | 0.205 | 4.614 | 60.294 | 74.259 | 196.96 | 173.222 | 7.441 |
| 5 | 33.051 | 424.549 | 0.189 | 9.207 | 39.382 | 49.084 | 316.682 | 48.674 | 3.138 |
| 6 | 102.956 | 363.854 | 0.178 | 5.146 | 71.731 | 64.929 | 247.589 | 148.686 | 6.813 |
| 7 | 95.421 | 369.313 | 0.185 | 11.238 | 50.278 | 75.52 | 286.727 | 53.148 | 3.015 |
| 8 | 129.566 | 284.723 | 0.237 | 6.015 | 87.693 | 65.774 | 165.261 | 126.318 | 9.432 |
| 9 | 84.012 | 422.808 | 0.192 | 5.305 | 73.943 | 62.683 | 239.118 | 208.537 | 6.249 |
| 10 | 79.081 | 354.272 | 0.256 | 6.988 | 58.602 | 31.585 | 278.365 | 128.449 | 4.912 |
| 11 | 36.78 | 382.783 | 0.142 | 10.189 | 35.884 | 44.263 | 174.7 | 256.871 | 4.111 |
| 12 | 94.138 | 284.341 | 0.218 | 10.237 | 44.125 | 48.241 | 210.124 | 158.738 | 3.225 |
| 13 | 64.621 | 316.446 | 0.144 | 3.07 | 55.637 | 64.486 | 150.87 | 185.25 | 4.47 |
| 14 | 101.855 | 338.586 | 0.21 | 5.507 | 60.538 | 105.386 | 200.102 | 129.879 | 7.731 |
| 15 | 99.539 | 316.927 | 0.211 | 8.232 | 34.788 | 50.011 | 246.324 | 159.393 | 4.308 |
| 16 | 156.599 | 304.163 | 0.205 | 6.907 | 59.68 | 65.925 | 222.897 | 157.066 | 6.907 |
| 17 | 79.715 | 382.693 | 0.255 | 7.698 | 18.52 | 222.234 | 165.645 | 66.92 | 3.985 |
| 18 | 148.424 | 297.141 | 0.191 | 5.842 | 44.031 | 29.02 | 243.223 | 171.917 | 5.783 |
| 19 | 108.916 | 287.656 | 0.184 | 6.237 | 52.169 | 36.165 | 119.37 | 205.256 | 1.862 |
| 20 | 215.757 | 279.379 | 0.195 | 8.01 | 120.032 | 87.585 | 208.67 | 87.041 | 6.371 |
| 21 | 116.651 | 340.618 | 0.214 | 5.253 | 19.113 | 28.154 | 262.832 | 162.963 | 6.074 |
| 22 | 78.89 | 351.791 | 0.188 | 8.517 | 46.036 | 59.301 | 225.703 | 169.499 | 6.532 |
| 23 | 171.298 | 285.875 | 0.251 | 5.186 | 61.691 | 101.368 | 180.709 | 90.164 | 6.773 |
| 24 | 131.046 | 282 | 0.229 | 5.471 | 65.723 | 86.496 | 249.611 | 52.84 | 11.689 |
| 25 | 129.676 | 316.831 | 0.226 | 6.312 | 56.517 | 88.868 | 235.361 | 116.791 | 8.545 |
| 26 | 122.394 | 310.071 | 0.275 | 5.315 | 62.072 | 76.241 | 229.876 | 111.597 | 13.178 |
| 27 | 157.226 | 301.344 | 0.261 | 10.372 | 109.58 | 35.675 | 184.179 | 176.744 | 6.61 |
| 28 | 174.401 | 274.875 | 0.207 | 9.586 | 159.743 | 35.745 | 156.233 | 107.137 | 3.785 |
| 29 | 77.584 | 302.552 | 0.226 | 3.86 | 72.329 | 58.341 | 137.252 | 140.817 | 9.934 |
| 30 | 231.463 | 330.746 | 0.209 | 12.092 | 106.34 | 23.693 | 226.54 | 161.803 | 5.431 |
| 31 | 81.628 | 327.439 | 0.251 | 4.165 | 60.557 | 70.911 | 168.534 | 166.432 | 11.202 |
| 32 | 144.217 | 336.406 | 0.273 | 6.316 | 85.804 | 85.667 | 209.341 | 143.877 | 11.761 |
| 33 | 221.628 | 294.729 | 0.259 | 10.933 | 78.17 | 118.097 | 209.424 | 103.907 | 11.349 |
| 34 | 85.677 | 354.134 | 0.18 | 7.776 | 84.317 | 54.948 | 229.375 | 99.756 | 6.116 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 35 | 93.967 | 337.857 | 0.178 | 3.926 | 81.401 | 60.044 | 180.483 | 149.994 | 6.513 |
| 36 | 155.066 | 294.983 | 0.22 | 8.219 | 47.062 | 43.735 | 233.734 | 186.361 | 6.252 |
| 37 | 118.168 | 369.407 | 0.209 | 5.582 | 75.63 | 78.496 | 263.974 | 134.895 | 8.877 |
| 38 | 155.287 | 430.204 | 0.299 | 7.051 | 112.47 | 105.948 | 266.957 | 139.941 | 13.331 |
| 39 | 223.944 | 283.096 | 0.186 | 8.244 | 14.875 | 109.965 | 62.685 | 131.78 | 6.642 |
| 40 | 102.089 | 280.436 | 0.205 | 4.905 | 59.532 | 78.519 | 187.906 | 101.779 | 9.14 |
| 41 | 131.127 | 365.442 | 0.214 | 6.744 | 85.824 | 73.366 | 204.212 | 207.116 | 6.291 |
| 42 | 94.432 | 368.091 | 0.229 | 8.505 | 79.859 | 100.083 | 230.688 | 88.693 | 4.363 |
| 43 | 205.957 | 282.545 | 0.227 | 6.06 | 53.012 | 27.052 | 333.867 | 115.225 | 7.527 |
| 44 | 116.617 | 326.074 | 0.231 | 8.274 | 30.466 | 42.9 | 289.771 | 156.866 | 3.087 |
| 45 | 193.806 | 236.212 | 0.175 | 5.151 | 40.999 | 5.203 | 304.792 | 114.665 | 4.191 |
| 46 | 73.233 | 486.438 | 0.22 | 3.46 | 279.037 | 0.428 | 28.666 | 27.217 | 9.76 |
| 47 | 108.967 | 349.154 | 0.252 | 6.276 | 66.064 | 67.984 | 191.266 | 206.572 | 9.231 |
| 48 | 161.773 | 549.27 | 0.257 | 6.58 | 63.333 | 86.147 | 167.996 | 280.677 | 16.237 |
| 49 | 179.098 | 354.372 | 1.313 | 12.878 | 51.656 | 107.739 | 228.967 | 57.192 | 77.482 |
| 50 | 95.447 | 321.75 | 0.264 | 11.692 | 17.836 | 6.684 | 204.33 | 321.243 | 4.704 |

Table 10. Target levels of input/output prices

| B | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.003 | 0.027 | 27.849 | 0.118 | 0.293 | 0.199 | 0.077 | 0.078 | 1 |
| 2 | 0.005 | 0.025 | 27.849 | 0.232 | 0.118 | 0.157 | 0.101 | 0.036 | 1 |
| 3 | 0.004 | 0.025 | 27.904 | 0.118 | 0.149 | 0.118 | 0.107 | 0.078 | 1 |
| 4 | 0.007 | 0.026 | 27.849 | 0.118 | 0.141 | 0.15 | 0.107 | 0.078 | 1 |
| 5 | 0.005 | 0.025 | 27.849 | 0.118 | 0.182 | 0.189 | 0.103 | 0.078 | 1 |
| 6 | 0.001 | 0.025 | 27.849 | 0.118 | 0.124 | 0.129 | 0.107 | 0.078 | 1 |
| 7 | 0.005 | 0.029 | 27.849 | 0.118 | 0.142 | 0.116 | 0.107 | 0.078 | 1 |
| 8 | 0.004 | 0.026 | 27.849 | 0.193 | 0.146 | 0.206 | 0.09 | 0.078 | 1 |
| 9 | 0.004 | 0.025 | 27.849 | 0.123 | 0.144 | 0.141 | 0.107 | 0.078 | 1 |
| 10 | 0.007 | 0.025 | 27.849 | 0.16 | 0.293 | 0.207 | 0.047 | 0.078 | 1 |
| 11 | 0.012 | 0.025 | 27.849 | 0.118 | 0.141 | 0.138 | 0.107 | 0.078 | 1 |
| 12 | 0.004 | 0.025 | 27.849 | 0.118 | 0.161 | 0.138 | 0.107 | 0.078 | 1 |
| 13 | 0.003 | 0.025 | 27.849 | 0.118 | 0.111 | 0.207 | 0.107 | 0.078 | 1 |
| 14 | 0.001 | 0.025 | 27.849 | 0.118 | 0.138 | 0.136 | 0.107 | 0.078 | 1 |
| 15 | 0.003 | 0.025 | 27.849 | 0.118 | 0.156 | 0.12 | 0.107 | 0.078 | 1 |
| 16 | 0.005 | 0.025 | 27.849 | 0.145 | 0.164 | 0.207 | 0.107 | 0.078 | 1 |
| 17 | 0.005 | 0.025 | 27.849 | 0.118 | 0.167 | 0.122 | 0.107 | 0.078 | 1 |
| 18 | 0.008 | 0.025 | 27.849 | 0.118 | 0.185 | 0.123 | 0.093 | 0.078 | 1 |
| 19 | 0.012 | 0.026 | 27.849 | 0.125 | 0.127 | 0.123 | 0.097 | 0.078 | 1 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 20 | 0.007 | 0.026 | 27.849 | 0.134 | 0.121 | 0.115 | 0.107 | 0.078 | 1 |
| 21 | 0.006 | 0.025 | 27.849 | 0.229 | 0.131 | 0.188 | 0.107 | 0.078 | 1 |
| 22 | 0.003 | 0.025 | 27.849 | 0.132 | 0.157 | 0.145 | 0.107 | 0.078 | 1 |
| 23 | 0.004 | 0.025 | 27.849 | 0.31 | 0.293 | 0.113 | 0.049 | 0.078 | 1 |
| 24 | 0.003 | 0.025 | 27.849 | 0.133 | 0.132 | 0.14 | 0.107 | 0.078 | 1 |
| 25 | 0.009 | 0.025 | 27.849 | 0.118 | 0.15 | 0.16 | 0.107 | 0.078 | 1 |
| 26 | 0.004 | 0.025 | 27.849 | 0.12 | 0.185 | 0.128 | 0.107 | 0.078 | 1 |
| 27 | 0.003 | 0.025 | 27.849 | 0.118 | 0.103 | 0.166 | 0.107 | 0.078 | 1 |
| 28 | 0.001 | 0.025 | 27.849 | 0.135 | 0.13 | 0.125 | 0.107 | 0.078 | 1 |
| 29 | 0.005 | 0.025 | 27.849 | 0.227 | 0.145 | 0.17 | 0.107 | 0.078 | 1 |
| 30 | 0.003 | 0.025 | 27.849 | 0.137 | 0.15 | 0.155 | 0.107 | 0.078 | 1 |
| 31 | 0.007 | 0.025 | 27.849 | 0.118 | 0.176 | 0.143 | 0.107 | 0.078 | 1 |
| 32 | 0.004 | 0.025 | 27.849 | 0.118 | 0.114 | 0.159 | 0.107 | 0.078 | 1 |
| 33 | 0.01 | 0.025 | 27.849 | 0.118 | 0.145 | 0.166 | 0.107 | 0.078 | 1 |
| 34 | 0.005 | 0.026 | 27.849 | 0.118 | 0.135 | 0.149 | 0.107 | 0.078 | 1 |
| 35 | 0.005 | 0.025 | 27.849 | 0.341 | 0.127 | 0.15 | 0.107 | 0.078 | 1 |
| 36 | 0.006 | 0.025 | 27.849 | 0.155 | 0.129 | 0.124 | 0.107 | 0.078 | 1 |
| 37 | 0.004 | 0.025 | 27.849 | 0.127 | 0.151 | 0.153 | 0.107 | 0.069 | 1 |
| 38 | 0.006 | 0.025 | 27.849 | 0.177 | 0.142 | 0.103 | 0.107 | 0.078 | 1 |
| 39 | 0.007 | 0.025 | 27.849 | 0.147 | 0.131 | 0.199 | 0.107 | 0.078 | 1 |
| 40 | 0.002 | 0.025 | 27.849 | 0.179 | 0.151 | 0.13 | 0.107 | 0.078 | 1 |
| 41 | 0.003 | 0.025 | 27.849 | 0.118 | 0.293 | 0.131 | 0.051 | 0.078 | 1 |
| 42 | 0.005 | 0.026 | 27.849 | 0.156 | 0.135 | 0.149 | 0.107 | 0.078 | 1 |
| 43 | 0.002 | 0.025 | 27.849 | 0.148 | 0.141 | 0.207 | 0.107 | 0.078 | 1 |
| 44 | 0.005 | 0.025 | 27.849 | 0.146 | 0.144 | 0.153 | 0.107 | 0.078 | 1 |
| 45 | 0.009 | 0.025 | 27.849 | 0.283 | 0.127 | 0.193 | 0.107 | 0.078 | 1 |
| 46 | 0.004 | 0.025 | 27.849 | 0.413 | 0.16 | 0.14 | 0.052 | 0.078 | 1 |
| 47 | 0.008 | 0.025 | 27.849 | 0.21 | 0.246 | 0.207 | 0.079 | 0.078 | 1 |
| 48 | 0.002 | 0.025 | 27.849 | 0.248 | 0.145 | 0.167 | 0.107 | 0.078 | 1 |
| 49 | 0.005 | 0.025 | 27.849 | 0.409 | 0.111 | 0.148 | 0.107 | 0.078 | 1 |
| 50 | 0.004 | 0.025 | 27.849 | 0.118 | 0.106 | 0.11 | 0.107 | 0.078 | 1 |

For instance, it can be seen in the $1-PE_o$ column in Table 8, DMU2 is profit inefficient. It is clear that; the contribution of price inefficiency is 0.56 and is greater than the amount of quantity inefficiency 0.27. It means that the market prices have the important contribution in the inefficiency of DMU2. In addition, the results show that DMU2 can remove only a small part of its profit inefficiency by the proportionate changes in the input/output prices. Further improvement can be accrued with altering the mix of input/output prices and also quantities.

Tables 9 and 10 show the target levels of input/output quantities and prices, respectively. For instance, in order to improve the profit efficiency, DMU2 must decrease the quantities of its inputs 1 and 4 respectively, to 73.579 and 6.424 and also decrease all input prices respectively, to 0.005, 0.025, 27.849 and 0.232. To further improvement in profit efficiency it must increase the quantities of outputs 1, 3 and 5 respectively, to 43.172, 153.721 and 6.255 and also increase all output prices except output 5, respectively, to 0.118, 0.157, 0.101 and 0.036.

## 6. Conclusion

In competitive markets, profit maximization may be accomplished by correcting the quantities of inputs and/or outputs. If DMUs have ability to vary the market prices, to maximize profit they can correct both the input/output quantities and prices. This paper developed a non-radial directional distance function approach to determine a degree of profit inefficiency of DMUs in non-competitive markets. The proposed measure satisfies the important properties, units invariance and translation invariance.

The current approach can help to units for examining their performances and identify the reasons of their inefficiency and remove its resources. The optimal solution of the proposed model presents the best input/output quantities and prices, and by using them, DMUs can have minimum cost and maximum revenue, and hence maximum profit. The profit inefficiency measure obtained by the optimal solution to the proposed model was decomposed into two components. This type of decomposition determines the amount of the profit inefficiency that is occurred because of the inappropriate selection of input/output quantities and input/output prices.

In fact, the approach proposed in the current paper yields information about profit efficiency and help decision makers to improve it by correcting the input/output quantities and prices simultaneously.

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