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# Determination of Right/Left return to Scale in Two-Stage Processes Based on Dual Simplex

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## Abstract

As a non-parametric method of relative efficient measurement of a group of decision making units ( $DMU_s$ ), Data Envelopment Analysis (DEA) is one of the most important tools in efficiency computation. One of the main concerns dealt with in DEA is dealing with return to scale in two-stage processes in which, produced outputs of the first stage inputs are used as inputs for the second stage. The outputs of the first stage are considered as the intermediate products. Therefore, the second stage uses these intermediate products for producing the outputs of the same stage. Based on this construction, the total process can be analyzed for efficiency production from two sub-processes. In this paper, a new model is proposed that eliminates the defections of the previous models and is used to determine the right and left return to scale in two-stage processes of decision making units.

**Keywords:** Efficiency, Two-stage processes, Return to scale.

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## 1. Introduction

Nowadays, units' calculation and comparison are one of the most important fields in economy that act cooperatively in the same field. Charnes et al. [1] have proposed a practical model to calculate and compare the efficiency of decision making that could calculate the efficiency of the units with various inputs and outputs. During 1970s, the field has attracted many attention and been developed to a great extent and different models have been proposed by experts in different areas. One of those areas is multi-stage decision making units. For example, Charnes et al. [2] faced a problem similar to a two-stage decision making, in which two-stage method must be used for army employment. Later on, Schinnar et al. [3], Sexton and Lewis [4], Chen and Zhu [5], and Liang et al. [6] applied two-stage DEA in mental health care, baseball, information technology, and evaluation the performance of supply chain, respectively. So far, various scientists have studied this field. For instance, Izadikhah and Saen [7], Kalhor and Kazemi matin [8]. Kao and Hwang [9] have proposed a different model by which the efficiency of the whole systems is decomposed into the efficiency of two systems. This model has been proposed based on the assumption of return to scale. Chen et al. [10] have proposed a model similar to Kao and Hwang [9], in which constant return to scale and variable return to scale were studied. Later, Chen et al. [11] and Kao and Hwang in [12] developed their proposed models.

Despotis et al. [13] proposed a new definition of system efficiency in two-stage process. It was based on weak link in supply chain, maximum flow algorithm, and minimal cut theorem for networks. Hatami-Marbini et al. [14] and Alirezaee et al. [15] introduced new approaches in return to scale in data envelopment analysis.

In this paper, it is purposed to introduce a new model for two-stage DEA. Considering both the importance of return to scale and the defects of the other mentioned proposed models in two-stage DEA, return to scale of two-stage model is discussed by the use of right and left return to scale that was studied by Golany and Yu [16] and Hadjicostas and Soteriou [17]. This study is done using the proposals of Zarepisheh and Soleimani-damaneh [18]. Other parts of this paper are as follows. Section 2 is devoted to literature review of two-stage processes. The organization of the paper is in the following way. Our recommended model is presented in section 3. In the section 4, return to scale in two-stage DEA is calculated with the proposed model. Finally, a numerical example is solved in section 5 and the section 6 is devoted to conclusions.

## 2. Review

Consider  $n$  decision making units that each  $DMU_j (j \in J = \{1, \dots, n\})$  generates the  $q$  outputs of  $z_{pj} (p = 1, \dots, q)$  using  $m$  inputs of  $x_{ij} (i = 1, \dots, m)$  and then the obtained  $q$  outputs would be the inputs for generating  $s$  outputs of  $y_{rj} (r = 1, \dots, s)$ .

Hwang and Kao [19] have used two-stage DEA of Seiford and Zhu [20] to measure the total efficiency as well as the efficiency of the first and the second stage. Also, they have introduced the efficiency of the first and the second stage by assuming constant return to scale as follows:

$$\begin{aligned}
 E_o^1 &= \max \quad \sum_{p=1}^q w_p z_{po} / \sum_{i=1}^m v_i x_{io} \\
 s. t. \quad &\sum_{p=1}^q w_p z_{pj} / \sum_{i=1}^m v_i x_{ij} \leq 1, \\
 &j = 1, \dots, n, \\
 &w_p, v_i \geq \varepsilon, p = 1, \dots, q; i = 1, \dots, m,
 \end{aligned} \tag{1}$$

And the efficiency score of the second stage has been introduced as follows:

$$\begin{aligned}
 E_o^2 &= \max \quad \sum_{r=1}^s u_r y_{ro} / \sum_{p=1}^q w_p z_{po} \\
 s. t. \quad &\sum_{r=1}^s u_r y_{rj} / \sum_{p=1}^q w_p z_{pj} \leq 1,
 \end{aligned}$$

$$\begin{aligned} j &= 1, \dots, n, \\ u_r, w_p &\geq \varepsilon, r = 1, \dots, s; p = 1, \dots, q. \end{aligned} \quad (2)$$

In which,  $\varepsilon$  was a small non-Archimedean number and  $w_p, v_i$  and  $u_r$  were weight of inputs, intermediate products and outputs, respectively. The efficiency of each stage in models (1) and (2) were calculated independently. Kao and Hwang [9] have proposed their new model as a relational model by using intermediate products and assuming constant return to scale and the equality of weights in intermediate products of data envelopment. The overall efficiency score of the relational model was calculated as follows:

$$\begin{aligned} E_o &= \max \sum_{r=1}^s u_r y_{ro} / \sum_{i=1}^m v_i x_{io} \\ s. t. & \sum_{r=1}^s u_r y_{rj} / \sum_{i=1}^m v_i x_{ij} \leq 1, \\ & j = 1, \dots, n, \\ & \sum_{p=1}^q w_p z_{pj} / \sum_{i=1}^m v_i x_{ij} \leq 1, \\ & j = 1, \dots, n, \\ & \sum_{r=1}^s u_r y_{rj} / \sum_{p=1}^q w_p z_{pj} \leq 1, j = 1, \dots, n, \\ & u_r, v_i, w_p \geq \varepsilon, \quad r = 1, \dots, s; \\ & i = 1, \dots, m; \quad p = 1, \dots, q. \end{aligned} \quad (3)$$

The set of limitations in model (3) were in fact the set of limitations of models (1) and (2).

So, the overall efficiency was  $E_o = E_o^1 \times E_o^2$ . Indeed, in their relational model, the overall efficiency score was the multiplication of the first and the second stages. But, as multiplication of two numbers between 0 and 1 leads to a smaller one, the result was not a true representation of efficiency score. Moreover, in Hwang and Kao independent model [19], two models are independently solved and this is not logical for the decision making units with a common decision maker. As the proposed model by Kao and Hwang [9] was in the form of constant return to scale, Chen et al. [10] introduced their model in form of constant return to scale and variable return to scale. Considering

the complex computation of this model and the large number of variables and constants, calculating the return to scale of decision making units is a difficult task. In the next section, a new model is proposed that is less complex.

### 3. Proposed model

According to what mentioned above, a new model is proposed here to calculate the efficiency of decision making units with different stages. Similar to the dependent model introduced by Kao Hwang [9], in this model, intermediate production is considered equal without taking the roles of the inputs and outputs into account and is entered into the model as one intermediate production. In this paper it is tried to calculate the efficiency of each stage simultaneously by considering the roles of the input and the output. The envelopment form of the model based on the output oriented is as follows:

$$\begin{aligned} \max & \quad \alpha \\ s. t. & \sum_{j=1}^n \lambda_j x_{ij} \leq x_{io}, \quad i = 1, \dots, m, \\ & \sum_{j=1}^n \lambda_j z_{pj} \geq \alpha z_{po}, \quad p = 1, \dots, q, \\ & \sum_{j=1}^n \lambda_j = 1, \\ & \sum_{j=1}^n \mu_j z_{pj} \leq z_{po}, \\ & \quad p = 1, \dots, q, \\ & \sum_{j=1}^n \mu_j y_{rj} \geq \alpha y_{ro}, \quad r = 1, \dots, s, \\ & \sum_{j=1}^n \mu_j = 1, \\ & \lambda_j \geq 0, \mu_j \geq 0, \quad j = 1, \dots, n. \end{aligned} \quad (4)$$

Considering the corresponding variables of  $t_p, u_r, w_p$  and  $v_i$  with constraints of model (4), the dual model is presented as follows:

$$\begin{aligned} \text{Min} & \quad \sum_{i=1}^m v_i x_{io} + \sum_{p=1}^q w_p z_{po} + \hat{u} + \bar{u} \\ s. t. & \sum_{i=1}^m v_i x_{ij} - \sum_{p=1}^q w_p z_{pj} + \hat{u} \geq 0, \\ & j = 1, \dots, n, \\ & \sum_{p=1}^q t_p z_{pj} - \sum_{r=1}^s u_r y_{rj} - \bar{u} \geq 0, \\ & j = 1, \dots, n, \\ & \sum_{p=1}^q w_p z_{po} + \sum_{r=1}^s u_r y_{ro} = 1, \\ & u_r \geq 0, v_i \geq 0, w_p \geq 0, t_p \geq 0, \end{aligned} \quad (5)$$

$r = 1, \dots, s, i = 1, \dots, m, p = 1, \dots, q.$

**4. Return to scale in two-stage DEA**

In this section, it is tried to develop the proposed model using the presented suggestions by Zarepisheh and Soleimani-damaneh [18].

In this model,

$$\begin{aligned} X_j &= (x_{1j}, x_{2j}, \dots, x_{mj}), Z_j \\ &= (z_{1j}, z_{2j}, \dots, z_{qj}), \text{ and } Y_j \\ &= (y_{1j}, y_{2j}, \dots, y_{sj}) \end{aligned}$$

are defined as the vectors of input, intermediate and output of  $DMU_j$ , respectively. Also,

$$\begin{aligned} X &= [x_1, x_2, \dots, x_n], Z \\ &= [z_1, z_2, \dots, z_n], \text{ and } Y \\ &= [y_1, y_2, \dots, y_n] \end{aligned}$$

are  $m \times n, q \times n,$  and  $s \times n$  matrices of inputs, intermediate productions and outputs, respectively. According to the proposed model, production possibility is defined as follows:

$$T_v = \left\{ (x, z, y) \in R_+^{m+s+q} \left| \begin{array}{l} \exists (\lambda, \mu) \in R^n: \\ \sum_{j=1}^n \lambda_j x_{ij} \leq x_{io}, \\ \sum_{j=1}^n \lambda_j z_{pj} \geq \alpha z_{po}, \\ \sum_{j=1}^n \lambda_j = 1, \\ \lambda \geq 0, \\ \sum_{j=1}^n \mu_j z_{pj} \leq z_{po}, \\ \sum_{j=1}^n \mu_j y_{rj} \geq \alpha y_{ro}, \\ \sum_{j=1}^n \mu_j = 1, \\ \mu \geq 0. \end{array} \right. \right\}$$

Where,  $o$  and  $e$  are two vectors with zero and one component, respectively. Based on the proposed model, the efficiency of the decision making unit, like  $DMU_o$  in output oriented, is evaluated as follows:

**Definition 1.**  $DMU_o$  is defined as the technical efficient unit in output oriented if  $\alpha^* = 1$  (where ‘\*’ shows optimality). For the inefficient units, according to Banker et al. [21], they can be used in RTS evaluation by being projected in the efficient frontier. Here, the projection points are defined as follows:

$$\hat{X}_o = X_o, \hat{Z}_o = \alpha^* Z_o, \hat{Y}_o = \alpha^* Y_o$$

As a result, in order to determine constant return to scale,  $(X_o, Z_o, Y_o)$  must be replaced by  $(\hat{X}_o, \hat{Z}_o, \hat{Y}_o)$ ; therefore, to determine RTS of the proposed model, the following limits can be used (see Hadjicostas, Soteriou [17]).

$$\begin{aligned} \rho_o^+ &= \lim_{\beta \rightarrow 1^+} \frac{\alpha(\beta)-1}{\beta-1}, \\ \rho_o^- &= \lim_{\beta \rightarrow 1^-} \frac{\alpha(\beta)-1}{\beta-1} \end{aligned} \tag{6}$$

The function  $\alpha(\beta)$  corresponding to  $DMU_o$  is

$$\alpha(\beta) = \max\{\alpha | (\beta X_o, \alpha Z_o, \alpha Y_o) \in T_v\} \tag{7}$$

**Notification 1.** According to model (4) and  $\alpha$  definition,  $\alpha(\beta)$  can be supposed as the optimal value of model (4) in which  $\beta X_o$  is placed in input as a substitution of  $X_o$ . By accepting this issue and model (7), it is clear that if  $\beta = 1$ , the researcher wants to solve model (4) for vectors of the inputs and outputs  $(X_o, Z_o, Y_o)$ . Therefore, considering the projection points  $(\hat{X}_o, \hat{Z}_o, \hat{Y}_o)$  and the fact that  $\alpha$  is optimal for model (4) along with  $(X_o, \alpha Z_o, \alpha Y_o)$  vector has been projected on the efficient frontier, it can be concluded that  $\alpha(1) = 1$ .

**Assumption 1.** As mentioned in Hadjicostas and Soteriou [17],  $\eta \in [0,1), \gamma \geq 0$  exists such that  $(\eta X_o, \gamma Z_o, \gamma Y_o) \in T_v$ .

It means that reduction of input  $s$  with the same relation from  $DMU_o$  in  $T_v$  is possible.

**Lemma 1.** If  $\alpha(1) = 1$ ,  $\rho_o^+$  exists and is finite.

**Proof:** To prove the following assumption, there is:

Assumption 1: convexity

$$\text{if } (x, y) \in T, (x', y') \in T, \Rightarrow (\lambda x + (1 - \lambda)x', \lambda y + (1 - \lambda)y') \in T$$

Assumption 2: Monotonicity

$$\text{if } (x, y) \in T, x' \geq x, 0_s \leq y' \leq y \text{ for } (x', y') \in R^m \times R^s \Rightarrow (x', y') \in T$$

Assumption 3: Inclusion of observations  $n \in N^+$  exists and  $n, DMU_s$  are exist too such that for each  $j = 1, \dots, n$  the observation of  $(x_j, y_j) \in T$  are related to  $DMU_j$ .

Assumption 4: Minimum extrapolation

If a set preserves the possibility of  $T'$  production of assumptions 1, 2, and 3 (for  $n$  observations  $((x_j, y_j), j = 1, \dots, n)$ , then  $T \subseteq T'$ . Now, if 3 and 4 are appointed,  $\rho_o^+$  exists and is finite.

**Lemma 2:** If  $\alpha(1) = 1$  and assumption 1 is determined,  $\rho_o^-$  exists and is finite.

**Proof:** If  $\alpha(1) = 1$  and assumption 1 is determined, the following limits exist and are finite.

$$\rho_o^+ = \lim_{\beta \rightarrow 1^+} \frac{\alpha(\beta) - 1}{\beta - 1}, \rho_o^- = \lim_{\beta \rightarrow 1^-} \frac{\alpha(\beta) - 1}{\beta - 1}$$

Here, assumption 1 is determined. The proposed model is convex because it is the products of two conceive sets.

**Definition 2:** RTS in  $DMU_o$  is increasing from right (constant, decreasing) if  $\rho_o^+ > 1 (= 1, < 1)$  and is decreasing if  $\rho_o^- > 1 (= 1, < 1)$ .

If assumption 1 is not determined, it means that reduction with the same proportion of  $DMU_o$  inputs in  $T_v$  is impossible. In this situation,  $DMU_o$  contraction cannot be discussed and, consequently, RTS cannot be defined for left.

Now, in order to calculate  $\rho_o^+$  and  $\rho_o^-$  according to Hadjicostas and Soteriou [17], we can act as follows.

First model (4) must be solved. Then,  $\rho_o^+$  and  $\rho_o^-$  can be calculated after the linear program model has been solved.

$$\begin{aligned} \rho_o^+ &= 1 - \alpha^* \\ \alpha^* &= \max \hat{u} + \bar{u} \\ \text{s.t. } &\sum_{i=1}^m v_i x_{ij} - \sum_{p=1}^q w_p z_{pj} + \hat{u} \geq 0, \\ &j = 1, \dots, n, \\ &\sum_{p=1}^q t_p z_{pj} - \sum_{r=1}^s u_r y_{rj} - \bar{u} \geq 0, \\ &j = 1, \dots, n, \\ &\sum_{p=1}^q w_p \hat{z}_{po} + \sum_{r=1}^s u_r \hat{y}_{ro} = 1, \\ &\hat{u} + \bar{u} + \sum_{i=1}^m v_i \hat{x}_{io} + \sum_{p=1}^q w_p \hat{z}_{po} = 1, \\ &\hat{u} \geq 0, \bar{u} \geq 0. \end{aligned} \quad (8)$$

And  $\rho_o^- = 1 - \beta^*$  is defined as follows:

$$\begin{aligned} \beta^* &= \text{Min } \hat{u} + \bar{u} \\ \text{s.t. } &\sum_{i=1}^m v_i x_{ij} - \sum_{p=1}^q w_p z_{pj} + \hat{u} \geq 0, \\ &j = 1, \dots, n, \\ &\sum_{p=1}^q t_p z_{pj} - \sum_{r=1}^s u_r y_{rj} - \bar{u} \geq 0, \\ &j = 1, \dots, n, \\ &\sum_{p=1}^q w_p \hat{z}_{po} + \sum_{r=1}^s u_r \hat{y}_{ro} = 1, \\ &\hat{u} + \bar{u} + \sum_{i=1}^m v_i \hat{x}_{io} + \sum_{p=1}^q w_p \hat{z}_{po} = 1, \\ &\hat{u} \geq 0, \bar{u} \geq 0. \end{aligned} \quad (9)$$

$\hat{x}_o, \hat{z}_o$ , and  $\hat{y}_o$  were introduced before.

**Theorem 1:** If  $\alpha(1) = 1$  and matrix B is an optimal base for model (4) as:

$$\left( B^{-1} \begin{pmatrix} x_o \\ Z_o \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 1 \\ 1 \end{pmatrix}, B^{-1} \begin{pmatrix} x_o \\ Z_o \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 0 \\ 0 \end{pmatrix} \right) \geq 0, \quad (10)$$

$$\text{So, } \rho_0^+ = C_B B^{-1} \begin{pmatrix} x_0 \\ Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 0 \\ 0 \end{pmatrix} \text{ in that } C_B$$

shows the target function coefficient vector corresponding to basis B in model (4).

**Proof:** Put

$$\bar{b} = B^{-1} \begin{pmatrix} x_0 \\ Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 1 \\ 1 \end{pmatrix},$$

$$\hat{b} = B^{-1} \begin{pmatrix} x_0 \\ Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 0 \\ 0 \end{pmatrix}$$

Define  $\bar{\delta} = \infty$  if  $\hat{b} \geq 0$ ; otherwise,  
 $\bar{\delta} = \min \left\{ \frac{-\hat{b}_i}{\bar{b}_i} : i \in \{1, \dots, m + s + 1\}, \hat{b}_i < 0 \right\}$

Because  $(\bar{b}, \hat{b}) \geq 0$ , we have  $\bar{\delta} > 0, \bar{b} + \delta \hat{b} \geq 0$ .

For each  $\delta \in (0, \bar{\delta})$  and

$$B^{-1} \begin{pmatrix} (1 + \delta)x_0 \\ (1 + \delta)Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 1 \\ 1 \end{pmatrix} \geq 0,$$

for each  $\delta \in (0, \bar{\delta})$ , it means that B is an optimal basis for model (4) that corresponds to  $((1 + \delta)x_0, (1 + \delta)Z_0, Y_0)$ . Optimality of basis is separated from RHS vector and, as a result, B is the optimal basis for model (4) that is corresponding to  $((1 + \delta)x_0, (1 + \delta)Z_0, Y_0)$ . Put  $\beta = 1 + \delta, \bar{\beta} = 1 + \bar{\delta}$ .

According to notification 1, for  $\beta \in (1, \bar{\beta})$ , it is concluded that

$$\alpha(\beta) = C_B B^{-1} \begin{pmatrix} \beta x_0 \\ \beta Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 1 \\ 1 \end{pmatrix}.$$

The equality of  $\alpha(1) = 1$  leads to

$$C_B B^{-1} \begin{pmatrix} x_0 \\ Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 1 \\ 1 \end{pmatrix} = 1, \text{ so:}$$

$$\alpha(\beta) = C_B B^{-1} \begin{pmatrix} (1 + \beta - 1)x_0 \\ (1 + \beta - 1)Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 1 \\ 1 \end{pmatrix}$$

$$= C_B B^{-1} \begin{pmatrix} x_0 \\ Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 1 \\ 1 \end{pmatrix} +$$

$$(\beta - 1) C_B B^{-1} \begin{pmatrix} x_0 \\ Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 0 \\ 0 \end{pmatrix} \Rightarrow$$

$$\alpha(\beta) = 1 + (\beta - 1) C_B B^{-1} \begin{pmatrix} x_0 \\ Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \frac{\alpha(\beta) - 1}{\beta - 1} = C_B B^{-1} \begin{pmatrix} x_0 \\ Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 0 \\ 0 \end{pmatrix} \text{ For each}$$

$\beta \in (1, \bar{\beta}) \Rightarrow$

$$\rho_0^+ = C_B B^{-1} \begin{pmatrix} x_0 \\ Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 0 \\ 0 \end{pmatrix}$$

**Theorem 2:** if  $\alpha(1) = 1$  and matrix B is an optimal basis for model (4) as

$$\begin{pmatrix} B^{-1} \begin{pmatrix} x_0 \\ Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 1 \\ 1 \\ -x_0 \\ -Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 0 \\ 0 \end{pmatrix} \\ B^{-1} \begin{pmatrix} -x_0 \\ -Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 0 \\ 0 \end{pmatrix} \end{pmatrix} \geq 0, \quad (11)$$

so,  $\rho_0^- = C_B B^{-1} \begin{pmatrix} -x_0 \\ -Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 1 \\ 1 \end{pmatrix}$ .

**Proof:** The proof of this theorem is similar to the previous one. In this case, it

should be  $\hat{b} = B^{-1} \begin{pmatrix} -x_0 \\ -Z_0 \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 0 \\ 0 \end{pmatrix}$ .

Figure 1 shows the different stages of the proposed method for obtaining left and right return to scale (RTS).

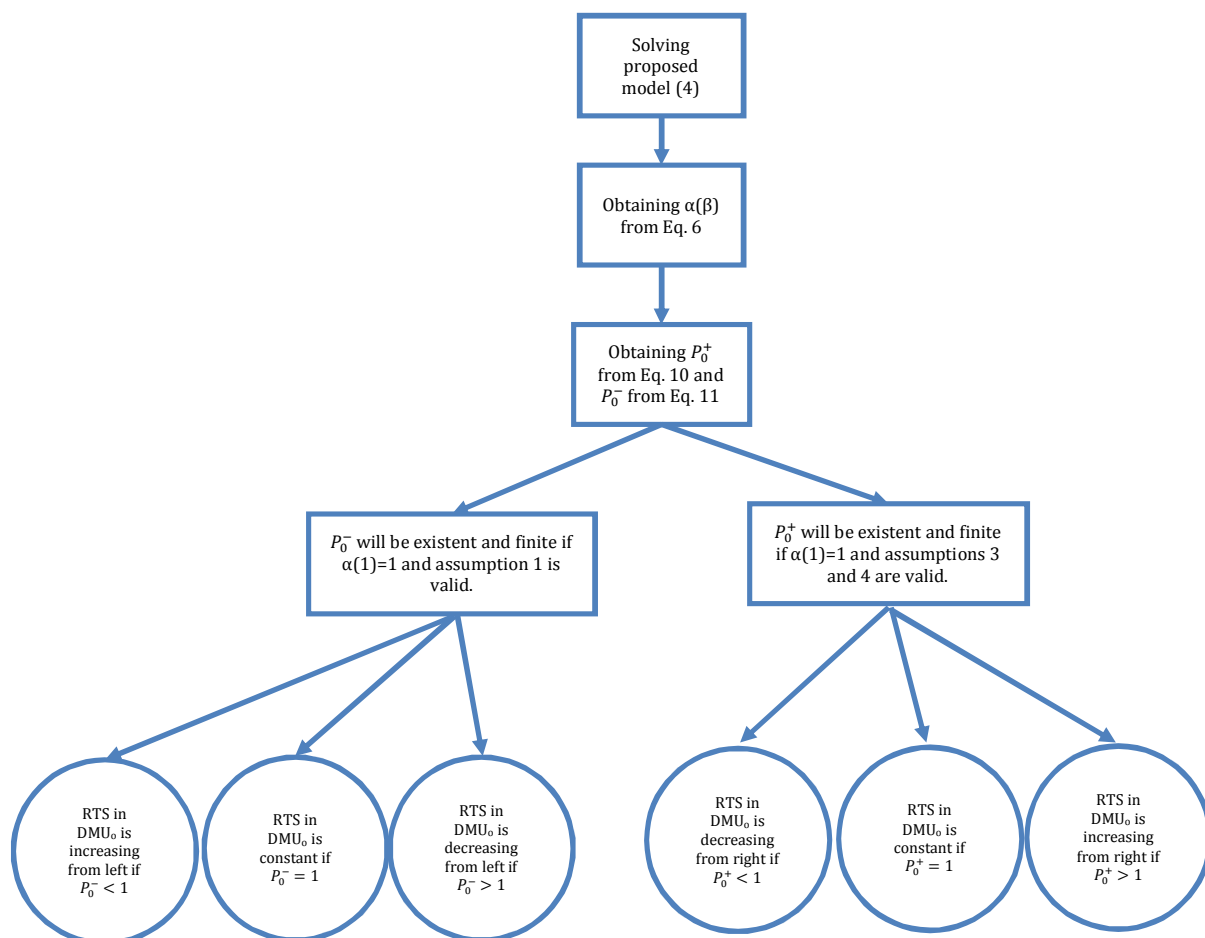


Figure 1: The algorithm to obtain left and right RTS based on the proposed model

**5. Numerical Example**

5.1 Example 1

Table 1 shows the data of 24 branches of none-life insurance company of Taiwan from Hwang and Kao [19]. The inputs of the first process are operation and insurance costs. On the other hand, the outputs of the first process that are intermediate production are direct written premiums and reinsurance premiums that are the inputs of the second stage. Finally, the outputs of the second stage are underwriting profit and investment profit. Table 2, shows the amount of the

efficiency of Hwang and Kao [19], independent model Kao and Hwang [9] dependent model, and the proposed model. In Kao and Hwang [9] dependent model, none of units are efficient. In the independent model, on the other hand, some of the units are efficient, while none of sub-processes are efficient. As it can be seen in the proposed model, most of the units are efficient and the inefficient units have similar efficiency, that leads to a better outcome compared to independent and dependent models.

**Table1: Input (x), intermediate production (z), and output (y) of 24 branches of none-life insurance company of Taiwan**

DMU	Operation expenses (x1)	Insurance expenses (x2)	Direct written premiums (z1)	Reinsurance premiums (z2)	Underwriting profit (y1)	Investment profit (y2)
1. Taiwan Fire	1178744	673512	7451757	856735	984143	681687
2. Chung Kuo	1381822	1352755	10020274	1812894	1228502	834754
3. Tai Ping	1177494	592790	4776548	560244	293613	658428
4. China Mariners	601320	594259	3174851	371863	248709	177331
5. Fubon	6699063	3531614	37392862	1753794	7851229	3925272
6. Zurich	2627707	668363	9747908	952326	1713598	415058
7. Taian	1942833	1443100	10685457	643412	2239593	439039
8. Ming Tai	3789001	1873530	17267266	1134600	3899530	622868
9. Central	1567746	950432	11473162	546337	1043778	264098
10. The First	1303249	1298470	8210389	504528	1697941	554806
11. Kuo Hua	1962448	672414	7222378	643178	1486014	18259
12. Union	2592790	650952	9434406	1118489	1574191	909295
13. Shingkong	2609941	1368802	13921464	811343	3609236	223047
14. South China	1396002	988888	7396396	465509	140100	332283
15. Cathay Century	2184944	651063	10422297	749893	3355197	555482
16. Allianz President	1211716	415071	5606013	402881	854054	197947
17. Newa	1453797	1085019	7695461	342489	3144484	371984
18. AIU	757515	547997	3631484	995620	692731	163927
19. North America	159422	182338	1141950	483291	519121	46857
20. Federal	145442	53518	316829	131920	355624	26537
21. Royal&Sun alliance	84171	26224	225888	40542	51950	6491
22. Asia	15993	10502	52063	14574	82141	4181
23. AXA	54693	28408	245910	49864	0.1	18980
24. Mitsui Sumitomo	163297	235094	476419	644816	142370	16976
25. Mean	1544215	828963	7832893	667964	1602873	477733



**Table 2: The amount of the efficiency of independent, dependent, and proposed models for 24 branches of none-life Company.**

DMU	Suggestion two-stage model	Relational two-stage model	Independent two-stage model
1	1	0.699	0.984
2	1	0.625	1.000
3	1	0.960	0.988
4	0.7243	0.304	0.488
5	1	0.767	1.000
6	0.9753	0.390	0.594
7	0.8032	0.277	0.470
8	0.8404	0.275	0.415
9	1	0.223	0.327
10	0.8616	0.46	0.781
11	0.7580	0.164	0.283
12	1	0.760	1.00
13	0.8873	0.208	0.353
14	0.7246	0.289	0.470
15	1	0.614	0.979
16	0.9099	0.320	0.472
17	1	0.360	0.635
18	1	0.259	0.427
19	1	0.411	0.822
20	1	0.547	0.935
21	0.8950	0.20	0.333
22	1	0.590	1.000
23	0.9855	0.420	0.599
24	1	0.135	0.257

5-2 Example 2

To investigate the proposed method for left/right return to scale, we use the following illustrative example:

Suppose that we have one input, one intermediate measure, and one output (m=s=q=1). Data is given in Table 3.

First, we consider the  $DMU_A$  as the under evaluation unit. The corresponding model (4) to this DMU is as follows and optimal

solution of this problem is shown in table of (4):

$$\begin{aligned}
 &max \ z = \alpha \\
 &s. \ t. \ 2\lambda_1 + 3\lambda_2 + 4\lambda_3 + 5\lambda_4 \leq 2, \\
 &2\lambda_1 + 4\lambda_2 + 3\lambda_3 + 5\lambda_4 \geq 2\alpha, \\
 &\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1, \\
 &2\mu_1 + 4\mu_2 + 3\mu_3 + 5\mu_4 \leq 2, \\
 &2\mu_1 + 5\mu_2 + 3\mu_3 + 5\mu_4 \geq 2\alpha, \\
 &\mu_1 + \mu_2 + \mu_3 + \mu_4 = 1, \\
 &\mu_j \geq 0, \lambda_j \geq 0, \ j = 1, \dots, 4, \alpha \text{ is free.}
 \end{aligned}$$

**Table 3: Input, output, and intermediate data**

DMU	X	Z	Y
A	2	2	2
B	3	4	5
C	4	3	3
D	5	5	5

**Table 4: Optimal Simplex table from solving model (4) for  $DMU_A$**

	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$	A	$\lambda_5$	$\lambda_6$	$\mu_1$	$\mu_2$	$\mu_3$	$\mu_4$	$\mu_5$	$\mu_6$	RHS	SRHS
Z	0	0	0	0	0	0	0	0	0	1/4	3/4	3/4	1/2	1	3/2
$\lambda_4$	0	0	1	1	0	2/3	1/3	0	0	-1/6	-1/2	-1/2	-1/3	0	1/3
$\alpha$	0	0	0	0	1	0	0	0	0	1/4	3/4	3/4	1/2	1	3/2
$\lambda_1$	1	0	1	0	0	1/3	2/3	0	0	-1/3	-1	-1	-2/3	1	-4/3
$\mu_2$	0	0	0	0	0	0	0	0	1	1/2	3/2	1/2	0	0	1
$\lambda_2$	0	1	-1	0	0	-1	-1	0	0	1/2	3/2	3/2	1	0	1
$\mu_1$	0	0	0	0	0	0	0	1	0	1/2	-1/2	-1/2	0	1	-1

Table 4 is the latest simplex Table and in the other words, is simplex optimal table in solving model (4) for  $DMU_A$ . Considering Table 4, and the latest right column, we obtain  $\rho_o^+$  from Eq. 10 and the following formula:

$$\rho_o^+ = C_B B^{-1} \begin{pmatrix} x_o \\ Z_o \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 0 \\ 0 \end{pmatrix}$$

Also, to obtain  $\rho_o^-$  using Eq. 11, the last column of Table 4 is multiplied by (-1).

$$\text{And } \rho_o^- = C_B B^{-1} \begin{pmatrix} -x_o \\ -Z_o \\ 0_{s \times 1} \\ 0_{q \times 1} \\ 1 \\ 1 \end{pmatrix}$$

will be resulted.

Notice that  $\lambda_5, \lambda_6, \mu_5$  and  $\mu_6$  are the covariants of 1, 2, 3, 4 and 5 limitations in Table (4).  $\lambda_7$  and  $\mu_7$  are the artificial variables of equal constrains. In table (4) it can be seen that, if we consider RHS and SRHS columns, we have condition (10) and we have  $\rho^+ = 3/2$ . Also, in order to calculate  $\rho^-$ , we consider table (4), multiplying SRHS in(-1), the result would be table (5). In this table, it can be seen that the corresponding lines to  $\lambda_4, \mu_2$  and  $\lambda_2$  are orderly (0,-1/3), (0,-1) and (0,-1) and we don't have the condition (11) therefore, it is not possible to apply the dual simplex.

Now we consider the  $DMU_C$ . Table (6) shows the obtained optimal table of the model (4), which corresponds to this DMU.

**Table 5: Multiplication the SRHS in Table 4 by -1 to obtain  $\rho^-$**

	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$	A	$\lambda_5$	$\lambda_6$	$\mu_1$	$\mu_2$	$\mu_3$	$\mu_4$	$\mu_5$	$\mu_6$	RHS	SRHS
Z	0	0	0	0	0	0	0	0	0	1/4	3/4	3/4	1/2	1	3/2
$\lambda_4$	0	0	1	1	0	2/3	1/3	0	0	-1/6	-1/2	-1/2	-1/3	0	-1/3
A	0	0	0	0	1	0	0	0	0	1/4	3/4	3/4	1/2	1	-3/2
$\lambda_1$	1	0	1	0	0	1/3	2/3	0	0	-1/3	-1	-1	-2/3	1	4/3
$\mu_2$	0	0	0	0	0	0	0	0	1	1/2	3/2	1/2	0	0	-1
$\lambda_2$	0	1	-1	0	0	-1	-1	0	0	1/2	3/2	3/2	1	0	-1
$\mu_1$	0	0	0	0	0	0	0	1	0	1/2	-1/2	-1/2	0	1	1

**Table 6: Optimal Simplex table from solving model (4) for DMU<sub>c</sub>**

	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$	$\alpha$	$\lambda_5$	$\lambda_6$	$\mu_1$	$\mu_2$	$\mu_3$	$\mu_4$	$\mu_5$	$\mu_6$	RHS	SRHS
Z	0	0	0	0	0	0	1/3	0	0	1/6	1/2	1/2	1/3	7/6	2
$\lambda_4$	0	0	3/2	3/2	0	1	0	0	0	-1/4	-3/4	3/4	-1/2	11/4	7
A	0	0	0	0	1	0	1/3	0	0	1/6	1/2	1/2	1/3	7/6	2
$\lambda_1$	1	0	1/2	-1/2	0	0	0	0	0	-1/4	-3/4	3/4	-1/2	7/4	3
$\mu_2$	0	0	0	0	0	0	0	0	1	1/2	3/2	1/2	0	1/2	2
$\lambda_2$	0	1	1/2	3/2	0	0	0	0	0	1/4	3/4	3/4	1/2	3/4	3
$\mu_1$	0	0	0	0	0	0	0	1	0	1/2	-1/2	-1/2	0	1/2	-2

**Table 7: Multiplication the SRHS in Table 6 by -1 to obtain  $\rho^-$**

	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$	$\alpha$	$\lambda_5$	$\lambda_6$	$\mu_1$	$\mu_2$	$\mu_3$	$\mu_4$	$\mu_5$	$\mu_6$	RHS	SRHS
Z	0	0	0	0	0	0	1/3	0	0	1/6	1/2	1/2	1/3	7/6	2
$\lambda_4$	0	0	3/2	3/2	0	1	0	0	0	-1/4	-3/4	3/4	-1/2	11/4	-7
A	0	0	0	0	1	0	1/3	0	0	1/6	1/2	1/2	1/3	7/6	-2
$\lambda_1$	1	0	1/2	-1/2	0	0	0	0	0	-1/4	-3/4	3/4	-1/2	7/4	-3
$\mu_2$	0	0	0	0	0	0	0	0	1	1/2	3/2	1/2	0	1/2	-2
$\lambda_2$	0	1	1/2	3/2	0	0	0	0	0	1/4	3/4	3/4	1/2	3/4	-3
$\mu_1$	0	0	0	0	0	0	0	1	0	1/2	-1/2	-1/2	0	1/2	+2

It is obvious that, considering RHS and SRHS columns, we have the condition (10) and also  $\rho^+ = 2$ . To obtain the  $\rho^-$ , we consider the table (6). Multiplying to (-1) we try to have the condition (11). The result is the table (7) with result  $\rho^- = 2$ .

### 6- Conclusion

In this paper, different methods have been shown for measuring efficiency and return to scale in two-stage processes.

So far, many methods for efficiency evaluation and network return to scale have been proposed, each contains pros and cons. One of the problems in determining network return to scale is the feasibility of the proposed method. For example, the proposed method by Golany and Yu, [16] is not always feasible, but our approach is always feasible for all DMUs under evaluation.

In addition, in many other network approaches, two independent models are considered to evaluate the efficiency of

network processes. Such as, independent model of Kao and Hwang [9] which two independent models will result in two independent efficiency frontiers. Indeed, we can hardly find a common approach for return to scale of units between these two frontiers. Although Kao and Hwang [9] evaluated the whole process efficiency, their model was only considered in constant return to scale condition.

Kao and Hwang [9] have computed efficiency in an overall process, but their model was only true assuming constant return to scale. Chen et al. [10] have proposed their models under constant and variable return to scale, but their model can be used only for determining efficiency and falls short to compute return to scale class. As addressed earlier, measuring return to scale in two-stage processes is not an easy task due to the unknown possible space for generating in the PPS. Therefore, in this paper we tried

to measure left and right return to scale in two-stage processes by proposing a new model and also applying suggested algorithm by Zarepisheh, Soleimani-damaneh, [18]. While the proposed method is with high computational complexity, considering theorems and examples, it can be accomplished with acceptable results. Moreover due to the proposed model, the joint border problem of the two stages has been solved.

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