Available online at http://ijdea.srbiau.ac.ir

Int. J. Data Envelopment Analysis (ISSN 2345-458X)

Vol.8 , No. 2, Year 2020 Article ID IJDEA-00422, 12 pages
Research Article

International Journal of Data Envelopment Analysis

Science and Research Branch (IAU)

Generalization of the RDM model in Data Envelopment Analysis

H. Taherzadeh[*1], G. Tohidi[2], B. Hsiao[3]

(1) Young Researchers and Elite Club, Mashhad Branch, Islamic Azad University, Mashhad, Iran.
(2) Islamic Azad University, Department of Mathematics, Faculty of Science, Central Tehran Branch, Islamic Azad University, Tehran, Iran.
(3) Department of Information Management, Chang Jung Christian University, Tainan, Taiwan.

Received 21 January 2020, Accepted 19 April 2020.

Abstract

The current study extends and provides a generalization of the range directional model (RDM). The proposed generalized RDM (GRDM) model utilizes an ideal point (IP) and anti-ideal point (AIP) simultaneously to evaluate the efficiency score. It is evident that approaching to the IP, not necessarily, leads to moving away from the AIP. This obviously happens when the IP, AIP, and DMU lie on a common line; however, the depicted situation usually occurs hardly ever. On the other side, there are loads of situations in which a DMU requires not only to approach the IP but also to move away from the AIP, simultaneously. The GRDM model imposes two criteria (i.e., approaching to IP and moving away from AIP) to asses DMUs. Therefore, the efficiency score, when GRDM is used, is less than or equal to the efficiency score obtained by using the RDM model; consequently, the discrimination power of the GRDM model is better than that of the RDM model due to finding more inefficiency regarding both IP and AIP. The GRDM model is unit- and translation-invariant. A numerical example is applied to demonstrate the applicability of the proposed model in comparison with the RDM model.

Keywords: RDM model; Ideal and Anti Ideal Point; Inefficiency; Data Envelopment Analysis (DEA)

[*] Corresponding author: Email: h.taherzadeh@hotmail.com

1. Introduction

Data envelopment analysis (DEA), which was introduced by Farrell [1] and developed by Charnes et al. [2], is a non-parametric linear programming (LP) methodology used to evaluate the efficiency score of a number of similar processing decision-making units (DMUs).

All inputs and outputs are assumed to be positive in many applications of DEA. However, this assumption is inapplicable to several situations, such as analyses of financial statements (e.g, Smith [3] and Feroz et al. [4]) and the rating of mutual funds (Murthi et al. [5]), Therefore, the problem of handling negative data has elicited the attention of researchers.

Negative inputs and/or outputs have been investigated, and data transformation has been applied to measure the efficiency score. In this manner, all values of a given variable can be translated, and all negative data can be turned to positive data (Pastor [6] and Lovell [7]). The transformation of negative data may affect the solution of DEA results (Seiford et al. [8]). Negative data can be subdivided into the two types. The first type contains naturally negative data, which are measured on a ratio scale with a natural zero (Portela *et al.* [9]). The second type holds avoidably negative data, which include those measured on an interval or ordinal scale without any natural zero (Ueda et al. [10]). One of the main approaches for dealing with negative input and output is that of research of Portela et al. [9]. They introduced the RDM, which is a non-oriented model and looks simultaneously for input contraction and output expansion. RDM is based on the directional distance function proposed by Chung et al. [11]. The RDM model is unit- and translation-invariant but never identifies all sources of inefficiency. Additionally, there are some recent studies dealing with negative data in other aspects.

In many real-life practical situations, like the banking industry, the main target is to move away from the worst possible case (i.e. AIP) rather than move toward the best possible case (i.e. IP). This can happen when decision makers do not have technically the full potential for approaching to the best case, while they are able to keep themselves far from the worst case. In this study, a new approach is developed to handle the negative data. We propose a generalization form of RDM that is formulated based on moving toward the IP while moving away from AIP. The new approach increases the discrimination power of RDM. Moreover, the proposed method is a suitable approach in the situation in which we have a plan to both move away and move toward the AIP and IP, respectively. For instance, in the banking industry, all banks have a strong desire to move away just from the worst possible situations rather than move toward the best possible case in order to prevent Bankruptcy. These situations could be account balance, bank loans, non-current receivables and so forth.

The rest of this paper is organized as follows. Section 2 reviews the RDM model. Section 3 introduces the proposed GRDM. Section 4 discusses the application of the proposed model. The last section provides the conclusion.

2. RDM

We consider n DMUs that are indexed by $j\ (j=1,...,n)$. The performance of each DMU is characterized by a production process of consuming m inputs $\left(x_{ij};\ i=1,\ldots,m\right)$ to yield s outputs $\left(y_{rj}; r=1,\ldots,s\right)$. Chung et al. [11] presented an LP model called the generic directional distance model to measure the DEA efficiency score of a specific DMU as follows Model (1).

$$\text{Max} \quad \beta \tag{1}$$

s.t.

$$\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} - \beta g_{x_i}, \quad i = 1,\dots,m, \tag{1.1}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} + \beta g_{y_r}, \quad r = 1,\dots,s, \tag{1.2}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \tag{1.3}$$

$$\begin{array}{l} g_{y_r} \ge 0, g_{xi} \ge 0, \lambda_j \ge 0, \\ j = 1,\dots,n, \ i = 1,\dots,m, \ r = 1,\dots,s. \end{array} \tag{1.4}$$

where (g_{x_i}, g_{y_r}) is the directional vector. A popular choice in the presence of positive data is the observed input and output levels. In the negative data scenario, Model (1) needs to be modified. Portela et al. [9] developed the idea of Model (1) and defined an IP as follows:

$$\begin{cases} ith \text{ IP's input}=\left(\min\limits_{1\le j\le n}\{x_{ij}\}\right), \quad i = 1,2,\dots,m, \\ rth \text{ IP's output}=\left(\max\limits_{1\le j\le n}\{y_{rj}\}\right), \ r = 1,2,\dots,s. \end{cases} \tag{2}$$

where the *ith* IP's input component is the lowest value among the observed input vector $\mathbf{x}_i$. The maximum value is considered for the r^{th} IP's output. The purpose of choosing IP is to identify the direction vectors (g_{x_i}, g_{y_r}) for each DMU (Figure 1). Therefore, (g_{x_i}, g_{y_r}) is selected on the basis of the under evaluation DMU_o as follows:

$$\begin{cases} g_{io} = x_{io} - \min\limits_{1\le j\le n}\{x_{ij}\}, \quad i = 1,...,m, \\ g_{ro} = \max\limits_{1\le j\le n}\{y_{rj}\} - y_{ro}, \quad r = 1,...,s. \end{cases} \tag{3}$$

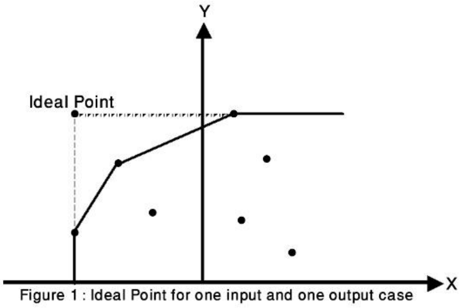

Figure 1 : Ideal Point for one input and one output case

Portela et al. [9] modified the generic directional distance model in the presence of negative data. The modified model, RDM, is defined as Model (4).

$$z_1 = \text{Max} \ \beta \tag{4}$$

s.t.

$$\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} - \beta R_{io}, \quad i = 1,\dots,m, \tag{4.1}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} + \beta R_{ro}, \quad r = 1,\dots,s, \tag{4.2}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \tag{4.3}$$

$$\lambda_j \ge 0, \quad j = 1,\dots,n. \tag{4.4}$$

where $R_{io} = x_{io} - \left(\min_{1\le j\le n}\{x_{ij}\}\right)$ and $R_{ro} = \left(\max_{1\le j\le n}\{y_{rj}\}\right) - y_{ro}$ are parameters in Model (4) and defined as follows:

$$\begin{cases} R_{io} = x_{io} - \min\limits_{1\le j\le n}\{x_{ij}\}, \quad i = 1,...,m, \\ R_{ro} = \max\limits_{1\le j\le n}\{y_{rj}\} - y_{ro}, \quad r = 1,...,s. \end{cases} \tag{5}$$

The optimal objective value of the RDM model shows the maximum inefficiency of under evaluation DMU_o. The advantage of the RDM model over the generic directional distance model, which can deal with negative data, is that it is unit- and translation-invariant and yields inefficiency scores between 0 and 1 (Portela et al. [9].)

3. GRDM

AIP is as crucial as IP. AIP is a point that is created by the observed DMUs in the worst case. Mathematically,

$$\begin{cases} ith \text{ AIP's input} = \left(\max\limits_{1\le j\le n}\{x_{ij}\}\right), & i=1,\dots,m, \\ rth \text{ AIP's output} = \left(\min\limits_{1\le j\le n}\{y_{rj}\}\right), & r=1,\dots,s. \end{cases} \quad (6)$$

The RDM model uses IP to project DMUs onto the efficient frontier. This purpose aims to move toward IP. Generally, moving toward IP does not mean moving away from AIP. This may occur when IP, AIP, and the observed DMU_o do not lie on a unique line. Therefore, the RDM model should be modified to incorporate the AIP. In our proposed model (GRDM), the RDM model is modified by including AIP. Precisely, DMUs move toward the IP while moving away from AIP in the GRDM model. In other words, another criterion (moving away from AIP) is added to measure the efficiency score of the RDM model. Figure 2 shows the IP and AIP geometrically. DMU_p is considered for evaluation. The RDM model suggests a movement (dotted line) to attain the efficient frontier, and the GRDM model suggests the combination of two different movements (solid lines) to attain the efficient frontier. Targets Q and R are on the efficient frontier. However, R is achieved based on two criteria (moving away from AIP and moving toward IP, simultaneously), and Q is just obtained based on moving toward IP.

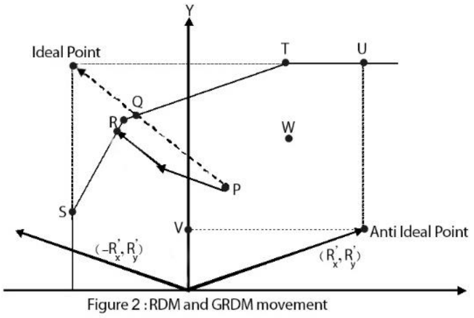

Figure 2 : RDM and GRDM movement

According to the aforementioned considerations, our proposed model, GRDM, is formulated as Model (7).

$$z_2 = \max \ \beta + \alpha \quad (7)$$

s.t.

$$\sum_{j=1}^{n}\lambda_j x_{ij} \le x_{io} - \beta R_{io} - \alpha R'_{io}, \ i=1,\dots,m, \quad (7.1)$$

$$\sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro} + \beta R_{ro} + \alpha R'_{ro}, \quad r=1,\dots,s, \quad (7.3)$$

$$\sum_{j=1}^{n}\lambda_j = 1, \quad (7.2)$$

$$\lambda_j, \alpha, \beta \ge 0, \quad j=1,\dots,n. \quad (7.3)$$

where R_{io}, R_{ro}, R'_{io} and R'_{ro} are parameters and defined as Eqs. 8-11:

$$R_{io} = x_{io} - \left(\min_{1\le j\le n}\{x_{ij}\}\right), \quad (8)$$

$$R_{ro} = \left(\max_{1\le j\le n}\{y_{rj}\}\right) - y_{ro}, \quad (9)$$

$$R'_{io} = \left(\max_{1\le j\le n}\{x_{ij}\}\right) - x_{io}, \quad (10)$$

$$R'_{ro} = y_{ro} - \left(\min_{1\le j\le n}\{y_{rj}\}\right) \quad (11)$$

To show that the generalized model provides the maximum inefficiency of DMU_o, we prove that $\alpha^* + \beta^* \le 1$.

Theorem 1: The optimal objective value of Model (7) is not greater than 1, $\alpha^* + \beta^* \le 1$.

Proof: The dual form of the GRDM model is as Model (12):

$$f_1 = \min \ \sum_{i=1}^{m} v_i x_{io} - \sum_{r=1}^{s} u_r y_{ro} + u_o \quad (12)$$

s.t.

$$\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} + u_o \ge 0, \ j=1,\dots,n, \quad (12.1)$$

$$\sum_{i=1}^{m} v_i R_{io} + \sum_{r=1}^{s} u_r R_{ro} = 1, \quad (12.2)$$

$$\sum_{i=1}^{m} v_i R''_{io} + \sum_{r=1}^{s} u_r R''_{ro} = 1, \quad (12.3)$$

$$v_i, u_r \geq 0,\ i=1,\ldots,m,\ r=1,\ldots,s, \quad (12.4)$$
$$u_o \text{ is free.} \quad (12.5)$$

For proof, we use the connection between the objective value of dual and primal models. By considering Models (7) and (12) and using the weak duality theorem, for any feasible solution of the primal and dual models, the following holds.

$$\beta+\alpha \leq \sum_{i=1}^{m} v_i x_{io} - \sum_{r=1}^{s} u_r y_{ro} + u_o, \quad (13)$$

To prove that $\alpha^*+\beta^* \leq 1$, we make a feasible solution with the objective value of one in Model (12). Toward this end, Model (12) can be rewritten as $u_r=0,(r=1,\ldots,s)$ by adding a new constraint $\sum_{i=1}^{m} v_i x_{io}+u_o=1$, that is,

$$f_2=\min \quad \sum_{i=1}^{m} v_i x_{io}+u_o \quad (14)$$

s.t.

$$\sum_{i=1}^{m} v_i x_{ij}+u_o \geq 0, \quad j=1,\ldots,n, \quad (14.1)$$
$$\sum_{i=1}^{m} v_i R_{io}=1, \quad (14.2)$$
$$\sum_{i=1}^{m} v_i R''_{io}=1, \quad (14.3)$$
$$\sum_{i=1}^{m} v_i x_{io}+u_o=1, \quad (14.4)$$
$$v_i \geq 0, \quad i=1,\ldots,m, \quad (14.5)$$
$$u_o \text{ is free.} \quad (14.6)$$

The dual form of Model (14) is expressed using real values s, p, q and a nonnegative vector $\lambda=(\lambda_1,...,\lambda_n)$ as Model (15).

$$f_3=\max \quad p+q+s \quad (15)$$

s.t.

$$\sum_{j=1}^{n} \lambda_j x_{ij} + pR_{io} + qR''_{io} + sx_{io} = x_{io}, i=1,...,m, \quad (15.1)$$
$$\sum_{j=1}^{n} \lambda_j + s = 1, \quad (15.2)$$
$$\lambda_j \geq 0, \quad j=1,\ldots,n. \quad (15.3)$$

Thus, $s=1, q=0, p=0, \lambda=0$ is a feasible solution of Model (15). The objective value for this feasible solution is one. This solution is optimal according to the fundamental theorem of duality. Therefore, at least one optimal solution exists for Model (14) with $f_2=1$, that is, $(v_1^*,....,v_m^*,u_o^*)$.
According to this solution, $(v_1,\ldots v_m,u_1,\ldots u_m,u_o)=(v_1^*,\ldots,v_m^*,0,\ldots,0,u_o^*)$ is a feasible solution of Model (12) with $f_1=1$. By the Eq.(13), $\alpha^*+\beta^* \leq 1$.■

Definition1: $DMU_j(j=1,\ldots,n)$ is called GRDM efficient if and only if $\alpha^*+\beta^*=0$ and all constraints of Model (7) are satisfied in the equality (i.e., all slacks are 0). Otherwise, DMU_j is GRDM inefficient.

The efficiency of (x_o, y_o) for DMU_o can be improved by GRDM as Eqs. 16-17:

$$\bar{x}_{io}=\sum_{j=1}^{n} \lambda_j^* x_{ij} = x_{io} - \beta^* R_{io} - \alpha^* R'_{io} - s_i^{-*}, \quad (16)$$
$$i=1,\ldots,m,$$
$$\bar{y}_{ro}=\sum_{j=1}^{n} \lambda_j^* y_{rj} = y_{ro} + \beta^* R_{ro} + \alpha^* R'_{ro} + t_r^{+*}, \quad (17)$$
$$r=1,\ldots,s.$$

GRDM has several properties, which are presented in the following lemmas. RDM is a special case of the GRDM model.

Specifically, the RDM model can be obtained by setting $\alpha = 0$.

Lemmas 1, 2, and 3 have straightforward proof; hence, the proof is omitted here.

Lemma1: The inefficiency score of GRDM is greater than or equal to the inefficiency score of RDM (i.e. $z_2^* \geq z_1^*$).

Lemma2: GRDM is translation-invariant.

Lemma3: GRDM is unit-invariant.

Lemma4: The improved activity $(\overline{x}_{io}, \overline{y}_{ro})$ defined by Eqs 16-17 is GRDM-efficient.

Proof: Let an optimal maximum slack solution of GRDM for DMU_o be $(\alpha^*, \beta^*, \lambda^*, s^{-*}, t^{+*})$. According to Eqs. 16-17, the improved activity for DMU_o is:

$$\overline{x}_{io} = \sum_{j=1}^{n} \lambda_j^* x_{ij} = x_{io} - \beta^* R_{io} - \alpha^* R'_{io} - s_i^{-*}, \quad i = 1, \ldots, m, \tag{18}$$

$$\overline{y}_{ro} = \sum_{j=1}^{n} \lambda_j^* y_{rj} = y_{ro} + \beta^* R_{ro} + \alpha^* R'_{ro} + t_r^{+*}, \quad r = 1, \ldots, s. \tag{19}$$

The GRDM for evaluating the improved activity $(\overline{x}_{io}, \overline{y}_{ro})$ can be written as Model (20).

$$\max \quad \overline{\beta} + \overline{\alpha} \tag{20}$$

s.t.

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \overline{x}_{io} - \overline{\beta}\overline{R}_{io} - \overline{\alpha}\overline{R}'_{io}, \quad i = 1, \ldots, m, \tag{20.1}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq \overline{y}_{ro} + \overline{\beta}\overline{R}_{io} - \overline{\alpha}\overline{R}'_{io}, \quad r = 1, \ldots, s, \tag{20.2}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \tag{20.3}$$

$$\lambda_j, \alpha, \beta \geq 0, \quad j = 1, \ldots, n. \tag{20.4}$$

where

$\overline{R}_{io} = \overline{x}_{io} - \left(\min_{1\leq j\leq n}\{x_{ij}\}\right)$,

$\overline{R}_{ro} = \left(\max_{1\leq j\leq n}\{y_{rj}\}\right) - \overline{y}_{ro}$,

$\overline{R}'_{io} = \left(\max_{1\leq j\leq n}\{x_{ij}\}\right) - \overline{x}_{io}$

and $\overline{R}'_{ro} = \overline{y}_{ro} - \left(\min_{1\leq j\leq n}\{y_{rj}\}\right)$. Using Eqs. 18-19, we obtain

$$\begin{aligned}\sum_{j=1}^{n} \lambda_j x_{ij} &= \left(x_{io} - \beta^* R_{io} - \alpha^* R'_{io} - s_i^{-*}\right) - \\ &\overline{\beta}\left(\left(x_{io} - \beta^* R_{io} - \alpha^* R'_{io} - s_i^{-*}\right) - \left(\min_{1\leq j\leq n}\{x_{ij}\}\right)\right) \\ &- \overline{\alpha}\left(\left(\max_{1\leq j\leq n}\{x_{ij}\}\right) - \left(x_{io} - \beta^* R_{io} - \alpha^* R'_{io} - s_i^{-*}\right)\right) + \hat{s}_i \\ &= x_{io} - R_{io}\left(\beta^* - \beta^*\overline{\beta} + \overline{\beta} + \beta^*\overline{\alpha}\right) - \\ &R'_{io}\left(\alpha^* - \alpha^*\overline{\beta} + \overline{\alpha} + \overline{\alpha}\alpha^*\right) + \\ &\overline{\beta} s_i^{-*} - \overline{\alpha} s_i^{-*} - \overline{s_i^{-}} - s_i^{-*}.\end{aligned} \tag{21}$$

$$\begin{aligned}\sum_{j=1}^{n} \lambda_j y_{rj} = y_{ro} + &\\ &R_{io}\left(\beta^* - \beta^*\overline{\beta} + \overline{\beta} + \beta^*\overline{\alpha}\right) - \\ &R'_{io}\left(\alpha^* - \alpha^*\overline{\beta} + \overline{\alpha} + \overline{\alpha}\alpha^*\right) + \\ &\overline{\alpha} t_r^{+*} - \overline{\beta} t_r^{+*} - \overline{t_r^{+}} - t_r^{+*}.\end{aligned} \tag{22}$$

Three cases are in direct contradiction to the optimal solution of Model (7) for DMU_o. These cases are as follows:

Case1: $\overline{\alpha} = 0, \overline{\beta} > 0$,

$$\begin{aligned}&\left(\beta^* - \beta^*\overline{\beta} + \overline{\beta} + \beta^*\overline{\alpha}\right) + \left(\alpha^* - \alpha^*\overline{\beta} + \overline{\alpha} + \overline{\alpha}\alpha^*\right) \\ &= \beta^* + \alpha^* + \overline{\beta}(1 - \beta^* - \alpha^*) + \overline{\alpha}(1 + \beta^* + \alpha^*) \\ &= \beta^* + \alpha^* + \overline{\beta}\left(1 - (\alpha^* + \beta^*)\right) > \beta^* + \alpha^*.\end{aligned} \tag{23}$$

Case2: $\overline{\alpha} > 0, \overline{\beta} = 0$,

$$\begin{aligned}&\left(\beta^* - \beta^*\overline{\beta} + \overline{\beta} + \beta^*\overline{\alpha}\right) + \left(\alpha^* - \alpha^*\overline{\beta} + \overline{\alpha} + \overline{\alpha}\alpha^*\right) \\ &= \beta^* + \alpha^* + \overline{\beta}(1 - \beta^* - \alpha^*) + \overline{\alpha}(1 + \beta^* + \alpha^*) \\ &= \beta^* + \alpha^* + \overline{\alpha}\left(1 + \alpha^* + \beta^*\right) > \beta^* + \alpha^*.\end{aligned} \tag{24}$$

Case3: $\overline{\alpha} > 0, \overline{\beta} > 0$

$$\begin{aligned}&\left(\beta^* - \beta^*\overline{\beta} + \overline{\beta} + \beta^*\overline{\alpha}\right) + \\ &\left(\alpha^* - \alpha^*\overline{\beta} + \overline{\alpha} + \overline{\alpha}\alpha^*\right) \\ &= \alpha^* + \beta^* + \overline{\beta}\left(1 - (\alpha^* + \beta^*)\right) + \\ &\overline{\alpha}\left(1 + \alpha^* + \beta^*\right) > \alpha^* + \beta^*\end{aligned} \tag{25}$$

Consequently, the only possible case is $(\overline{\alpha}, \overline{\beta}) = (0, 0)$. Furthermore, if $\overline{\alpha} = \overline{\beta} = 0$, then

$$0 \leq \sum_{i=1}^{m} \overline{s_i^-} + \sum_{i=1}^{m} s_i^{-*} + \sum_{r=1}^{s} \overline{t_r^+} + \sum_{r=1}^{s} t_r^{+*} \leq \sum_{i=1}^{m} s_i^{-*} + \sum_{r=1}^{s} t_r^{+*} \quad (26)$$

Hence, $\sum_{i=1}^{m} \overline{s_i^-} + \sum_{r=1}^{s} \overline{t_r^+} = 0$, which implies that $\overline{s_i^-} = \overline{t_r^+} = 0, (i = 1,\ldots,m, r = 1,\ldots,s)$. Thus, the conditions of the GRDM efficiency are satisfied and achieved as claimed. ■

Notably, weights that correspond to the relative importance of the item can be assigned to α and β as follows:

$$\text{Max} \quad w_\beta \beta + w_\alpha \alpha \quad (27)$$

s.t.

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} - \beta R_{io} - \alpha R_{io}^{'}, \quad i = 1,\ldots,m, \quad (27.1)$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \beta R_{io} + \alpha R_{io}^{'}, \quad r = 1,\ldots,s, \quad (27.2)$$

$$\sum_{j=1}^{n} \lambda_j = 1, \quad (27.3)$$

$$\lambda_j, \alpha, \beta \geq 0; \; j = 1,\ldots,n \quad (27.4)$$

where $w_\alpha + w_\beta = 1$ and $w_\alpha, w_\beta \geq 0$.

4. Numerical Illustration

We consider a dataset that consists of 13 DMUs, and each DMU has two inputs, namely x_1 and x_2, and three outputs, namely y_1, y_2 and y_3. The inputs are Cost and Effluent, and the outputs are Saleable, CO_2, and Methane (Sharp et al. [10]). Table 1 shows the data for the 13 DMUs. Models (4) and (7) are applied to the dataset of Table 1. Table 2 shows the results. Three DMUs (DMU2, DMU4, and DMU12) have notably different efficiency scores when evaluated by Models (4) and (7). This means that these DMUs have an intention to move away from the AIP while approaching to the IP. This intention has never seen among other DMUs.

Tables 3 and 4 show the projection points generated by GRDM and RDM, respectively.

In the optimal solutions, while $\alpha^* = 0$ (for DMUs 3, 7, 8, 9, 11 and 13), the optimal solution of the GRDM model is as the RDM model. In this case, the projection point of under evaluation DMU is the same. This confirms that the RDM model is a particular case of the GRDM model.

According to the last rows of Tables 3 and 4, the overall input consumption used by the projected DMUs obtained by GRDM is higher than the overall input consumption used by the projected DMUs with the RDM model. Moreover, the overall output productions produced by the projected DMUs using the GRDM model, are more than the overall output production produced by the projected DMUs using the RDM model.

Table 1. Data set of 13 DMUs

DMU	x_1	x_2	y_1	y_2	y_3
DMU 1	1.03	-0.05	0.56	-0.09	-0.44
DMU 2	1.75	-0.17	0.74	-0.24	-0.31
DMU 3	1.44	-0.56	1.37	-0.35	-0.21
DMU 4	10.8	-0.22	5.61	-0.98	-3.79
DMU 5	1.3	-0.07	0.049	-1.08	-0.34
DMU 6	1.98	-0.1	1.61	-0.44	-0.34
DMU 7	0.97	-0.17	0.82	-0.08	-0.43
DMU 8	9.82	-2.32	5.61	-1.42	-1.94
DMU 9	1.59	0	0.52	0	-0.37
DMU 10	5.96	-0.15	2.14	-0.52	-0.18
DMU 11	1.29	-0.11	0.57	0	-0.24
DMU 12	2.38	-0.25	0.57	-0.67	-0.43
DMU 13	10.3	-0.16	9.56	-0.58	0

Table 2: GRDM and RDM optimal solutions

	GRDM			RDM	
	Efficiency	α^*	β^*	Efficiency	β^*
DMU 1	0.9648	0	0.0351	0.0964	0.0351
DMU 2	0.9177	0.001463	0.0808	0.9181	0.0818
DMU 3	1	0	0	1	0
DMU 4	0.3045	0.6954	0	0.7352	0.2647
DMU 5	0.9242	0	0.0757	0.9242	0.0757
DMU 6	0.9708	0	0.02917	0.9708	0.02917
DMU 7	1	0	0	1	0
DMU 8	1	0	0	1	0
DMU 9	0.9944	0	0.0055	0.9944	0.0055
DMU 10	0.8595	0	0.1404	0.8595	0.1404
DMU 11	1	0	0	1	0
DMU 12	0.8255	0.05214	0.1223	0.8448	0.1551
DMU 13	1	0	0	1	0

Table3: Projection points obtained by GRDM and optimal solution for Lambdas

	$xp(i1)$	$xp(i2)$	$yp(r1)$	$yp(r2)$	$yp(r3)$	Optimal Lambdas
DMU 1	1.02	-0.16	0.78	-0.07	-0.40	$\lambda_3^* = 0.01527, \lambda_7^* = 0.97929, \lambda_{13}^* = 0.00544$
DMU 2	1.36	-0.32	0.94	-0.16	-0.23	$\lambda_3^* = 0.4691, \lambda_7^* = 0.34314, \lambda_{11}^* = 0.1408, \lambda_{13}^* = 0.04696$
DMU 3	1.44	-0.56	1.37	-0.35	-0.21	$\lambda_3^* = 1$
DMU 4	10.27	-0.30	9.31	-0.63	-0.12	$\lambda_8^* = 0.09861, \lambda_{13}^* = 0.90139$
DMU 5	1.29	-0.18	0.71	-0.06	-0.25	$\lambda_3^* = 0.64895, \lambda_7^* = 0.35108,$
DMU 6	1.70	-0.55	1.61	-0.36	-0.20	$\lambda_3^* = 0.94238, \lambda_{13}^* = 0.05762$
DMU 7	0.97	-0.17	0.82	-0.08	-0.43	$\lambda_7^* = 1$
DMU 8	9.82	-2.32	5.61	-1.42	-1.94	$\lambda_8^* = 1$
DMU 9	1.59	0.00	0.52	0.00	-0.37	$\lambda_{11}^* = 1$
DMU 10	2.71	-0.50	2.54	-0.38	-0.18	$\lambda_3^* = 0.73678, \lambda_{13}^* = 0.26322$
DMU 11	1.29	-0.11	0.57	0	-0.24	$\lambda_{11}^* = 1$
DMU 12	1.41	-0.48	1.22	-0.29	-0.22	$\lambda_3^* = 0.96292, \lambda_{13}^* = 0.03708$
DMU 13	10.30	-0.16	9.56	-0.58	0	$\lambda_{13}^* = 1$
Sum	45.17	-5.81	35.56	-4.38	-4.79	

Table4: Projection points obtained by RDM and optimal solution for Lambdas

	$xp(i1)$	$xp(i2)$	$yp(r1)$	$yp(r2)$	$yp(r3)$	Optimal Lambdas
DMU 1	0.97	-0.17	0.82	-0.08	-0.43	$\lambda_3^* = 0.01779, \lambda_7^* = 0.97796, \lambda_{13}^* = 0.00425$
DMU 2	1.21	-0.30	0.96	-0.16	-0.31	$\lambda_3^* = 0.5689, \lambda_7^* = 0.39528, \lambda_{11}^* = 0.03581$
DMU 3	1.44	-0.56	1.37	-0.35	-0.21	$\lambda_3^* = 1$
DMU 4	7.44	-1.00	5.61	-0.77	-0.81	$\lambda_3^* = 0.2699, \lambda_8^* = 0.20583, \lambda_{13}^* = 0.52427$
DMU 5	1.12	-0.14	0.70	-0.04	-0.34	$\lambda_3^* = 0.64895, \lambda_7^* = 0.35105$
DMU 6	1.70	-0.55	1.61	-0.36	-0.20	$\lambda_3^* = 0.94238, \lambda_{12}^* = 0.05762$
DMU 7	0.97	-0.17	0.82	-0.08	-0.43	$\lambda_7^* = 1$
DMU 8	9.82	-2.32	5.61	-1.42	-1.94	$\lambda_8^* = 1$
DMU 9	1.59	0.00	0.52	0.00	-0.37	$\lambda_{11}^* = 1$
DMU 10	2.71	-0.50	2.54	-0.38	-0.18	$\lambda_3^* = 0.75806, \lambda_{13}^* = 0.24194$
DMU 11	1.29	-0.11	0.57	0	-0.24	$\lambda_{12}^* = 1$
DMU 12	1.35	-0.49	1.27	-0.30	-0.25	$\lambda_3^* = 1$
DMU 13	10.3	-0.16	9.56	-0.58	0	$\lambda_{13}^* = 1$
Sum	41.91	-6.47	31.96	-4.52	-5.71	

Table 5 demonstrates the optimal solution obtained by applying Model (27), the weighted GRDM, to the dataset of Table 1. The intention of decision-makers to get away from AIP is assumed to be four times higher than their intention to move toward IP. Table 5 shows that the projection points differ for RDM and GRDM. Consequently, the weighted GRDM model can be used for different management points of view and situations.

Table 5: Optimal solutions and projection points obtained by weighted GRDM when $w_\alpha = 0.8, w_\beta = 0.2$

	Efficiency	α^*	β^*	$xp(i1)$	$xp(i2)$	$yp(r1)$	$yp(r2)$	$yp(r3)$
DMU 1	0.991527	0.003316	0.0291	1	-0.18	0.82	-0.08	-0.42
DMU 2	0.973224	0.02184	0.04652	1.52	-0.36	1.16	-0.2	-0.22
DMU 3	1	0	0	1.44	-0.56	1.37	-0.35	-0.21
DMU 4	0.443671	0.695411	0	10.25	-0.37	9.17	-0.66	-0.19
DMU 5	0.982767	0.003693	0.071394	1.24	-0.4	1.14	-0.24	-0.3
DMU 6	0.977641	0.027948	0	1.73	-0.55	1.64	-0.36	-0.2
DMU 7	1	0	0	0.97	-0.17	0.82	-0.08	-0.43
DMU 8	1	0	0	9.82	-2.32	5.61	-1.42	-1.94
DMU 9	0.998894	0	0.005531	1.29	-0.11	0.57	0	-0.24
DMU 10	0.96898	0.012194	0.106325	5.37	-0.38	5	-0.45	-0.12
DMU 11	1	0	0	1.29	-0.11	0.57	0	-0.24
DMU 12	0.933826	0.052142	0.122303	1.77	-0.55	1.67	-0.36	-0.2
DMU 13	1	0	0	10.3	-0.16	9.56	-0.58	0
Sum				47.99	-6.22	39.1	-4.78	-4.71

5. Conclusion

In many situations, for instance, the banking industry, decision-maker prefers to move away from the worst possible case (or AIP) rather than move toward IP. In particular, in all sports leagues, some weak teams struggle to not lie among the last there or four teams because they have to compete in the lower league for the next season. In another case, all banks require account balance to be increased by their customers. If not, the bank would be about to bankruptcy in the not-too-distant future. The GRDM model can readily deal with these sorts of problems. Our proposed model is formulated based on two main objectives. First, try to move toward the best possible situation (IP); the RDM model also considers this issue. Second, try to move away from the worst possible situation (AIP) not taken into consideration by the RDM model. Therefore, the GRDM model can quickly transform to RDM model by excluding the AIP from the model. Consequently, the RDM model is a particular case of the GRDM model.

It should be mentioned that our proposed method is still suffering from excluding the environmental factor constraints, which can be considered to utilize the proposed model in a real-life situation; it can, therefore, be addressed as a future study. Moreover, investigating the proposed model in the presence of non-homogenous DMUs would be the future study. Furthermore, big data in DEA has not been considered in the presence of the negative data yet and is highly recommended to interested researchers.

References

[1] Farrell M (1957). The measurement of productive efficiency, Journal of the Royal Statistical Society Series A: General,120(3), 253–281.

[2] Charnes A, Cooper WW and Rhodes E (1978), Measuring efficiency of decision making units, European Journal of Operational Research, 2, 429–444.

[3] Smith P (1990). Data Envelopment Analysis Applied to Financial Statements," Omega, 18(2), 131-138.

[4] Feroz E, Kim S and Rab R (2003). Financial Statement Analysis: A Data Envelopment Analysis Approach, Journal of the Operational Research Society, 54(1), 48-58.

[5] Murthi B, Choi Y and Desai P (1997). Efficiency of Mutual Funds and Portfolio Performance Measurement: A Non-Parametric Approach, European Journal of Operational Research, 98(2), 408-418.

[6] Pastor J T (1994). How to discount environmental effects in DEA: An application to Bank branches. Working paper No. 011/94, Depto. De Estadistica e Investigacion Operativa, Universidad de Alicante, Spain.

[7] Lovell CAK (1995). Measuring the macroeconomic performance of the Taiwanese economy, International Journal of Production Economics, 39, 165–178.

[8] Seiford L M and Zhu J (2002). Modeling undesirable factors in efficiency evaluation, European journal of operational research, 142, 16–20.

[9] Silva Portela M, Thanassoulis E and Simpson G (2004). Negative data in DEA: a directional distance approach applied to bank branches, Journal of the Operational Research Society, 55(10), 1111–1121.

[10] Ueda T and Hoshiai Y (1997). Application of component analysis for parsimonious summarization of DEA inputs and/or outputs, Journals of operational research society Japan, 40: 466–478.

[11] Chung Y, Fare R and Grosskopf S (1997). Productivity and undesirable outputs: a directional distance function approach. Journal of environmental management, 51, 229–240.

[12] Sharp J A, Liu W B and Meng W (2006). A modified slacks-based measure model for data envelopment analysis with 'natural' negative outputs and inputs. Journal of Operational Research Society, 57, 1–6. 2006.