Available online at http://ijdea.srbiau.ac.ir

Int. J. Data Envelopment Analysis (ISSN 2345-458X)

Vol. 10, No. 2, Year 2022 Article ID IJDEA-00422, pages 51-62
Research Article

International Journal of Data Envelopment Analysis

Science and Research Branch (IAU)

Centralized resource allocation SBM proposed method

M. Seyfpanah[*]

Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran

Received 25 December 2021, Accepted 28 March 2022

Abstract

In this paper, a new method for centralized resource allocation based on the proposed SBM model of data envelopment analysis has been introduced. Models of DEA search targets separately for each DMU, but in this proposed model, by solving one model searches targets for all DMUs and projects all of them on the efficient frontier where this method will find a better image point rather than the previous methods. The goal is to reduce inputs and increase outputs to the non-radial. Finally, it is compared with previous methods by illustrating some examples and it seems that this method is for better.

Keywords: Data Envelopment Analysis, Centralized resource allocation, Efficiency MPSS.

[*] Corresponding author: Email: M.seyfpanah@gmail.com

1. Introduction

Data envelopment analysis (DEA) is a linear programming (LP) and it is also a non-parametric technique that evaluates the relative performance of decision making units (DMUs) introduced by [1]. Evaluating efficiency of DMUs is one of the most important efforts which is made in DEA and most of the authors have presented the models for the complete and perfect evaluation of efficiency of DMUs with respect to multi inputs and multi outputs. In classic models of DEA, these DMUs usually set their input and output targets to recognize their autonomy separately; consequently, when analyzing their relative performance, the DMUs are each separately projected onto the efficient frontier [2]. In many real cases in the world, there are situations in which all the DMUs fall under dominion of a centralized Decision Maker (DM) that controls them. This type of situation occurs whenever all of the units belong to the same organization (public or private), which provides the units with the necessary resources to obtain their outputs. Many DEA applications (such as those by bank branches, hospitals, university departments, supermarket chains and police stations) fall into this category [2]. In such a decision-making environment, the centralized decision-maker instead of considering and minimizing the consumption of inputs for each DMU separately, aims to minimize the overall input consumption or to maximize the overall output production [3-5]. For example, Korhonen developed an interactive formal approach based on DEA and multiple-objective linear programming (MOLP) and applied it to a resource-allocation problem that typically appears in organizations with a centralized decision-making environment [3]. The approach is illustrated via a supermarket dataset of 25 supermarkets situated in Finland that belong to the same chain. The aim of the chain’s management is to allocate available resources among the supermarkets in such a way that the total amount of output will be maximized simultaneously. Similar to Korhonen, Du developed a DEA-based production planning approach in a centralized decision-making environment [3,5]. Their objective is to simultaneously maximize the total output produced and minimize the total input consumed by all units. The approach is illustrated via a set of 20 fast food restaurants in the city of Hefei, Anhui Province belonging to the same chain, which has a central decision making team of several members who supervise the operations of all branches and make future sales plans. Recently, Lozano presented two centralized resource allocation BCC (CRA-BCC) models in a decision-making environment [2]. One type of model searches radial reductions of the total consumption of each input by all units, while the other type searches separate reductions for each input according to a preference structure. Asmild reconsidered one of the centralized models proposed by Lozano and suggested modifying it to only consider adjustments of previously inefficient units for at least three reasons [6]. They also showed how this new model formulation relates to standard BCC models, namely, as the analysis of the mean inefficient point. Mar-Molinero et al. (2012) developed a simplified version of the CRA-BCC model by Lozano and Villa (2004), which makes the model easier to implement in many situations [7]. In addition, the most efficient units can be identified. Other extensions to the basic centralized resource allocation model by [8-10].

The remainder of this paper is organized as follows. In section 2, SBM model, CRA model and CRA enhanced Russell model are discussed and in section 3, our CRASBM model is introduced. In what follows, we compare our approximation with the previous ones and numerical examples are used to illustrate the proposed approaches, and finally section 5 concludes the paper.

2- Background

In this section, the SBM (Tone, 2001) and CRA enhanced Russell model (Hosseinzadeh Lotfi et al., 2010) and CRA Lozano and Villa are briefly discussed. Consider a case of n DMUs, each dial SBM model, introduced by Tone (2001), is as follows [2,9,11]:

$$Min \quad W = \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}}}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{ro}}} \qquad (1)$$

$$\begin{aligned} s.t. \quad & \sum_{j=1}^{n}\lambda_j x_{ij} + s_i^- = x_{io} \qquad i=1,2,....,m \\ & \sum_{j=1}^{n}\lambda_j y_{rj} - s_r^+ = y_{ro} \qquad r=1,2,....,s \\ & s_i^- \geq 0, \ i=1,2,....,m \\ & s_r^+ \geq 0, \ r=1,2,....,s \\ & \lambda_j \geq 0., \ j=1,......,n \end{aligned}$$

Let j,k=1,2,…,n be the indices for DMUs; i=1,2,…,m be the indices for inputs; r=1,2,…,s be the for outputs; x_{ij} be the amount of input I consumed by DMU j; y_{rj} be the quantity of output r produced by DMU j; θ be the radial contraction of the total input vector and $\lambda_k = (\lambda_{1k}, \lambda_{2k}, ..., \lambda_{nk})$ be the vector for projecting DMU r; the CRA model proposed by Lozano and Villa(2004) is as follows [2]:

$$\begin{aligned} \min \quad & \theta \\ s.t. \quad & \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk} x_{ij} \leq \theta \sum_{j=1}^{n} x_{ij}, \quad i=1,...,m, \\ & \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk} y_{rj} \geq \sum_{j=1}^{n} y_{rj}, \quad r=1,...,s, \\ & \sum_{j=1}^{n}\lambda_{jk} = 1 \ \ k=1,...,n, \\ & \lambda_{jk} \geq 0, \ k=1,...,n, \ j=1,...,n. \end{aligned}$$

The above-mentioned model has three aims:

(a) The inefficient DMUs can be projected on the frontier of efficient DMUs by solving one model, instead of solving a model for each DMU separately.
(b) An existing technically efficient DMU may be projected onto a different point on the efficient frontier while it should be projected onto itself in conventional DEA models.
(c) The total consumption of the DMUs can be reduced, instead of reducing the inputs of any of the DMUs.

The above model has n^2+1 variables and m+s+n constraints [9].

And the CRA enhanced Russell model proposed by Hosseinzadeh Lotfi et al. (2010) is as follows [9]:

$$\min \quad \gamma = \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i}{\frac{1}{s}\sum_{r=1}^{s}\varphi_r} \qquad (2)$$

$$\begin{aligned} s.t. \quad & \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk} x_{ij} \leq \theta_i \sum_{j=1}^{n} x_{ij}, \quad i=1,...,m, \\ & \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk} y_{rj} \geq \varphi_r \sum_{j=1}^{n} y_{rj}, \quad r=1,...,s, \\ & \sum_{j=1}^{n}\lambda_{jk} = 1 \ \ k=1,...,n, \\ & \lambda_{jk} \geq 0, \ k=1,...,n, \ j=1,...,n, \\ & \theta_i \leq 1, \ i=1,...,m \\ & \varphi_r \geq 1, \ r=1,...,s. \end{aligned}$$

The advantage of CRA enhanced Russell model over Lozano and Villa model is that there is no need to select preference coefficients for reduction in the ith input consumed and increase in the rth output product. On the other hand, since all the constraints in the proposed model are binding at optimality, it does not need to solve the phase-II problem. In radial models, decrease in all inputs is equal and these models provide the highest amount of decrease in all inputs. This is while in non-radial models, input components are not all deceased equally.

3- Centralized resource allocation SBM proposed method

The centralized resource allocation SBM proposed method is as follows.

$$\min \quad \omega = \frac{1-\dfrac{1}{m}\sum_{i=1}^{m}\dfrac{s_i^-}{\sum_{j=1}^{n} x_{ij}}}{1+\dfrac{1}{s}\sum_{r=1}^{s}\dfrac{s_r^+}{\sum_{j=1}^{n} y_{rj}}} \qquad (3)$$

$$\begin{aligned} s.t.\quad & \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}x_{ij}+s_i^- = \sum_{j=1}^{n}x_{ij}, \quad i=1,\dots,m,\\ & \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}y_{rj}-s_r^+ = \sum_{j=1}^{n}y_{rj}, \quad r=1,\dots,s,\\ & \sum_{j=1}^{n}\lambda_{jk}=1 \quad k=1,\dots,n,\\ & \lambda_{jk}\ge 0,\ k=1,\dots,n,\ j=1,\dots,n,\\ & s_i^- \, and\ s_r^+ \ free\ for\ all\ i,r \end{aligned}$$

Theorem1. Model (3) is feasible.

Proof. We put $\tilde{\lambda}_{jk}=1, j=k$ and $\tilde{\lambda}_{jk}=0, j\ne k, j=1,\dots,n,$ $k=1,\dots,n,\ \tilde{s}_i^-=0, i=1,\dots,n,$ $\tilde{s}_r^+=0, r=1,\dots,s$, then $\left(\tilde{s}^-,\tilde{s}^+,\tilde{\lambda}\right)$ is a feasible solution of (3).

Once model (3) is solved, the corresponding vector $\lambda_k^*=\left(\lambda_{1k}^*,\lambda_{2k}^*,\dots,\lambda_{nk}^*\right)$ for each DMU r is the operating point at which it should aim. The inputs and outputs of any such point can be computed as

$$\begin{cases} \hat{x}_{ik}=\sum_{j=1}^{n}\lambda_{jk}^* x_{ij} \\ \hat{y}_{rj}=\sum_{j=1}^{n}\lambda_{jk}^* y_{rj} \end{cases}, \ i=1,\dots,m,\ k=1,\dots,n,\ r=1,\dots,s.$$

Theorem 2. For any DMU k, the operating point onto which it is projected by model (3) is pareto efficient.

Proof. By contraction, suppose $\left(\hat{x}_{1k},\dots,\hat{x}_{mk},\hat{y}_{1k},\dots,\hat{y}_{sk}\right)$ is not parato efficient, and then exists a vector $\bar{\lambda}_k=\left(\bar{\lambda}_{1k},\bar{\lambda}_{2k},\dots,\bar{\lambda}_{nk}\right)$ satisfying $\sum_{j=1}^{n}\bar{\lambda}_{jk}=1$ that defines an operating point

$$\bar{x}_{ik}=x_{ik}-\bar{s}_i^-=\sum_{j=1}^{n}\bar{\lambda}_{jk}x_{ij}\le\hat{x}_{ik} \quad i=1,\dots,m,$$

$$\bar{y}_{rj}=y_{rj}+\bar{s}_r^+=\sum_{j=1}^{n}\bar{\lambda}_{jk}y_{rj}\ge\hat{y}_{rj} \quad r=1,\dots,s.$$

Such that the previous inequality is strict at least for one input or one output. Without the loss of generality, suppose

$$\bar{x}_{tk}=x_{tk}-\bar{s}_t^-=\sum_{j=1}^{n}\bar{\lambda}_{jk}x_{tj}<\hat{x}_{tk}=x_{tk}-s_t^{*-}$$

So define $x_{tk}-\bar{s}_t^-<x_{tk}-s_t^{*-}$ and for the rest $x_{ik}-\bar{s}_i^-\le x_{ik}-s_i^{*-}, i=1,\dots,m\ i\ne t.$

Hence $s_t^{*-}<\bar{s}_t^-$

$$s_i^{*-}=\bar{s}_i^-\ \ i=1,\dots,m,\ i\ne t,$$

$$\frac{\sum_{k=1}^{n}x_{tk}-\bar{s}_t^-}{\sum_{k=1}^{n}x_{tk}}<\frac{\sum_{k=1}^{n}x_{tk}-s_t^{*-}}{\sum_{k=1}^{n}x_{tk}},$$

$$\frac{\sum_{k=1}^{n}x_{ik}-\bar{s}_i^-}{\sum_{k=1}^{n}x_{ik}}\le\frac{\sum_{k=1}^{n}x_{ik}-s_i^{*-}}{\sum_{k=1}^{n}x_{ik}}$$

We define $\tilde{\lambda}_j = \lambda_j^*,\ j = 1,...,n, j \neq k$ and $\tilde{\lambda}_k = \bar{\lambda}_k$, $\bar{s}_t^- = \tilde{s}_t^-$ $\bar{s}_i^- = \tilde{s}_i^-\ i = 1,...,m,\ i \neq t,$. We have $\tilde{\lambda} = \left(\tilde{\lambda}_1, \tilde{\lambda}_2, ..., \tilde{\lambda}_n\right)$, $\tilde{s}^+ = \left(\tilde{s}_1^-, \tilde{s}_2^-, ..., \tilde{s}_m^-\right)\ \tilde{s}^+ = \left(\tilde{s}_1^+, \tilde{s}_2^+, ..., \tilde{s}_s^+\right)$, then $\left(\tilde{s}^-, \tilde{s}^+, \tilde{\lambda}\right)$ is a feasible solution of model (3), and we have

$$\frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{\tilde{s}_i^-}{\sum_{j=1}^{n}x_{ij}}}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{\tilde{s}_r^+}{\sum_{j=1}^{n}y_{rj}}} < \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^{*-}}{\sum_{j=1}^{n}x_{ij}}}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^{*+}}{\sum_{j=1}^{n}y_{rj}}}$$

Having a lower objective function value than optimum of model (3), this is a contradiction. The proof is thus completed.

Theorem 3. The optimal value of the objective of model (3) is $0 < \omega^* \leq 1$.

Proof. First, to prove $\omega^* \leq 1$, we put $\tilde{\lambda}_{jk} = 1, j = k$ and $\tilde{\lambda}_{jk} = 0,\ j \neq k,\ j = 1,...,n, k = 1,...,n,\ \tilde{s}_i^- = 0, i = 1,...,n,$ $\tilde{s}_r^+ = 0, r = 1,...,s$, then $\left(\tilde{s}^-, \tilde{s}^+, \tilde{\lambda}\right)$ is a feasible solution of (3). The objective function value for this solution is 1, and regarding minimization we have $\omega^* \leq 1$.

Now we have to show that $\omega^* \neq o$. By contradiction $\omega^* = o$, then

$1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{\sum_{j=1}^{n}x_{ij}} = 0,$ we have

$$\sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}x_{ij} + s_i^- = \sum_{j=1}^{n}x_{ij},\ \ i = 1,...,m,$$

hence

$$\sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}\frac{x_{ij}}{\sum_{j=1}^{n}x_{ij}} + \frac{s_i^-}{\sum_{j=1}^{n}x_{ij}} = 1,\ \ i = 1,...,m,$$

so $\sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}\frac{x_{ij}}{\sum_{j=1}^{n}x_{ij}} = 0,\ \ i = 1,...,m,$

then $\tilde{\lambda}_{jk} = 0,$

For all j=1,.., n , k=1,…,n. this is a contradiction because we $\sum_{j=1}^{n}\lambda_{jk} = 1, k = 1,...,n.$ The proof is therefore completed.

By introducing the positive variable of B, model (3) can be like the following:

$$\min\quad \beta - \frac{1}{m}\sum_{i=1}^{m}\frac{\beta s_i^-}{\sum_{j=1}^{n}x_{ij}} \tag{4}$$

$$\begin{aligned}
s.t.\quad & \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}x_{ij} + s_i^- = \sum_{j=1}^{n}x_{ij},\ \ i = 1,...,m,\\
& \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}y_{rj} - s_r^+ = \sum_{j=1}^{n}y_{rj},\ \ r = 1,...,s,\\
& \sum_{j=1}^{n}\lambda_{jk} = 1\ \ k = 1,...,n,\\
& \beta + \frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{\sum_{j=1}^{n}y_{rj}} = 1\\
& \lambda_{jk} \geq 0,\ \ k = 1,...,n,\ j = 1,...,n, \beta \geq \varepsilon\\
& s_i^- and\ s_r^+\ free\ for\ all\ i, r
\end{aligned}$$

Now, we will define:

$S_i^- = \beta S_i^-,\ \ S_r^+ = \beta S_r^+,\ \ \mu_j = \beta\lambda_j$

By such change of variables, model of (3) will be converted as follows:

$$\min \quad \beta - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{\sum_{j=1}^{n} x_{ij}} \qquad (5)$$

$$\begin{aligned}
s.t. \quad & \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}x_{ij} + s_i^- = \beta\sum_{j=1}^{n}x_{ij}, \quad i=1,\dots,m, \\
& \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}y_{rj} - s_r^+ = \beta\sum_{j=1}^{n}y_{rj}\ , \quad r=1,\dots,s, \\
& \sum_{j=1}^{n}\lambda_{jk} = \beta \quad k=1,\dots,n, \\
& \beta + \frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{\sum_{j=1}^{n}y_{rj}} = 1 \\
& \lambda_{jk} \ge 0,\ k=1,\dots,n,\ j=1,\dots,n, \beta \ge \varepsilon \\
& s_i^- \, and \, s_r^+ \ free \ for \, all \, i, r
\end{aligned}$$

4- A numerical example

In this section, we describe the numerical examples by Lozano and Villa, Hosseinzade Lotfi et.al.

Example1: Let consider 7 DMUS that by using an input, produce an output. The data and results have been shown in table (1).

We can see under the method of Lozano and Villa, units of 1, 6, 5, 4 and 7 are projected to unit 2 and the deficient unit 3 is projected to non-cephalic unit of 3 on the efficient frontier. By the method of Hosseinzadeh et.al Units of 3, 2, 1 and 7 are projected on units of (4.13, 8.26) on the efficient frontier. The units of 4, 5 and 6 also are projected to unit (5.16, 10.32) on the efficient frontier as same. In proposed method, all the units of 6, 5, 4, 3, 2, 1 and 7 will be projected to the unit of (3.84, 7.68). on the efficient frontier. It can be seen that this method has the same image for all DMUs. Ratio of total outputs produce to total inputs consume at the image point in proposed method is also equal to 2, while in the method of Lozano and Villa, it is equal to 1.957 and in method of Hosseinzadeh et.al. it is equal to 2. According to the figures (1), (2) , (3) and (4), the proposed method also seems that is better.

Table 1: Input and Output and the results models

DMUs			Lozano and villa approach		Hosseinzade lotfi(2010) approach		Hosseinzade lotfi(2014) approach		Proposed approach	
	X	Y	X	Y	X	Y	X	Y	X	Y
1	3	3	4	8	4.13	8.26	5	10	3.84	7.68
2	4	8	4	8	4.13	8.26	5	10	3.84	7.68
3	5	5	3.6	6	4.13	8.26	5	10	3.84	7.68
4	5	10	4	8	5.16	10.32	5	10	3.84	7.68
5	6	8	4	8	5.16	10.32	5	10	3.84	7.68
6	7	11	4	8	5.16	10.32	5	10	3.84	7.68
7	8	9	4	8	4.13	8.26	5	10	3.84	7.68
Total	38	54	27.6	54	32	64	35	70	26.88	53.76

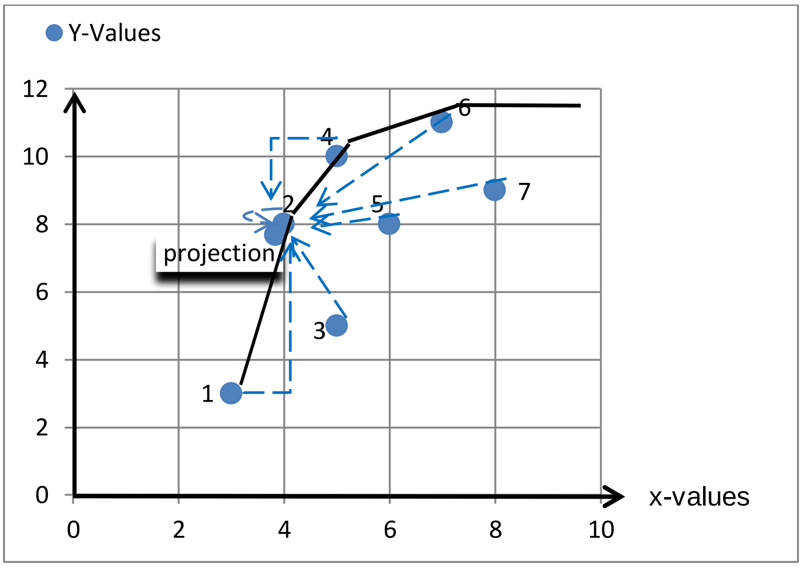

Fig. 1. Illustration of a proposed method

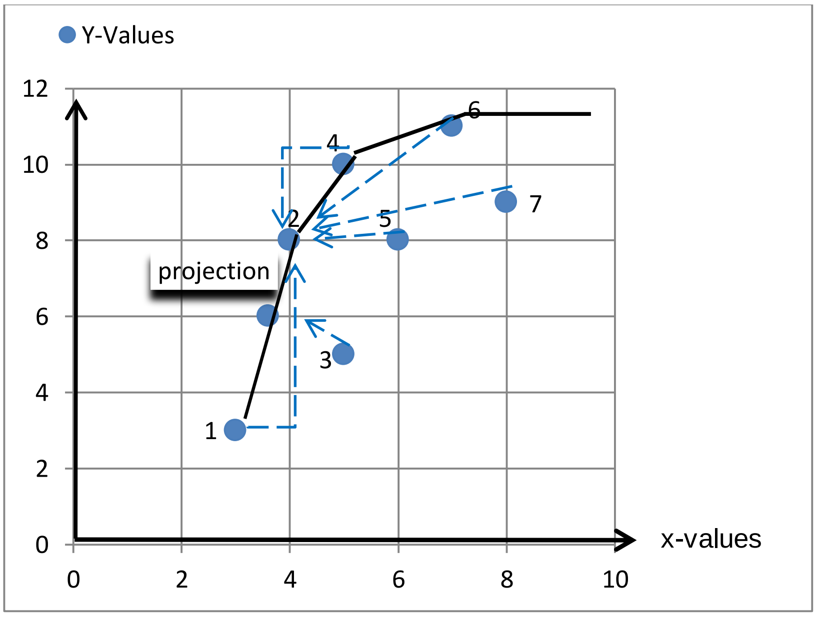

Fig. 2. Illustration of a Lozano and Villa

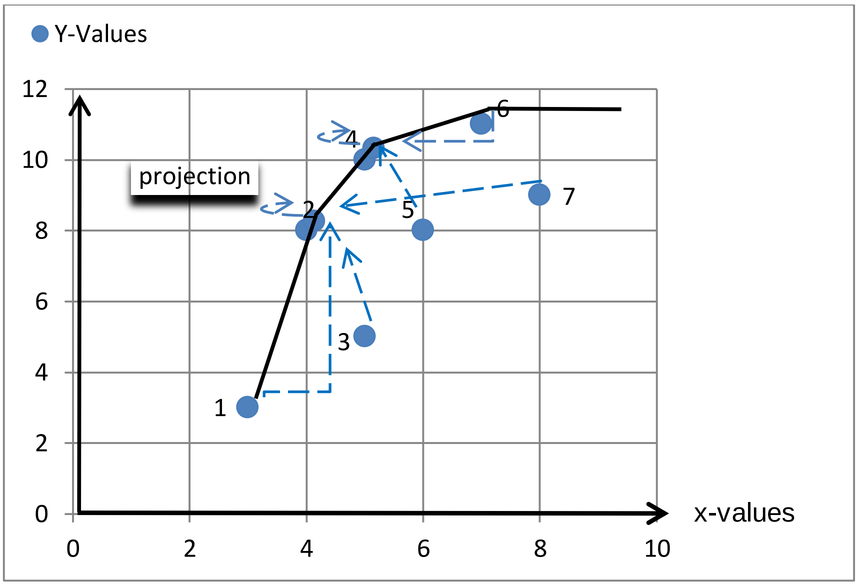

Fig. 3. Illustration of a Hosseinzadeh et al (2010)

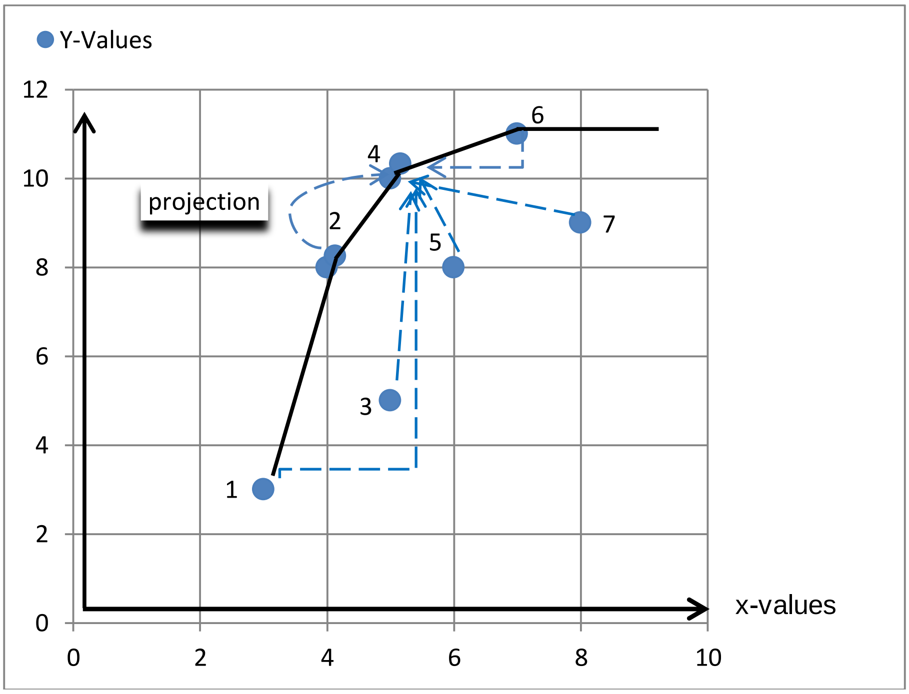

Fig. 4. Illustration of a Hosseinzadeh et al (2014)

Example 2: Let consider 7 DMUs with 2 inputs and one output. The data and results by method of Lozano and Villa and Hosseinzadeh Lotfi et.al and the proposed method have been shown in table (2).

In method of Lozano and Villa, units 4, 6 and 7 are projected to unit 2 and unit 5 to unit 4, unit 1 to unit non-extreme on the efficient frontier. Under method of Hosseinzadeh et.al units 1, 2 and 5 are projected on unit 4, and units 3 and 7 are projected on unit 2 and units 4 and 6 are projected on unit 3. In the proposed method, units of 2, 5 and 6 will be projected to unit 3 and units 4 and 7 are projected on unit 2 and unit 1 is projected on unit 4. See the figures (4) and (5) and (6).

Table 2: Inputs and Output and the results models

DMUs				Lozano and Villa approach			Hosseinzade lotfi approach			Proposed approach		
	X1	**X2**	**Y**	**X1**	**X2**	**Y**	**X1**	**X2**	**Y**	**X1**	**X2**	**Y**
1	**6**	**2**	**1**	**3.5**	**3.5**	**1**	**2**	**5**	**1**	**2**	**5**	**1**
2	**4**	**3**	**1**	**4**	**3**	**1**	**2**	**5**	**1**	**3**	**4**	**1**
3	**3**	**4**	**1**	**3**	**4**	**1**	**4**	**3**	**1**	**3**	**4**	**1**
4	**2**	**5**	**1**	**4**	**3**	**1**	**3**	**4**	**1**	**4**	**3**	**1**
5	**4**	**7**	**1**	**2**	**5**	**1**	**2**	**5**	**1**	**3**	**4**	**1**
6	**5**	**5**	**1**	**4**	**3**	**1**	**3**	**4**	**1**	**3**	**4**	**1**
7	**5**	**3**	**1**	**4**	**3**	**1**	**4**	**3**	**1**	**4**	**3**	**1**
Total	**29**	**29**	**7**	**24.5**	**24.5**	**7**	**20**	**29**	**7**	**22**	**27**	**7**

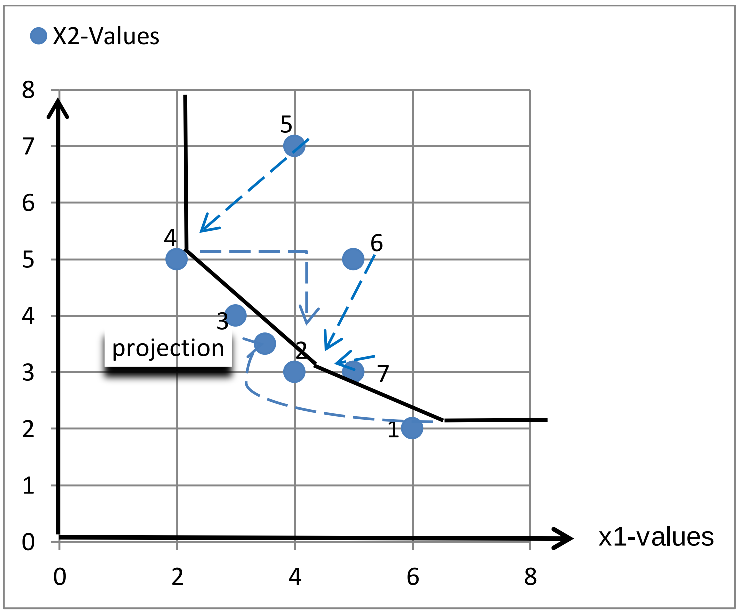

Fig. 4. Illustration of a Lozano and Villa

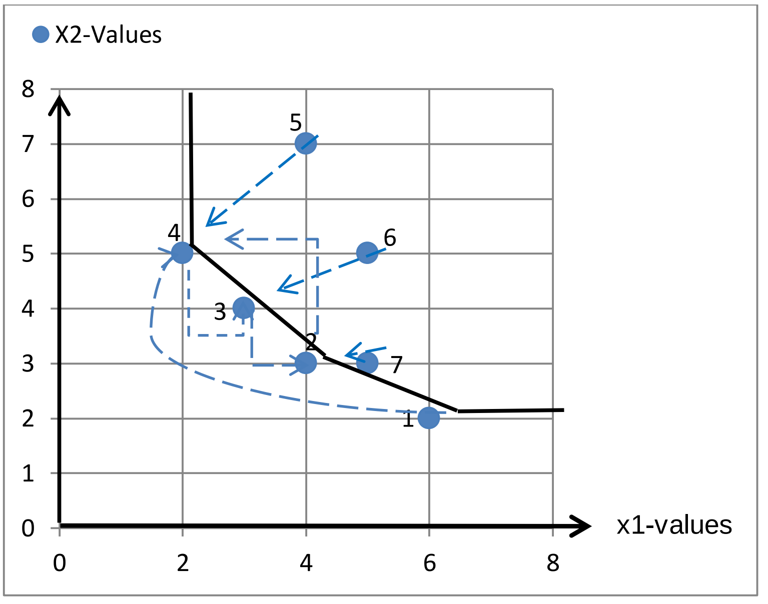

Fig. 5. Illustration of a Hosseinzadeh et al

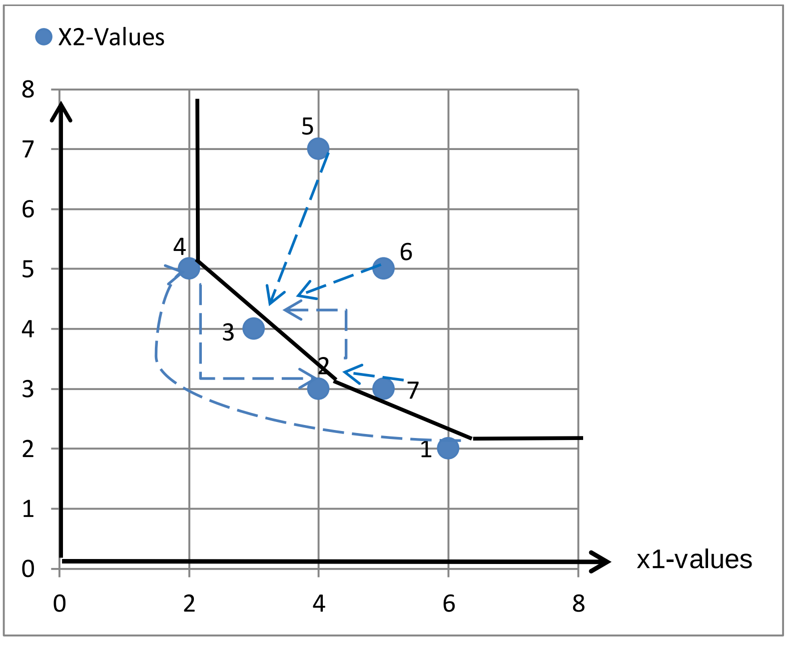

Fig. 6. Illustration of a proposed method

Example 3: let consider 10 DMUs with 2 inputs and 2 outputs. The data and results by method of Lozano and Villa and the proposed method have been shown in table (3).

In the presented method, according to the obtained results from solving the numerical examples, it can be seen that by more reduction in inputs, less output would be reduced. If cost of each one of the inputs is a few more than price of the outputs, in this case it is clear that further reduction in input rather than less reduction in output will be effective and in general, the unit will earn more profit. As an instance, consider example number 3. In this example the total sum of the first and second inputs of DMUs are 98 and 90,

Table 3: Inputs and Outputs and the results by method of Lozano and Villa and the proposed method

					Lozano and villa approach				Proposed approach			
DMUs	**x1**	**x2**	**y1**	**y2**	**X1**	**X2**	**Y1**	**Y2**	**X1**	**X2**	**Y1**	**Y2**
1	**9**	**9**	**2**	**1**	**6**	**10**	**5**	**3**	**6.98**	**5.81**	**3.49**	**3.49**
2	**12**	**8**	**3**	**1**	**6.71**	**9.12**	**4.82**	**3.18**	**6.98**	**5.81**	**3.49**	**3.49**
3	**7**	**12**	**2**	**2**	**6**	**10**	**5**	**3**	**6.98**	**5.81**	**3.49**	**3.49**
4	**6**	**10**	**5**	**3**	**6**	**10**	**5**	**3**	**6.98**	**5.81**	**3.49**	**3.49**
5	**10**	**5**	**4**	**4**	**6**	**10**	**5**	**3**	**6.98**	**5.81**	**3.49**	**3.49**
6	**8**	**10**	**3**	**3**	**10**	**5**	**4**	**4**	**6.98**	**5.81**	**3.49**	**3.49**
7	**12**	**10**	**6**	**6**	**10**	**5**	**4**	**4**	**6.98**	**5.81**	**3.49**	**3.49**
8	**14**	**6**	**8**	**2**	**10**	**5**	**4**	**4**	**6.98**	**5.81**	**3.49**	**3.49**
9	**12**	**12**	**1**	**6**	**10**	**5**	**4**	**4**	**6.98**	**5.81**	**3.49**	**3.49**
10	**8**	**8**	**3**	**5**	**10**	**5**	**4**	**4**	**6.98**	**5.81**	**3.49**	**3.49**
Total	**98**	**90**	**37**	**33**	**80.71**	**74.12**	**44.82**	**35.18**	**69.8**	**58.1**	**34.9**	**34.9**
%change					**-%17.64**	**-%17.64**	**%21.13**	**%6.6**	**-%28.77**	**-%35.44**	**-%5.67**	**%5.75**

respectively and also the total sum of the outputs of DMUs are 37 and 33, respectively. In the first the method, sum of the first and second inputs of DMUs has reduced to a rate of 17.29 and 15.88 units, respectively but the first and second outputs of DMUs has increased to a rate of 7.82 and 2.18 units, respectively. Similarly, in the proposed method, at the first and second inputs of DMUs, respectively, 28.2 and 31.9 decrease is obtained and in the first and second outputs 2.1 and 1.9 increase is obtained, respectively. The amount of reduction the total input or increases in the total output of DMUs for these two methods have been shown in the following table.

As it shows, the amount of reduction in the first and second inputs is more than the first method but in proportion to the savings, we have not observed any reduction in output. In total, there are 26.93 unit reductions in two inputs and there are 10.20 unit reductions in total outputs; therefore, that the proportion of these two numbers shows that the second method seems better

Table 4: results reduce the total input or increase the total output

	reduction in the first input	reduction in the second input	Increase in the first output	Increase in the second output
Lozano and villa approach	17.29	15.88	7.82	2.18
Proposed approach	28.20	31.90	-2.1	1.90
Difference	10.91	16.02	9.92	0.28

4- Conclusion

In this paper, we have presented a new non-radial centralized resource allocation. In models of data envelopment analysis, for evaluating n DMUs and projecting all units on the efficiency frontier, N programming models have been solved. In the proposed method, by solving just one model, all the decision making units can be projected on efficient frontier that this method will find a better image point rather than the previous methods. In this method, the percent of decreasing inputs is more than decreasing inputs in previous methods. It can be used for indefinite or phase data.

References

[1] Charnes, A., Cooper, W.W., Rhodes, E., 1978. Measuring efficiency of decision making units. European Journal of Operational Research 2 (6), 429–444.

[2] Lozano, S., Villa, G., 2004. Centralized resource allocation using data envelopment analysis. Journal of Productivity Analysis 22, 143–161.

[3] Korhonen, P., Syrjanen, M., 2004. Resource allocation based on efficiency analysis. Management Science 50 (8), 1134–1144.

[4] Fang, Lei, A generalized DEA model for centralized resource allocation European Journal of perational Research 228 (2013) 405–412.

[5] Du, J., Liang, L., Chen, Y., Bi, G., 2010. DEA-based production planning. Omega 38,105–112.

[6] Asmild, M., Paradi, J.C., Pastor, J.T., 2009. Centralized resource allocation BCC models. Omega 37, 40–49.

[7] Mar-Molinero, C., Prior, D., Segovia, M.M., Portillo, F., 2012. On centralized resource utilization and its reallocation by using DEA. Annals of Operations Research, 11.http://dx.doi.org/10.1007/s10479-012-1083-8.

[8] Lozano, S., Villa, G., 2005. Centralized DEA models with the possibility of downsizing. Journal of the Operational Research Society 56 (4), 357–364.

[9] Hosseinzadeh Lotfi, F., Noora, A.A., Jahanshahloo, G.R., Geramia, J., Mozaffari, M.R.,2010. Centralized resource allocation for enhanced Russell models. Journal of Computational and Applied Mathematics 235 (1), 1–10.

[10] Yu, M.M., Chern, C.C., Hsiao, B., 2013. Human resource rightsizing using centralized data envelopment analysis: evidence from Taiwan's airports. Omega 41, 119–130.

[11] Tone, K. (2001). "A slacks-based measure of efficiency in data envelopment analysis". *European Journal of Operational Research.* 130, 498-509.