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Comparison of ranking methods in Data envelopment analysis using Rough set theory

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Abstract

Data envelopment analysis is an efficiency evaluation method for assessing DMUs. In this context, ranking efficient DMUs is very important. So far, various models have been provided for this purpose. According to the results of these methods, it could be seen that the ranks obtained for each DMU will vary. Due to this issue, we cannot provide a specific method to specify ranking method to DM. In order to solve this problem, this paper employs the Rough Set theory to determine the compatibility or incompatibility of these methods together and thus provide the appropriate ranking among the existing methods.

Keywords: Data envelopment analysis (DEA), Ranking, Rough set theory (RST).

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1. Introduction

It is often necessary in real performance assessment practice to rank a group of decision making units (DMUs) in terms of their efficiencies. Data envelopment analysis (DEA) developed by Charnes et al. [1] has been universally recognized as a useful tool of performance assessment, but very often more than one DMU is evaluated as DEA efficient, which makes DEA efficient units unable to be compared or ranked.

To rank DEA efficient units, quite a lot of research has been done and many ranking methodologies have been suggested in the DEA literature. For example, Andersen and Petersen [2] proposed a procedure that was later referred to as the super-efficiency method for ranking DEA efficient units. Super-efficiency refers to the DEA efficiency measured by excluding the DMU under evaluation from the constraints of DEA models and has been deeply researched in the literature [3, 4, 5 and 6]. This paper employs the Rough Set theory to determine the compatibility or incompatibility of these methods together and thus provide the appropriate ranking among the existing methods.

The following is organized as follows. In Section 2, we present a literature review on DEA ranking. In section 3 we introduce DEA and CCR model. In section4, we explain rough set theory. In section 5 we have proposed method. Finally, in section 6 we have conclusion.

2. Literature review

Jahanshahloo et al. [7] proposed a ranking system to rank DEA efficient units in terms of their influences on DEA inefficient units. The influence of a DEA efficient unit on DEA inefficient units is measured by the efficiency change of DEA inefficient units before and after the DEA efficient unit is excluded from their reference set. The DEA efficient unit that can cause the biggest efficiency change of DEA inefficient units when it is removed from their reference set is deemed as the most important DMU.

In the first stage, DEA was performed to distinguish between DEA efficient and inefficient units. In the second stage, the discriminant analysis was performed to provide a score function for the two sets, efficient and inefficient, based on the linear combination of all inputs and outputs.

Friedman and Sinuany-Stern [8] utilized the canonical correlation analysis to provide a single weight vector for inputs and outputs, respectively, common to all DMUs. They first constructed a composite input variable as a linear combination of inputs and a composite output variable as a linear combination of outputs, and then maximized the coefficient of correlation between the composite input and the composite output to generate a common set of weights.

The efficiencies of DMUs were defined with the obtained common weights as the ratio of the composite output to the composite input. Based upon the efficiencies defined with the common weights, all DMUs were ranked.

3. CCR model

Suppose that there are n DMUs to be evaluated in terms of m inputs and s outputs. Let $x_{ij}(i=1,\dots,m)$ and $y_{rj}(j=1,\dots,n)$ be the input and output values of DMUj (j=1,…,n). Then the efficiency of DMUj can be defined as

$$\begin{aligned} & minimize \quad \theta \\ s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io} \quad , \; i=1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \; , \; r=1,\dots,s \quad (1) \\ & \lambda_j \ge 0 \quad , \quad j=1,\dots,n \end{aligned}$$

The above model is introduced as input oriented CCR model. LP model (1) is solved n times in total, each time for one DMU. As a result, at least one DMU is evaluated as DEA efficient, but very often more than one DMU proves to be DEA efficient. How to distinguish between

DEA efficient units has long been a research topic and attracted considerable interest in the DEA literature. In the following section, we introduce Rough Set Theory (RST) combining whit DEA, thus provide the appropriate ranking among the existing methods.

4. Rough Set Theory

4.1. Information system

Formally, an *information system* IS (or an approximation space) could be seen as a system $IS(U, A)$[9, 10 and 11]. Where U is the universe (a finite set of objects, U= $\{x_1, x_2, \dots, x_n\}$) and A is the set of attributes (features, variables). Each attribute a ∈A (attribute a belonging to the considered set of attributes A. defines an information function $U \rightarrow V_a$, where V_a is the set of values of a, called the domain attribute a. For example, measurements performed for 4 objects. The results can be organized in a matrix (4 × 3).

$$\begin{matrix} 2 & 1 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \\ 2 & 2 & 3 \end{matrix}$$

Using the terminology of the rough sets theory, this data set can be considered as an information system IS= (*U, A)*, where universe U and attributes A correspond to the set of objects and to the set of variables, respectively:

$$U = \{x_1, x_2, x_3, x_4\}$$
$$A = \{a_1, a_2, a_3\}$$

The domains of the particular attributes are:

$$V_1 = \{2,3\}$$
$$V_2 = \{1,2\}$$
$$V_3 = \{1,3\}.$$

i.e., the domain of each attribute is the set of values of this attribute. The information function f_a for this system is presented in Table 1.

Table 1: Information system

U	a_1	a_2	a_3
x_1	2	1	3
x_2	3	2	1
x_3	2	1	3
x_4	2	2	3

4.2. Indiscernibility relation

For every set of attributes $B \subset A$, an *indiscernibility relation* Ind (*B)* is defined in the following way:

Two objects, x_1 and x_2 , are indiscernible by the set of attributes *B* in *A*, if $b(x_i) = b(x_j)$ for every $b \subset B$. The equivalence class of Ind (B) is called *elementary set* in *B* because it represents the smallest discernible groups of objects. For any element x_1 of *U*, the equivalence class of x_1 in relation Ind (*B)* is represented as $[x_i]_{Ind(B)}$. The construction of elementary sets is the first step in classification with rough sets.

4.3. Lower and upper approximations

Let *X* denote the subset of elements of the universe $U(X \subset U)$. The lower approximation of *X* in $B(B \subseteq A)$ denoted as $\underline{BX}$, is defined as the union of all these elementary sets which are contained in *X*. More formally:

$$\underline{BX} = \{x_i \in U | [x_i]_{Ind(B)} \subset X\}$$

The above statement is to be read as: the lower approximation of the set *X* is a set of objects x_i, which belongs to the elementary sets contained in *X* (in the space *B*).

The upper approximation of the set *X*, denoted as BX, is the union of these elementary sets, which have a non-empty intersection with *X*:

$$BX = \{x_i \in U | [x_i]_{Ind(B)} \cap X \neq \emptyset\}$$

For any object *x* of the lower approximation of *X* (i.e.,$x_i \subset \underline{BX}$), it is certain that it belongs to *X*. For any object

x of the upper approximation of X (i.e., $x_i \subset BX$), we can only say that x_i may belongs to X. The difference:

$$BNX = BX - \underline{BX}$$

is called a boundary of X in U.

If the lower and upper approximation are identical (i.e.,$BX = \underline{BX}$), then set X is definable, otherwise, set X is indefinable in U. There are four types of indefinable sets in U:

1. If $\underline{BX} \neq \emptyset$ and BX≠U, X is called roughly definable in U;
2. If $\underline{BX} \neq \emptyset$ and BX=U, X is called externally indefinable in U;
3. If $\underline{BX} = \emptyset$ and BX≠U, X is called internally indefinable in U;
4. If $\underline{BX} = \emptyset$ and BX=U, X is called totally indefinable in U,

Where $\emptyset$ denotes an empty set.

4.4. Independence of attributes

In order to check, whether the set of attributes is independent or not, one checks for every attribute whether its removal increases the number of elementary sets in the IS or not.

If Ind (A) =Ind (A-a_i) then the attribute a_i is called superfluous. Otherwise, the attribute a_i is in-dispensable in A.

4.5. *Core and reduct of attributes*

If the set of attributes is dependent, one can be interested in finding all possible minimal subsets of attributes, which lead to the same number of elementary sets as the whole set of attributes (*reducts*) and in finding the set of all indispensable attributes (*core*).

The concepts of core and reduct are two fundamental concepts of the rough sets theory. The reduct is the essential part of an IS, which can discern all objects discernible by the original IS. The core is the common part of all reducts. To compute reducts and core, the *discernibility matrix* is used. The discernibility matrix has the dimension n×n, where n denotes the number of elementary sets and its elements are defined as the set of all attributes which discern elementary sets $[x]_i$and $[x]_j$.

Boolean function, constructed in the following way: to each attribute from the set of attributes, which discern two elementary sets, (e.g.,$\{a_1, a_2, a_3\}$), we assign a Boolean variable 'a', and the resulting Boolean function attains the form $(a_1 + a_2 + a_3)$ or it can be presented as $(a_{1\vee} a_{2\vee} a_3)$ If the set of attributes is empty, we assign to it the Boolean constant 1.

Table2: Discernibility matrix

	Set1	**Set2**	**Set3**	**Set 4**	**Set 5**
Set 1					
Set 2	a_1,a_2,a_3				
Set 3	a_2	a_1,a_3			
Set 4	a_1,a_3	a_1,a_2,a_3	a_1,a_2,a_3		
Set 5	a_1,a_3	a_1,a_2,a_3	a_1,a_2,a_3	a_3	

For the discernibility matrix presented in Table 2, the discernibility function has the following form:

$$\begin{aligned} f(A) = {} & (a_1 + a_2 + a_3)a_2(a_1 + a_3)(a_1 + a_3) \\ & \times (a_1 + a_3)(a_1 + a_2 + a_3)(a_1 + a_2 + a_3) \\ & \times (a_1 + a_2 + a_3)(a_1 + a_2 + a_3) \\ & \times a_3 \quad . \end{aligned}$$

To calculate the final form of f (A), the absorption law is applied. According to the absorption law, if the elementary set 1 differs from the elementary set 2 due to the attributes a_1, a_2, a_3 , and from the elementary set 3 due to the attribute a_2 , it is enough to take into the account the attribute a_2 only, which discerns this set from both set 2 and set 3, i.e.:

$$(a_1 + a_2 + a_3)a_2 = a_2.$$

One has to take into account the following sets of attributes:

$\{a_2\}$, $\{a_1, a_2, a_3\}$, $\{a_1, a_3\}$, $\{a_1, a_3\}$

Then, the discernibility function attains the following form:
$(a_2)(a_1+a_2+a_3)(a_1+a_3)(a_1+a_3) = a_2(a_1+a_3) = a_2a_1 + a_2a_3$

Then reducts are as follows:
$\{a_2, a_1\}$
$\{a_2, a_3\}$
It means that minimal sets of attributes contain: a_2, a_1 or a_2, a_3.
And the core contains: a_2.

5. Proposed method

In this method we have used five ranking methods for 50 DMUs using 3 inputs to produce 4 outputs. Ranking methods that we discuss contains AP, MAJ, LJK, Super SBM and $l_1 - norm$. These models are as follows:

1) AP model:

$$\begin{aligned} & Minimize \quad \theta \\ & s.t \quad \sum_{\substack{j=1\\j\neq 0}}^{n} \lambda_j x_{ij} \le \theta x_{io}, \quad i=1,\dots,m \\ & \quad \sum_{\substack{j=1\\j\neq 0}}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r=1,\dots,s \qquad (1) \\ & \quad \lambda_j \ge 0, \quad j=1,\dots,n, \quad j\neq 0 \end{aligned}$$

2) MAJ model:

$$\begin{aligned} & Minimize \quad 1+w_0 \\ & s.t \quad \sum_{\substack{j=1\\j\neq 0}}^{n} \lambda_j x_{ij} \le x_{io}+w_0, \quad i=1,\dots,m \\ & \quad \sum_{\substack{j=1\\j\neq 0}}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r=1,\dots,s \qquad (2) \\ & \quad \lambda_j \ge 0, \quad j=1,\dots,n, \quad j\neq 0 \end{aligned}$$

3) LJK model:

$$\begin{aligned} & Minimize \quad 1+\frac{1}{m}\sum_{i=1}^{m}\frac{s_{i2}^{+}}{R_i^{-}} \\ & s.t \quad \sum_{\substack{j=1\\j\neq 0}}^{n} \lambda_j x_{ij} \le x_{io}+S_{i2}^{+}, \quad i=1,\dots,m \\ & \quad \sum_{\substack{j=1\\j\neq 0}}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r=1,\dots,s \qquad (3) \\ & \quad \lambda_j, S_{i2}^{+} \ge 0, \quad j=1,\dots,n, j\neq 0 \end{aligned}$$

4) Super SBM model:

$$\begin{aligned} & Minimize \quad \frac{1}{m}\sum_{i=1}^{m}\frac{\overline{x_{io}}}{x_{io}} \\ & s.t \quad \sum_{\substack{j=1\\j\neq 0}}^{n} \lambda_j X_j \le \bar{X} \qquad (4) \\ & \quad \sum_{\substack{j=1\\j\neq 0}}^{n} \lambda_j Y_j \ge \bar{Y} \\ & \quad X_0 \le \bar{X} \quad and \quad Y_0 = \bar{Y} \\ & \quad \lambda \ge 0. \end{aligned}$$

5) $l_1 - norm$ model:

$$\begin{aligned} & \text{Minimize} \quad \Gamma_c^0(X,Y) = \sum_{i=1}^{m} x_i - \sum_{r=1}^{s} y_r + \alpha \\ & s.t \quad \sum_{\substack{j=1\\j\neq 0}}^{n} \lambda_j x_{ij} \le x_i, \quad i=1,\dots,m \quad n \\ & \quad \sum_{\substack{j=1\\j\neq 0}}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r=1,\dots,s \qquad (5) \\ & \quad x_i \ge x_{io}, \quad i=1,\dots,m \\ & \quad 0 \le y_r \le y_{ro}, \quad r=1,\dots,s \\ & \quad \lambda_j \ge 0, \quad j=1,\dots,n, \ j\neq 0 \end{aligned}$$

Data are illustrated in table 3. Using inputs and outputs of table 3 for all DMUs to the ranking methods in the input oriented and linear forms, the results are illustrated in table 4.

Table3: Inputs and outputs

	I_1	I_2	I_3	O_1	O_2	O_3	O_4
DMU_1	6.8176	73.3712	27.514	11.6299	40.112	131.481	72.962
DMU_2	6.4759	58.1469	25.9553	7.3154	64.4024	117.684	59.2133
DMU_3	7.0749	37.5742	18.2163	14.6737	77.4713	179.39	50.0939
DMU_4	9.6416	54.9204	23.1552	18.5383	37.3602	150.28	65.9272
DMU_5	5.1673	28.0567	19.9884	5.3657	89.1125	109.616	77.1309
DMU_6	7.9737	62.1998	29.7654	16.3677	77.056	153.742	86.5724

DMU_{7}	9.1299	60.4314	15.4367	16.4505	72.9396	174.541	33.0599
DMU_{8}	9.0335	53.2831	18.4318	14.7432	34.2023	122.852	79.5042
DMU_{9}	7.9733	20.6806	11.7678	10.6197	41.0915	89.073	62.9989
DMU_{10}	10.4028	74.5141	27.2702	6.3051	42.2523	90.194	41.21
DMU_{11}	10.535	40.5029	15.175	18.1169	42.2251	202.929	22.1656
DMU_{12}	7.5216	78.951	20.0106	6.1621	65.3723	209.954	61.1574
DMU_{13}	10.2736	69.4222	16.6756	4.8153	56.614	67.067	21.223
DMU_{14}	5.4337	70.8021	20.5875	16.1399	80.1004	173.738	40.7647
DMU_{15}	5.5442	41.7202	21.0456	3.4914	65.7695	205.279	68.7887
DMU_{16}	11.5346	34.8871	24.418	15.6982	76.0163	69.049	31.9358
DMU_{17}	8.933	52.9791	21.6323	23.3369	54.541	138.702	38.92
DMU_{18}	7.4159	39.0299	15.9866	19.5988	44.7819	68.487	60.3163
DMU_{19}	9.0213	30.8964	26.7954	19.6459	60.7929	134.65	55.9545
DMU_{20}	8.8467	18.9441	7.5775	3.1479	76.7046	186.468	23.3778
DMU_{21}	7.3275	51.6098	18.4259	23.1998	58.5752	149.995	21.6579
DMU_{22}	10.325	74.2512	5.6157	9.4975	83.4719	84.576	22.4143
DMU_{23}	9.5811	32.9693	25.6034	5.2755	41.0002	104.01	39.4723
DMU_{24}	8.032	51.9184	6.3562	22.0525	66.9434	113.452	41.1571
DMU_{25}	7.4556	76.921	11.3744	5.3221	68.5176	155.559	81.0507
DMU_{26}	9.8495	51.2309	12.1209	15.4141	36.4931	103.63	74.5299
DMU_{27}	7.3196	35.4983	11.3819	6.3377	73.7703	104.143	83.5581
DMU_{28}	7.6452	54.2627	11.4367	8.4409	59.7899	189.691	69.9514
DMU_{29}	9.676	27.3091	20.3611	11.618	76.1819	178.174	34.5995
DMU_{30}	9.0625	23.1278	16.393	19.1899	87.4956	146.727	61.6551
DMU_{31}	8.6929	74.421	15.2862	23.9441	38.5084	134.786	46.5696
DMU_{32}	7.3124	53.2943	7.2791	18.9503	43.6247	101.13	60.5453
DMU_{33}	9.7168	27.9953	17.0014	4.5951	60.9729	125.273	66.0234
DMU_{34}	7.8431	43.9414	20.2221	19.8725	58.922	187.581	78.1689
DMU_{35}	5.003	59.3118	11.6088	3.8512	83.8574	155.887	28.5481
DMU_{36}	6.9213	31.869	12.2079	4.6227	45.8385	156.968	51.5049
DMU_{37}	8.0415	46.3159	11.65	14.0104	79.3883	136.409	57.0493
DMU_{38}	10.6827	59.8838	5.601	23.0456	43.3045	91.963	22.8577
DMU_{39}	5.9878	67.3408	23.15	4.803	85.8556	144.941	59.8431
DMU_{40}	8.9922	79.2689	16.2452	16.9509	41.2918	144.971	33.9221
DMU_{41}	6.0944	71.5173	19.6405	7.3298	56.5796	172.643	42.2892
DMU_{42}	8.9529	17.0637	24.825	7.3806	45.0912	194.714	77.228
DMU_{43}	4.9999	54.8167	14.5489	19.4217	51.442	95.339	39.5136
DMU_{44}	7.7706	62.1049	17.7911	23.4604	37.3905	117.852	21.63
DMU_{45}	12.2963	62.3651	12.2726	21.9064	60.0036	128.66	39.4824
DMU_{46}	8.4139	15.9147	8.7871	9.5354	49.1705	173.095	86.3274
DMU_{47}	8.6348	21.5584	6.9205	23.6945	74.6365	154.293	51.2345
DMU_{48}	7.7384	26.3013	13.6584	10.2546	61.2353	130.634	39.7003
DMU_{49}	10.3548	48.7574	27.7103	17.812	79.1982	87.971	83.4978
DMU_{50}	10.2744	77.9046	8.2335	21.9547	39.4692	110.23	79.6506

We used Rough Set Theory for scores of ranking methods as attributes and DMUs as objectives.
Table 4 illustrates scores of ranking methods for these DMUs. We used these methods for all efficient and inefficient DMUs.

Table 4: Results

	AP	MAJ	LJK	SUPER SBM	L_1
DMU_1	0.87	1	1	1	0
DMU_2	0.72	1	1	1	0
DMU_3	1.01	1.09	1	1	0.09
DMU_4	0.73	1	1	1	0
DMU_5	1.56	4.83	1.12	1.28	4.37
DMU_6	0.92	1	1	1	0
DMU_7	0.81	1	1	1	0
DMU_8	0.83	1	1	1	0
DMU_9	0.82	1	1	1	0
DMU_{10}	0.36	1	1	1	0
DMU_{11}	0.86	1	1	1	0
DMU_{12}	0.89	1	1	1	0
DMU_{13}	0.44	1	1	1	0
DMU_{14}	1.23	2.23	1.03	1.08	1.23
DMU_{15}	1.4	3.71	1.08	1.17	2.83
DMU_{16}	0.62	1	1	1	0
DMU_{17}	0.83	1	1	1	0
DMU_{18}	0.96	1	1	1	0
DMU_{19}	0.84	1	1	1	0
DMU_{20}	1.2	4.57	1.06	1.16	6.16
DMU_{21}	1	1	1	1	0
DMU_{22}	1.38	3.12	1.02	1.13	2.12
DMU_{23}	0.47	1	1	1	0
DMU_{24}	1.01	1.06	1	1	0.09
DMU_{25}	1.04	1.31	1.01	1.02	0.39
DMU_{26}	0.8	1	1	1	0
DMU_{27}	1.11	2.02	1.03	1.07	1.63
DMU_{28}	1.03	1.26	1.01	1.01	0.33
DMU_{29}	0.87	1	1	1	0
DMU_{30}	1.08	2.5	1.01	1.03	2.28
DMU_{31}	0.84	1	1	1	0
DMU_{32}	1.1	1.72	1.02	1.04	0.87

DMU_{33}	0.67	1	1	1	0
DMU_{34}	1.1	1.88	1.02	1.04	0.91
DMU_{35}	1.35	3.23	1.08	1.2	3.06
DMU_{36}	0.88	1	1	1	0
DMU_{37}	0.91	1	1	1	0
DMU_{38}	1.19	2.04	1.01	1.06	1.04
DMU_{39}	0.89	1	1	1	0
DMU_{40}	0.7	1	1	1	0
DMU_{41}	0.85	1	1	1	0
DMU_{42}	1.06	1.84	1.01	1.04	1.35
DMU_{43}	1.3	2.63	1.05	1.11	1.69
DMU_{44}	0.87	1	1	1	0
DMU_{45}	0.62	1	1	1	0
DMU_{46}	1.85	12	1.18	1.63	23
DMU_{47}	1.96	12	1.19	1.62	14
DMU_{48}	0.79	1	1	1	0
DMU_{49}	0.79	1	1	1	0
DMU_{50}	1.13	2.07	1.01	1.04	1.07

Using rough set theory for this information system we obtained reduct as bellow:

Reduct: $\{AP, MAJ\}$

It means that we can reduce set of attributes (ranking methods) and the reduct demonstrates minimal set of ranking methods. So AP and MAJ method are incompatible with each other so we can give to DM these two methods for ranking instead of all five methods. For this situation core and reducts are the same, since we have just one reduct.

6. Conclusion

This paper employed the Rough Set theory to determine the compatibility or incompatibility of some of ranking methods together and thus provided the appropriate ranking among the existing methods. We reduced set of attributes (ranking methods). So we demonstrated that AP and MAJ method are incompatible with each other so we can give to DM these two methods for ranking instead of all five methods. For this situation core and reducts are the same, since we have just one reduct.

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