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A New Three-Stage Robust Data Envelopment Analysis Model with application in diary industrial

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Abstract
Data Envelopment Analysis (DEA) is a non-parametric mathematical programming technique widely used for evaluating the relative efficiency of homogeneous decision-making units (DMUs) that use multiple inputs to produce multiple outputs. The DMUs may consist of several sub-processes that interact and perform various operations. The conventional DEA treats DMUs as "black boxes" and internal structure of the DMU is not taken into consideration. The Network DEA (NDEA) methods are capable of reflecting accurately the DMUs' internal structure and considered the DMU as a network of interconnected sub-units. On the other hand, many real-world applications face with uncertain data which the optimal solutions of models may even become infeasible and the ranking of DMUs can be invalid. Robust DEA (RDEA) is the last uncertain DEA approach that is applied for performance assessment of DMUs in the presence of uncertain data.
In this study we propose a new Robust Network DEA (RNDEA) model. We calculate the efficiency of this model by considering a series of intermediate measures and robust constraints. We present a case study in the dairy industry with three series stages to exhibit the efficacy of the approach and demonstrate the applicability of the proposed model.

Keywords: Network DEA, robust optimization, multi-stage systems, Shyster approach

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1. Introduction

Data Envelopment Analysis (DEA), a non-parametric Linear Programming (LP) based method developed by Charnes et al. (1978) for relative efficiency measurement of Decision-Making Units (DMUs), is a generalization of the Farrell's (1957) single-input single-output technical efficiency measure to the multiple-input multiple-output situations [1,2]. DEA gives relational performance efficiency by using a ratio of the weighted sum of outputs to the weighted sum of inputs and operations it with a mathematical programming. Unlike parametric methods, DEA does not require an explicit functional form relating inputs and outputs. This method for efficiency measurement is known as DEA constant returns to scale (CRS model), with an assumption that all firms are operating at an optimal scale. Later Banker et al. (2004) extended DEA to include Variable Returns to Scale (VRS) [3].

However, DEA has been widely accepted as a qualified methodology to estimate the efficiency of a system, but in many real-life cases, DMUs have network internal structure and are composed of several – divisions that related to each other. Individual divisions have their inputs and outputs respectively. An intermediate output from one division becomes an intermediate input for another division [4-6]. Traditional DEA models look DMUs as "black box" that consume a set of inputs to produce a set of outputs which may result in inaccurate efficiency score [7].

To open the "black box" and get greater insight into the production process, the Network DEA (NDEA) model is generalized to analyze the network structure of systems by researchers, such as [4,6,8,9,], first introduced network DEA model, which was improved and extended by other researchers. Lewis and Sexton (2004) propose a network DEA model for multistage system which is an extension of the two-stage DEA model [9]. Their studies solve a DEA model for each node independently. Tone and Tsutsui (2009) propose a network slacks-based measure model (NSBM) to evaluate efficiency when inputs and outputs might change no proportionally [10].

The literature on Network DEA has increased substantially in the last few years, both as regards to theoretical aspects. For systems composed of two processes connected in series, Kao and Hwang (2008) developed a DEA model to measure the efficiencies of the system and component processes at the same time. An interesting finding is that the efficiency of the system is the product of those of the two processes [11]. Their model can be extended to more than two processes. Kao (2014) reviews studies on network DEA by examining the models used and the structures of the network system of the problem being studied. This review highlights some directions for future studies from the methodological point of view, and is inspirational for exploring new areas of application from the empirical point of view [12]. Tsihrintzis et al. (2019) described the underlying notions of network DEA methods and their advantages over the classical DEA ones. They also conducted a critical review of the state-of-the art methods in the field and provided a thorough categorization of a great volume of network DEA literature in a unified manner [12,13].

One of the most important issues associated with DEA is the uncertainty associated with data. Since the resulted problem formulation of DEA technique is in form of linear programming, when all input data are subject to uncertainty, it is practically impossible to use old fashion methods to handle uncertainty. Soyster (1973) introduced robust optimization to handle uncertainty. He investigated a very simple approach to robust optimization. In this approach the column vectors of the constraint were assumed to belong to ellipsoidal uncertainty sets [14]. Peykani et al. (2020) reviewed the milestone approaches for handling uncertainty in data envelopment analysis (DEA). This paper presents the detailed classifications of robust data envelopment analysis (RDEA). RDEA is appropriate for measuring the efficiencies of decision-making units (DMUs) in the presence of the data and distributional uncertainties. This paper reviews scenario-based and uncertainty set of DEA models. It covers 73 studies from 2008 to 2019. The paper concludes with suggestions about the guidelines for future researches in the field of RDEA [15].

In this paper, we propose a new robust optimization formulation for Network DEA. Our suggested robust Network DEA model is based on Soyster approach and Kao and Hwang (2008) model. We first illustrate Network DEA model presented by Kao and Hwang in section 2. In section 3, robust DEA with uncertain data proposed by Soyster are discussed. Our Robust Network DEA (RNDEA) models present in section 4. In section 5, we give a numerical example and compare with solutions of three-stage fuzzy DEA model.

2. Multi-stage Data Envelopment Analysis

Let introduce the following basic notation:

$j \in J = \{1,...,n\}$: The index set of the n DMUs.

$k \in \{1,...,n\}$: Denotes the index of evaluated DMU.

$X_j = \left(x_{ij}, i = 1,...,m\right)$: The vector of first stage external inputs used by DMU_j.

$Z_j^t = \left(z_{dj}^t, d = 1,...,D; t = 1,...,h-1\right)$: The vector of stage-t intermediate outputs produced by DMU_j.

$Y_j = \left(y_{rj}, r = 1,...,s\right)$: The vector of latest stage final outputs produced by DMU_j.

$v = \left(v_i, i = 1,...,m\right)$: The vector of weights for the first stage external inputs in Kao et al. model.

$w^t = \left(w_d^t, d =,...,D; t = 1,...,h-1\right)$: The vector of weights for stage-t intermediate outputs in Kao et al. model.

$u = \left(u_r, r = 1,...,s\right)$: The vector of weights for the latest stage final outputs in Kao et al. model.

$\lambda^t = \left(\lambda_j^t, j = 1,...,n; t = 1,...,h\right)$: The vector of weights for DMU_j in stage t in Chen et al. model.

E_k: The overall efficiency of DMU k.

to have the same multiplier no matter how it is used while the former allows a factor to have different multipliers when it is used in different places. An interesting result of the relational model is that the system efficiency is the product of the two process efficiencies. Figure 1 is a pictorial expression of the series system with two-stage.

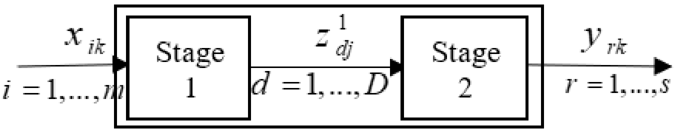

Figure 1. Two- stage series system

2.1 Two-stage DEA model by Kao and Hwang (2008)

The system efficiency of DMU k is calculated by the input orientation model from the series system of Kao and Hwang:

Kao and Hwang presented an input orientation model for calculating the system efficiency of DMU k. this model had an additional system constraint, by removing the additional system constraint in this model, the following reduced two-step model is obtained:

$$
\begin{aligned}
&E_k = \max \sum_{r=1}^{s} u_r y_{rk} \qquad (1)\\
&s.t. \quad \sum_{i=1}^{m} v_i x_{ik} = 1\\
&\sum_{d=1}^{D} w_d^1 z_{dj}^1 - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j=1,\dots,n\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d^1 z_{dj}^1 \le 0, \quad j=1,\dots,n\\
&u_r \ge 0, v_i \ge 0, \quad r=1,\dots,s, i=1,\dots,m,\\
&w_d^1 \ge 0, \qquad d=1,\dots,D.
\end{aligned}
$$

The efficiency score of model (1) is between zero and one ($0 < E_k \le 1$). DMU k is efficient under this model if and only if its efficiency score be equal to one. The dual of model (1) is as follow

$$
\begin{aligned}
&E_k = \min \theta \qquad (2)\\
&\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{ik}, \quad i=1,\dots,m\\
&\sum_{j=1}^{n} \left(\lambda_j^1 - \lambda_j^2\right) z_{dj}^1 \ge 0, \quad d=1,\dots,D\\
&\sum_{j=1}^{n} \lambda_j^2 y_{rj} \ge y_{rk}, \quad r=1,\dots,s\\
&\lambda_j^t \ge 0, \quad \text{t=1,2} \quad j=1,\dots,n
\end{aligned}
$$

Chen et al (2010) shown that entirely efficiency scores of Kao and Hwang can't indicate the amount of input reductions or output increases in inefficient DMUs [16]. They represented a two-stage approach, which determined the DEA frontier or DEA projection for inefficient DMU under the framework of Kao and Hwang that expressed as

$$
\begin{aligned}
&E_k = \min \theta \qquad (3)\\
&\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{ik}, \quad i=1,\dots,m\\
&\sum_{j=1}^{n} \lambda_j^1 z_{dj}^1 \ge \overline{z}_{dj}^1, \quad d=1,\dots,D\\
&\sum_{j=1}^{n} \lambda_j^2 z_{dj}^1 \le \overline{z}_{dj}^1, \quad d=1,\dots,D\\
&\sum_{j=1}^{n} \lambda_j^2 y_{rj} \ge y_{rk}, \quad r=1,\dots,s\\
&\lambda_j^t \ge 0, \quad \text{t=1,2}, \ j=1,\dots,n\\
&\overline{z}_{dj}^1 \ge 0, \quad \text{t=1,2}, \ d=1,\dots,D.
\end{aligned}
$$

2.2 A Multi-Stage DEA Model by Kao and Hwang (2008)

Consider a series system of h processes. As in the preceding section, let x_{ij} and y_{rj} be defined as the inputs and outputs of the system, respectively. Denote z_{dj}^{t} as the d^{th} intermediate product, $d=1,\dots,D$, of process t, t = 1,...,h-1, for DMUj. The intermediate products of process t are the outputs of process t as well as the inputs of process t + 1. Note that the intermediate products of the last process h are the outputs of the system. The number of intermediate products, D, can be different for each process. Here, it is assumed that they are the same for all processes just for simplification of notation. Figure 2 is a pictorial expression of the series system.

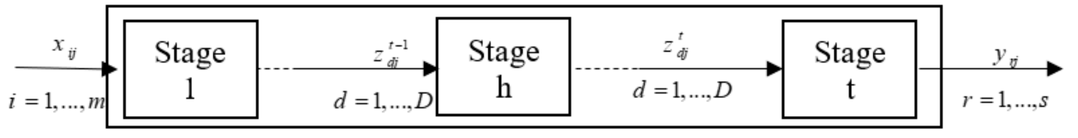

Figure 2. Multi- stage series system

Denote w_d^t as the multiplier, or the importance, associated with the d^{th} intermediate product of process t. As in the previous section, after removing the useless system constraint, the system efficiency of DMU k is calculated by the following model generalized of model (1).

$$E_k^{K-NDEA} = \max \sum_{r=1}^{s} u_r y_{rk} \tag{4}$$

$$\begin{aligned}
s.t.\ & \sum_{i=1}^{m} v_i x_{ik} = 1, && \mathrm{i}=1,\dots,m \\
& \sum_{d=1}^{D} w_d^1 z_{pj}^1 - \sum_{i=1}^{m} v_i x_{ij} \le 0, && j=1,\dots,n \\
& \sum_{d=1}^{D} w_d^t z_{dj}^t - \sum_{d=1}^{D} w_d^{t-1} z_{dj}^{t-1} \le 0, && \begin{cases} j=1,\dots,n \\ t=2,\dots,h-1 \end{cases} \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d^{h-1} z_{dj}^{h-1} \le 0, && j=1,\dots,n \\
& u_r \ge 0, v_i \ge 0, \quad r=1,\dots,s, \quad i=1,\dots,m \\
& w_d^t \ge 0, && d=1,\dots,D,\ t=1,\dots,h-1.
\end{aligned}$$

Dual of the above model can be represented as follow.

$$E_k^{dK-NDEA} = \min \theta \tag{5}$$

$$\begin{aligned}
\text{s.t.} & \sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{ik}, && i=1,\dots,m \\
& \sum_{j=1}^{n} \left(\lambda_j^t - \lambda_j^{t+1}\right) z_{dj}^t \ge 0, && \begin{cases} d=1,\dots,D, \\ t=1,\dots,h-1 \end{cases} \\
& \sum_{j=1}^{n} \lambda_j^h y_{rj} \ge y_{rk}, && r=1,\dots,s \\
& \lambda_j^t \ge 0,\ j=1,\dots,n,\ \ t=1,\dots,h-1.
\end{aligned}$$

Azizi and Kazemi Matin (2010) developed two-stage model of Chen et al (2010) to a multi-stage network model as follow [17].

$$E^{Ch-NDEA} = \min \theta \quad (6)$$

$$\text{s.t. :}\sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{ik},\ i = 1,\ldots,m$$

$$\sum_{j=1}^{n} \lambda_j^t z_{dj}^t \ge \overline{z}_{dk}^t,\ \ \mathrm{d} = 1,\ldots,D,\ t = 1,\ldots,h-1$$

$$\sum_{j=1}^{n} \lambda_j^{t+1} z_{dj}^t \le \overline{z}_{dk}^t, \mathrm{d} = 1,\ldots,D,\ t = 1,\ldots,h-1$$

$$\sum_{j=1}^{n} \lambda_j^h y_{rj} \ge y_{rk},\ \mathrm{r} = 1,\ldots,s$$

$$\lambda_j^t \ge 0, \qquad j = 1,\ldots,n,\ \mathrm{t} = 1,\ldots,h-1$$

$$\overline{\mathrm{z}}_{\mathrm{dk}}^{\mathrm{t}} \ge 0, \qquad d = 1,\ldots,D, t = 1,\ldots,h-1.$$

2.3 Robust DEA with Soyster formulation

Soyster (1973) consider the following nominal linear optimal problem [14]:

$$\max\ cx \quad (7)$$

s.t.

$$\tilde{A}x \le b$$

$$l \le x \le u.$$

In above formulation, assume that only elements of matrix $\tilde{A} = \left[\tilde{a}_{ij}\right]$ are uncertain. Without losing the generality, suppose object function c don't be uncertain while we can use maximize object z, and add constraint $z - cx \le 0$ and so included this constraint in $\tilde{A}x \le b$ [18].

He introduced the $m \times m$ matrix $\overline{A}$ as $\overline{A} = \left(\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n\right)$. For each *j*, he defined the column $\overline{a}_j$ whose i^{th} component is equal to $\delta^*\left(e_i \middle| K_j\right) = \sup_{a_j \in K_j} a_{ij}$. The auxiliary linear programming to (7) will be denoted $LP\left(\overline{A}\right)$ as follows:

$$LP\left(\overline{A}\right): \max cx \quad (8)$$

s.t.

$$\overline{A}x \le b$$

$$x \ge 0.$$

Soyster showed that the optimal solution to $LP\left(\overline{A}\right)$ is also the optimal solution to (7).

2.4 DEA Counterpart Based on Soyster Approach

Model (1) with uncertain outputs, $\tilde{y}_{rj} = y_{rj} + \eta_{rj}^y \hat{y}_{rj}$, $\hat{y}_{rj} \ge 0, \eta_{rj}^y \in [-1,1]$, will be as follow:

$$E_k = \max\ w$$

$$\text{s.t.:}\sum_{i=1}^{m} v_i x_{ik} = 1$$

$$w - \sum_{r=1}^{s} u_r y_{rk} - \sum_{r=1}^{s} u_r \eta_{rk}^y \hat{y}_{rk} \le 0, \forall \eta_{rk}^y \in [-1,1]$$

$$\sum_{r=1}^{s} u_r y_{rj} + \sum_{r=1}^{s} u_r \eta_{rj}^y \hat{y}_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0,$$

$$j = 1,\ldots,n, \forall \eta_{rj}^y \in [-1,1]$$

$$v_r, u_i \ge 0, \quad i = 1,\ldots,m, \quad \mathrm{r} = 1,\ldots,s$$

By using the Soyster robustness method,

$$E_k = \max\ w$$

$$\text{s.t.: }\sum_{i=1}^{m} v_i x_{ik} = 1$$

$$w - \sum_{r=1}^{s} u_r y_{rk} + \sup_{\eta_{rk}^y \in [-1,1]} \left(-\sum_{r=1}^{s} u_r \eta_{rk}^y \hat{y}_{rk} \right) \le 0,$$

$$\begin{cases} \sum_{r=1}^{s} u_r y_{rj} + \sup_{\eta_{rk}^y \in [-1,1]} \left(\sum_{r=1}^{s} u_r \eta_{rj}^y \hat{y}_{rj} \right) \\ -\sum_{i=1}^{m} v_i x_{ij} \le 0,\ \ j = 1,\ldots,n \end{cases}$$

$$v_r, u_i \ge 0, \quad i = 1,\ldots,m, \quad \mathrm{r} = 1,\ldots,s$$

In the first constraint we have $\sup\limits_{\eta_{rk}^{y}\in[-1,1]}\left(-\sum_{r=1}^{s}u_r\eta_{rk}^{y}\hat{y}_{rk}\right)=-\min\limits_{\eta_{rk}^{y}\in[-1,1]}\left(\sum_{r=1}^{s}u_r\eta_{rk}^{y}\hat{y}_{rk}\right)$.

Since $y_{rk}\geq 0,\hat{y}_{rk}\geq 0$ and $\eta_{rk}^{y}\in[-1,1]$, then $\min\limits_{\eta_{rk}^{y}\in[-1,1]}\left(\sum_{r=1}^{s}u_r\eta_{rk}^{y}\hat{y}_{rk}\right)=-\sum_{r=1}^{s}u_r\hat{y}_{rk}$.

So, its robust counterpart constraint is $E_k-\sum_{r=1}^{s}u_r y_{rk}+\sum_{r=1}^{s}u_r\hat{y}_{rk}\leq 0$. For the same reason in the second constraint, the robust counterpart constraint be obtained as $\sum_{r=1}^{s}u_r y_{rk}+\sum_{r=1}^{s}u_r\hat{y}_{rk}-\sum_{i=1}^{m}v_i x_{ij}\leq 0$.

As a result, the Soyster Robust DEA model (SRDEA) with uncertainty in outputs is as follows:

$$
\begin{aligned}
&\max\ E_k\\
&\text{s.t.: }\sum_{i=1}^{m}v_i x_{ik}=1\\
&E_k-\sum_{r=1}^{s}u_r y_{rk}+\sum_{r=1}^{s}u_r\hat{y}_{rk}\leq 0\\
&\sum_{r=1}^{s}u_r y_{rj}+\sum_{r=1}^{s}u_r\hat{y}_{rk}-\sum_{i=1}^{m}v_i x_{ij}\leq 0,\ j=1,\dots,n\\
&v_r,u_i\geq 0\qquad i=1,\dots,m\qquad r=1,\dots,s
\end{aligned}
$$

3. Robust Network DEA (RNDEA)

We assume that the input data, x_{ij}, are a definite value, but the values of intermediate products, z_{ij}^{t}, and finished products, y_{ij}, are subject to uncertainty data, that's mean

$$
\begin{aligned}
&\tilde{y}_{rj}=y_{rj}+\eta_{rj}^{y}\hat{y}_{rj},\ \hat{y}_{rj}\geq 0,\eta_{rj}^{y}\in[-1,1]\\
&\tilde{z}_{dj}^{t}=z_{dj}^{t}+\eta_{dj}^{zt}\hat{z}_{dj}^{t},\ \ \hat{z}_{dj}\geq 0,\eta_{dj}^{z}\in[-1,1]\\
&r=1,\dots,s,\ \ j=1,\dots,n,\qquad t=1,\dots,h
\end{aligned}
$$

One should note that $\hat{y}_{rj}$ and $\hat{z}_{dj}^{t}$ are a percentage of y_{rj} and z_{dj}^{t}, respectively. Then we have $\hat{y}_{rj}<y_{rj}$ and $\hat{z}_{dj}^{t}<z_{dj}^{t}$, so $.t=1,\dots,h-1\ \tilde{z}_{dj}^{t}>0,$ and $\tilde{y}_{rj}>0$

In the following, at the first, we produce Robust Network DEA models for two-stage systems and then develop those to multi-stage systems.

3.1 Two-stage robust network DEA model

Consider two-stage system (Figure 1) where $x_{ij}, i=1,\dots,m$, and y_{rj}, $r=1,\dots,s$ are inputs and final outputs and $z_{dj}^{1}, d=1,\dots,D$, as intermediate products, are the output value of the first stage and input values of second stage for DMU_j, $j=1,\dots,n$.

3.1.1 Two-stage robust model based on Kao and Hwang (model (1))

In this case, by replacing assumptions (*) in model (1), it's uncertain two-stage DEA model is as following:

$$
\begin{aligned}
&\max E_k \qquad\qquad (9)\\
&\text{s.t.}\quad \sum_{i=1}^{m}v_i x_{ik}=1\\
&E_k-\sum_{r=1}^{s}u_r y_{rj}-\sum_{r=1}^{s}u_r\eta_{rj}^{y}\hat{y}_{rj}\leq 0\\
&\sum_{d=1}^{D}w_d^{1}z_{dj}^{1}+\sum_{d=1}^{D}w_d^{1}\eta_{dj}^{z}\hat{z}_{dj}^{1}-\sum_{i=1}^{m}v_i x_{ij}\leq 0,\quad j=1,\dots,n\\
&\sum_{r=1}^{s}u_r y_{rj}+\sum_{r=1}^{s}u_r\eta_{rj}^{y}\hat{y}_{rj}-\sum_{d=1}^{D}w_d^{1}z_{dj}^{1}-\sum_{d=1}^{D}w_d^{1}\eta_{dj}^{z}\hat{z}_{dj}^{1}\leq 0,\\
&\qquad j=1,\dots,n\\
&u_r,v_i,w_d^{1}\geq 0,\ \ \mathrm{r}=1,\dots,s,\ \ i=1,\dots,m,\ \ d=1,\dots,D.
\end{aligned}
$$

As the previous section, the counterpart relation for second constraint is equal to $E_k-\sum_{r=1}^{s}u_r y_{rj}+\sum_{r=1}^{s}u_r\hat{y}_{rj}\leq 0$. For the same reason, the robust counterpart constraint for third constraint is equal to $\sum_{d=1}^{D}w_d^{1}z_{dj}^{1}+\sum_{d=1}^{D}w_d^{1}\hat{z}_{dj}^{1}-\sum_{i=1}^{m}v_i x_{ij}\leq 0$, and as a result the robust counterpart DEA model for model (9), Based on Soyster approach, is as follows:

$$\max E_k \tag{10}$$
$$\text{s.t.} \quad \sum_{i=1}^{m} v_i x_{ik} = 1$$
$$E_k - \sum_{r=1}^{s} u_r y_{rk} + \sum_{r=1}^{s} u_r \hat{y}_{rk} \le 0$$
$$\sum_{d=1}^{D} w_d^1 z_{dj}^1 + \sum_{d=1}^{D} w_d^1 \hat{z}_{dj}^1 - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1,\dots,n$$
$$\sum_{r=1}^{s} u_r y_{rj} + \sum_{r=1}^{s} u_r \hat{y}_{rj} - \sum_{d=1}^{D} w_d^1 z_{dj}^1 + \sum_{d=1}^{D} w_d^1 \hat{z}_{dj}^1 \le 0, \quad j = 1,\dots,n$$
$$u_r, v_i, w_d^1 \ge 0, \text{r} = 1,\dots,s, \; i = 1,\dots,m, d = 1,\dots,D$$

3.1.2 Two-stage robust model based on dual of Kao and Hwang (model (2))

According model (2), with replacement uncertain data presented in (*), uncertain model is obtained as follows:

$$E_k = \min \theta \tag{11}$$
$$s.t. \quad \sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{ik}, \qquad i = 1,\dots,m$$
$$\sum_{j=1}^{n} \left(\lambda_j^1 - \lambda_j^2\right)\left(z_{dj}^1 + \eta_{dj}^z \hat{z}_{dj}^1\right) \ge 0, \qquad \text{d} = 1,\dots,D$$
$$\sum_{j=1}^{n} \lambda_j^2 \left(y_{rj} + \eta_{rj}^y \hat{y}_{rj}\right) \ge \left(y_{rk} + \eta_{rk}^y \hat{y}_{rk}\right), \text{r} = 1,\dots,s$$
$$\lambda_j^t \ge 0 \qquad \text{t=1,2}, \qquad j = 1,\dots,n$$

By the Soyster method, the robust counterpart model could write as follow.

$$E_k = \min \theta$$
$$s.t. \quad \sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{ik}, \qquad i = 1,\dots,m$$
$$\begin{cases} \sum_{j=1}^{n} \left(\lambda_j^1 - \lambda_j^2\right) z_{dj}^1 \ge \sup_{\eta_{dj}^z \in [-1,1]} \left(-\sum_{j=1}^{n} \lambda_j^1 \eta_{dj}^z \hat{z}_{dj}^1 \right) \\ + \sup_{\eta_{dj}^z \in [-1,1]} \left(\sum_{j=1}^{n} \lambda_j^2 \eta_{dj}^z \hat{z}_{dj}^1 \right), \; \text{d} = 1,\dots,D \end{cases}$$
$$\begin{cases} \sum_{j=1}^{n} \lambda_j^2 y_{rj} \ge y_{rk} + \sup_{\eta_{rk}^y \in [-1,1]} \left(\eta_{rk}^y \hat{y}_{rk} \right) \\ + \sup_{\eta_{rj}^y \in [-1,1]} \left(-\sum_{j=1}^{n} \lambda_j^2 \eta_{rj}^y \hat{y}_{rj} \right), \text{r} = 1,\dots,s \end{cases}$$
$$\lambda_j^t \ge 0 \qquad \text{t=1,2}, \; j = 1,\dots,n$$

In second constraint, with the assumption that $\lambda_j^t \ge 0, \hat{z}_{dj}^1 \ge 0$,

$\sup_{\eta_{dj}^z \in [-1,1]} \left(-\sum_{j=1}^{n} \lambda_j^1 \eta_{dj}^z \hat{z}_{dj}^1 \right) = \sum_{j=1}^{n} \lambda_j^1 \hat{z}_{dj}^1$ and $\sup_{\eta_{dj}^z \in [-1,1]} \left(\sum_{j=1}^{n} \lambda_j^2 \eta_{dj}^z \hat{z}_{dj}^1 \right) = \sum_{j=1}^{n} \lambda_j^2 \hat{z}_{dj}^1$, and so

$$\sum_{j=1}^{n} \left(\lambda_j^1 - \lambda_j^2\right) z_{dj}^1 \ge \sum_{j=1}^{n} \lambda_j^1 \hat{z}_{dj}^1 + \sum_{j=1}^{n} \lambda_j^2 \hat{z}_{dj}^1$$

or

$$\sum_{j=1}^{n} \left(\lambda_j^1 - \lambda_j^2\right) z_{dj}^1 - \sum_{j=1}^{n} \left(\lambda_j^1 + \lambda_j^2\right) \hat{z}_{dj}^1 \ge 0 .$$

Similarly, in third constraint, $\sup_{\eta_{rj}^y \in [-1,1]} \left(-\sum_{j=1}^{n} \lambda_j^2 \eta_{rj}^y \hat{y}_{rj} \right) = \sum_{j=1}^{n} \lambda_j^2 \hat{y}_{rj}$ we have $\sum_{j=1}^{n} \lambda_j^2 y_{rj} \ge y_{rk} + \hat{y}_{rk} + \sum_{j=1}^{n} \lambda_j^2 \hat{y}_{rj}$.

Then, the counterpart robust linear model of model (11) will be as follows:

$$E_k = \min \theta \qquad (12)$$
$$s.t. \quad \sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{ik}, \qquad i = 1,\dots,m$$
$$\sum_{j=1}^{n} \left(\lambda_j^1 - \lambda_j^2\right)\left(z_{dj}^1 - \hat{z}_{dj}^1\right) \ge 0 \qquad d = 1,\dots,D$$
$$\sum_{j=1}^{n} \lambda_j^2 y_{rj} - \sum_{j=1}^{n} \lambda_j^2 \hat{y}_{rj} \ge y_{rk} + \hat{y}_{rk}, r = 1,\dots,s$$
$$\lambda_j^t \ge 0, \qquad \text{t=1,2}, \qquad j = 1,\dots,n$$

3.1.3 Two-stage robust model based on Chen et al. (model (3))

As one can see, in model (3), by replacing the given uncertain values of problem and applying Soyster method, the robust model of uncertain two-stage model can be written as follows:

$$E_k = \min \theta \qquad (13)$$
$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{ik}, \qquad i = 1,\dots,m$$
$$\sum_{j=1}^{n} \lambda_j^1 z_{dj}^1 \ge \overline{z}_{dk}^1 + \sup_{\eta_{dj}^z \in [-1,1]} \left(-\sum_{j=1}^{n} \lambda_j^1 \eta_{dj}^z \hat{z}_{dj}^1 \right), \ \mathrm{d} = 1,\dots,D$$
$$\sum_{j=1}^{n} \lambda_j^2 z_{dj}^1 + \sup_{\eta_{dj}^z \in [-1,1]} \left(\sum_{j=1}^{n} \lambda_j^2 \eta_{dj}^z \hat{z}_{dj}^1 \right) \le \overline{z}_{dk}^1, \ \mathrm{d} = 1,\dots,D$$
$$\left\{ \begin{array}{l} \sum_{j=1}^{n} \lambda_j^2 y_{rj} \ge y_{rk} + \sup_{\eta_{rk}^y \in [-1,1]} \eta_{rk}^y \hat{y}_{rk} \\ + \sup_{\eta_{rj}^y \in [-1,1]} \left(-\sum_{j=1}^{n} \lambda_j^2 \eta_{rj}^y \hat{y}_{rj} \right), \end{array} \right. \quad ,\mathrm{r} = 1,\dots,s$$
$$\lambda_j^t, \overline{z}_{dk}^1 \ge 0, \quad j = 1,\dots,n, \ \ \mathrm{t} = 1,2, \quad d = 1,\dots,D.$$

For second constraint, because of $\lambda^1 \ge 0, \hat{z}_{dj}^1 \ge 0$, we have

$$\sup_{\eta_{dj}^z \in [-1,1]} \left(-\sum_{j=1}^{n} \lambda_j^1 \eta_{dj}^z \hat{z}_{dj}^1 \right) = \max_{\eta_{dj}^z \in [-1,1]} \left(-\sum_{j=1}^{n} \lambda_j^1 \eta_{dj}^z \hat{z}_{dj}^1 \right) = \sum_{j=1}^{n} \lambda_{j\,dj}^{1z} \hat{z}_{dj}^1$$

and so $\sum_{j=1}^{n} \lambda_j^1 z_{dj}^1 - \sum_{j=1}^{n} \lambda_j^1 \hat{z}_{dj}^1 \ge \overline{z}_{dk}^1$.

In third constraint, for the same reason $\sup_{\eta_{dj}^z \in [-1,1]} \left(\sum_{j=1}^{n} \lambda_j^2 \eta_{dj}^z \hat{z}_{dj}^1 \right) = \sum_{j=1}^{n} \lambda_{j\,dj}^{2z} \hat{z}_{dj}^1$.
Then its robust counterpart constraint is equal to $\sum_{j=1}^{n} \lambda_j^2 z_{dj}^1 + \sum_{j=1}^{n} \lambda_j^2 \hat{z}_{dj}^1 \le \overline{z}_{dk}^1$.

In a similar way of previous model, the counterpart robust model based Soyster approach for model (13) is obtained as follows.

$$E_k = \min \theta \qquad (14)$$
$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{ik}, \qquad i = 1,\dots,m$$
$$\sum_{j=1}^{n} \lambda_j^1 z_{dj}^1 - \sum_{j=1}^{n} \lambda_j^1 \hat{z}_{dj}^1 \ge \overline{z}_{dk}^1, \qquad \mathrm{d} = 1,\dots,D$$
$$\sum_{j=1}^{n} \lambda_j^2 z_{dj}^1 + \sum_{j=1}^{n} \lambda_j^2 \hat{z}_{dj}^1 \le \overline{z}_{dk}^1, \qquad \mathrm{d} = 1,\dots,D$$
$$\sum_{j=1}^{n} \lambda_j^2 y_{rj} - \sum_{j=1}^{n} \lambda_j^2 \hat{y}_{rj} \ge y_{rk} + \hat{y}_{rk}, \ \mathrm{r} = 1,\dots,s$$
$$\lambda_j^t, \overline{z}_{dk}^1 \ge 0, \ \mathrm{t} = 1,2, \ d = 1,\dots,D, \ j = 1,\dots,n.$$

3.2 Multi-stage robust network DEA model

Consider the *h*-stage series system as depicted in Fig.2. Denote x_{ij} as the i^{th} input to the first stage. Here, at each stage $t \ (t = 1,\dots,h-1)$, z_{dj}^t, intermediate products, indicates the values of output of *t* stage that becomes input to (*t*+1) stage. Also y_{rj} represents output of final stage. In fact, y_{rj} is considered as final product.

3.2.1 Multi-stage robust model based on Kao and Hwang model (model (4))

On the base of relations (*) and the Soyster method by using the methods mentioned in the previous subsection, non-linear robust model of model (4) is the following:

$$\max\ E_k \qquad (15)$$
$$\text{s.t.}\quad \sum_{i=1}^{m} v_i x_{ik} = 1,$$
$$E_k + \sum_{r=1}^{s} u_r y_{rk} + \sup_{\eta_{rk}^{y}\in[-1,1]}\left(-\sum_{r=1}^{s} u_r \eta_{rk}^{y}\hat{y}_{rk}\right) \le 0,$$
$$\begin{cases}\sum_{d=1}^{D} w_d^1 z_{dj}^1 + \sup_{\eta_{rk}^{y}\in[-1,1]}\left(\sum_{d=1}^{D} w_d^1 \eta_{dj}^{1z}\hat{z}_{dj}^1\right) \\ -\sum_{i=1}^{m} v_i x_{ij} \le 0,\end{cases}, j = 1,\dots,n$$
$$\begin{cases}\sum_{d=1}^{D} w_d^t z_{dj}^t + \sup_{\eta_{dj}^{tz}\in[-1,1]}\left(\sum_{d=1}^{D} w_d^t \eta_{dj}^{tz}\hat{z}_{dj}^t\right) \\ -\sum_{d=1}^{D} w_d^{t-1} z_{dj}^{t-1} + \sup_{\eta_{dj}^{tz}\in[-1,1]}\left(-\sum_{d=1}^{D} w_d^{t-1}\eta_{dj}^{(t-1)z}\hat{z}_{dj}^{t-1}\right) \le 0 \\ j = 1,\dots,n, t = 2,\dots,h-1\end{cases}$$
$$\sum_{r=1}^{s} u_r y_{rj} + \sup_{\eta_{rj}^{y}\in[-1,1]}\left(\sum_{r=1}^{s} u_r \eta_{rj}^{y}\hat{y}_{rj}\right) - \sum_{d=1}^{D} w_d^{h-1} z_{dj}^{h-1}$$
$$\sup_{\eta_{dj}^{(h-1)z}\in[-1,1]}\left(-\sum_{d=1}^{D} w_d^{h-1}\eta_{dj}^{(h-1)z}\hat{z}_{dj}^{h-1}\right) \le 0,\ \ j = 1,\dots,n$$
$$u_r, v_i \ge 0, \qquad \mathrm{r} = 1,\dots,s, \qquad i = 1,\dots,m$$
$$w_d^t \ge 0, \qquad d = 1,\dots,D, \quad t = 1,\dots,h-1$$

By implying a method similar to the method used in robust two-stage model, robust counterpart model of model (15) is written as follows:

$$E^{K-RNDEA} = \max E_k \qquad (16)$$
$$\text{s.t.}\quad \sum_{i=1}^{m} v_i x_{ik} = 1,$$
$$E_k - \sum_{r=1}^{s} u_r y_{rk} + \sum_{r=1}^{s} u_r \hat{y}_{rk} \le 0,$$
$$\begin{cases}\sum_{d=1}^{D} w_d^1 z_{dj}^1 + \sum_{d=1}^{D} w_d^1 \hat{z}_{dj}^1 \\ -\sum_{i=1}^{m} v_i x_{ij} \le 0,\ j = 1,\dots,n\end{cases}$$
$$\begin{cases}\sum_{d=1}^{D} w_d^t z_{dj}^t + \sum_{d=1}^{D} w_d^t \hat{z}_{dj}^t - \sum_{d=1}^{D} w_d^{t-1} z_{dj}^{t-1} \\ +\sum_{d=1}^{D} w_d^{t-1}\hat{z}_{dj}^{t-1} \le 0,\ j = 1,\dots,n, t = 2,\dots h-1\end{cases}$$
$$\begin{cases}\sum_{r=1}^{s} u_r y_{rj} + \sum_{r=1}^{s} u_r \hat{y}_{rj} - \sum_{d=1}^{D} w_d^{h-1} z_{dj}^{h-1} \\ \sum_{d=1}^{D} w_d^{h-1}\hat{z}_{dj}^{h-1} \le 0,\ j = 1,\dots,n\end{cases}$$
$$u_r, v_i \ge 0, \qquad \mathrm{r} = 1,\dots,s, \qquad i = 1,\dots,m$$
$$w_d^t \ge 0, \qquad d = 1,\dots,D, \qquad t = 1,\dots,h.$$

3.2.2 Multi-stage robust model based on dual of Kao and Hwang model (model (5))

According model (5), replacing uncertain data equations in (*) and with Soyster assumptions the uncertain model is obtained as follows.

$$E_k = \min\theta \qquad (17)$$
$$\text{s.t.}\quad \sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{ik}, \qquad i = 1,\dots,m$$
$$\begin{cases}\sum_{j=1}^{n}\left(\lambda_j^t - \lambda_j^{t+1}\right) z_{dj}^t + \sup_{\eta_{dj}^{tz}\in[-1,1]}\left(\sum_{j=1}^{n}\lambda_j^t \eta_{dj}^{tz}\hat{z}_{dj}^t\right) \\ \ge \sup_{\eta_{dj}^{tz}\in[-1,1]}\left(\sum_{j=1}^{n}\lambda_j^{t+1}\eta_{dj}^{tz}\hat{z}_{dj}^t\right),\end{cases}$$
$$\mathrm{d} = 1,\dots,D, t = 2,\dots,h-1$$
$$\begin{cases}\sum_{j=1}^{n}\lambda_j^h y_{rj} \ge y_{rk} + \sup_{\eta_{rk}^{y}\in[-1,1]}\left(\eta_{rk}^{y}\hat{y}_{rk}\right) \\ + \sup_{\eta_{dj}^{tz}\in[-1,1]}\left(-\sum_{j=1}^{n}\lambda_j^h \eta_{rj}^{y}\hat{y}_{rj}\right)\end{cases}, \mathrm{r} = 1,\dots,s$$
$$\lambda_j^t \ge 0,\ \ j = 1,\dots,n,\ \ t = 2,\dots,h.$$

Similar to method used in model (12), counterpart robust model will be obtained as follows

$$E^{dK-RNDEA} = \min\theta \qquad (18)$$

$$\text{s.t.}\sum_{j=1}^{n}\lambda_j^1 x_{ij} \le \theta x_{ik}, \qquad i=1,\dots,m$$

$$\begin{cases}\sum_{j=1}^{n}\left(\lambda_j^t-\lambda_j^{t+1}\right)\left(z_{dj}^t-\hat{z}_{dj}^t\right)\ge 0\\ \mathrm{d}=1,\dots,D \quad , \quad t=2,\dots,h-1\end{cases}$$

$$\sum_{j=1}^{n}\lambda_j^h\left(y_{rj}-\hat{y}_{rj}\right)\ge y_{rk}+\hat{y}_{rk}, \ \mathrm{r}=1,\dots,s$$

$$\lambda_j^t \ge 0, \qquad j=1,\dots,n, \qquad t=2,\dots,h.$$

3.2.3 Multi-stage robust model based on Chen et al. model (model (6))

In the similar way, by substituting given uncertain variables in (*) and based on Soyster method, the model (6) would be as follow:

$$E_k = \min\theta \qquad (19)$$

$$\text{s.t.} \quad \sum_{j=1}^{n}\lambda_j^1 x_{ij} \le \theta x_{ik}, \quad i=1,\dots,m$$

$$\begin{cases}\sum_{j=1}^{n}\lambda_j^t z_{dj}^t \ge \overline{z}_{dk}^t\\ +\sup_{\eta_{dj}^{zt}\in[-1,1]}\left(-\sum_{j=1}^{n}\lambda_j^t\eta_{dj}^{zt}\hat{z}_{dj}^t\right)\end{cases}, \boxed{\begin{matrix}\mathrm{d}=1,\dots,D\\ t=1,\dots,h-1\end{matrix}}$$

$$\begin{cases}\sum_{j=1}^{n}\lambda_j^{t+1}z_{dj}^t+\sup_{\eta_{dj}^{zt}\in[-1,1]}\left(\sum_{j=1}^{n}\lambda_j^{t+1}\eta_{dj}^{zt}\hat{z}_{dj}^t\right)\le\overline{z}_{dk}^t\\ \mathrm{d}=1,\dots,D, t=1,\dots,h-1\end{cases}$$

$$\begin{cases}\sum_{j=1}^{n}\lambda_j^h y_{rj}\ge y_{rk}+\sup_{\eta_{rk}^y\in[-1,1]}\left(\eta_{rk}^y\hat{y}_{rk}\right)\\ +\sup_{\eta_{dj}^{tz}\in[-1,1]}\left(-\sum_{j=1}^{n}\lambda_j^h\eta_{rj}^y\hat{y}_{rj}\right), \mathrm{r}=1,\dots,s\end{cases}$$

$$\lambda_j^t, \overline{z}_{dk}^t \ge 0, \quad j=1,\dots,n,$$
$$d=1,\dots,D, \ t=1,\dots,h.$$

With a similar way used in model (14), robust model will be obtained as follows:

$$E^{Ch-RNDEA} = \min\theta \qquad (20)$$

$$\text{s.t.}\sum_{j=1}^{n}\lambda_j^1 x_{ij} \le \theta x_{ik}, \qquad i=1,\dots,m$$

$$\begin{cases}\sum_{j=1}^{n}\lambda_j^t\left(z_{dj}^t-\hat{z}_{dj}^t\right)\ge\overline{z}_{dk}^t,\\ \mathrm{d}=1,\dots,D, t=1,\dots,h-1\end{cases}$$

$$\begin{cases}\sum_{j=1}^{n}\lambda_j^{t+1}\left(z_{dj}^t+\hat{z}_{dj}^t\right)\le\overline{z}_{dk}^t,\\ \mathrm{d}=1,\dots,D, t=1,\dots,h-1\end{cases}$$

$$\sum_{j=1}^{n}\lambda_j^h\left(y_{rj}-\hat{y}_{rj}\right)\ge y_{rk}+\hat{y}_{rk}, \ \mathrm{r}=1,\dots,s$$

$$\lambda_j^t, \overline{z}_{dk}^t \ge 0, \ j=1,\dots,n, d=1,\dots,D, t=1,\dots,h.$$

4. The feasibility of the counterpart model and their equivalence

In this section, first, we prove that robust counterpart models are equivalent with their uncertain Kao and Hwang multi-stage model. For this purpose, we prove that model (15) and model (16) are equivalent. Next, we show that models are feasible. For this we define an interval efficiency score for them.

4.1 Equivalence of models

Let u_r^0, v_i^0 and w_d^{t0} for $r=1,\dots,s$, $i=1,\dots m$, $d=1,\dots,D$ and $t=1,\dots h$ be arbitrary feasible solution for uncertain model (15). Now we have

$$E_k-\sum_{r=1}^{s}u_r^0\left(y_{rk}+\eta_{rk}^y\hat{y}_{rk}\right)\le 0 \qquad \text{or}$$

$$E_k-\sum_{r=1}^{s}u_r^0 y_{rk}-\sum_{r=1}^{s}u_r^0\eta_{rk}^y\hat{y}_{rk}\le 0 \qquad \text{for}$$

each $\eta_{rk}^y\in[-1,1]$, then for $\eta_{rk}^y=-1$, we will have

$$E_k-\sum_{r=1}^{s}u_r^0 y_{rk}-\sum_{r=1}^{s}u_r^0\eta_{rk}^y\hat{y}_{rk}$$
$$=E_k-\sum_{r=1}^{s}u_r^0 y_{rk}+\sum_{r=1}^{s}u_r^0\hat{y}_{rk}.$$

And so $E_k-\sum_{r=1}^{s}u_r^0 y_{rk}+\sum_{r=1}^{s}u_r^0\hat{y}_{rk}\le 0$.

For the third constraint, let be $\eta_{dj}^{1z}=1$, then $\sum_{d=1}^{D}w_d^{10}z_{dj}^1+\sum_{d=1}^{D}w_d^{10}\hat{z}_{dj}^1-\sum_{i=1}^{m}v_i x_{ij}\le 0$.

At the same way, for the fourth constraint

$$\begin{cases}\sum_{d=1}^{D}w_d^{t0}z_{dj}^t+\sum_{d=1}^{D}w_d^{t0}\eta_{dj}^{tz}\hat{z}_{dj}^t-\sum_{d=1}^{D}w_d^{(t-1)0}z_{dj}^{t-1}\\ -\sum_{d=1}^{D}w_d^{(t-1)0}\eta_{dj}^{(t-1)z}\hat{z}^{t-1}\le 0.\end{cases}$$

Suppose $\eta_{dj}^{tz}=1,\eta_{dj}^{(t-1)z}=-1$, so

$$\sum_{d=1}^{D}w_d^{t0}z_{dj}^t+\sum_{d=1}^{D}w_d^{t0}\hat{z}_{dj}^t-\sum_{d=1}^{D}w_d^{(t-1)0}z_{dj}^{t-1}$$
$$+\sum_{d=1}^{D}w_d^{(t-1)0}\hat{z}^{t-1}\le 0.$$

In the final constraint, assuming that $\eta_{rj}^y=1,\eta_{dj}^{(h-1)z}=-1$, so

$$\sum_{r=1}^{s}u_r y_{rj}+\sum_{r=1}^{s}u_r\hat{y}_{rj}-\sum_{d=1}^{D}w_d^{(h-1)0}z_{dj}^{h-1}$$
$$+\sum_{d=1}^{D}w_d^{(h-1)0}\hat{z}^{h-1}\le 0.$$

Then u_r^0,v_i^0 and w_d^{t0} are feasible solutions for counterpart model (16).

On the other hand, suppose u_r^0,v_i^0 and w_d^{t0} for $r=1,\ldots,s$, $i=1,\ldots m$, $d=1,\ldots,D$ and $t=1,\ldots h$ be arbitrary feasible solution for counterpart model (16). In second constraint, $E_k-\sum_{r=1}^{s}u_r^0\left(y_{rk}-\hat{y}_{rk}\right)\le 0$, because of $-1\le\eta_{rk}^y$, $\left(y_{rk}-\hat{y}_{rk}\right)\le\left(y_{rk}+\eta_{rk}^y\hat{y}_{rk}\right)$ and then $E_k-\sum_{r=1}^{s}u_r^0\left(y_{rk}+\eta_{rk}^y\hat{y}_{rk}\right)\le E_k-\sum_{r=1}^{s}u_r^0\left(y_{rk}-\hat{y}_{rk}\right)\le 0.$

In third constraint, we have $\sum_{d=1}^{D}w_d^{10}z_{dj}^1+\sum_{d=1}^{D}w_d^{10}\hat{z}_{dj}^1-\sum_{i=1}^{m}v_i x_{ij}\le 0$. Since $\eta_{dj}^{1z}\le 1$, then $\left(z_{dj}^1+\eta_{dj}^{1z}\hat{z}_{dj}^1\right)\le\left(z_{dj}^1+\hat{z}_{dj}^1\right)$ and so $\sum_{d=1}^{D}w_d^{10}z_{dj}^1+\sum_{d=1}^{D}w_d^{10}\eta_{dj}^1\hat{z}_{dj}^1-\sum_{i=1}^{m}v_i x_{ij}\le 0$ or $\sum_{d=1}^{D}w_d^{10}\left(z_{dj}+\eta_{dj}^{1z}\hat{z}_{dj}\right)-\sum_{i=1}^{m}v_i x_{ij}\le 0$.

In the next constraint $\sum_{d=1}^{D}w_d^{t0}\left(z_{dj}^t+\hat{z}_{dj}^t\right)-\sum_{d=1}^{D}w_d^{(t-1)0}\left(z_{dj}^{t-1}-\hat{z}_{dj}^{t-1}\right)\le 0,$ while $\eta_{dj}^{tz}\le 1$ and $-1\le\eta_{dj}^{(t-1)z}$, so $\left(z_{dj}^t+\eta_{dj}^{tz}\hat{z}_{dj}^t\right)\le\left(z_{dj}^t+\hat{z}_{dj}^t\right)$ and $\left(z_{dj}^{t-1}-\hat{z}_{dj}^{t-1}\right)\le\left(z_{dj}^{t-1}+\eta_{dj}^{tz}\hat{z}_{dj}^{t-1}\right)$, as a result

$$\sum_{d=1}^{D}w_d^{t0}\left(z_{dj}^t+\eta_{dj}^{tz}\hat{z}_{dj}^t\right)$$
$$-\sum_{d=1}^{D}w_d^{(t-1)0}\left(z_{dj}^{t-1}+\eta_{dj}^{tz}\hat{z}_{dj}^{t-1}\right)\le 0.$$

Finally, like previous, one can see if $\sum_{d=1}^{D}u_r\left(y_{rj}+\hat{y}_{rj}\right)-\sum_{d=1}^{D}w_d^{(h-1)0}\left(z_{dj}^{h-1}-\hat{z}_{dj}^{t-1}\right)\le 0,$ because $\eta_{rk}^y\le 1$ and $-1\le\eta_{dj}^{(h-1)z}$, then $\sum_{d=1}^{D}u_r\left(y_{rj}+\eta_{rj}^y\hat{y}_{rj}\right)-\sum_{d=1}^{D}w_d^{(h-1)0}\left(z_{dj}^{h-1}+\eta_{dj}^{(h-1)z}\hat{z}_{dj}^{t-1}\right)\le 0.$

According to the previous section, one can see that u_r^0,v_i^0 and w_d^{t0} for $r=1,\ldots,s$, $i=1,\ldots m$, $d=1,\ldots,D$ and $t=1,\ldots h$ are feasible solution for uncertain model (15).

4.2 Interval Efficiency

In this research, we assume that output and intermediate values are unknown exactly but lie in bounded intervals, namely $\tilde{y}_{rj} = y_{rj} \pm \hat{y}_{rj}$ $(r = 1,...,s,\ j = 1,...,n)$ and $\tilde{z}_{dj} = z_{dj} \pm \tilde{z}_{dj}$ $(d = 1,...,D,\ j = 1,...,n)$ where the upper and lower bounds of intervals are positive constants. If some of the data are in other forms of imprecise data, e.g., ordinal data or ratio bounded data, we can use the technique in Zhu (2003) to convert them into interval bounded data firstly.

We note that under robust data, robust model of Kao and Hwang and robust model of dual of Kao and Hwang model are no longer equivalent and dual models. In fact, the efficiency score obtained from robust of Kao and Hwang model is always lesser than that from robust of dual of Kao and Hwang model. This is due to the fact that

$$\max_{\substack{\tilde{y}_{rj} \in [y_{rj} - \hat{y}_{rj},\, y_{rj} + \hat{y}_{rj}] \\ \tilde{z}^t_{dj} \in [z^t_{dj} - \hat{z}^t_{dj},\, z^t_{dj} + \hat{z}^t_{dj}]}} E \le \max_{\substack{y_{rj} \\ z_{dj}}} E \le \min_{\substack{y_{rj} \\ z_{dj}}} \theta \le \min_{\substack{\tilde{y}_{rj} \in [y_{rj} - \hat{y}_{rj},\, y_{rj} + \hat{y}_{rj}] \\ \tilde{z}^t_{dj} \in [z^t_{dj} - \hat{z}^t_{dj},\, z^t_{dj} + \hat{z}^t_{dj}]}} \theta$$

When imprecise data is considered in network DEA approach, the optimal efficiency scores become to interval data too, where the multiplier-type model yields the lower efficiency bound, and the envelopment-type models yields the upper efficiency bound.

5. Numerical Example

This section is provided to show the right approach and compare the results of models with a numerical example has used it. The proposed Robust NDEA models were coded using LINGO 14.0 software. The codes of proposed mathematical models were executed on a laptop with Core i7 and Windows 8.1.

5.1 Three stage DEA example

Khalili-damghani and taghavifard (2012) used a fuzzy three stage DEA model for evaluating performance efficiency for a set of 40 dairy supply chain [20]. We use the conceptual model (Figure 5) and data used in that research and convert the fuzzy data to robust data for run the models suggested in our research. The Upper and Lower bound of data presented in Table 1 and table2. We Consider $x_i = \left(x_i^L + x_i^U\right)/2$ and $\hat{x}_i = \left(x_i^U - x_i^L\right)/2$. The upper and lower bounds of data have been represented in Table 1 and Table 2, respectively.

At the first, the problem is solved without considering any uncertainty in data by a three-stage network DEA model based on K-NDEA model (model 4) and results illustrated in third column in Table 3. Running K-RNDEA model (model 16), DK-RNDEA model (model 18) and CH-RNDEA model (model 20) yielded lower and upper bounds of efficiency scores for all DMUs. The result has been represented in second, fourth and fifth columns in Table 3. In Figure 4 the result of K-NDEA model (blue line), K-RNDEA model (orange line), DK-RNDEA model (yellow line) and CH-RNDEA model (gray line) have been plotted. As one can see from Table 3 and Figure 4, K-RNDEA model create a lower bound and DK-RNDEA and CH-RNDEA models give upper bounds for K-NDEA model that the first is a better upper bound.

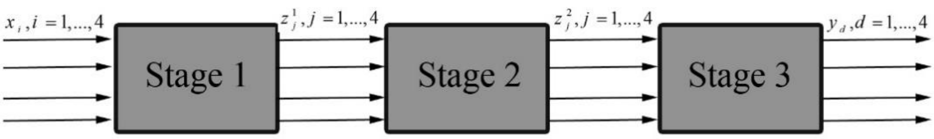

Figure 3. A three-stage Network System

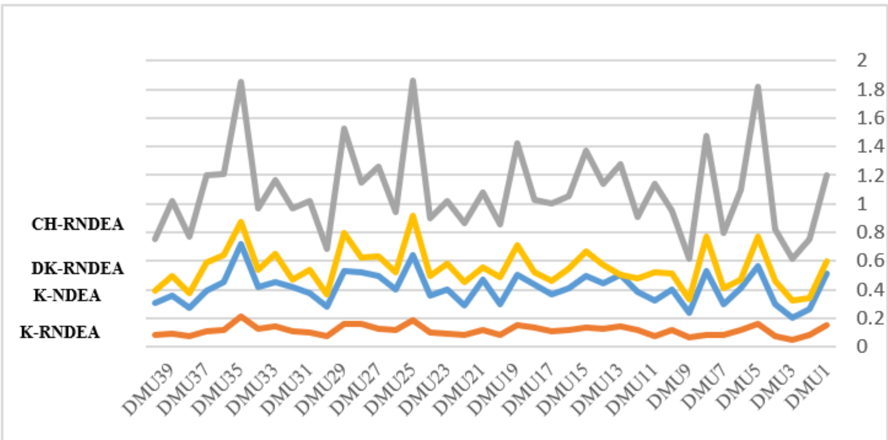

Figure 4. Chart Of efficiency scores of nominal and robust models

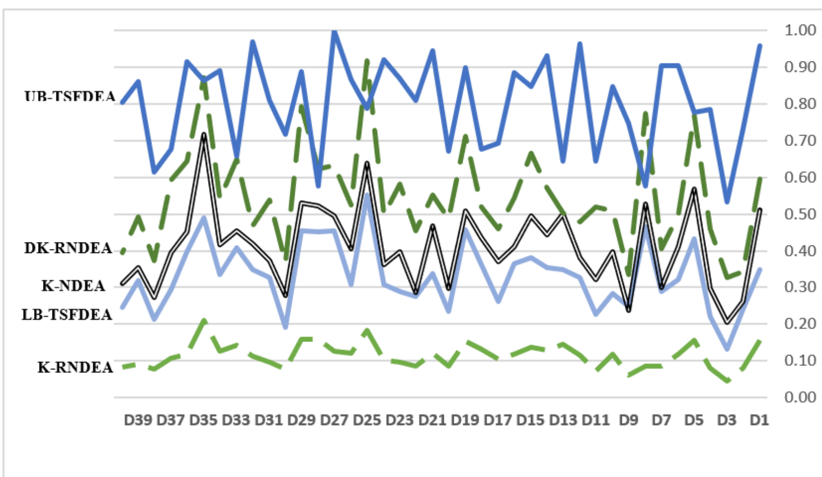

Figure 5. Chart of comparison TSFDEA and TS-SRDEA results

Simply, it is seen from Table 3 and Figure 4 that the K-RNDEA model is a lower bound and DK-RNDEA and CH-RNDEA models are the upper bounds for the efficiency scores of the K-NDEA model. Clearly seen that the upper bound obtained by the DK-RNDEA model than the value of the CH-RNDEA model is more accurate. Therefore, we can use the results of K-RNDEA model and DK-RNDEA model to define an efficiency interval for efficiency scores of DMUs.

5.2 Comparison of RNDEA models with TSFDEA models

Khalili-Damghani and Taghavifard (2012) solved this problem by using a three-stage fuzzy DEA model (TSFDEA) and calculated its lower and upper bound values of efficiency scores. The Table 4 shows the upper and lower bound values calculated by TSFDEA (column 5, 6) and the efficiency scores calculated by our three-stage Soyster robust DEA (K-RNDEA, DK-RNDEA) models (column 3, 4) [20].

It can be seen from Table 4 and Figure 5, al mostly, the upper bound obtained from DK-RNDEA for efficiency interval is more accurate than the TSFDEA model. On the other hand, the lower bound obtained from K-RNDEA for efficiency interval is less accurate than the TSFDEA model, al mostly.

To compare the accuracy of the values obtained from TSFDEA models and RNDEA models, the Mean squared error method has been used. For this purpose, square values for the results of the models are given in columns 7 to 10 of Table 4. The Mean squared error of the upper and lower bound values in our proposed RNDEA model is 0.116742 and for the TSFDEA model is 0.181763. Also, the total length of the intervals obtained by the proposed method is equal to 17.42910008 and the total length of the intervals obtained by the TSFDEA method is equal to 18.781213. Therefore, it seems that the proposed method in this article is more accurate. Of course, more experiments need to be done for certainty.

6. Conclusion

Performance efficiency of most of real systems cannot be evaluated with conventional DEA models, because of complicated structure and uncertain data. In this paper a brief review of some of series network models are presented. Then some Network DEA models are presented which can be used for efficiency evaluation of all the series Network structures that produce uncertain products. We use Soyster approach to overcome uncertainty. A set of Robust NDEA models have been proposed for assessing relative efficiency score of overall of multi-level serial processes.

A numerical example used to illustrate the approaches. The proposed approaches were applied in assessment of efficiency of a three-stage model of top-forty Iranian dairy supply chains. Three linear crisp DEA model were produced for each DMU. The results are acceptable and reliable. The results were promising and computations were straightforward. More formally, the proposed approach can be utilized to distinguish the relative efficiency scores of overall and sub-processes of a complicated process. So, the proposed approach can be assumed as a proper framework for handling multi-stage processes mixed with uncertainty in several areas of management and engineering.

Table 1. The upper bounds of inputs, intermediate measures, and outputs [20]

	X_1^u	X_2^u	X_3^u	X_4^u	Z_1^{1u}	Z_2^{1u}	Z_3^{1u}	Z_4^{1u}	Z_1^{2u}	Z_2^{2u}	Z_3^{2u}	Z_4^{2u}	Y_1^u	Y_2^u	Y_3^u	Y_4^u
D1	0.8	0.6	0.6	0.5	0.4	0.7	0.3	0.7	0.9	0.7	0.3	0.5	0.9	0.5	0.4	0.8
D2	0.98	0.8	0.6	0.9	0.9	0.3	0.9	0.6	0.7	0.8	0.4	0.7	0.4	0.9	0.8	0.4
D3	0.5	0.5	0.6	0.7	0.74	0.4	0.8	0.9	0.7	0.9	0.9	0.5	0.4	0.4	0.4	0.4
D4	0.7	0.9	0.5	0.7	0.3	1	0.7	0.33	0.7	0.3	0.53	0.9	0.4	0.7	0.7	0.63
D5	0.5	0.3	0.4	0.85	0.46	1	0.9	0.6	0.74	0.3	0.3	0.67	0.7	0.7	0.96	0.6
D6	0.6	0.5	0.8	0.5	0.63	0.63	0.3	0.64	0.3	0.5	0.9	0.4	0.6	0.9	0.8	0.6
D7	0.8	0.6	0.74	0.98	0.6	0.6	0.5	0.3	0.7	0.8	0.9	0.9	0.7	0.5	0.4	0.63
D8	0.4	0.96	0.9	0.5	0.7	0.7	0.7	0.6	0.84	0.5	0.24	0.73	0.84	1	1	0.4
D9	0.94	1	0.94	0.5	0.5	0.95	0.83	0.5	0.42	0.7	0.7	0.3	0.5	0.4	0.3	0.5
D10	0.9	0.87	0.6	0.6	0.4	0.8	0.5	0.8	0.7	0.9	0.73	0.34	0.8	0.6	0.6	0.8
D11	0.6	0.6	0.4	0.7	0.9	0.98	0.7	0.93	0.6	0.8	0.7	0.6	0.5	0.6	0.95	0.42
D12	0.8	0.6	1	0.5	0.76	0.6	0.7	0.3	0.86	0.9	0.83	0.65	0.9	0.7	0.6	0.9
D13	0.4	0.7	0.9	0.9	0.7	0.7	0.9	0.9	0.4	0.85	0.4	0.7	0.4	1	1	0.4
D14	1	0.64	0.7	0.4	0.96	0.56	0.3	0.53	0.9	0.8	1	0.42	0.7	0.8	0.75	0.7
D15	0.4	0.4	0.6	0.94	0.4	0.4	0.3	0.3	0.63	0.74	0.6	0.4	0.6	0.7	0.6	0.6
D16	0.95	0.7	0.4	0.6	0.3	0.3	0.5	0.9	0.4	0.5	0.5	0.7	0.7	0.6	0.5	0.6
D17	0.5	0.4	0.95	0.3	0.67	0.6	0.8	0.8	0.9	0.8	0.6	0.5	0.5	0.7	0.6	0.5
D18	0.7	0.8	0.7	0.6	0.54	0.4	0.75	0.4	0.73	0.73	0.6	0.9	0.9	0.5	0.5	0.85
D19	0.98	0.4	0.84	0.6	0.4	0.4	0.6	0.6	0.3	0.4	1	0.83	0.8	0.5	0.94	0.8
D20	0.7	0.7	0.6	0.7	0.7	0.66	0.9	0.4	0.4	1	0.87	0.7	0.5	0.83	0.5	0.8
D21	0.7	0.7	0.86	0.5	0.94	0.97	0.53	0.75	0.9	0.75	0.3	0.85	0.9	0.4	0.9	0.84
D22	0.8	0.98	0.6	0.9	0.9	0.3	0.9	0.56	0.7	0.8	0.4	0.7	0.93	0.7	0.23	0.9
D23	0.5	0.5	0.6	0.7	0.74	0.4	0.8	0.9	0.7	0.89	0.9	0.41	0.7	0.6	0.7	0.83
D24	0.7	0.9	0.5	0.7	0.3	1	0.7	0.53	0.62	0.3	0.63	0.9	0.7	0.7	0.63	0.7
D25	0.5	0.3	0.94	0.95	0.48	1	0.9	0.6	0.54	0.3	0.3	0.87	0.9	0.84	0.9	0.7
D26	0.6	0.6	0.7	0.5	0.86	0.63	0.3	0.6	0.3	0.5	0.9	0.4	0.74	0.5	0.5	0.9
D27	0.97	0.6	0.5	0.98	0.6	0.6	0.5	0.3	0.6	0.7	0.87	0.9	1	0.8	0.92	0.84
D28	0.4	0.96	0.9	0.6	0.7	0.7	0.7	0.6	0.4	0.5	0.74	0.83	0.4	0.5	0.4	1
D29	0.4	1	0.4	0.6	0.5	0.95	0.83	0.53	0.4	0.6	0.6	0.3	0.6	0.9	0.65	0.63
D30	0.9	0.7	0.7	0.6	0.32	0.8	0.5	0.8	0.7	0.9	0.4	0.36	0.6	0.4	0.6	0.6
D31	0.7	0.7	0.4	0.97	0.9	0.98	0.7	0.23	0.6	0.8	0.7	0.66	0.7	0.7	0.64	0.6
D32	0.98	0.76	1	0.5	0.86	0.6	0.7	0.3	0.76	0.81	0.86	0.5	1	0.6	0.9	0.56
D33	0.5	0.7	0.9	0.8	0.7	0.7	0.9	0.9	0.4	0.5	0.4	0.6	0.87	0.76	0.8	1
D34	1	0.94	0.7	0.4	0.96	0.96	0.3	0.53	0.9	0.7	1	0.4	0.7	0.4	0.7	0.8
D35	0.31	0.4	0.6	0.4	0.4	0.4	0.3	0.75	0.64	0.4	0.64	0.4	0.6	0.9	0.75	0.7
D36	0.5	0.8	0.4	0.6	0.3	0.3	0.5	0.9	0.4	0.5	0.5	0.7	0.7	0.8	0.6	0.6
D37	0.5	0.41	0.95	0.4	0.64	0.6	0.8	0.8	0.9	0.8	0.5	0.5	0.84	0.6	0.4	0.7
D38	0.7	0.8	0.7	0.6	0.42	0.4	0.85	0.4	0.73	0.83	0.69	0.9	0.5	0.7	0.4	0.5
D39	0.9	0.4	0.74	0.85	0.4	0.4	0.6	0.6	0.3	0.4	1	0.63	0.6	0.5	0.5	0.42
D40	0.7	0.7	0.6	0.7	0.66	0.64	0.9	0.4	0.4	1	0.8	0.7	0.7	0.6	0.5	0.6

Table 2. The lower bounds of inputs, intermediate measures, and outputs [20]

	X_1^u	X_2^u	X_3^u	X_4^u	Z_1^{1u}	Z_2^{1u}	Z_3^{1u}	Z_4^{1u}	Z_1^{2u}	Z_2^{2u}	Z_3^{2u}	Z_4^{2u}	Y_1^u	Y_2^u	Y_3^u	Y_4^u
D1	0.6	0.5	0.5	0.3	0.3	0.6	0.2	0.6	0.7	0.6	0.2	0.4	0.7	0.3	0.3	0.7
D2	0.7	0.7	0.4	0.8	0.7	0.2	0.8	0.4	0.5	0.6	0.2	0.5	0.2	0.7	0.7	0.2
D3	0.4	0.3	0.5	0.6	0.3	0.3	0.7	0.7	0.5	0.8	0.7	0.3	0.2	0.2	0.2	0.2
D4	0.6	0.8	0.3	0.5	0.2	0.8	0.6	0.2	0.5	0.2	0.2	0.7	0.2	0.5	0.5	0.2
D5	0.3	0.2	0.3	0.4	0.3	0.8	0.8	0.4	0.3	0.2	0.2	0.6	0.5	0.5	0.5	0.5
D6	0.4	0.4	0.6	0.3	0.5	0.2	0.2	0.5	0.2	0.4	0.7	0.3	0.4	0.7	0.7	0.4
D7	0.6	0.4	0.3	0.7	0.5	0.5	0.4	0.2	0.5	0.6	0.7	0.7	0.5	0.3	0.3	0.5
D8	0.2	0.5	0.7	0.4	0.6	0.5	0.5	0.4	0.3	0.4	0.2	0.12	0.3	0.8	0.8	0.3
D9	0.3	0.8	0.2	0.4	0.4	0.4	0.2	0.4	0.3	0.5	0.5	0.2	0.3	0.2	0.2	0.3
D10	0.8	0.6	0.5	0.4	0.2	0.6	0.4	0.6	0.5	0.7	0.2	0.2	0.6	0.4	0.4	0.6
D11	0.5	0.5	0.2	0.6	0.7	0.7	0.6	0.2	0.4	0.6	0.5	0.5	0.3	0.4	0.4	0.3
D12	0.7	0.4	0.8	0.4	0.5	0.5	0.6	0.2	0.5	0.7	0.7	0.4	0.7	0.5	0.5	0.7
D13	0.3	0.5	0.7	0.7	0.5	0.5	0.8	0.7	0.3	0.4	0.3	0.5	0.2	0.8	0.8	0.2
D14	0.8	0.3	0.6	0.2	0.5	0.5	0.2	0.2	0.7	0.6	0.8	0.3	0.5	0.6	0.6	0.5
D15	0.2	0.3	0.5	0.3	0.3	0.3	0.2	0.2	0.5	0.3	0.5	0.3	0.4	0.5	0.5	0.4
D16	0.4	0.6	0.2	0.5	0.2	0.2	0.4	0.7	0.3	0.4	0.4	0.3	0.5	0.4	0.4	0.5
D17	0.4	0.3	0.4	0.2	0.5	0.5	0.6	0.7	0.7	0.6	0.4	0.3	0.3	0.5	0.5	0.3
D18	0.5	0.7	0.5	0.4	0.3	0.3	0.4	0.3	0.2	0.2	0.5	0.7	0.7	0.3	0.3	0.7
D19	0.7	0.2	0.3	0.4	0.3	0.3	0.5	0.5	0.2	0.3	0.8	0.2	0.6	0.3	0.3	0.6
D20	0.6	0.5	0.4	0.6	0.5	0.5	0.7	0.2	0.2	0.8	0.6	0.5	0.3	0.2	0.3	0.6
D21	0.6	0.5	0.5	0.3	0.3	0.6	0.2	0.6	0.7	0.6	0.2	0.4	0.7	0.2	0.7	0.3
D22	0.7	0.7	0.4	0.8	0.7	0.2	0.8	0.4	0.5	0.6	0.2	0.5	0.2	0.5	0.2	0.7
D23	0.4	0.3	0.5	0.6	0.3	0.3	0.7	0.7	0.5	0.8	0.7	0.3	0.5	0.4	0.5	0.2
D24	0.6	0.8	0.3	0.5	0.2	0.8	0.6	0.2	0.5	0.2	0.2	0.7	0.5	0.5	0.5	0.5
D25	0.3	0.2	0.3	0.4	0.3	0.8	0.8	0.4	0.3	0.2	0.1	0.6	0.7	0.3	0.7	0.5
D26	0.4	0.4	0.6	0.3	0.5	0.2	0.2	0.5	0.2	0.4	0.7	0.3	0.3	0.3	0.3	0.7
D27	0.6	0.4	0.3	0.7	0.5	0.5	0.4	0.2	0.5	0.6	0.7	0.7	0.8	0.6	0.8	0.3
D28	0.2	0.5	0.7	0.4	0.6	0.5	0.5	0.4	0.3	0.4	0.26	0.11	0.2	0.3	0.2	0.8
D29	0.3	0.8	0.2	0.4	0.4	0.4	0.2	0.4	0.3	0.5	0.5	0.2	0.4	0.7	0.4	0.2
D30	0.8	0.6	0.5	0.4	0.2	0.6	0.4	0.6	0.5	0.7	0.2	0.2	0.4	0.2	0.4	0.4
D31	0.5	0.5	0.2	0.6	0.7	0.7	0.6	0.2	0.4	0.6	0.5	0.5	0.5	0.5	0.5	0.4
D32	0.7	0.4	0.8	0.4	0.5	0.5	0.6	0.2	0.5	0.7	0.7	0.4	0.8	0.4	0.8	0.5
D33	0.3	0.5	0.7	0.7	0.5	0.5	0.8	0.7	0.3	0.4	0.3	0.5	0.6	0.5	0.6	0.8
D34	0.8	0.3	0.6	0.2	0.5	0.5	0.2	0.2	0.7	0.6	0.8	0.3	0.5	0.3	0.5	0.6
D35	0.2	0.3	0.5	0.3	0.3	0.3	0.2	0.2	0.5	0.3	0.5	0.3	0.4	0.7	0.4	0.5
D36	0.4	0.6	0.2	0.5	0.2	0.2	0.4	0.7	0.3	0.4	0.4	0.6	0.5	0.6	0.5	0.4
D37	0.4	0.3	0.4	0.2	0.5	0.5	0.6	0.7	0.7	0.6	0.4	0.3	0.3	0.4	0.3	0.5
D38	0.5	0.7	0.5	0.4	0.3	0.3	0.4	0.3	0.2	0.2	0.5	0.7	0.3	0.5	0.3	0.3
D39	0.7	0.2	0.3	0.4	0.3	0.3	0.5	0.5	0.2	0.3	0.8	0.2	0.4	0.4	0.4	0.3
D40	0.6	0.5	0.4	0.6	0.5	0.5	0.7	0.2	0.2	0.8	0.6	0.5	0.5	0.4	0.4	0.4

Table 3. Efficiency scores for three stage sample

	K-RNDEA	K-NDEA	DK-RNDEA	CH-RNDEA
D1	0.15544	0.51110	0.59553	0.59553
D2	0.08049	0.26079	0.34244	0.34244
D3	0.04502	0.20451	0.32648	0.32648
D4	0.07842	0.29629	0.45968	0.45968
D5	0.15620	0.56706	0.77022	0.77022
D6	0.11880	0.41157	0.49821	0.49821
D7	0.08567	0.30066	0.40595	0.40595
D8	0.08567	0.52659	0.77417	0.77417
D9	0.06151	0.23635	0.33358	0.33358
D10	0.11826	0.39739	0.50930	0.50930
D11	0.07293	0.31998	0.52054	0.52054
D12	0.11636	0.38202	0.47954	0.47954
D13	0.14569	0.50001	0.50448	0.50448
D14	0.12901	0.44361	0.57234	0.57234
D15	0.13647	0.49390	0.66604	0.66604
D16	0.11751	0.41110	0.54479	0.54479
D17	0.10502	0.36896	0.46018	0.46018
D18	0.13255	0.43594	0.52028	0.52028
D19	0.15273	0.50740	0.71324	0.71324
D20	0.08447	0.29594	0.48614	0.48614
D21	0.11934	0.46716	0.55249	0.55249
D22	0.08604	0.28570	0.45350	0.45350
D23	0.09572	0.39680	0.58229	0.58229
D24	0.10072	0.36134	0.49722	0.49722
D25	0.18388	0.63788	0.91869	0.91869
D26	0.12062	0.40464	0.52521	0.52521
D27	0.12605	0.49562	0.63436	0.63436
D28	0.15828	0.52280	0.62272	0.62272
D29	0.15879	0.53088	0.79246	0.79246
D30	0.07593	0.27722	0.36673	0.36673
D31	0.09730	0.37374	0.53910	0.53910
D32	0.11258	0.41978	0.46807	0.46807
D33	0.14282	0.45565	0.64997	0.64997
D34	0.12635	0.41675	0.54152	0.54152
D35	0.20873	0.71652	0.87222	0.87222
D36	0.11878	0.45194	0.64448	0.64448
D37	0.10705	0.39455	0.59274	0.59274
D38	0.07738	0.27255	0.37194	0.37194
D39	0.09142	0.35516	0.49268	0.49268
D40	0.08230	0.30924	0.39591	0.39591

Table 4. Comparison results of RNDEA models and FTSDEA model

	K-RNDEA	LB-TSFDEA	K-NDEA	DK-RNDEA	UB-TSFDEA
D1	0.15544	0.3479595	0.51110	0.59553	0.958486404
D2	0.08049	0.2403438	0.26079	0.34244	0.732388614
D3	0.04502	0.130298	0.20451	0.32648	0.533159083
D4	0.07842	0.219912	0.29629	0.45968	0.784724982
D5	0.15620	0.4335226	0.56706	0.77022	0.776165434
D6	0.11880	0.3211291	0.41157	0.49821	0.906117657
D7	0.08567	0.289095	0.30066	0.40595	0.906117657
D8	0.08567	0.4659895	0.52659	0.77417	0.575780282
D9	0.06151	0.2486869	0.23635	0.33358	0.746330186
D10	0.11826	0.2819946	0.39739	0.50930	0.847015167
D11	0.07293	0.2270316	0.31998	0.52054	0.644190479
D12	0.11636	0.3276272	0.38202	0.47954	0.964517485
D13	0.14569	0.3473709	0.50001	0.50448	0.645035033
D14	0.12901	0.3528425	0.44361	0.57234	0.932113829
D15	0.13647	0.3797547	0.49390	0.66604	0.846589218
D16	0.11751	0.3638269	0.41110	0.54479	0.886526503
D17	0.10502	0.2623749	0.36896	0.46018	0.692678146
D18	0.13255	0.3620847	0.43594	0.52028	0.676876741
D19	0.15273	0.4579346	0.50740	0.71324	0.899787961
D20	0.08447	0.2355745	0.29594	0.48614	0.670920786
D21	0.11934	0.3386424	0.46716	0.55249	0.946499051
D22	0.08604	0.2754563	0.28570	0.45350	0.810064371
D23	0.09572	0.2882303	0.39680	0.58229	0.870195759
D24	0.10072	0.3078927	0.36134	0.49722	0.920122078
D25	0.18388	0.5517692	0.63788	0.91869	0.787821803
D26	0.12062	0.3072108	0.40464	0.52521	0.868243405
D27	0.12605	0.4531317	0.49562	0.63436	1
D28	0.15828	0.4528382	0.52280	0.62272	0.575780282
D29	0.15879	0.4548555	0.53088	0.79246	0.888358693
D30	0.07593	0.1921567	0.27722	0.36673	0.718053002
D31	0.09730	0.326676	0.37374	0.53910	0.810097928
D32	0.11258	0.347212	0.41978	0.46807	0.968854176
D33	0.14282	0.4085115	0.45565	0.64997	0.659232528
D34	0.12635	0.3352685	0.41675	0.54152	0.891927157
D35	0.20873	0.4888391	0.71652	0.87222	0.865318629
D36	0.11878	0.398574	0.45194	0.64448	0.914430012
D37	0.10705	0.2937327	0.39455	0.59274	0.675561156
D38	0.07738	0.213271	0.27255	0.37194	0.614393154
D39	0.09142	0.3196149	0.35516	0.49268	0.860523435
D40	0.08230	0.2444869	0.30924	0.39591	0.803938629

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