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Ranking Decision Making Units with Fuzzy Data Using Cross-efficiency with use of ranking function

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Abstract

Data covering analysis is a technique for evaluating the performance of homogeneous decision-making units. In evaluating the performance of units, it is necessary the efficiency and ranking of units be done (be calculated). Ranking decision-making units is based on efficiency. According to the existence of various efficacies in the applied examples, different techniques have been developed for ranking, that in this study, cross efficiency method has been used. This technique has been developed for fuzzy input and output modes. While data are fuzzy, cross efficiency table has been calculated inaccurately and table entries are fuzzy numbers, so the average (mean) of calculated efficiency in the table is also a fuzzy number. Since comparing the averages of obtained efficiencies ranks the decision-making units, it is necessary to propose a solution for this comparison. In this research, after providing the required grounds with assistance of models and doing the necessary changes in them, the averages of efficiency are compared by assistance of fuzzy ranking function. And with the knowledge that every decision-making unit that has better average (mean) is more efficient, decision-making units are ranked.

Keywords: Data Envelopment Analysis (DEA); Fuzzy- Data Envelopment Analysis (F-DEA); Ranking; Fuzzy ranking approach

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1. Introduction

In the world we live in, many of the definitions and concepts are imprecise and uncertain and relative. Here, it is necessary that imprecise data be compared with each other. In this comparison the decision-making individual faces a kind of uncertainty that is related to lack of precise and strong bordering of concepts. These concepts are not arguable, inferable and capable of decision-making by Aristotelian logic, which its necessity is accurate and quantitative data. ([1-3]).

The theory of fuzzy sets, applying special models, is capable to give mathematical format to many concepts and systems that are imprecise and vague and provides the ground for inferring and decision-making in uncertain conditions. ([4-15])

The theory of fuzzy sets was proposed for the first time in 1965 by Askarzadeh, Iranian born scientist and professor of Berkley University in America. After him, many researchers tried to develop and generalize this theory and applied it in different fields; in this research, we have tried to move in this direction. We evaluate the decision-making units at the state that the input and outputs are fuzzy and in this evaluation, we realize our purpose by assistance of cross efficiency model. In the third section, we have studied ranking with fuzzy input and output by assistance of cross efficiency model without using α-shear model (that all entries of cross table are fuzzy numbers) and also the mean of all efficiencies of DMUs are fuzzy, and thereby we present a new research to be remained in the memories. ([17-22]).

2. Fuzzy data envelopment analysis

Applying DEA models to evaluate the relative efficiency of DMUs, usually more than one efficient DMU is evaluated. Ranking efficient decision-making units has certain importance. One of the methods to determine the efficiency and ranking of decision-making units (DMU) is Sexton (cross) method which is described briefly as follows. This model is as following at non-fuzzy states.

$$e_o = max\ \frac{U\,Y_o}{VX_o} \qquad o \in \{1,2,\dots,n\}$$

$$S.T\ \frac{U\,Y_j}{VX_j} \le 1 \qquad j = 1,\dots,n \qquad (1)$$

$$U \ge 1\varepsilon\ , V \ge 1\varepsilon$$

Where, {1,2, ..., n} are the sets of indices of decision-making units and $o \in \{1,2,\dots,n\}$is the under evaluation unit. $X_j = (X_{1j},\dots,X_{mj})$ is the input vector of j[th] unit and $Y_j = (Y_{1j},\dots,Y_{sj})$ is the output vector of unit j and is assumed to be $Y_j \neq 0, Y_j \ge 0,\ X_j \neq 0, X_j \ge 0$.

And by solving the above model, the answer (u_o^*, v_o^*) is obtained. The efficiencies of all the units are calculated as follows based on this measure (weight).

$$\theta_{io} = \frac{u_o^* Y_i}{v_o^* X_i} \qquad i = 1,\dots,n$$

The Sexton table is as follows at non-fuzzy mode.

Table 1: Cross efficiencies at non-fuzzy mode

	DMU1	*DMU2*	…	*DMUn*	*Mean*
DMU1	$\theta_{11} = \theta_1^*$	$\theta_{12} = \theta_2^*$	…	$\theta_{1n} = \theta_n^*$	$\frac{1}{n}\sum_{j=1}^{n}\theta_{1j}$
DMU2	$\theta_{21} = \theta_1^*$	$\theta_{22} = \theta_2^*$	…	$\theta_{2n} = \theta_n^*$	$\frac{1}{n}\sum_{j=1}^{n}\theta_{2j}$
…	…	…	…	…	…
DMUn	$\theta_{n1} = \theta_1^*$	$\theta_{n2} = \theta_2^*$	…	$\theta_{nn} = \theta_n^*$	$\frac{1}{n}\sum_{j=1}^{n}\theta_{nj}$

The Sexton method a method for measuring the efficiency of units, in addition, it performs ranking of DMUs. Each DMU that is better in terms of mean is more efficient.
One of the major advantages of the Sexton method (cross efficiency) is that it ranks all DMUs.
Membership function: Suppose X is a non-empty set, a fuzzy subset of X is determined by a function

$$\mu_{\tilde{A}(x)} : X \rightarrow [0,1]$$

This is called the membership function.
$\mu_{\tilde{A}(x)}$ denotes the degree of x belonging to the fuzzy set $\tilde{A}$.

Triangular fuzzy number

If the fuzzy number LR, L and R, are linear functions as shown below; we call them triangular and are shown as sorted triple of $\tilde{N} = (l, m, u)$, where m represents the central value or the possible value of $(\mu_{\tilde{N}}(m) = 1\ )$, and l and u are the right and left ranges respectively.

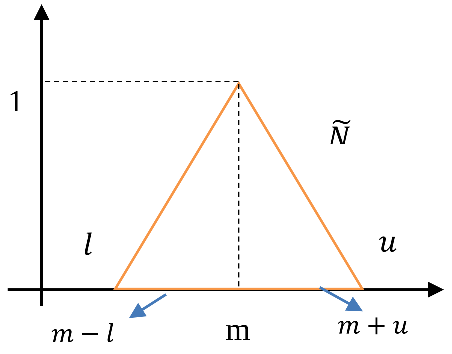

Figure 1. Triangular fuzzy number

Triangular fuzzy number, $\tilde{N} = (l, m, u)$, is also shown as $\tilde{N} = (X^l, X^m, X^u)$; where, x^m is a central value or a maximum value of $(\mu_{\tilde{N}}(X^m) = 1)$. x^l is the pessimistic value or the minimum value of $(\mu_{\tilde{N}}(X^l) = 0)$ and x^u is the optimistic value or the maximum value of $((\mu_{\tilde{N}}(X^u) = 0)$. The membership function of a triangular fuzzy number like $\tilde{N}$ is as follows.

$$\mu_{i(X)} = \begin{cases} \dfrac{X - l}{m - \alpha} & l \leq X \leq m \\ \dfrac{u - X}{u - m} & m \leq X \leq u \\ 0 & \text{other points} \end{cases}$$

The ordered tetrad, $\tilde{M} = (m_1, m_2, l, u)$; where, m_1 and m_2 are the most possible value, that is, $\forall x \in [m_1, m_2]; \mu_{\tilde{M}}(X) = 1$; l and u are the right and left ranges of $\tilde{M}$ respectively. Trapezoidal fuzzy number is also shown as the ordered tetrad, $\tilde{M}$=(X^{m1}, X^{m2}, X^l, X^u); where, X^{m1} and X^{m2} are the most possible value, that is, $\forall x \in [X^{m1}, X^{m2}]; \mu_{\tilde{M}}(x) = 1$; x^l is the pessimistic value or the minimum value of $(\mu_{\tilde{M}}(x^l) = 0)$ and x^u is the optimistic value or the maximum value of $(\mu_{\tilde{M}}(x^u) = 0)$.

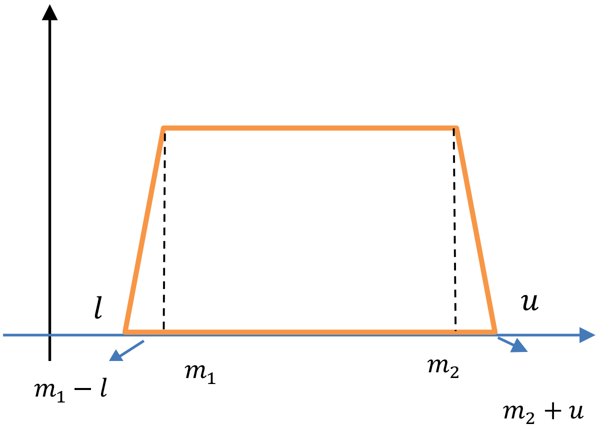

Figure 2. Trapezoidal fuzzy number

Trapezoidal fuzzy number, $\tilde{\mu}$ =(m_1, m_2, l, u), is a membership function for the trapezoidal fuzzy number such as $\tilde{M}$ as follows.

$$M_i(x) = \begin{cases} 0 & x \leq l \\ \dfrac{m_1 - x}{m_1 - l} & l \leq x \leq m_1 \\ 1 & m_1 \leq x \leq m_2 \\ \dfrac{x - m_2}{u - m_2} & m_2 \leq x \leq u \\ 0 & x \geq u \end{cases}$$

3. Ranking with fuzzy input and output in the cross efficiency

Suppose the inputs and outputs of decision making units are a fuzzy number; then the vector of inputs and outputs of jth unit will be as follows.

$\widetilde{X}_J = (\tilde{X}_{1j}, \ldots, \tilde{X}_{mj})$

$\widetilde{Y}_J = (\tilde{Y}_{1j}, \ldots, \tilde{Y}_{sj})$

For (j=1,…,n) DMU_J in the fuzzy state, we have m to fuzzy input and s to fuzzy output.

Sexton method (cross efficiency) is as follows at fuzzy mode.

First, the efficiency of units with fuzzy data is calculated as follows:

$$e_o = max\ \frac{U\widetilde{y_o}}{V\widetilde{x_o}} \quad o \in \{1,2,\ldots,n\}$$

$$\text{s.t } \frac{U\widetilde{Y_J}}{V\widetilde{X_J}} \le 1 \ \ j = 1,\ldots,n \qquad (2)$$

$$U \ge 1\varepsilon\ ,\ \ V \ge 1\varepsilon$$

The model (2) is as follows in linear fuzzy model:

$$\begin{aligned} &max\ u\ \widetilde{Y_o} \\ &st\ U\widetilde{Y_J} - V\widetilde{X_J} \le 0 \quad j = 1,\ldots,n \\ &V\widetilde{X_o} = 1 \\ &U \ge 1\varepsilon\ ,\ \ V \ge 1\varepsilon \end{aligned}$$

The dual model (2) is as follows in linear fuzzy model:

$$\begin{aligned} &\min \tilde{\theta} \\ &s.t\ \textstyle\sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \le \tilde{\theta}\,\widetilde{x_{io}},\ i = 1,\ldots,m \qquad (3) \\ &\textstyle\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \ge \tilde{y}_{ro}, \qquad r = 1,\ldots,s \\ &\lambda_j \ge 0, \qquad j = 1,\ldots,n \end{aligned}$$

In this research, we follow how to form the cross table (Sexton model) for fuzzy data, and try to consider fuzzy numbers triangular and finally rank DMUs by assistance of the ranking fuzzy function and to reach this purpose, we consider:

$$\forall_i \forall_j\ \tilde{X}_{ij} = (L_{ij}^x, m_{ij}^x, u_{ij}^x)$$

$$i = 1,\ldots,m\ ,\ \ j = 1,\ldots,n$$

$$\forall_r \forall_j\ \tilde{Y}_{rj} = (L_{rj}^y, m_{rj}^y, u_{rj}^y)$$

$$r = 1,\ldots,s$$

That is, all inputs and outputs are fuzzy triangular numbers. In this study, we define the relations (actions) among fuzzy numbers as follows.

We assume: $\tilde{A} = (l, m, u)$ and $\tilde{B} = (\acute{l}, \acute{m}, \acute{u})$ are LR fuzzy numbers and r is a scalar.

The sum of fuzzy numbers: $\tilde{A} + \tilde{B} = (l, m, u) + (\acute{l}, \acute{m}, \acute{u}) = (l + \acute{l}, m + \acute{m}, u \acute{+} u)$

The subtraction of fuzzy numbers: $\tilde{A} - \tilde{B} = (l, m, u) - (\acute{l}, \acute{m}, \acute{u}) = (l + u', m - m', u + l')$

The multiplication of fuzzy numbers:

$$\tilde{A} \times \tilde{B} \cong \begin{cases} (m\acute{l} + \acute{m}l, m\acute{m}, + ml' + \acute{m'}u) & \tilde{A} > 0, \tilde{B} > 0 \\ (ml' - m'u, mm', ml' - m'l) & \tilde{A} > 0, \tilde{B} < 0 \\ (-m'u - ml', mm', -m'l - ml') & \tilde{A} < 0, \tilde{B} < 0 \end{cases}$$

The division of fuzzy numbers:

$$\frac{\tilde{A}}{\tilde{B}} \cong \frac{(l, m, u)}{(l', m', u')} = \left(\frac{ml' + m'l}{m'^2}, \frac{m}{m'}, \frac{ml' + m'u}{m'^2}\right)$$

Scalar multiplication:

$$r\tilde{A} = \begin{cases} (rl, rm, ru) & r > 0 \\ (rl, -ru, -rl) r < 0 \end{cases}$$

Now, we consider the following fuzzy model:

$$\begin{aligned} &\max E\tilde{Y} \qquad (4) \\ &s.t \quad W\tilde{Y}_j - V\tilde{X}_j \le 0 \quad j = 1,\ldots,n \\ &V\tilde{X}_p = 1 \\ &W \ge 0, V \ge 0 \end{aligned}$$

We consider a ranking function and know:

$$(l, m, u) \le (\acute{l}, \acute{m}, \acute{u}) \Leftrightarrow \begin{cases} l \le \acute{l} \\ m \le \acute{m} \\ u \le \acute{u} \end{cases}$$

We expand model (4) as follows:

$$\max\left(\textstyle\sum_{r=1}^{s} w_r l_{rp}^y, \sum_{r=1}^{s} w_r m_{rp}^y, \sum_{r=1}^{s} w_r u_{rp}^y\right)$$

$$s.t\left(\textstyle\sum_{r=1}^{s} w_r l_{rj}^y - \sum_{i=1}^{m} v_i u_{ij}^x, \sum_{r=1}^{s} w_r m_{rj}^y\right) - \left(\sum_{i=1}^{m} v_i m_{ij}^x, \sum_{r=1}^{s} w_r u_{rj}^y - \sum_{i=1}^{m} v_i l_{ij}^x\right) \le (0,0,0) \ \ j = 1,\ldots,n$$

$$\left(\textstyle\sum_{i}^{m} v_i l_{ip}^x, \sum_{i}^{m} v_i m_{ip}^x, \sum_{i}^{m} v_i u_{ip}^x\right) = (1,1,1)$$

$$u \ge 0, v \ge 0\,.$$

Model (4) is a covering dual form that is the covering form, θ ≥ 0; thus, the corresponding constraint can be≤ 1, that is, $v \tilde{\times}\ p \le 1$.

We consider model (6) as following:

$$\max \lambda_1\left(\sum_{r=1}^{s} w_r\, l_{rp}^{y}\right) + \lambda_2\left(\sum_{r=1}^{s} w_r\, m_{rp}^{y}\right) + \lambda_3\left(\sum_{r=1}^{s} w_r\, u_{rp}^{y}\right)$$

$$s.t \qquad (6)$$

$$\sum_{r=1}^{s} w_r\, l_{rj}^{y} - \sum_{i=1}^{m} v_i\, l_{ij}^{x} \le 0 \quad j = 1,\dots,n$$

$$\sum_{r=1}^{s} w_r\, m_{rj}^{y} - \sum_{i=1}^{m} v_i\, m_{ij}^{x} \le 0 \quad j = 1,\dots,n$$

$$\sum_{r=1}^{s} w_r\, u_{rj}^{y} - \sum_{i=1}^{m} v_i\, u_{ij}^{x} \le 0 \quad j = 1,\dots,n$$

$$\sum_{i=1}^{m} v_i\, l_{ip}^{x} \le 1$$

$$\sum_{i=1}^{m} v_i\, m_{ip}^{x} \le 1$$

$$\sum_{i=1}^{m} v_i\, u_{ip}^{x} \le 1,\ U \ge 0,\ V \ge 0$$

We solve model (6) and assume (w_p^*, v_p^*) is an optimum solution. Then, we have:

$$\theta_{tp} = \frac{w_p^*\, \tilde{y}_t}{v_p^*\, \tilde{x}_t} = \frac{\left(\sum_{r=1}^{s} w_{rp}^*\, l_{rt}^{y}, \sum_{r=1}^{s} w_{rp}^*\, m_{rt}^{y}, \sum_{r=1}^{s} w_{rp}^*\, u_{rt}^{y}\right)}{\left(\sum_{i=1}^{m} v_{ip}^*\, l_{it}^{x}, \sum_{i=1}^{m} v_{ip}^*\, m_{it}^{x} \sum_{i=1}^{m} v_{ip}^*\, u_{it}^{x}\right)} \cong \left(l_{tp}^{\theta}, m_{tp}^{\theta}, u_{tp}^{\theta}\right)$$

Therefore, we have:

tp entry of fuzzy cross efficiency table = $(l_{tp}^{\theta}, m_{tp}^{\theta}, u_{tp}^{\theta})$

With regard to the above calculation of fuzzy cross efficiency table is as follows.

Table 2: fuzzy cross efficiency table

	DMU$_1$	…	*DMU*$_n$	*Fuzzy average (mean)*
DMU$_1$	$(l_{11}^{\theta}, m_{11}^{\theta}, u_{11}^{\theta})$	…	$(l_{1n}^{\theta}, m_{1n}^{\theta}, u_{1n}^{\theta})$	$(\frac{1}{n}\sum_j l_{1j}^{\theta}, \frac{1}{n}\sum_j m_{1j}^{\theta}, \frac{1}{n}\sum_j u_{1j}^{\theta})$
DMU$_2$	$(l_{21}^{\theta}, m_{21}^{\theta}, u_{21}^{\theta})$	…	$(l_{2n}^{\theta}, m_{2n}^{\theta}, u_{2n}^{\theta})$	$(\frac{1}{n}\sum_j l_{2j}^{\theta}, \frac{1}{n}\sum_j m_{2j}^{\theta}, \frac{1}{n}\sum_j u_{2j}^{\theta})$
…	…	…	…	…
DMU$_n$	$(l_{n1}^{\theta}, m_{n1}^{\theta}, u_{n1}^{\theta})$	…	$(l_{nn}^{\theta}, m_{nn}^{\theta}, u_{nn}^{\theta})$	$(\frac{1}{n}\sum_j l_{nj}^{\theta}, \frac{1}{n}\sum_j m_{nj}^{\theta}, \frac{1}{n}\sum_j u_{nj}^{\theta})$

The above table that all its entries are triangular fuzzy numbers shows the efficiency of DMUs in the fuzzy mode. As can be seen, the average is also triangular fuzzy. The triangular fuzzy averages should be compared with each other and to compare them and determine the largest number, we use fuzzy ranking function.

4. Applied example

Ten commercial banks have been studied and the required data from these ten banks were obtained that all are triangle fuzzy numbers. Then using introduced models in this chapter, we have formed the table of cross efficiency and by comparing fuzzy averages in the table, rating (ranking) of banks was done. The details of study and needed calculations are as following.

Table 3: xL(j,i)

	i1	i2	i3
DMU1	5.42	166965005	347912609
DMU2	6.5	1364254263	321087157
DMU3	5.13	1021540167	439622053
DMU4	7.58	1023094065	247470622
DMU5	3.89	244442242	28332000
DMU6	4.44	150114017	175107405
DMU7	2.69	41603512	55843067
DMU8	2.26	1025368685	5079356
DMU9	3.77	1259611949	321956067
DMU10	5.6	1720212885	58700000

Table 4: xM(j,i)

	i1	i2	i3
DMU1	11.505	3204527225	6267251735
DMU2	17.77571429	5420093131	13974801379
DMU3	14.77714286	4062997330	2430817731
DMU4	14.93928571	5811247992	3264590272
DMU5	14.79285714	3728154627	8935005782
DMU6	11.85928571	4825940830	5148882350
DMU7	10.82642857	4557896111	7333046577
DMU8	10.1457142	3925577955	4346972893
DMU9	15.90214286	5262472780	7062857806
DMU10	13.45428571	4451249734	8880600175

Table 5: xU(j,i)

	i 1	i2	i3
DMU1	21.61	5826283949	30033076818
DMU2	56.23	15713640424	94870216509
DMU3	38.59	11969476089	9958384916
DMU4	31.43	30435770419	17958774018
DMU5	26.41	11450174945	82231846069
DMU6	19.31	15105382313	29728030018
DMU7	25.01	11461622508	36434239083
DMU8	22.8	12424040548	19571582641
DMU9	31.9	18113748298	57849361275

DMU10	33.24	12448647772	91314625872

Table 6: yL(j,r)

	o1	o2	o3	o4	o5
DMU1	7098487595	28948043462	848671179	38368691	5162000
DMU2	7949322656	23631050649	566162650	42883893	4405000
DMU3	14995970790	38631972139	864134766	36411899	7300000
DMU4	18514914833	29432773959	895838606	77239155	18275000
DMU5	17332785899	19275628277	18843288	1431607	3925000
DMU6	20385936597	21779585799	4024246	56474	11749913
DMU7	7452604318	9327588934	569445	391780	1106670
DMU8	6146639414	28111168328	221934579	25128335	650000
DMU9	27850207872	20439226024	1367871203	25582456	17733000
DMU10	10096103316	28454116922	517976795	27773398	3500000

Table 7: yM(j,r)

	o1	o2	o3	o4	o5
DMU1	52171726468	55729852938	3957105421	158252484.1	209362113
DMU2	92582493774	91501177806	5900502096	431321754.4	1907797026
DMU3	80053680037	72004902720	5333368785	309486691.8	1347623292
DMU4	80184379532	93783934491	5620799538	516295053.6	2014151023
DMU5	89211750661	84640654618	4911634657	168888314.4	401399286.7
DMU6	80255164183	72319356101	5875395233	351730379.7	1530606378
DMU7	69565403708	64608470345	4198035611	196957135.3	632104134.3
DMU8	50638122921	59838074926	3005501778	207056005.1	676723365
DMU9	81830300111	88236906513	5600393718	223554326.6	542498353.9
DMU10	86094189632	85908459121	5128502987	354636065.6	2959881112

Table 8: yU(j,r)

	o1	o2	o3	o4	o5
DMU1	122497919047	121751849411	6914738665	305467932	747392532
DMU2	378785621113	329970969669	22330988268	2306896375	15347945440
DMU3	144962760424	167941180111	14534600915	1057234112	5720495923
DMU4	320467175205	353170012187	24239290627	1683454030	17734125040
DMU5	286782666394	390987804421	19805543528	457407119	1939809308
DMU6	311973279912	211466703060	35304269319	2688388239	11142908153
DMU7	266434914435	124678248906	15460389283	792206863	3956028971
DMU8	188565753715	140587061901	8069534218	570261939	6055939136
DMU9	232634001693	259004083181	20717137949	517953877	1841137172
DMU10	443720575872	355691625890	21626923578	2316300360	30782513082

According to calculations, the following table is obtained.

Table 9. efficiency results.

Decision-making Unit	Related fuzzy averages
DMU_1	$(0.0100,0.0552,0.2579)=A_1$
DMU_2	$(0.0034,0.0654,0.7503)=A_2$
DMU_3	$(0.0079,0.0636,0.4389)=A_3$
DMU_4	$(0.0091,0.0793,0.5899)=A_4$
DMU_5	$(0.0065,0.0645,0.9688)=A_5$
DMU_6	$(0.0103,0.0814,0.9576)=A_6$
DMU_7	$(0.0032,0.0716,0.8634)=A_7$
DMU_8	$(0.0087,0.0666,0.8574)=A_8$
DMU_9	$(0.0076,0.0619,0.7519)=A_9$
DMU_{10}	$(0.0064,0.0791,0.9532)=A_{10}$

Here, we should compare to each other the fuzzy averages related to DMUs. In this comparison, each DMU which has the better average is more efficient.

To compare fuzzy numbers, we use the following number.

Suppose $A=(a_1 , a_2, a_3)$ and $B=(b_1,b_2,b_3)$ are triangular fuzzy numbers.

We define that: $D(A) = a_1 + \frac{1}{4(a_3-a_2)}$

Thus, we have:

$A < B \Leftrightarrow D(A) < D(B)$

$A \leq B \Leftrightarrow D(A) \leq D(B)$

$A = B \Leftrightarrow D(A) = D(B)$

According to the above definition and table 9, we have:

$D(A_1)=0.0100+\frac{1}{4(0.2579-0.0552)} = 1.243349$

$D(A_2)=0.0034+\frac{1}{4(0.7503-0.0.0654)} = 0.368416$

$D(A_3)=0.0079+\frac{1}{4(0.4389-0.0636)} = 0.674033$

$D(A_4)=0.0091+\frac{1}{4(0.5899-0.0793)} = 0.498720$

$D(A_5)=0.0065+\frac{1}{4(0.9688-0.0645)} = 0.977355$

$D(A_6)=0.0103+\frac{1}{4(0.9576-0.0814)} = 3.081553$

$D(A_7)=0.0032+\frac{1}{4(0.8634-0.0716)} = 0.318936$

$D(A_8)=0.0087+\frac{1}{4(0.8574-0.0666)} = 0.394835$

$D(A_9)=0.0076+\frac{1}{4(0.7519-0.0619)} = 1.323389$

$D(A_{10})=0.0064+\frac{1}{4(0.9532-0.0791)} = 0.299408$

By considering the above calculations, we observe:

$D(A6)>D(A9)>D(A1)>D(A3)>D(A4)>D(A2)>D(8)>D(A7)>D(A10)>D(A5)$

Ranking of decision-making units are as followings:

Table 10: Ranking of DMUs

DMU	1	2	3	4	5	6	7	8	9	10
Rank	3	6	4	5	10	1	8	7	2	9

As we can see, table 10 shows the rank of decision making units that each decision making units indicates a branch of a commercial bank.

Dmu_6 has obtained the rank 1; by considering the input and output tables, it is observed that this *Dmu* comparing with the other *Dmu*s has the least input and the most output, thus, obtaining rank 2 is not far from reality.

Dmu_5 has obtained rank 10, that is, among ten decision making units is the weakest unit. by considering the input and output tables, it is observed that this *Dmu* comparing with the other *Dmu*s, in average, has the most input and the least output, thus, obtaining rank 10 is not far from reality. And in this way, the other *Dmu*s can be interpreted by comparing them with each other.

5. Conclusion

In this paper, Sexton Model (Cross efficiency) was used to rank decision making units (*Dmu*), while the inputs and outputs all are fuzzy numbers. By manipulation related fuzzy model and doing necessary calculations, we obtained the entries of table of cross efficiency that all are triangular fuzzy numbers, then calculated the fuzzy average (mean) for each *Dmu*. The fuzzy average (mean) of all *Dmu*s is comparable by fuzzy ranking function. Each *Dmu* which has higher fuzzy mean is more efficient. We conclude that Sexton model has the capability of evaluation of *Dmu*s and ranking them in fuzzy mode. To make this study an applied one, ten branches of a commercial bank were studied; real data of these banks were used and the banks were ranked by Cross Efficiency Method. In this ranking, in addition to determining the strongest and weakest banking branches, also the branches have been interpreted that the results are in agreement with reality.

References

[1] Cooper, W. W., Charnes, A., & Thrall, R. M. (1991). A structure for classifying and characterizing efficiency and inefficiency in data envelopment analysis. *Journal of Productivity Analysis*, *2*(3), 197-237.

[2] Cooper, W.W., Park, K.S., Yu, G. (1999). IDEA and AR-IDEA models for dealing with imprecise data in DEA. *Manage. Sci.*, *45*, 597–607.

[3] Cooper, W. W., Seiford, L. M., & Tone, K. (2007). DATA ENVELOPMENT ANALYSIS A Comprehensive Text with Models, Applications, References and DEA-Solver Software Second Edition. *Springer*, *ISBN*, *387452818*, 490.

[4] Adler, N., Friedman, L., & Sinuany-Stern, Z. (2002). Review of ranking methods in the data envelopment analysis context. *European Journal of Operational Research*, *140*(2), 249-265.

[5] Saati, S., Hatami-Marbini, A., & Makui, A. (2009). An Application of Fuzzy Numbers Ranking in Performance Analysis. *Journal of Applied Sciences*, *9*(9).

[6] Sexton, T. (1979). The single vehicle many to many routing and scheduling problem, Ph.D. Thesis, State University of New York at Stony Brook.

[7] Wang, Y. M., Greatbanks, R., & Yang, J. B. (2005). Interval efficiency assessment using data envelopment analysis. *Fuzzy sets and Systems*, *153*(3), 347-370.

[8] Garcia, P. A. A., & Schirru, R. (2005). A fuzzy data envelopment analysis approach for FMEA. *Progress in Nuclear Energy*, *46*(3), 359-373.

[9] Jahanshahloo, G. R., Soleimani-Damaneh, M., & Nasrabadi, E. (2004). Measure of efficiency in DEA with fuzzy input–output levels: a methodology for assessing, ranking and imposing of weights restrictions. *Applied Mathematics and Computation*, *156*(1), 175-187.

[10] Allahviranloo, T., Saneifard, R., Hosseinzadeh, F., & Mikaeilvand, N. (2007). Euclidean Ranking DMUs with fuzzy data in DEA. *Applied Mathematical Sciences*, *60*, 2989-2998.

[11] Hatami-Marbini, A., Emrouznejad, A., & Tavana, M. (2011). A taxonomy and review of the fuzzy data envelopment analysis literature: two decades in the making. *European Journal of Operational Research*, *214*(3), 457-472.

[12] Kao, C., & Liu, S. T. (2000). Fuzzy efficiency measures in data envelopment analysis. *Fuzzy sets and systems*, *113*(3), 427-437.

[13] Lertworasirikul, S., Fang, S. C., A Joines, J., & LW Nuttle, H. (2003). Fuzzy data envelopment analysis (DEA): a possibility approach. *Fuzzy sets and systems*,*139*(2), 379-394.

[14] Jensen, J. R., & Ji, M. (1999). Effectiveness of subpixel analysis in detecting and quantifying urban imperviousness from Landsat Thematic Mapper imagery. *Geocarto International*, *14*(4), 33-41.

[15] Khoshfetrat, S., & Daneshvar, S. (2011). Improving weak efficiency frontiers in the fuzzy data envelopment analysis models. *Applied Mathematical Modelling*,*35*(1), 339-345.

[16] Zerafat Angiz L, M., Mustafa, A., & Emrouznejad, A. (2010a). Ranking efficient decision-making units in data envelopment analysis using fuzzy concept. *Computers & Industrial Engineering*, *59*(4), 712-719.

[17] Yücel, A., & Güneri, A. F. (2011). A weighted additive fuzzy programming approach for multi-criteria supplier selection. *Expert Systems with Applications*,*38*(5), 6281-6286.

[18] Zerafat Angiz L, M., Emrouznejad, A., Mustafa, A., & Al-Eraqi, A. S. (2010b). Aggregating preference ranking with fuzzy data envelopment analysis. *Knowledge-Based Systems*, *23*(6), 512-519.

[19] Qin, R., Liu, Y. K., & Liu, Z. Q. (2011). Methods of critical value reduction for type-2 fuzzy variables and their applications. *Journal of Computational and Applied Mathematics*, *235*(5), 1454-1481.

[20] Wu, D. D. (2009a). Performance evaluation: an integrated method using data envelopment analysis and fuzzy preference relations. *European Journal of Operational Research*, *194*(1), 227-235.

[21] Wu, D. D. (2009b). Supplier selection in a fuzzy group setting: A method using grey related analysis and Dempster–Shafer theory. *Expert Systems with Applications*, *36*(5), 8892-8899.

[22] Wen, M., You, C., & Kang, R. (2010). A new ranking method to fuzzy data envelopment analysis. *Computers & Mathematics with Applications*, *59*(11), 3398-3404.