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Meta cost Malmquist Index with variable relative importance as a function of time in different period and FDH models of DEA

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Abstract

Meta Cost Malmquist Index explains change of cost productivity of Decision Making Unit (DMUs) in several time periods. The Trade Offs approach is an advanced tool for the improvement of the discrimination of Data Envelopment Analysis (DEA) models. They used CRS models in DEA for computing this index, since the convexity assumption is strong condition for computing, so for solving this problem in this paper we use Free Disposal Hull (FDH) models in DEA for computing Meta Cost Malmquist Index. Also in this paper Meta Cost Malmquist Index is evaluated considering in fact that relative importance of input and outputs id different periods are different. In the papers concerning Meta Cost Malmquist Index this fact is not considered, which is very important from managerial point of you. The main advantage this index is that, it is circular.

Keywords: Trade Offs, Meta Cost Malmquist Index, Meta Cost Frontier, Variable Relative, Function of Time, Free Disposal Hull (FDH) Model

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1. Introduction

Data Envelopment Analysis (DEA) is a mathematical programming technique that measures the relative efficiency of Decision Making Units (DMUs) with multiple inputs and outputs. Charnes and et al. (1978) first proposed DEA as an evaluation tool to measure and compare the relative efficiency of DMUs. Their model assumed Constant Returns to Scale (CRS, the CCR model) and the model with Variable Return to Scale (VRS, the BCC model) was developed by Banker and et al (1984). Podinovski suggests the incorporation of production Trade Offs in to DEA models, under this circumstance (Podinavski 2004), when we use Trade Offs in our models; the original technology expands to include the new area, Podinovski and et al (2004) show that the Production Possibility Set (PPS), generated by the traditional DEA axioms, may not include all the producible production points, the PPS generated by the DEA models is only the subset of the PPS with Trade Offs.

Also Maria Portella and Thanassoulis (2008), defined Meta Efficiency and based on Meta Malmquist Index, they computed Meta Malmquist Index in CRS and VRS models of DEA. Meta Malmquist was computed based on change of Meta Efficiency.

The structure of the paper is as follows: In section (2) we explain Cost Efficiency for DMUs in different models of DEA. In section (3) we described FDH Models of DEA. In section (4) we explain Meta Cost Efficiency for DMUs in different models of DEA (CRS, VRS, TO) with Variable Relative Importance as a Function of Time in Different Period and we use FDH Models. In the next section (5), we define Meta Cost Malmquist Index with Variable Relative by using FDH Models. In section (6), (7), we compute Meta Cost Malmquist Index, by using the Meta Cost Frontier to compare the productivities of units and we explain the advantage of circular Meta Cost Malmquist Index with Variable Relative Importance as a Function of Time in Different Period and by using FDH Models. The last section summarizes and concludes.

2. Cost Efficiency in Different Models of DEA (CRS, VRS, TO)

Assuming that there are n DMUs each with m inputs and s outputs, we evaluate the Cost Efficiency of DMU_o, $o \in \{1,...,n\}$ in the following way:

$$CX^{(CRS)} = \min \sum_{i=1}^{m} C_{io} X_i \qquad (1)$$

$$\begin{aligned} s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le x_i, && i = 1,...,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, \ r = 1,...,s \\ & \lambda_j \ge 0, && j = 1,...,n \\ & x_i \ge 0, && i = 1,...,m \end{aligned}$$

Where j is the DMU index $j = 1,...,n$, r the output index, $r = 1,...,s$ and i the input index $i = 1,...,m$, y_{rj} value of the rth output for the jth DMU, x_{ij} the value of the ith input for the jth DMU_o and $C = (C_1,...,C_m)$ is the common unit input price or unit cost vector. Let the optimal solution obtained from solving model (1) be (X^*, λ^*), then the Cost Efficiency is defined in ratio from:

$$E_c^{(CRS)}(\text{CostEfficiency}) = \frac{CX \quad CRS}{CX_o} = \frac{\sum_{i=1}^{m} c_{io} x_i}{\sum_{i=1}^{m} c_{io} x_{io}} \qquad (2)$$

It is alleged that $0 \le EC \le 1$; moreover, $DMU_o = (x_o, y_o)$ is Cost Efficient if and only if EC = 1. By similar way, we can compute the Cost Efficiency of DMU_o in VRS model of DEA by addition a constraint $\sum_{j=1}^{n} \lambda_j = 1$ to model (1).

Supposing there are trade-offs, we shall represent the trade-offs in form (P_{if}, Q_{rf}) where $i=1,...,m$, $r=1,...,s$ and $f=1,...,l$ (for more details about trade-offs model of DEA see Podinovski (2004)).

We evaluate the Cost Efficiency of DMU_o, $o \in \{1,...,n\}$ in trade-offs model of DEA according to the following model:

$$CX^{(TO)} = \min \sum_{i=1}^{m} C_{io} x_i \qquad (3)$$

$$\begin{aligned} s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + \sum_{f=1}^{l} \pi_f p_{if} \le x_i, && i=1,...,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} + \sum_{f=1}^{l} \pi_f q_{rf} \ge y_{ro}, && r=1,...,s \\ & \lambda_j \ge 0, && j=1,...,n \\ & x_i \ge 0, && i=1,...,m \\ & \pi_f \ge 0, && f=1,...,l \end{aligned}$$

Therefore the cost efficiency of DMU_o in Trade Offs model of DEA is:

$$E_c^{(T0)}(\text{Cost Efficiency}) = \frac{CX \quad To}{CX_o} = \frac{\sum_{i=1}^{m} c_{io} x_i}{\sum_{i=1}^{m} c_{io} x_{io}}$$

3. Free Disposal Hull (FDH) Models

Considering the observed output vector as $Y_j \in R^s$ and the input vector as, $X_j \in R^m$, we assume that the inputs and outputs are nonnegative and $X_j \ne 0$, $Y_j \ne 0$ for DMU_j, $j=1,...,n$.

The basic motivation for introducing FDH model is to make sure that the efficiency evaluation is effected from only actually observed performances. For using FDH in DEA models, Deprins, Simar and Tulknes make some assumptions and extends the axioms of PPS in the following manner (for more details about FDH Models see [5,6]).

Assumption:

1-The main point for making production possibility set is removing convexity axiom.

Extended axioms:

1- (Nonempty). The observed; $(X_j, Y_j) \in T$, $j=1,...,n$

2- (Proportionality). If $(X, Y) \in T$, then $(\lambda X, \lambda Y) \in T$ for all $\lambda \ge 0$.

3- (Free disposability). If $(X, Y) \in T$, $\bar{X} \le X$, $\bar{Y} \ge Y$, then $(X, Y) \in T$.

4- (Minimum extrapolation). T is the smallest set that satisfies axiom 1-3. (Where T is, $T = \{(X, Y)\}$ output vector $Y \ge 0$ can produced from input vector $X \ge 0$ }).

Now, the PPS can be defined on the basis of the following the minimal PPS (PPSFDH−CRS) that satisfies axioms (1-4) is:

$$PPSFDH - CCR = U_{i=1}^{m} \{(x, y) \mid x \ge \lambda_j^i x_j, y \le \lambda_j^i y_j, \lambda_j^i \ge 0, (j=1,...,n)\}$$

Based on, PPSFDH−CCR for assessing the efficiency of $DMU_k, k=1,...,n$ that is defined from this PPS, we have following model:

DEA model with FDH technology and input orientation:

$$MinMax\, \theta^k \qquad (5)$$

$$\begin{aligned} s.t \quad & \lambda_{jk} x_{ij} \le \theta^k x_{ik}, && i=1,...,m && (a) \\ & \lambda_{jk} y_{ri} \ge y_{ik}, && r=1,...,s && (b) \\ & \lambda_{jk} \ge 0, && j,k=1,...,n \end{aligned}$$

By computing λ_{jk} from constraint (b) we will have:

$$\lambda_{jk} \ge \frac{y_{ik}}{y_{rj}}, \qquad r=1,...,s$$

Let $\lambda_{lk} = Max \left\{ \frac{y_{ik}}{y_{rj}} \middle| r=1,...,s \right\} = \frac{y_{1k}}{y_{1j}}$ (6)

So

$$\begin{cases} \theta_j^k \geq \dfrac{1k^* x_{ij}}{x_{ik}}, & i=1,\ldots,m \\ \theta_j^{*k} = \max\left\{\dfrac{1k^* x_{ij}}{x_{ik}}\right\}, & i=1,\ldots,m \end{cases} \quad (7)$$

Therefore $\theta^{*k} = \min \theta_j^{*k}, j=1,\ldots,n \quad (8)$

Similarly, we can compute efficiency of DMUk in VRS model of FDH, by following way:

$$\begin{aligned} \min \quad & \theta \qquad\qquad (9) \\ s.t \quad & X\lambda \leq \theta X_k \\ & Y\lambda \geq Y_k \\ & 1\lambda = 1,\ \lambda_j \in \{0,1\} \end{aligned}$$

Model (5) is mix integer programming, λ is integer variable and θ is free variable.

4. Meta Cost Efficiency in Different Models of DEA with Variable Relative Importance as a Function of Time in Different Period by using FDH Models in DEA

Consider $\text{DMUs}(1,\ldots,n)$ observed over time period $t\,(1,2,\ldots,n)$, T so that Meta period covers T periods. Let $\left(X_{ij}^{t} Y_{ij}^{t}\right)$ be respectively the i-th input and r-th output level of DMU_j in period t within the Meta period, The Meta Cost Efficiency of DMU_o and $o \in (1,2,\ldots,n)$ observed in some period $r \in (1,2,\ldots,t)$ is $CX_{oc}^{m(FX-CRS)}$:

$$\begin{aligned} CX_{oc}^{m(FX-CRS)} = \min \theta_j^{oc} = \sum_{i=1}^{m} c_{io}^{c} \alpha_{jo}^{c} \alpha_{jo}^{c},\ j=1,\ldots,n \\ s.t \quad \lambda_{jo}^{t} \alpha_{ij}^{t} x_{ij}^{t} \leq x_i^c, \qquad i=1,\ldots m, t=1,\ldots T\ (a) \\ \lambda_{jo}^{t} \beta_{rj}^{t} y_{rj}^{t} \geq \beta_{ro}^{c} y_{ro}^{c}, \quad o=1,\ldots n, t=1,\ldots T\ (b) \\ \lambda_{jo}^{t} \geq 0, \qquad o=1,\ldots n, t=1,\ldots T\ (10) \\ \lambda_{jo}^{t} \geq 0, \qquad i=1,\ldots m, r=1,\ldots T \end{aligned}$$

Where α^t the variation of multiplier of the i-th input for DMU_j in period t and β_{ij}^{t} the variation of multiplier of r-th output for DMUjin period t.

By computing λ^t from constraint (b) we have:

$$\lambda_{jo}^{t} \geq \frac{\beta_{ro}^{c} y_{ro}^{c}}{\beta_{rj}^{t} y_{rj}^{t}},\quad r=1,\ldots,s, t=1,\ldots,T$$

Let

$$\lambda_{lo}^{*t} = \max\left\{ \frac{\beta_{ro}^{r} y_{ro}^{r}}{\beta_{rj}^{t} y_{rj}^{t}} \middle| r=1,\ldots,s, t=1,\ldots,T \right\} (11)$$

So

$$x_i^c \leq \lambda_{jo}^{*t} \alpha_{ij}^{t} x_{ij}^{t}\quad, i=1,\ldots m, r=1,\ldots T$$

$$x_i^{*c} = \min\left\{ \lambda_{jo}^{*t} \alpha_{ij}^{t} x_{ij}^{t} \middle| i=1,\ldots m, r=1,\ldots T \right\} (12)$$

Therefore

$$CX_{oc}^{m(FX-CRS)} = \min \theta_j^{oc}, j=1,\ldots,n\ (13)$$

Model (10) relates to FDH technologies. Let the optimal solution from obtained by solving model (5) be $\left(x^{*\tau}, \lambda^*\right)$ then the Meta Cost Efficiency is defined in ratio from as:

$$E_{co}\tau^{m(FDH-CRS)}(\text{CostEfficiency}) =$$

$$\frac{CX_{o\tau}^{m(FDH-CRS)}}{CX_{o\tau}} = \frac{\sum_{i=1}^{m} io^{c^\tau} io^{\tau} i^{x^\tau}}{\sum_{i=1}^{m} c_{io}^{\tau} io^{\tau} x_{io}^{\tau}} \quad (14)$$

Now let $CX^{m(FDH-CRS)}$ be the Meta Cost Efficiency of unit j as observe in period t and was computed by using a model such as (10), Then we have:

Meta Cost Efficiency of unit j observed in period t = within period t Cost Efficiency of unit $j \times T$ technological Gap between period t boundary and the Meta Cost Frontier.

Putting the foregoing decomposition in symbols we have $E_{jt}^{m(FDH-CRS)} = E_{jt}^{T(FDH-CRS)} \times TG_{jt}^{c(FDH-CRS)}$ where $CX_{it}^{T(FDH-CRS)}$ is obtained for each unit j as the optimal value of $CX_{jt}^{T(FDH-CRS)}$ in Model (10) after dropping all instances apart from those occurring in period t, that we can compute by solving models (15), (16):

$$CX_{oc}^{T(FDH-CRS)} = \min \theta_j^{oc} = \sum_{i=1}^{m} c_{io}^{c} \alpha_j^{oc} \alpha_j^{oc}, \; j = 1,\dots,n$$

$$\begin{aligned} s.t \quad & \lambda_{jo}^{T} \alpha_{ij}^{T} x_{ij}^{T} \le x_i^{c}, && i = 1,\dots,m \\ & \lambda_{jo}^{t} \beta_{rj}^{t} y_{rj}^{t} \ge \beta_{ij}^{c} y_{ro}^{c}, && r = 1,\dots,S \\ & \lambda_{jo}^{t} \ge 0, \quad j = 1,\dots n && (15) \\ & x_i^{c} \ge 0, \quad i = 1,\dots m \end{aligned}$$

So

$$Eco_c^{T(FDH-CRS)} = \frac{CX_{or}^{T\ FDH\ CRS}}{CX_{o\tau}} = \frac{\sum_{i=1}^{m} c_{io}^{r} \alpha_{io}^{r} i^{x^r}}{\sum_{i=1}^{m} c_{io}^{r} \alpha_{io}^{r} x_{io}^{\tau}} \quad (16)$$

And TG_{jt}^{c} is retrieved residually as $TG_{jt}^{c(CRS)} = \frac{E_{jt}^{m(CRS)}}{E_{jt}^{T(CRS)}}$,by similar way we can compute the Meta Cost Malmquist Index of DMU_O in VRS model of DEA, we evaluate the Meta Cost Efficiency of $\mathrm{DMU}_o, o \in \{1, 2, \dots, n\}$ in trade-offs model of DEA as follows:

$$CX_{oc}^{m(FDH-TO)} = \min \theta_j^{oc} = \sum_{i=1}^{m} c_{io}^{c} \alpha_j^{oc} \alpha_j^{oc}, \; j = 1,\dots,n$$

$$\begin{aligned} & s.t \; \lambda_{jo}^{t} \alpha_{ij}^{t} x_{ij}^{t} + \sum_{f=1}^{l} \pi_{fi} \alpha_{if}^{t} P_{if}^{t} \le x_i^{c}, i = 1,\dots,m, t = 1,\dots,T \\ & \lambda_{jo}^{t} \beta_{rj}^{t} y_{rj}^{t} + \sum_{f=1}^{l} \pi_{fi} \beta_{if}^{t} Q_{if}^{t} \ge \beta_{ij}^{c} y_{ro}^{c}, r = 1,\dots,S, t = 1,\dots,T \\ & \lambda_{jo}^{t} \ge 0 \quad j = 1,\dots n, t = 1,\dots,T \quad (17) \\ & x_i^{c} \ge 0, \quad i = 1,\dots m, t = 1,\dots,T \\ & \pi_{fi} \ge 0, \quad f = 1,\dots l, t = 1,\dots,T \end{aligned}$$

So

$$Eco_c^{T(FDH-CRS)} = \frac{CX_{or}^{T\ FDH\ CRS}}{CX_{o\tau}} = \frac{\sum_{i=1}^{m} c_{io}^{r} \alpha_{io}^{r} i^{x^r}}{\sum_{i=1}^{m} c_{io}^{r} \alpha_{io}^{r} x_{io}^{\tau}} \quad (18)$$

5. Meta Cost Malmquist Index in Different Models of DEA with Variable Relative Importance as a Function of Time in Different Period by using FDH Models in DEA

Cost productivity change of unit j between period t and $t+1 CMI_{t,t+1}^{j(FDH-CRS)} = \frac{E_{jt+1}^{m(FDH-CRS)}}{E_{jt}^{m(FDH-CRS)}}$.

Where $E_{jt}^{m(FDH-CRS)}$ is the Meta Cost Efficiency of unit j as observed in period t. Note that when $CMI_{t,t+1}^{j}$ (Meta Cost Malmquist Index) is greater than 1, the Cost productivity of unit j has increased from t to $t+1$ (Since its Meta Cost Efficiency in $t+1$ is higher than that in period t), Cost productivity decline happens when $CMI_{t,t+1}^{j}$ is less than 1. Therefore:

$$CMI_{t,t+1}^{j(FDH-CRS)} = \frac{E_{jt+1}^{m(FDH-CRS)}}{E_{jt}^{m(FDH-CRS)}} = \frac{E_{jt+1}^{T+1(FDH-CRS)}}{E_{jt}^{T(FDH-CRS)}} \times \frac{TG_{jt+1}^{c(FDH-CRS)}}{TG_{jt}^{c(FDH-CRS)}} \quad (19)$$

$$CMI_{t,t+1}^{j(FDH-TO)} = \frac{E_{jt+1}^{m(FDH-TO)}}{E_{jt}^{m(FDH-TO)}} = \frac{E_{jt+1}^{T+1(FDH-TO)}}{E_{jt}^{T(FDH-TO)}} \times \frac{TG_{jt+1}^{c(FDH-TO)}}{TG_{jt}^{c(FDH-TO)}} \quad (20)$$

The term $\frac{E_{jt+1}^{T+1}}{E_{jt}^{T}}$ in (19) captures the Cost Efficiency change of unit j from year t to year $t+1$ as in the traditional Cost Malmquist Index of cost productivity change. The term $\frac{TG_{jt+1}}{TG_{jt}}$ in (19) captures Cost Frontier shift between period t and $t+1$.

To see this note that $\frac{TG_{jt+1}^{c(FDH-CRS)}}{TG_{jt}^{c(FDH-CRS)}} = \frac{E_{jt+1}^{m(FDH-CRS)}}{E_{jt}^{m(FDH-CRS)}} = \frac{E_{jt+1}^{T(FDH-CRS)}}{E_{jt}^{T+1(FDH-CRS)}}$.

Since:

$$E_{jt}^{m(FDH-CRS)} = E_{jt}^{T(FDH-VRS)} \times \frac{E_{jt}^{T(FDH-CRS)}}{E_{jt}^{T(FDH-VRS)}} \quad (21)$$

$$\times \frac{E_{jt}^{m(FDH-VRS)}}{E_{jt}^{T(FDH-VRS)}} \times \frac{E_{jt}^{m(FDH-CRS)} / E_{jt}^{m(FDH-VRS)}}{E_{jt}^{T(FDH-CRS)} / E_{jt}^{T(FDH-VRS)}}$$

The expression in (21) can be simplified to:

$$E_{jt}^{m(FDH-CRS)} = E_{jt}^{T(FDH-VRS)} \times \frac{E_{jt}^{m(FDH-VRS)}}{E_{jt}^{m(FDH-VRS)}}$$

$$\times \frac{E_{jt}^{m(FDH-CRS)}}{E_{jt}^{m(FDH-VRS)}} = TGV_{jt} \times MSCE_{jt} \quad (22)$$

That is Meta Cost Efficiency decomposes in to within period Cost Efficiency in relation to a VRS frontier $E_{jt+1}^{T(FDH-VRS)}$, Technological Gap between the VRS Cost Frontier in t and the VRS Meta Cost Frontier $\left(\frac{E_{jt}^{m(FDH-VRS)}}{E_{jt}^{T(FDH-VRS)}}\right)$ labeled TGV and Meta Scale Cost Efficiency $\left(\frac{E_{jt}^{m(FDH-CRS)}}{E_{jt}^{m(FDH-VRS)}}\right)$ labeled MSCE. Note MSCE captures the distance between the CRS and VRS Meta Cost Frontier at the input-output mix of unit jas observed in period t. By similar way we can compute $E_{jt}^{m(FDH-TO)}$.

$$E_{jt}^{m(FDH-TO)} = E_{jt}^{T(FDH-VRS)} \times \left(\frac{E_{jt}^{m(FDH-VRS)}}{E_{jt}^{T(FDH-VRS)}}\right) \quad (23)$$

$$\times \frac{E_{jt}^{m(FDH-TO)}}{E_{jt}^{m(FDH-VRS)}} = E_{jt}^{T(FDH-VRS)} \times TGV_{jt} \times MSCE_{jt}$$

By using (22), (23) the Meta Cost Malmquist Index, defined as $CMI_{t,t+1}^{j(CRS)} = \frac{E_{jt+1}^{m(CRS)}}{E_{jt}^{m(CRS)}}$ can be computed as shown in (24), (25):

$$CMI_{t,t+1}^{j(FDH-CRS)} = \frac{E_{jt+1}^{T+1(FDH-VRS)}}{E_{jt}^{T(FDH-VRS)}} \times \frac{TGV_{jt+1}}{TGV_{jt}}$$

$$\times \frac{MSCE_{jt+1}}{MSCE_{jt}} \quad (24)$$

$$CMI_{t,t+1}^{j(FDH-TO)} = \frac{E_{jt+1}^{T+1(FDH-VRS)}}{E_{jt}^{T(FDH-VRS)}} \times \frac{TGV_{jt+1}}{TGV_{jt}}$$

$$\times \frac{MSCE_{jt+1}^{F}}{MSCE_{jt}^{F}} \quad (25)$$

That is the index can be decomposed in to pure technical Cost Efficiency change $\frac{E_{jt+1}^{T+1(FDH-VRS)}}{E_{jt}^{T(FDH-VRS)}}$ Frontier shift between VRS frontier $\frac{TGV_{jt+1}}{TGV_{jt}}$, and Meta Scale Cost Efficiency change $\frac{MSCE_{jt+1}}{MSCE_{jt}}$.

6. Using the Meta Cost Frontier to compare productivities of units with Variable Relative Importance as a Function of Time in Different Period by using FDH Model DEA

With using now, the unit specific boundaries we can compute two Cost Efficiency scores for each unit instance unit jt (unit j as observed in period t). One Cost Efficiency will be relative to the Meta Cost Frontier as before and denoted E_{jt}^{m} while the second will be relative to unit specific boundary as defined above, and it is denoted E_{jt}^{Uj}, where the index U_j relates to the unit specific boundary of unit $jE_{jo\tau}^{Uj}$ in relation to unit $jo \in \{1,...,n\}$ observed in period $\tau \in \{1,...,T\}$ obtained by solving models (26), (27):

$$CX_{joc}^{U_j(CRS)} = \min \sum_{i=1}^{m} c_{ijo}^{c} \alpha_{\alpha ij}^{o} x_i^c$$
$$s.t\ \lambda_{jot}^{t} \alpha_{ijo}^{t} x_{ijo}^{t} \le x_i^c, i = 1,\dots,m, t = 1,..,T$$
$$\lambda_{jot}^{t} \beta_{rjo}^{t} y_{ijo}^{t} \ge \beta_{rjo}^{c} y_{rjo}^{c}, r = 1,\dots,S, t = 1,..,T$$
$$\lambda_{jot} \ge 0 \qquad j = 1,\dots n, t = 1,..,T \qquad (26)$$
$$x_i^c \ge 0, \qquad i = 1,\dots m, t = 1,..,T$$

So:

$$E_{Cjoc}^{U_j(FDH-CRS)} = \frac{Cx_{Jor}^{U_j(FDR-CRS)}}{CX_{jor}} = \frac{\sum_{i=1}^{m} c_{ijo}^{r}\ \alpha_{ijo}^{r}\ i^{x^{*}r}}{\sum_{i=1}^{m} c_{ijo}^{r}\ \alpha_{ijo}^{r}\ x_{ijo}^{r}}$$
(27)

Note now that we have $E_{jt}^{m(FDH-CRS)} = E_{joc}^{U_j(FDH-CRS)} \times UG_{jt}^{(FDH-CRS)}$ where UG_{jt} is retrieved residually and it measured the distance from the unit specific Cost Frontier to the Meta Cost Frontier. We shall refer to UG_{jt} as the unit Cost Frontier Gap for unit j measured at the units input output mix in period t.

Now, we compute $E_{joc}^{U_j(FDH-TO)}$:

$$CX_{joc}^{U_j(FDH-TO)} = \min \sum_{i=1}^{m} c_{ijo}^{c} \alpha_{\alpha ij}^{o} x_i^c$$
$$s.t\ \lambda_{jot}^{t} \alpha_{ijo}^{t} x_{ijo}^{t} + \sum_{f=1}^{1} \pi_f^t \alpha_{if}^{t} p_{if}^{t} \le x_i^c, i = 1,\dots,m, t = 1,..,T$$
$$\lambda_{jot}^{t} \beta_{rjo}^{t} y_{ijo}^{t} + \sum_{f=1}^{1} \pi_f^t \beta_{if}^{t} q_{if}^{t} \le \beta_{rjo}^{c} y_{rjo}^{c}, r = 1,\dots,S, t = 1,..,T$$
$$\lambda_{jot} \ge 0 \qquad j = 1,\dots n, t = 1,..,T \qquad (28)$$
$$x_i^c \ge 0, \qquad i = 1,\dots m, t = 1,..,T$$
$$\pi_{fi}^{t} \ge 0, \qquad f = 1,\dots l, t = 1,..,T$$

So:

$$E_{Cjoc}^{U_j(FDH-TO)} = \frac{CX_{jor}^{U_j(FDH-TO)}}{CX_{jor}} = \frac{\sum_{i=1}^{m} c_{ijo}^{r} \alpha_{ijo}^{r} i^{x^r}}{\sum_{i=1}^{m} c_{ijo}^{r} \alpha_{ijo}^{r} x_{ijo}^{r}} \qquad (29)$$

Therefore:

$$CMI_{t,t+1}^{j(FDH-CRS)} = \frac{E_{jt+1}^{T+1(FDH-CRS)}}{E_{jt}^{T(FDH-CRS)}} = \frac{E_{jt+1}^{U_j(CRS)}}{E_{jt}^{U_j(CRS)}} \times \frac{UC_{jt+1}^{c(FDH-CRS)}}{UC_{jt}^{c(FDH-CRS)}} \qquad (30)$$

$$CMI_{t,t+1}^{j(FDH-CRS)} = \frac{E_{jt+1}^{m(FDH-TO)}}{E_{jt}^{m(FDH-CRS)}} = \frac{E_{jt+1}^{U_j(FDH-TO)}}{E_{jt}^{U_j(FDH-TO)}} \times \frac{UC_{jt+1}^{c(FDH-TO)}}{UC_{jt}^{c(FDH-TO)}} \qquad (31)$$

7. Advantage of the circular Meta Cost Malmquist Index with Variable Relative Importance as aFunction of Time in Different Period by using FDH Models in DEA

One key advantage of the Meta Cost Malmquist Index is that it is circular. To see this note that $\frac{E_{jt+2}^{m}}{E_{jt}^{m}} = \frac{E_{jt+2}^{m}}{E_{jt+1}^{m}} \times \frac{E_{jt+1}^{m}}{E_{jt}^{m}}$ that is the cost productivity change between periods t and $t+2$ is the product of the successive Cost productivity change from period t to $t+1$.

Thus, for example Cost Efficiency change from period t to $t+2$ is $\frac{E_{jt+2}^{T+2}}{E_{jt}^{T}} = \frac{E_{jt+2}^{T+2}}{E_{jt+1}^{T+1}} \times \frac{E_{jt+1}^{T+1}}{E_{jt}^{T}}$ the RHS being the product of the successive Cost Efficiency change indices from period t to $t+1$ and from period $t+1$ to $t+2$. Similarly boundary shift from period t to $t+2$ is $\frac{TG_{jt+2}}{TG_{jt}} = \frac{TG_{jt+2}}{TG_{jt+1}} \times \frac{TG_{jt+1}}{TG_{jt}}$ the RHS being the product of the successive boundary shift from period t to $t+1$ and from period $t+1$ to t + 2. Also $\frac{UG_{jt+2}}{UG_{jt}} = \frac{UG_{jt+2}}{UG_{jt+1}} \times \frac{UG_{jt+1}}{UG_{jt}}$ and $\frac{E_{jt+2}^{U_j}}{E_{jt}^{U_j}} = \frac{E_{jt+2}^{U_j}}{E_{jt+1}^{U_j}} \times \frac{E_{jt+1}^{U_j}}{E_{jt}^{U_j}}$.

8. Conclusion

Considering the variation of relative importance and incorporation them as multipliers in the models shows that,the results for real data have superiority to the other models, the reason is that the cost of inputs and outputs in some data that can be cast in money every with inflation, should be consider seriously, and this should be takinginto account in evaluating the Meta Cost Malmquist index, in different period of results shows in fact. The results of show the validity of circularity. The main reason using FDH Models in DEA for computing Meta CostMalmquist Index is that, many of natural agents are not convex.

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