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CO_2 Reduction in a Centralized System with a Bounded Adjusted Measure model in DEA

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Abstract
The high consumption of energy and the production of greenhouse gases have forced the government to impose restrictions on energy consumption and reduce environmental pollutants in industries. In this research, a model based on Bounded adjusted measure (BAM) and centralized resource allocation is presented, which reduces pollution among all units by solving only one model. The presented model can be useful for pollution control in specific geographical or industrial areas where the central manager wants to reduce pollution to a certain level among all units.

Keywords: Data Envelopment Analysis, Centralized resource allocation, eco-Efficiency, co_2 reduction

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1. Introduction

Performance measurement and evaluation are fundamental to management planning and control activities, and accordingly, have received considerable attention by both management researchers and theorists [1]. Facing a new round of industrialization and urbanization during the 12th five-year plan 2011-2015, the pressure of natural resource shortages has already become the most important concern on economic growth and social development in China [2]. In order to balance logical utilization of natural resources and sustainable development, many resource rules have been strengthened by the central Chinese government. For example, the Chinese government set the targets of reducing the energy consumption of per unit of GDP by 20% and 16% in the 11th and 12th five-year plans effective from 2006-2010 and 2011-2015 respectively [3,4]. Therefore, it is important for the Chinese government to determine the resource allocation among China's regions. Data envelopment analysis (DEA) was firstly developed by Charnes et al. [5]. As a non-parametric method, DEA is used to evaluate the relative performance of a group of homogenous decision-making units (DMUs), especially a group with multiple inputs and multiple outputs [6-8]. In addition to efficiency evaluation, DEA has been widely used in solving the problem of resource allocation (see: [9-11]). Athanassopoulos [12] integrated resource allocation and target setting in multilevel planning problems to allocate central grants to Greek local authorities. Beasley [13] proposed an alternative DEA-based approach by maximizing the average efficiency of all units and adding additional constraints to obtain a unique allocation scheme. Korhonen and Syrjänen [9] proposed a DEA-based multiple-objective linear programming (MOLP) method to maximize the values of multiple output variables when allocating fixed resources. Their approach is based on two assumptions: one is that all units are able to modify their production in the current production possibility set formed by efficient units; the other is that the units can modify their production plans without changing their efficiency. Asmild et al. [14] suggested modifying one of the centralized models to only consider adjustment of inefficient units. Hosseinzadeh Lotfi et al. [15] proposed a centralized resource allocation (CRA) model for the enhanced Russell model. In their paper, All the DMUs can be easily projected onto the efficient frontier by solving only one model. This projection can be made by transforming the proposed model to a linear programming problem. In their paper, instead of non-radially increasing or decreasing the inputs or outputs individually, increase or decrease non-radially all of the inputs and outputs at the same time. Fang [16] extended Lozano and Villa's and Asmild et al.'s models to a more general case. Amirteimoori and Tabar [17] proposed an approach to allocate resources based on the assumption that output targets are set beforehand. Fang and Li [18] formulated a centralized model to reallocate resources based on an extended revenue model under a centralized decision-making environment, Fang [18] proposed a new approach for resource allocation based on efficiency analysis. Wu et al. [19] proposed a DEA-based approach by considering both economic and environmental factors for resource allocation. Hosseinzadeh Lotfi et al. [20]' paper is to present the Stochastic Centralized Resource Allocation (SCRA) in order to allocate centralized resources where inputs and outputs are stochastic. Zhou et al. [21] study the CO2 emissions performance of China's transport sector consisting of 30 administrative regions from 2003 to 2009 using the data envelopment analysis (DEA) methodology. Zhou et al. [22] presented a non-radial DEA model based on input orientation to evaluate the efficiency of natural resource utilization for each of 26 provincial regions of mainland China. Then, in order to save natural resources, they proposed a DEA-based approach to allocate the natural resources among provincial regions. The general attitude in the performance evaluation of units is to minimize the inputs and maximizing the outputs, as done in conventional CCR and BCC models. But it should be noted that organizations are not always looking to maximize output and minimize input because outputs and inputs can be desirable or undesirable [23]. In this paper, by categorizing inputs and outputs as energy and non-energy, desirable and undesirable. Moreover, considering discretionary and non-discretionary factors, a new model for re-allocation of resources is presented in such a way that only by solving one model, it considers co_2 reduction among all DMUs.

This paper is structured as follows: Section 2 reviews the centralized resource allocation and BAM model. In the section 3, we present the centralize resource allocation model by classifying inputs and outputs as energy, non-energy, desirable, undesirable and discretionary, non-discretionary based on BAM model. In section 4, to show applicability of the proposed models, we apply the models on a real case. Finally, conclusions are given in section 5.

2- Background

Suppose that we have n DMUs, DMUj; j = 1, ..., n, consuming various amounts of m inputs to produce s outputs. Suppose $X_j = (x_{1j}, ..., x_{mj})^T$, $Y_j = (y_{1j}, ..., y_{sj})^T$, $X_j \geq 0, Y_j \geq 0, X_j \neq 0, Y_j \neq 0$ are the input and output vectors, respectively.

The centralized resource allocation model provided by Lozano and villa [10] is as follows.

$$
\begin{aligned}
\min \quad & \theta \qquad\qquad (1)\\
s.t. \quad & \sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} x_{ij} \leq \theta \sum_{r=1}^{n} x_{ir} \qquad i = 1, ..., m\\
& \sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} x_{kj} \leq \sum_{r=1}^{n} x_{kr} \qquad k = 1, ..., s\\
& \sum_{j=1}^{n} \lambda_{jr} = 1 \qquad r = 1, ..., n\\
& \lambda_{jr} \geq 0 \qquad j = 1, ..., n \quad , r = 1, ..., n
\end{aligned}
$$

The above-mentioned model has three aims: 1- The inefficient DMUs can be projected on the efficient frontier of by solving one model, instead of solving a model for each DMU separately. 2- An existing technically efficient DMU may be projected onto a different point on the efficient frontier. 3- The total consumption of the DMUs can be reduced, instead of reducing the inputs of any of the DMUs.

The BAM model proposed by Cooper et al. [24] is as follows:

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{m} \beta_{io}{}^{x} \eta_i^{x} + \sum_{r=1}^{s} \beta_{ro}{}^{y} \eta_r{}^{y} \qquad\qquad (2)\\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + \eta_i^{x} = x_{io} \qquad i = 1, ..., m\\
& \sum_{j=1}^{n} \lambda_j y_{rj} - \eta_r^{y} = y_{ro} \qquad r = 1, ..., s\\
& \sum_{j=1}^{n} \lambda_j = 1\\
& \eta_i{}^{x} \geq 0, \eta_r{}^{y} \geq 0, \lambda_j \geq 0
\end{aligned}
$$

η_i^x , η_r^y are slack variables for inputs and outputs.

They considered lower-sided ranges for inputs and upper-sided ranges for outputs. The weights of the model and the upper and lower ranges of the outputs and inputs are defined as follows:

$$\beta_{io}^{x} = \frac{1}{(m+s)L_{io}^{-}} \ , \ \beta_{ro}^{y} = \frac{1}{(m+s)U_{ro}^{+}} \qquad (3)$$

$$L_{io}^{-} = x_{io} - \underline{x}_i \qquad i = 1,...,m$$

$$U_{ro}^{+} = \bar{y}_r - y_{ro} \qquad r = 1,...,s$$

$$\underline{x}_i = \min\{x_{ij} \mid j = 1,...,n\}$$

$$\bar{y}_r = \max\{y_{rj} \mid j = 1,...,n\}$$

If the corresponding slack become zero ($\eta_i^x = 0$), then β_{io}^x will be zero, i.e., corresponding input is not possible to improve. Similarly, if $\eta_r^y = 0$, then β_{ro}^y =0, i.e., there is no output shortfalls.

Due to the increasing emission of greenhouse gases (GHG), climate change and global warming has become a major policy issue in the world. Many models have been proposed to measure eco-efficiency in DEA technology. There have been a variety of approach dealing with undesirable outputs, which can be mainly divided into two categories. The first is based on the strong disposability assumption, the second is based on the weak disposability assumption for undesirable outputs. Wu et al. [25] used the non-radial model based on variable return to scale to evaluate energy and eco-efficiency. They considered inputs in two types; energy and non-energy, also, the outputs are divided into two groups; desirable and undesirable. Moreover, in their model, undesirable outputs are treated like inputs. Fare et al. [26] stated that considering undesirable outputs as inputs contradicts of production possibility. They considered the assumption of weak disposability in model. Weak disposability means that it is possible to reduce undesirable outputs by decreasing the production activity. Korhonen and Luptacik [27] treated undesirable outputs as inputs, because they wanted to reduce undesirable outputs. This behavior is the principle of strong disposability assumption. We consider DEA models based on variable return to scale (VRS) and strong disposability assumption, that calculates the amount of energy saving and reduction of pollution in all of DMU_j.

Rashidi and Farzipoor [28] incorporated input and output separations to deal with energy and non-energy inputs; desirable and undesirable outputs. Furthermore, the inputs and outputs are separated into discretionary and non-discretionary factors for calculate the eco-efficiency scores of each DMU based on the BAM model. Suppose:

$$x = x^{E} \cup x^{NE} \ , \quad x^{E} = x^{DE} \cup x^{NDE} \ ,$$

$$x^{NE} = x^{DNE} \cup x^{NDNE} \ , \quad x^{E} \cap x^{NE} = \Phi$$

$$\left|x^{DE}\right| = I_1 \ , \quad \left|x^{NDE}\right| = I_2 \ , \quad \left|x^{DNE}\right| = I_3 \ , \quad \left|x^{NDNE}\right| = I_4$$

$$y = y^{g} \cup y^{b} \ , \quad y^{g} = y^{Dg} \cup y^{NDg} \ , \quad y^{b} = y^{Db} \cup y^{NDb}$$

$$\left|y^{Dg}\right| = R_1 \ , \quad \left|y^{NDg}\right| = R_2 \ , \quad \left|y^{Db}\right| = R_3 \ , \quad \left|y^{NDb}\right| = R_4$$

That:

x^{DE} : discretionary and energy input

x^{NDE} : non-discretionary and energy input

x^{DNE} : discretionary and non-energy input

x^{NDNE} : non-discretionary and non-energy input

y^{Dg} : discretionary and desirable output

y^{NDg} : non-discretionary and desirable output

y^{Db} : discretionary and undesirable output

y^{NDb} : non-discretionary and undesirable output

3- Centralized resource allocation BAM proposed method

The proposed model ensures that all units can be improved in terms of eco-efficiency after reallocation of resources. This model considers all the DMUs in one model and there is no need to solve the model for each DMU. The advantage of the model is that the total pollution should be reduced to the extent that the central manager wants, rather than individual units having to reduce their pollution. The central allocation resources model with energy and non-energy, discretionary and non-discretionary inputs for production desirable and undesirable, discretionary and non-discretionary according to BAM model, is presented as follows.

$$\max \quad \left(\sum_{i\in DE} \frac{\sum_{k=1}^{n}\delta_{ik}{}^{DE}}{(I_1+I_3+R_1+R_3)(\sum_{k=1}^{n}L_{ik}{}^{DE^-})} + \sum_{r\in Db} \frac{\sum_{k=1}^{n}\delta_{rk}{}^{Db}}{(I_1+I_3+R_1+R_3)\left(\sum_{k=1}^{n}L_{rk}{}^{Db^-}\right)} - \sum_{i\in DNE} \frac{\sum_{k=1}^{n}\delta_{ik}{}^{DNE}}{(I_1+I_3+R_1+R_3)(\sum_{k=1}^{n}L_{ik}{}^{DNE^-})} - \sum_{r\in Dg} \frac{\sum_{k=1}^{n}\delta_{rk}{}^{Dg}}{(I_1+I_3+R_1+R_3)\left(\sum_{k=1}^{n}u_{rk}{}^{Dg^+}\right)} \right) \tag{4}$$

$$\begin{aligned}
s.t.\quad & \\
& \sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}x_{ik} \le \sum_{k=1}^{n}(x_{ik}-\delta_{ik}{}^{DE}) && i\in DE \\
& \sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}x_{ik} \le \sum_{k=1}^{n}\left(x_{ik}-\delta_{ik}{}^{DNE}\right) && i\in DNE \\
& \sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}x_{ik} \le \sum_{k=1}^{n}x_{ik} && i\in ND \\
& \sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}y_{rk}^{g} \ge \sum_{k=1}^{n}\left(y_{rk}{}^{g}-\delta_{rk}{}^{Dg}\right) && r\in Dg \\
& \sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}y_{rk}^{g} \ge \sum_{k=1}^{n}y_{rk}{}^{g} && r\in NDg \\
& \sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}y_{rk}{}^{b} \le \sum_{k=1}^{n}(y_{rk}{}^{b}-\delta_{rk}{}^{Db}) && r\in Db \\
& \sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}y_{rk}{}^{b} \le \sum_{k=1}^{n}y_{rk}{}^{b} && r\in NDb \\
& \sum_{k=1}^{n}\lambda_{jk}=1 && j=1,\dots,n \\
& \delta_{ik}{}^{DE} \le \alpha_i x_{ik}{}^{DE} && i\in DE,\ j=1,\dots,n \\
& \delta_{rk}{}^{Db} \le \beta_r y_{rk}{}^{Db} && r\in Db,\ j=1,\dots,n \\
& \sum_{k=1}^{n}\delta_{rk}{}^{Db} \ge q_r && r\in Db \\
& \delta_{ik}{}^{DE}\ge 0, \delta_{ik}{}^{DNE}, \delta_{rk}{}^{Dg}, \delta_{rk}{}^{Db} \ge 0, \lambda_{jk}\ge 0,
\end{aligned}$$

$\delta_{ik}{}^{DE}$, $\delta_{ik}{}^{DNE}$ Represent the amount of i th energy and non-energy discretionary input reduction in DMU_k , and $\delta_{rk}{}^{Db}$, $\delta_{rk}{}^{Dg}$ denote the reduction amounts of r th undesirable and desirable discretionary output in DMU_k .also:

$$L_{ik}^{DE^-} = x_{ik} - \underline{x}_i \qquad i \in DE$$

$$L_{ik}^{DNE^-} = x_{ik} - \underline{x}_i \qquad i \in DNE$$

$$L_{rk}^{Db^-} = y_{rk} - \underline{y}_r \qquad r \in Db$$

$$U_{rk}^{Dg^+} = \bar{y}_r - y_{rk} \qquad r \in Dg$$

In the model (4), due to the reduction of energy consumption and environmental pollutants, we considered maximum reducing variables for discretionary energy inputs and undesirable outputs moreover, minimum reduction variables for discretionary non-energy inputs and desirable outputs. Because undesirable outputs have been treated like inputs, the objective function for undesirable outputs is considered as a lower range. The $\delta_{ik}{}^{DE} \le \alpha_i x_{ik}$ constraint in the model indicates that the amount of discretionary energy input reduction should be a percentage of its. This is so that the reductions are reasonable, not that some energy inputs have a significant reduction and others have no reduction or a small reduction. Theis $\delta_{rk}{}^{Db} \le \beta_r y_{rk}{}^{Db}$ described in the same way. The $\sum_{k=1}^{n}\delta_{rk}{}^{Db} \ge q_r$ constraint is added to the model by the central manager to control pollution reduction caused by production pollution in all DMUs.

4. Practical example

In this part, a numerical example is given to implement the model and analyze it. Table1 shows a number of OECD member countries, the inputs and outputs related to these countries. Inputs include labor force, consumption of petroleum products,

consumption of coal and average annual precipitation. The outputs include the co2 emission and GDP (Gross Domestic Product). The division of these inputs and outputs is as follows:
Labor force: discretionary and energy input (100,000)
Coal consumption: discretionary and energy input (million tons)
Petroleum consumption: discretionary and energy input (ten thousand barrels per day)
Precipitation average: discretionary and non-energy input (mm per year)
Co2 emission: discretionary and undesirable output (trillion)
GDP: discretionary and desirable output (kt)

Table1. Inputs and outputs of 20 OECD countries

	Countries	Labor force (100,000)	Coal consumption	petroleum consumption	precipitation	GDP	CO_2 emission
1	Australia	109.11	155.56	89.90	533	848.31	380.30
2	Austria	43.12	7.12	30.20	1112	370.99	68.09
3	Belgium	75.65	7.11	65.00	846	460.32	106.42
4	Canada	170.44	65.68	227.98	536	1425.01	566.40
5	The Czech Republic	52.01	63.29	22.30	678	181.31	121.78
6	Denmark	31.23	9.80	20.00	700	321.30	48.90
7	Finland	30.05	9.3	21.80	532	239.98	64.10
8	France	277.2	21.97	198.10	868	2581.87	378.31
9	Germany	413.1	279.36	242.50	699	3329.90	784.47
10	Greece	50.17	75.90	45.30	649	304.50	97.24
11	Hungary	43.25	12.99	16.01	586	135	56.49
12	Iceland	1.93	0.23	2.12	1941	21.30	2.53
13	Italy	251.28	28.01	171.99	831	2321.27	466.11
14	Japan	675.8	206.99	504.10	1671	4257.40	1253.11
15	Luxemburg	3.52	0.12	6.20	941	53.41	11.31
16	Netherlands	887.95	15.47	112.2	779	779.84	1172.27
17	Norway	26.07	1.43	24.00	1415	401.12	44.89
18	Poland	171.07	150.36	52.10	601	443.72	314.76
19	Portugal	55.99	5.43	31.20	849	253.19	61.75
20	Slovak	266.93	8.74	9.10	831	85.90	37.10

Table2. Re-allocated values for inputs and outputs

Countries	Labor force	Coal consumption	petroleum consumption	GDP	CO_2 emission
1	109.1100	124.4480	71.9200	848.3100	266.2100
2	43.1200	5.6960	24.1600	370.9900	47.6630
3	75.6500	5.6880	52.0000	460.3200	74.6630
4	170.4400	52.5440	182.3840	1425.0100	396.4800
5	52.0100	50.6320	17.8400	181.3100	85.2460
6	31.2300	7.8400	16.0000	321.3000	34,2300
7	30.0500	7.4400	17.4400	239.9800	44.8700
8	277.2000	17.5760	158.4800	2581.8700	264.8170
9	413.1000	223.4880	194.0000	3324.9000	549.1290
10	50.1700	60.7200	36.2400	304.5000	68.0680
11	43.2500	10.3920	12.8080	135.0000	39.5330
12	1.9300	0.1840	1.6960	21.3000	1.7710
13	251.2800	22.4080	137.5920	2321.2700	326.2770
14	675.8000	165.5920	403.2800	4257.4000	877.1770
15	3.5200	0.0960	4.9600	53.4100	7.9170
16	887.9500	12.3760	89.7600	779.8400	120.5890
17	26.0700	1.1440	19.2000	401.1200	31.4230
18	171.0700	120.2880	41.6800	443.7200	220.3320
19	55.9900	4.3440	24.9600	253.1900	43.2250
20	266.9300	6.1920	7.2800	85.9000	25.9700

The results obtained from the implementation of model by setting $\alpha_i = 0.2 \quad \forall i \in DE$, $\beta_r = 0.3 \quad \forall r \in Db$, $q_r = 1000$ are given in table2. The results show that a reduction of 224.772 in the consumption of coal and 2158.44 in the consumption of petroleum products in all DMUs, will reduce pollution by 1510.899 among all units, without labor force and GDP have a reduction.

5. Conclusion

This paper has presented a method based on the BAM model and centralize resources allocation that tries to reduce co_2 pollution and reduce energy consumption at the general level. The model considered energy and non-energy inputs and outputs as desirable and undesirable. In addition to, considered discretionary and non-discretionary factors. The model is designed so that the central manager can control pollution among all DMUs. In previous studies, models for calculating eco-efficiency were presented with the BAM model, which evaluated each DMU efficiency separately. But in the presented model in this paper, there is no need to solve the model for each unit, and the central manager has wanted to reduce pollution and reduce energy consumption at the massive level among all DMUs. The model has been implemented for a number of OECD countries. The amount of energy saving and co_2 reduction among all DMUs is considered. Therefore, by solving only one model, the amount of energy reduction can be calculated in individual units as well as in the whole. According to this article, it is suggested to expand the model by keeping the efficiency constant after re-allocation.

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