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Estimating Most Productive Scale Size of the provinces of Iran in the Employment sector using Interval data in Imprecise Data Envelopment Analysis (IDEA)

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Abstract

Unemployment is one of the most important economic problems in Iran, so that many of its managers plan to increase employment rates. Increasing the employment rate needs to increase economic productivity which DEA is one of the most appropriate evaluation methods for estimating the productivity of similar organizations. Employment in the amount of data input and output can be just interval. In this study by solving two models, using one of which the upper bound for efficiency and using the other, the lower bound for decision making units efficiency is acquired, we provide a new model for Most productive scale size with interval data. The main purpose of this study is to determine the productivity of Iran and sensitive indicators to provide a fundamental solution to exit from unemployment. The economic sector managers can do more exact planning for economic growth.

Keywords: Employment, Data envelopment analysis, Efficiency ,Interval data, Most productive scale size (MPSS).

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1. Introduction

Productivity is often used as a performance benchmark to rank firms or countries or to measure the rate of performance improvement over time. Such rankings gained credibility once studies documented that productivity is positively correlated with other indicators of success such as profit, employment growth, export status, technology adoption, or mere survival. Many different ways to measure productivity exist, one of these methods is the non-parametric Data Envelopment Analysis, which is based on mathematical linear programming and has been used several times in evaluating homogeneous decision making units (DMU's) relative efficiency. This method's ability to compare similar units to each other and also the possibility to analyze its outcome has added more to its application in various areas. DEA started by Charnes, Cooper, and Rhodes with starting CCR model in (1978) and continued later by introducing BCC model by Banker, Charnes and Cooper in (1984). DEA is a non-parametric method to evaluate relative efficiency of decision making units based on multiple inputs and outputs. The original DEA method assumes that inputs and outputs are examined by exact amounts. The term imprecise data means that input and output data cannot be obtained accurately due to lack of confidence. Cooper et al (1999) discussed interval data. They considered the combination of ordinal data and interval data as imprecise data and a method in the form of IDEA (Imprecise Data Envelopment Analysis) was created by Cooper et al (1999). When the data is imprecise, efficiency size obtained from these data must also be imprecise. Despotisand Smirlis (2006) also entered bounded data in DEA. Zhu (2003) considered fuzzy DEA model with CCR approach with interval data, ranking data, and relative data. Jahanshahloo and Khodabakhshi (2003) obtained most productivity scale size determination in classic DEA, Khodabakhshi (2009) obtained most productivity scale size in stochastic DEA. In order to solve the stochastic model, equivalent deterministic is also provided in his study, although the equivalent deterministic is non-linear but it can be turned into quadratic programming. Eventually, the proposed point of view of the study was applied onto the software firm data and the evaluation was compared in two states of classic and stochastic. Eslami et al (2012) presented a model for estimating most productive scale size with imprecise-chance constrained input–output orientation. That research probe a realistic decision problem that contains fuzzy and uncertain data Wang and Lan (2013). In this paper we have estimated the MPSS by using the pessimistic DEA model and a double frontiers approach. At first, we estimated the MPSS from the pessimistic point of view.Though, all the papers about the MPSS in DEA are based on the optimistic point of view. The performances of decision making units (DMU's) can also be measured from the pessimistic point of viewWang and Yang (2007). Since, the results of MPSS application in different evaluation system might give different results, therefore by applying Double Frontiers and Hurwicz's Criterion, the performance of each unit is estimated in both optimistic and pessimistic point of view (see Hurwicz (1951), Hurwicz (1952) and Wang (2013)). Lee (2016) compared most productive scale size against demand fulfillment Lee (2016). The above study shows that the DEA is one of the most appropriate evaluation methods to measure the productivity and organization. But, so far the most size scale research productivity in the provinces Considering employment using imprecise data envelopment analysis period, was not done. So one of the main purposes of this study is to evaluate the productivity of the country's provinces sector employment and the question of research are: What are the employment sector productivity provinces? What are the sensitive indicators in employment? How would you rate their productivity in each province? In this research, through the design of two new models of imprecise data envelopment analysis, the above questions were answered, and some of the features of these two suggested models are presented below: "method for calculating the most productive scale size in DEA with interval data", "determining the MPSS score and ranking decision making units.","method for determining the units that have used all their inputs to produce outputs", and in the end providing a way to help managers in the economic sector to make more precise planning for economic growth. The rest of this paper is as follows: section 2 contains some preliminaries. In section 3, two models to identify MPSS is proposed with interval data. In section 4, estimating most productive scale size of the provinces of Iran in the employment sector, and finally in section 5: we conclude.

2. Preliminaries

We assume that n DMU (DMU_j; j=1,…,n) exist, all of which have m inputs and s outputs. The input's DMUj vector can be shown by $x_j = (x_{ij}\ ;\ i = 1,\dots,m)$ and the output vector can be shown by $y_j = (y_{rj}\ ;\ r = 1,\dots,s)$ and we assume that xij and yrj are positive for any amounts of i and r, which i=1,…, m and r= 1,…,s.Production Possibility Set (PPS) is defined as follows:

T={(x,y)| output vector y can be produced by input vector x}.

Definition 1: (Banker's et al (1984)): $(X_O, Y_O) \in T$ is most productive scale size (mpss) if and only if for every $(\theta X_O, \Phi Y_O) \in T$ we have $\Phi \geq \theta$.

According to this definition, Jahanshahlou and Khodabakhshi (2003) introduced the following linear model to determine MPSS.

$$\begin{aligned} &\text{Max}\ \phi_o - \theta_o \\ &\text{s.t}\ \sum_{j=1}^{n} \lambda_j x_j \leq \theta_o x_o \\ &\sum_{j=1}^{n} \lambda_j y_j \geq \phi_o y_o \\ &\sum_{j=1}^{n} \lambda_j = 1 \\ &s_r^+, s_i^-, \lambda_j \geq 0 \quad \forall r = 1,\dots\dots,s \\ &\forall i = 1,\dots\dots,m \quad \forall j = 1,\dots\dots,n \end{aligned} \tag{1}$$

Since $\theta_0 = 1$, $\Phi_0 = 1$, $\lambda_o=1$, (j=o) $\theta_o=1$, is a solution for which the value of objective function is zero, it is obvious that the optimal value of objective function is not negative. Thus $\Phi^*_0 - \theta_0^* \geq 0$. The ε model of model (1) can be written as follows:

$$\begin{aligned} &\text{Max}\ \ \Phi_o - \theta_o + \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) \\ &\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- - \theta_o x_{io} = 0 \\ &i = 1,\dots,m \\ \text{s. t.}\ &-\sum_{j=1}^{n} \lambda_j y_{rj} + s_r^+ + \Phi_o y_{ro} = 0 \\ &r = 1,\dots,s \\ &\sum_{j=1}^{n} \lambda_j = 1 \\ &\lambda_j, s_i^-, s_r^+ \geq 0,\ \forall r = 1,\dots\dots,s \\ &\forall i = 1,\dots\dots,m \quad \forall j = 1,\dots\dots,n \end{aligned} \tag{2}$$

In solving model (2), we first obtain Max $\Phi_0 - \theta_0$ without considering slacks and then we determine the slacks.

Theorem 1: In order for DMU0 to be MPSS, the two following conditions must apply:

a) Optimal value of objective function be equal to zero

$$s_i^{-*} = s_r^{+*} = 0$$

b) $\forall\ i = 1,2,\ldots\ldots,m$

$\forall\ r = 1,2,\ldots\ldots,s$

Proof: Refer to Jahanshahlou and Khodabakhshi (2003).

3. Interval Data

In this section we assume that inputs and outputs are indefinite numbers and are presented as interval. For $x_{ij} \in [x_{ij}^L, x_{ij}^U]$ instance and $y_{rj} \in [y_{rj}^L, y_{rj}^U]$. In which x_{ij}^L is the lower bound and x_{ij}^U is upper bound of the $[x_{ij}^L, x_{ij}^U]$, and also y_{rj}^L is the lower bound and y_{rj}^U is the upper bound of the $[y_{rj}^L, y_{rj}^U]$. The worst case occurs when the desired DMU0 unit has the most input i. e $x_{io} = x_{io}^U$., (i= 1,…,m) and the minimum output is $y_{ro} = y_{ro}^L$, (r= 1,…,s).

In this case, efficiency can be obtained as an interval. In a way that the upper bound of efficiency is related to optimistic case and the lower bound is related to pessimistic case. The following model can be used to calculate BCC model upper bound (optimistic).

$$\begin{aligned}
u_o^* = \ & \mathrm{Min}\,\theta_o - \varepsilon(1s^+ + 1s^-) \\
\text{s.t}\ & \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j y^l_{rj} + \lambda_o y_{ro}^u - s_r^+ = y^u_{ro} \qquad r = 1,\ldots\ldots,s \\
& \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j x^u_{ij} + \lambda_o x^l_{io} + s_i^- = \theta_o x^l_{io} \qquad i = 1,\ldots\ldots,m \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& s_r^+, s_i^-, \lambda_j \geq 0 \quad \forall\ i = 1,2,\ldots\ldots,m \\
& \forall\ r = 1,2,\ldots\ldots,s \quad \forall\ j = 1,2,\ldots\ldots,n
\end{aligned} \tag{3}$$

In which u_0* indicates DMU_0 best efficiency amount. The following model can be used to calculate DMU_0 lower bound, in which L_0* indicates the worst DMU_0 score.

$$\begin{aligned}
L_o^* = \ & \mathrm{Min}\,\theta_o - \varepsilon(1s^+ + 1s^-) \\
\text{s.t}\ & \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j y^u_{rj} + \lambda_o y_{ro}^l - s_r^+ = y^l_{ro} \qquad r = 1,\ldots\ldots,s \\
& \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j x^l_{ij} + \lambda_o x^u_{io} + s_i^- = \theta_o x^u_{io} \qquad i = 1,\ldots\ldots,m \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& s_r^+, s_i^-, \lambda_j \geq 0 \quad \forall\ i = 1,2,\ldots\ldots,m \\
& \forall\ r = 1,2,\ldots\ldots,s \quad \forall\ j = 1,2,\ldots\ldots,n
\end{aligned} \tag{4}$$

Based on the resulted efficiencies, the DMU's can be classified as follows:

1) If u_0* = L_0* = 1, then the DMU_0 is strong efficient.

2) If u_0* = 1 and L_0*<1, then the DMU_0 is weak efficient.

3) If u_0* < 1, then the DMU_0 is inefficient.

Considering the model 2 to evaluate the unit's MPSS and the method used in models 3 and 4, we propose the following two models for determining MPSS.

$$
\begin{aligned}
& z_{u_o}^{*} = \mathrm{Max}\,\phi_o - \theta_o + \varepsilon(1s^{+} + 1s^{-}) \\
& \text{s.t} \quad \sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j y^{l}_{rj} + \lambda_o y^{u}_{ro} - s_r^{+} - \phi_o y^{u}_{ro} = 0 \quad r = 1,\dots,s \\
& \sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j x^{u}_{ij} + \lambda_o x^{l}_{io} + s_i^{-} - \theta_o x^{l}_{io} = 0 \quad i = 1,\dots,m \qquad (5) \\
& \sum_{i=1}^{n} \lambda_j = 1 \\
& s_r^{+}, s_i^{-}, \lambda \geq 0 \quad \forall\, i = 1,2,\dots,m \\
& \forall\, r = 1,2,\dots,s \quad \forall\, j = 1,2,\dots,n
\end{aligned}
$$

and

$$
\begin{aligned}
& z_{L_o}^{*} = \mathrm{Max}\,\phi_o - \theta_o + \varepsilon(1s^{+} + 1s^{-}) \\
& \text{s.t} \quad \sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j y^{u}_{rj} + \lambda_o y^{l}_{ro} - s_r^{+} - \phi_o y^{l}_{ro} = 0 \quad r = 1,\dots,s \\
& \sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j x^{l}_{ij} + \lambda_o x^{u}_{io} + s_i^{-} - \theta_o x^{u}_{io} = 0 \quad i = 1,\dots,m \qquad (6) \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& s_r^{+}, s_i^{-}, \lambda_j \geq 0 \quad \forall\, i = 1,2,\dots,m \\
& \forall\, r = 1,2,\dots,s \quad \forall\, j = 1,2,\dots,n
\end{aligned}
$$

Definition 2: (Interval MPSS) DMUo is MPSS if and only if the two following conditions are satisfied for models (5) and (6):

a) The optimal value of the objective two function is zero.

b) The values of slacks in alternative optimal solutions are zero.

DMUo is not interval MPSS if $Z_{L_o}^{*} \neq Z_{u_o}^{*} \neq 0$, or if some slacks are non-zero

4. Estimating Most Productive Scale Size of the provinces of Iran in the employment sector

The present study was conducted in 2015-2016, and data was collected from library documentation, information technology and statistics departments of provincial governorates of Iran. According to the interviews with managers of the employment sector of the governorates of Iran, two input and output variables were presented below. The two inputs and the two outputs are defined as follows: X1 is 'The percent of those seeking job who have skills', X2 is 'The percent of those seeking job who don't have skills', Y1 is 'Employment rate' and Y2 is 'Economic participation rate'. Assuming that the inputs and outputs are interval data, the inputs and outputs are presented in table (1).

From tables (2) and (3) it can be concluded that there is a direct relationship between employment rate and economic growth in Iran. This relationship can provide a basic way out of the unemployment problem, as the decline in economic growth in 2015-2016 has led to a sharp increase in unemployment in the country's economy. What is the important question about these results? Which units are the most productive scale size among the units? According to the tenth column of tables (2) and (3), the Azarbaijan sharghi province has the most productive scale size. According to columns 6 and 9 of tables (2) and (3), the province was more sensitive to the indicators of "The percent of those seeking job who have skills" and "Economic participation rate". These indicators can be used as a measure for increasing the productivity of Iran in the employment sector. Therefore, managers of the economic sector by implementing specialized training programs can increase the skills of the labor and increase employment rates.

Table(1): Labor situation in 2015-2016

Row	province	Input		Output	
		The percent of those seeking job who have skills	The percent of those seeking job who don't have skills	Employment rate	Economic participation rate
		X1.	X2.	Y1	Y2
1	Azarbaijan sharghi	[35.05 , 45]	[54.99 , 64.94]	[90 , 93.2]	[42 , 49.3]
2	Azarbaijan gharbi	[44.99 , 58.33]	[41.66 , 55]	[89.3 , 89.7]	[48.4 , 49.6]
3	Ardebil	[43.19 , 44.99]	[55 , 56.80]	[88 , 90.1]	[42.1 , 45.2]
4	Esfahan	[42.32 , 45]	[55 , 57.67]	[88 , 90.6]	[40.6 , 42.6]
5	Ilam	[41.22 , 45]	[54.99 , 58.77]	[85.4 , 87.4]	[36.9 , 40.9]
6	Bushehr	[41.89 , 45]	[54.99 , 58.10]	[88.3 , 89.3]	[35.3 , 37.7]
7	Tehran	[45 , 60.98]	[39.01 , 54.99]	[88.1 , 89]	[37.6 , 38.2]
8	Chahar mahal bakhtiari	[35.70 , 45]	[54.99 , 64.29]	[83.6 , 85.9	[38.8 , 40.8]
9	Khorasan Jonubi	[40.76 , 45]	[54.99 , 59.23]	[89.9 , 93]	[37.3 , 41.6]
10	Khorasan Razavi	[38.25 , 45]	[55 , 61.74]	[88.8 , 91.7]	[43.1 , 43.1]
11	Khorasan shomali	[30.62 , 45]	[54.99 , 69.37]	[92.4 , 93.6]	[36.8 , 36.9]
12	Khuzestan	[45 , 52.54]	[47.45 , 55]	[87.5 , 86.9]	[36.3 , 38.3]
13	Zanjan	[44.99 , 63.53]	[36.46 , 55]	[90.5 , 91.6]	[45.7 , 47.4]
14	Semnan	[38.73 , 44.99]	[55 , 61.26]	[90.9 , 91.7]	[32.8 , 37.4]
15	Sistan va baluchestan	[39.92 , 44.99]	[55 , 60.07]	[86.9 , 88.8]	[27.1 , 28.8]
16	Fars	[44.99 , 45.98]	[54.01 , 55]	[85.6 , 87.8]	[31.9 , 36.4]
17	Ghazvin	[40.37 , 45]	[54.99 , 59.62]	[87.8 , 91.6]	[35 , 36.4]
18	Ghom	[40.17 , 45]	[54.99 , 59.82]	[89.2 , 90.1]	[34.3 , 34.4]
19	Kordestan	[44.99 , 49.94]	[50.05 , 55]	[87.5 , 89.2]	[39.1 , 40.6]
20	Kerman	[44.16 , 44.99]	[55 , 55.83]	[88.1 , 90.9]	[31.7 , 36.2]
21	Kermanshah	[45 , 50.72]	[49.27 , 54.99]	[86.2 , 87.9]	[34.5 , 37.5]
22	Kohgiluye Boyr Ahmad	[45 , 54.98]	[45.01 , 54.99]	[85.1 , 85.2]	[29.5 , 31.4]
23	Golestan	[44.99 , 53.18]	[46.81 , 55]	[91.7 , 92.9]	[41.2 , 41.3]
24	Gilan	[45 , 45.46]	[54.53 , 54.99]	[84.7 , 87]	[39.7 , 40.2]
25	Lorestan	[45 , 46.49]	[53.50 , 54.99]	[80.89 , 82.9]	[37.9 , 40]
26	Mazandaran	[44.99 , 64.77]	[35.22 , 55]	[91.9 , 92.6]	[36.5 , 37.4]
27	Markazi	[36.46 , 45]	[54.99 , 63.53]	[88.2 , 88.8]	[33.9 , 41.1]
28	Hormozgan	[44.99 , 45]	[54.99 , 55]	[91.6 , 93.1]	[26 , 31.9]
29	Hamedan	[41.58 , 45]	[54.99 , 58.34]	[82 , 86]	[40 , 42.7]
30	Yazd	[43.25 , 44.99]	[55 , 56.74]	[91 , 93.4]	[34.7 , 41.2]

Table (2): Computational results fordeterministic model, model (6).

DMU	province	${Z_{L_O}}^*$	ϕ^*	θ^*	${s^+_1}^*$	${s^+_2}^*$	${s^-_1}^*$	${s^-_2}^*$	MPSS
DMU1	Azarbaijan sharghi	0.000	1.000	1.000	0.000	0.000	0.000	0.000	MPSS
DMU2	Azarbaijan gharbi	0.272	1.988	0.735	0.000	0.000	3.738	0.000	
DMU3	Ardebil	0.219	1.053	0.855	0.000	0.000	0.000	4.320	
DMU4	Esfahan	0.150	1.023	0.899	0.000	0.000	0.000	5.133	
DMU5	Ilam	0.286	1.086	0.843	0.000	0.000	0.000	8.676	
DMU6	Bushehr	0.261	1.050	0.847	0.000	0.000	0.000	11.650	
DMU7	Tehran	0.371	1.042	0.713	0.000	0.000	0.000	8.488	
DMU8	bakhtiari Chahar mahal	0.329	1.112	0.812	0.000	0.000	0.000	5.871	
DMU9	Khorasan Jonubi	0.242	1.032	0.841	0.000	0.000	0.000	10.285	
DMU10	Khorasan Razavi	0.239	1.046	0.826	0.000	0.000	0.000	3.823	
DMU11	Khorasan shomali	0.248	1.008	0.785	0.000	0.000	0.000	12.146	
DMU12	Khuzestan	0.316	1.054	0.787	0.000	0.000	0.000	9.854	
DMU13	Zanjan	0.285	1.001	0.721	0.788	0.000	0.000	0.000	
DMU14	Semnan	0.269	1.021	0.836	0.000	0.000	0.000	15.370	
DMU15	Sistan va baluchestan	0.331	1.068	0.855	0.000	0.000	0.000	19.874	
DMU16	Fars	0.296	1.081	0.838	0.000	0.000	0.000	14.008	
DMU17	Ghazvin	0.277	1.057	0.837	0.000	0.000	0.000	11.807	
DMU18	Ghom	0.267	1.040	0.813	0.000	0.000	0.000	13.122	
DMU19	Kordestan	0.271	1.055	0.861	0.000	0.000	0.000	6.994	
DMU20	Kerman	0.266	1.051	0.805	0.000	0.000	0.000	15.272	
DMU21	Kermanshah	0.322	1.070	0.764	0.000	0.000	0.000	11.268	
DMU22	Kohgiluye Boyr Ahmad	0.398	1.082	0.781	0.000	0.000	0.000	16.050	
DMU23	Golestan	0.258	1.005	0.861	0.000	0.000	0.000	6.658	
DMU24	Gilan	0.257	1.093	0.849	0.000	0.000	0.000	5.142	
DMU25	Lorestan	0.319	1.143	0.685	0.000	0.000	0.000	5.118	
DMU26	Mazandaran	0.369	1.998	0.685	0.000	0.000	0.000	11.101	
DMU27	Markazi	0.304	1.054	0.816	0.000	0.000	0.000	13.261	
DMU28	Hormozgan	0.256	1.011	0.866	0.000	0.000	0.000	22.273	
DMU29	Hamedan	0.302	1.131	0.846	0.000	0.000	0.000	3.496	
DMU30	Yazd	0.229	1.018	0.856	0.000	0.000	0.000	13.313	
sum	-	-	-	-	0.788	0.000	0.000	267.564	

Table (3): Computational results for deterministic model, model (5).

DMU	province	${Z_{U_O}}^*$	ϕ^*	θ^*	${s^+_1}^*$	${s^+_2}^*$	${s^-_1}^*$	${s^-_2}^*$	MPSS
DMU1	Azarbaijan sharghi	0.000	1.000	1.000	0.000	0.000	0.000	0.000	MPSS
DMU2	Azarbaijan gharbi	0.000	1.000	1.000	0.000	0.000	0.000	0.000	
DMU3	Ardebil	0.000	1.000	0.000	0.000	0.000	0.000	0.000	
DMU4	Esfahan	0.010	1.000	1.000	0.000	0.857	0.000	1.500	
DMU5	Ilam	0.020	1.037	1.027	0.000	1.457	0.000	0.203	
DMU6	Bushehr	0.030	1.015	1.010	0.000	0.554	0.000	4.351	
DMU7	Tehran	0.000	1.000	1.000	0.000	0.000	0.000	0.000	
DMU8	bakhtiari Chahar mahal	0.000	1.000	1.000	0.000	0.000	0.000	0.000	
DMU9	Khorasan Jonubi	0.000	1.000	1.000	0.000	0.000	0.000	0.000	

DMU10	Khorasan Razavi	0.000	1.000	1.000	0.000	0.000	0.000	0.000	
DMU11	Khorasan shomali	0.000	1.000	1.000	0.000	0.000	0.000	0.000	
DMU12	Khuzestan	0.000	1.000	1.000	0.000	0.000	0.000	0.000	
DMU13	Zanjan	0.000	1.000	1.000	0.000	0.000	0.000	0.000	
DMU14	Semnan	0.000	1.000	1.000	0.000	0.000	0.000	0.000	
DMU15	Sistan va baluchestan	0.040	1.020	1.060	0.000	3.307	0.000	13.216	
DMU16	Fars	0.060	1.032	1.018	3.495	0.000	0.000	5.039	
DMU17	Ghazvin	0.000	1.000	0.000	0.000	0.000	0.000	0.000	
DMU18	Ghom	0.010	1.006	1.054	0.000	2.933	0.000	8.009	
DMU19	Kordestan	0.000	1.000	0.000	0.000	0.000	0.000	0.000	
DMU20	Kerman	0.040	0.997	1.000	1.840	0.000	0.000	6.519	
DMU21	Kermanshah	0.000	1.000	1.000	0.000	0.000	0.000	0.000	
DMU22	Kohgiluye Boyr Ahmad	0.000	1.000	1.000	0.000	0.000	0.000	0.000	
DMU23	Golestan	0.000	1.000	1.000	0.000	0.000	0.000	0.000	
DMU24	Gilan	0.050	1.041	1.009	3.068	0.000	0.000	0.737	
DMU25	Lorestan	0.070	1.065	1.028	3.942	0.000	2.311	0.000	
DMU26	Mazandaran	0.000	1.000	1.000	0.000	0.000	0.000	0.000	
DMU27	Markazi	0.000	1.000	1.000	0.000	0.000	0.000	0.000	
DMU28	Hormozgan	0.040	0.973	1.000	2.678	0.000	0.000	11.557	
DMU29	Hamedan	0.010	0.998	1.018	0.000	0.969	4.801	0.000	
DMU30	Yazd	0.000	1.000	1.000	0.000	0.000	0.000	0.000	
sum	-	-	-	-	15.023	10.077	7.112	51.131	

Based on amounts of obtained upper and lower bounds of MPSS, the DMU's can be classified as follows:a) From Table (2) and (3), it is observed that optimal solutions for DMU1 Azarbaijan Sharghi, is $Z_{L_o}{}^* = Z_{u_o}{}^* = 0$ and $s^-{}_i{}^* = s^+{}_r{}^* = 0$. Therefore, by Definition (2) this province is most productive scale size among the 30 province. It means that this unit has an optimal status from using resources and producing outputs point of view. These most productive scale size province is benchmarks for all DMUs. The rest of companies for which $Z_{L_o}{}^* \neq Z_{u_o}{}^* \neq 0$ are not most productive based on their scale sizes.b) For DMU_{17}, DMU_{19}, DMU_{21}, DMU_{22}, DMU_{23}, DMU_{26}, DMU_{27}, DMU_{30}, DMU_7, DMU_8, DMU_9, DMU_{10}, DMU_7, DMU_{11}, DMU_{12}, DMU_{13}, DMU_{14}, DMU_2, DMU_3, according to table (3), $Z_{u_o}{}^* = \phi^* - \theta^* = 1.0000 - 1.0000 = 0.0000$ i.e. this DMU has the upper bound of efficiency. But according to table (2), $Z_{L_o}{}^* = \phi^* - \theta^* \neq 0.0000$ meaning that this units are weak MPSS, and has not used all of its inputs to create outputs.

c) According to theorem (2), if $Z_{L_o}{}^* \neq Z_{u_o}{}^* \neq 0$, or if some slacks are non-zero, then the DMU_0 is not interval MPSS. In this example for DMU6, DMU15, DMU29, DMU28, DMU_{25}, DMU_{24}, DMU_{20}, DMU_{18}, DMU_{16}, DMU_4, and DMU_5, the MPSS is not interval. In other words, these provinces are parts of inefficient units, and cannot be selected as MPSS.

Table (4): Provincial ranking table based on MPSS score.

DMU	province	MPSS Score= $Z_{L_o}{}^* - Z_{u_o}{}^*$	Ranking based on MPSS Score
DMU1	Azarbaijan sharghi	0.000	1
DMU2	Azarbaijan gharbi	0.272	19
DMU3	Ardebil	0.219	5
DMU4	Esfahan	0.140	2
DMU5	Ilam	0.266	16
DMU6	Bushehr	0.231	8
DMU7	Tehran	0.371	29
DMU8	bakhtiari Chahar mahal	0. 329	27
DMU9	Khorasan Jonubi	0.242	11
DMU10	Khorasan Razavi	0.239	10
DMU11	Khorasan shomali	0.248	12
DMU12	Khuzestan	0.316	25
DMU13	Zanjan	0.285	21
DMU14	Semnan	0.269	17
DMU15	Sistan va baluchestan	0.291	22
DMU16	Fars	0.236	9
DMU17	Ghazvin	0.277	20
DMU18	Ghom	0.257	14
DMU19	Kordestan	0.271	18
DMU20	Kerman	0.220	6
DMU21	Kermanshah	0.322	26
DMU22	Kohgiluye Boyr Ahmad	0.398	30
DMU23	Golestan	0.258	15
DMU24	Gilan	0.207	3
DMU25	Lorestan	0.249	13
DMU26	Mazandaran	0.369	28
DMU27	Markazi	0.304	24
DMU28	Hormozgan	0.216	4
DMU29	Hamedan	0.292	23
DMU30	Yazd	0.229	7

From Table (4), it is observed that Azarbaijan Sharghi province is ranked one. Therefore, this unit has themost rate of productivity among the provinces of Iran. According to Table (2), for they province, $Z_{L_o}{}^* = 0.000$, $\phi^* = 1.000$, $\theta^* = 1.000$, $s^+{}_1{}^* = 0.000$, $s^+{}_2{}^* = 0.000$, $s^-{}_1{}^* = 0.000$, $s^-{}_2{}^* = 0.000$. It means that this unit has an optimal status from using resources and producing outputs point of view. In Table (3), This DMUis $Z_{u_o}{}^* = 0.000$, $\phi^* = 1.000$, $\theta^* = 1.000$, $s^+{}_1{}^* = 0.000$, $s^+{}_2{}^* = 0.000$, $s^-{}_1{}^* = 0.000$, $s^-{}_2{}^* = 0.000$. So by Definition (2), this province is the most productive scale size among the 30 province. For the province of Isfahan, it is ranked three, according to table (2). $Z_{L_o}{}^* = 0.150$, $\phi^* = 1.023$, $\theta^* = 0.899$, $s^+{}_1{}^* = 0.000$, $s^+{}_2{}^* = 0.000$, $s^-{}_1{}^* = 0.000$, $s^-{}_2{}^* = 5.133$ It means that this unit has MPSS Score = $Z_{L_o}{}^* - Z_{u_o}{}^* = 0.140$. $s^-{}_1{}^* = 0.000$ and $s^-{}_2{}^* = 5.133$ indicate that the province has a high unemployment and economic participation rate. The province of Kohgiluye Boyr Ahmad, which ranked thirty, has the lowest rank among the provinces. According to Table (2), $Z_{L_o}{}^* = 0.398$, $\phi^* = 1.082$, $\theta^* = 0.781$, $s^+{}_1{}^* = 0.000$, $s^+{}_2{}^* = 0.000$, $s^-{}_1{}^* = 0.000$, $s^-{}_2{}^* = 16.050$, and= $Z_{L_o}{}^* - Z_{u_o}{}^* = 0.398$. Therefore, by the Definition (1) this company is not most productive scale size. Due to $s^-{}_2{}^* = 16.050$, the province has a high unemployment rate. There is a inverse relationship between the most productive scale size and the unemployment rate, and economic managers in this province should benchmark the economic managers of the Sharghi Azerbaijan province, by implementing specialized training programs to increase the labor skills in province of Kohgiluye Boyr Ahmad to achieve economic growth.

5. Conclusion

The purpose of this study was to estimating most productive scale size of the provinces of Iran in the employment sector by using an imprecise data envelopment analysis (IDEA) with interval data in 2015-2016. The results of this study showed that the ability to identify the decision-making units that have the most productive scale size among the units under study is one of the important characteristics of data envelopment analysis. Many articles have been published on the most productive scale size, but due to the use of precise data or stochastic data, their results are not comparable to the results of this study. According to Tables (2) and (3) of the models used in this paper, for the first time, the most productive scale size can be determined by the extent that the economic sector managers can plan more completely for the provinces of Iran in the economic for and employment sector. Therefore, based on the results of this study and based on the new proposed model of productivity, Azarbaijan Sharghi has the most productivity and can be the reference unit for other provinces of the country in the employment sector. That is, the unit uses all the capacity at the inputs of "The percent of those who are looking for a job who have the skills", "The percentage of those seeking a job who does not have the skills". Thus, the sum of the sixth and ninth columns of Tables (2) and (3) of the indicators of the percentage of those seeking a job who have been able to have the economic participation rate was also introduced as the most sensitive indicators. As a way to improve productivity, managers of the economic sector in other provinces of the country will benchmark the economic managers of the Azarbaijan Sharghi province and increase their specialized skills training programs to provide expected economic growth. According to the fourth column of Table (4) in 2014-2015, the provinces of Isfahan and Guilan ranked second and third respectively. It is observed that the province of Kohgiluye Boyr Ahmad has the lowest rank, which according to the ninth column of Table (2), this province has a high unemployment rate, and the most productive scale size and unemployment rate is an inverse relationship. Another basic solution in the economic sector is that the model of the most productive scale size should be considered in the allocation of education and employment in different provinces of the country. By implementing this study, in addition to functional models, economic sector managers provided the possibility of more perfect planning for the growth of economic participation and resource savings. Applying the proposed approaches in different fields, practically, would be interesting for further research. Estimating most productive scale size by considering Ordinal data is, also, a direction for further research.

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