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An Extended of Multiple Criteria Data Envelopment Analysis Models for Ratio Data

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Abstract

One of the problems of the data envelopment analysis traditional models in the multiple form that is the weights corresponding to certain inputs and outputs are considered zero in the calculation of efficiency and this means that not all input and output components are utilized for the evaluation of efficiency, as some are ignored. The above issue causes the efficiency score of the under evaluation unit not to be calculated correctly. One of the ways to deal with the pseudo-inefficiency is to use data envelopment analysis models with multi-criteria structure. In this regard, we first investigate the models of data envelopment analysis with multi-criteria structure and further, with regard to the ability of the ratio-based data envelopment analysis models, we develop data envelopment analysis models with a multi-criteria structure for ratio data and the feasibility and the bounded condition of the above models and their efficiency intervals are described. By presenting a numerical example, we compare the efficiency scores obtained from the models presented with the previous models and we show that the proposed models can be used to deal with the pseudo-inefficiency and efficiency underestimation. Finally, we present the results.

Keywords: Multi-criteria data envelopment analysis; Discrimination power; Weight dispersion; DEA-R; Pseudo-inefficiency.

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1. Introduction

DEA is a technique based on mathematical programming, used for evaluating the relative efficiency of a set of decision-making units (DMUs). The efficiency of each DMU is determined by the efficiency frontier. The units on the efficiency frontier are assumed efficient, and the others are considered inefficient. Technically, DEA sets up a production possibility set (PPS) and considers its frontier as the efficient frontier, formed under the condition of non-domination. Extended by Charnes et al. (1978). DEA enjoys the advantage of weight selection, which was one of the most frequently studied topics in DEA literature.

Many studies have been conducted in the area of weights. Meanwhile, a weakness in DEA is lack of discrimination among efficient DMUs, which yields a large number of efficient units; this occurs when the number of units under evaluation is less than the sum of inputs and outputs. In such cases, the weights corresponding to certain inputs and outputs are considered zero for the efficient DMUs and this means that not all input and output components are utilized for the evaluation of efficiency, as some are ignored. As a result of the unrealistic weight distribution in DEA, some units may be considered efficient in the analysis, while not really having an efficient performance; this is due to assignment of an overly large weight to an output or an overly small weight to an input. Thompson et al. (1986, 1990) were the first to utilize weight restrictions for improving the discrimination power. Their work was extended later by other authors in the field; in this relation, the method of assurance region (AR) was proposed by Charnes et al. (1990) as well as Khalili et al. (2010).

Another method of weight restriction is the cone-ratio method, presented in (Dyson and Thanassouli, 1988; Mecit and Alp, 2013; Sarrico and Dyson, 2004; Thanassoulis and Allen, 1998). It is worth mentioning that all these models were presented in order to deal with the problem of unrealistic weights distribution. The problem with AR and cone-ratio methods was their dependence on the measurement of inputs and outputs, which sometimes led to the infeasibility of models and computational difficulties.

Super-efficiency models were presented in 1993 by Anderson and Peterson, aiming to overcome the problem of discrimination power and poor weights dispersion, as presented in (Andersen and Petersen, 1993; Chen et al., 2013; Chen and Liang, 2011). Another method for dealing with the issue of discrimination power was the cross-efficiency technique presented by Wang and Chin (2010, 2011). The problem with super-efficiency models was the infeasibility of some efficient units under variable returns to scale assumption. Though Chen studied the input- and output-oriented models, this problem of infeasibility was not overcome, as pointed out in the counterexamples presented by Soleimani-damaneh et al. (2006) regarding Chen's model. Lee et al. (2011, 2012) presented a two-stage process for dealing with the infeasibility problem through adjustments in input savings and output surpluses. Also, Lee et al. (2012) showed that in cases where some inputs and outputs had zero values, the problem still remained infeasible.

The main problems of cross-efficiency techniques included multiple weights and solutions in the related models, besides the large number of solutions, and the need to solve numerous linear and nonlinear programming problems (see Angiz et al., 2010; cooks et al., 2013).

One of the techniques used for improving the discrimination power was the utilization of multi-objective models in DEA, see (Chen et al., 2009; Foroughi et al., 2011; Li and Reeves 1999). The formulation presented by Li and Reeves (1999) was a tri-objective problem, in which the three objectives did not have any priority over each other; however, as mentioned by Ghasemi et al. (2014), the model proposed by Li and Reeves (1999) exhibited the problem of having zero weights in some input or output components, which led to those components not being involved in the evaluation of efficiency. They showed that multi-objective models do not solve the problem of discrimination power.

Bal et al. (2008, 2010) used goal programming to solve the MCDEA model and proposed the GPDEA model. They argued that the GPDEA model would resolve the discrimination power issue; meanwhile, Ghasemi et al. (2014) showed that the model still suffers the problems of previous models and the weights of certain input or output components may still be zero; so, these components may not be involved in the efficiency measurement and incorrect efficiency values may result. Ghasemi et al. (2014) presented the Bio-MCDEA model in order to deal with the problem of discrimination power and solve the problems with the previous models. Providing a bi-objective model based on the MCDEA model, they showed that the proposed model solves the previous problems, and it is more suitable from the viewpoint of computation. However, their suggested model is based on imposition of weight restrictions on input and output components, and if the restrictions are removed, some input and output weights may become zero and the above-mentioned problem would remain unsolved. However, it provides more realistic efficiency values.

Wei et al. (2011c, 2011a) presented an input-oriented ratio-based model (DEA-R-I model) based on the ratio of input to output components and showed that the model can be an eligible alternative for the CCR models. The above-mentioned (input-oriented) models, due to having a larger space for weight selection, enjoy greater efficiency levels than input-oriented CCR models; meanwhile; the CCR model does not consider weight restrictions and some weights may become zero when evaluated by this model, therefore, the corresponding weights might not be involved in the evaluation of efficiency; however, the DEA–R-I model obviates this problem to some extent. The above-mentioned (input-oriented) model can be a proper substitute for the input-oriented CCR model (CCR-I model) without weight restriction; thus, the discrimination power problem is resolved in the CCR-I model. Later, Wei et al. (2011a) demonstrated that the CCR model may not properly reflect the efficiency value of some units and face the problem of efficiency underestimation (US), which was proven to be more critical for efficient units, as it mostly involves them.

Wei et al. (2011c) compared the efficiency and super-efficiency values in CCR-I and DEA-R models and revealed their differences. They showed that the efficiency values in DEA–R-I models are greater than or equal to the corresponding weights in CCR-I models, which would reduce efficiency underestimation in DEA-R, as opposed to CCR-I. They showed that the inherent limitation in the CCR-I model leads to the inefficiency of units that are efficient in practice; in this regard, their values are ignored due to the zero weights in input and output components and the efficient DMUs are considered inefficient. Subsequently, they began analyzing their super-efficiency models and showed that the efficiency underestimation problem was present in the super-efficiency model as well. In a comparison of values obtained from CCR-I- and DEA-R-I-based super-efficiency models, Wei et al. (2011c) indicated that the super-efficiency values of DEA-R-I models are greater than or equal to their corresponding values in CCR-I; they also found that the super-efficiency values obtained from CCR-I models are not real values for some units. Besides, by keeping the output levels constant, super-efficiency DEA–R-I models provide lower input levels for the units to remain efficient, and this shows that the super-efficiency CCR-I models do not represent the exact super-efficiency values for some units and the super-efficiency values presented by DEA-R-I are more accurate and suitable for ranking extremely efficient units. This feature is rooted in the nature of DEA-R-I models, since DEA-R-I models enjoy a greater space for selecting weights and the mean weights of input and output components is greater in DEA-R-I models compared to their corresponding super-efficiency models in CCR-I.

Wei et al. (2011b) utilized the DEA–R-I model for evaluation of efficiency among hospitals, and raising the problem of pseudo-inefficiency, indicated that due to inherent weight restrictions, some efficient units in the CCR-I model are determined as inefficient; this phenomenon is called pseudo-inefficiency. They also demonstrated that the efficiency values of the DEA-R-I model are usually greater than or equal to the corresponding values in the CCR-I model. They showed that the same CCR inefficient units are DEA-R-I efficient. Therefore, using the DEA-R-I model is a proper approach for dealing with the phenomenon of the pseudo-inefficiency.

Given the importance of DEA-R-I compared to CCR-I models, the author decided to propose a new model based on DEA-R-I with a multi-objective programming framework in order to deal with the issue of discrimination power. In this regard, considering the characteristics of DEA-R models in weight selection and their larger space for weight selection compared to the CCR-I model, we can use these models to overcome the discrimination power issue. Therefore, with consideration to the problems of previous models, this article provides several new models within a multi-objective programming framework based on DEA-R-I in order to address the problem of discrimination power. Moreover, through comparison with previous models, we will demonstrate the advantages to these new models in dealing with the problem of pseudo-inefficiency and efficiency underestimation.

The rest of the paper is organized as follows: The second section is a brief description of GPDEA, MCDEA, bio-MCDEA and DEA-R-I models, as well as a discussion of their strengths and weaknesses. Then, the new models for improvement of discrimination power in MCDEA are provided and in the next section, a numerical example is set based on previous and new models. Finally, the results are analyzed and compared and in the end, the conclusions are drawn.

2. Improving Discrimination Power in Data Envelopment Analysis

2.1 Multiple Criteria Data Envelopment Analysis (MCDEA)

Consider n decision-making units that consume m inputs ($x_{ij},\ i=1,..,m,\ j=1,\dots,n$) in order to produce s outputs (y_{rj} $,r=1,..,s,\ j=1,\dots,n$). Li and Reeves (1999) presented the problem of MCDEA as follows:

Model 1: Multiple criteria data envelopment analysis (MCDEA)

$$
\begin{aligned}
&\min \quad (d_o \ or \ \max \quad \theta_0=\textstyle\sum_{r=1}^{s} u_r y_{r0})\\
&\min \quad M,\\
&\min \quad \textstyle\sum_{j=1}^{n} d_j,\\
&s.t. \quad \textstyle\sum_{i=1}^{m} v_i x_{i0}=1, \qquad (1)\\
&\textstyle\sum_{r=1}^{s} u_r y_{rj}-\sum_{i=1}^{m} v_i x_{ij}+d_j=0,\\
&j=1,\dots,n,\\
&M-d_j\geq 0,\ j=1,\dots,n,\\
&u_r\geq 0,\ \ r=1,\dots,s,\\
&v_i\geq 0,\ \ i=1,\dots,m,\ \ d_j\geq 0,\\
&j=1,\dots,n.
\end{aligned}
$$

In this model, d_0 is the measure of inefficiency, while $h_o=1-d_o$ is the measure of efficiency and $\sum_{r=1}^{s} u_r y_{ro}$ is the efficiency value in traditional DEA models. M indicates the maximum deviation and the last objective function shows the sum of all deviations; the constraint $M - d_j \geq 0, j = 1, \dots, n.$ optimizes all functions simultaneously. We use the adaptive weighted sum method to solve the MOLP problem above; to this end, we turn the problem into a single-objective problem. In this regard, we assume: $\min\ w_1\, d_o + w_2 M + w_3 \sum_{j=1}^{n} d_j$. The weight vectors (w_1, w_2, w_3) are selected based on the importance of each objective function according to the decision-maker's preferences. If we choose the weight vector $w = (0,0,1)$ to solve Model (1), the resulting model would be called Minsum. Li and Reeves (1999) did not propose a solution for their model which could optimize all objective functions at the same time. To solve the MCDEA problem, we can use the goal-programming approach, which optimizes the problem with respect to the ideals of the objective functions.

Bal et al. (2008, 2010) presented the following goal-programming model to solve the MCDEA problem provided by Li and Reeves (1999):

Model 2: Goal programming data envelopment analysis under CRS (GPDEA-CCR)

$$
\begin{aligned}
\min\ & a = \left\{\mathrm{d}_1^- + \mathrm{d}_1^+ + \mathrm{d}_2^+ + \textstyle\sum_{j=1}^{n} d_{3j}^- + \sum_{j=1}^{n} \mathrm{d_j}\right\} \\
\mathrm{s.t.}\ & \textstyle\sum_{\mathrm{i=1}}^{\mathrm{m}} v_i\, \mathrm{x_{i0}} + \mathrm{d}_1^- - \mathrm{d}_1^+ = 1, \\
& \textstyle\sum_{\mathrm{r=1}}^{\mathrm{s}} u_r \mathrm{y_{r0}} + \mathrm{d}_2^- - \mathrm{d}_2^+ = 1, \\
& \textstyle\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \quad j = 1, \dots, n, \qquad (2) \\
& M - d_j + d_{3j}^- - d_{3j}^+ = 0, \quad j = 1, \dots, n, \\
& u_r \geq 0,\ r = 1, \dots, s, \quad v_i \geq 0,\ i = 1, \dots, m, \\
& d_j \geq 0,\ \ j = 1, \dots, n,\ \ \mathrm{d}_1^-,\ \mathrm{d}_1^+,\ \mathrm{d}_2^-, \\
& \mathrm{d}_2^+ \geq 0,\ \ \mathrm{d}_{3\mathrm{j}}^-,\ \mathrm{d}_{3\mathrm{j}}^+ \geq 0,\ \ j = 1, \dots, n.
\end{aligned}
$$

In this model, d_1^- and d_1^+ are unwanted deviations for the goal, where the weighted sum of inputs under analysis equals to unity. Variables d_2^+ and d_2^- are wanted and unwanted deviation variables corresponding to the weighted sum of outputs in the unit under evaluation, which is less than or equal to unity. $\mathrm{d}_{3\mathrm{j}}^+$ and $\mathrm{d}_{3\mathrm{j}}^-$ are wanted and unwanted deviation variables for the goal $\mathrm{M} - \mathrm{d_j} \geq 0$. The weights of all deviations in the objective function are equal and deviations from the goal value are minimized. It should be noted that the main problem of GPDEA models, as mentioned by Ghasemi et al. (2014), is the zero values of some weights corresponding to some inputs and outputs in the optimal solution. Therefore, efficiency values are calculated inaccurately for those units and efficiency is not determined precisely. It is even possible that the obtained efficiency values of some units would be less than the actual values; i.e., pseudo-inefficiency would occur. Hence, GPDEA-CCR models cannot be utilized for dealing with the problem of dispersion and discrimination power in DEA. Another model in this relation is the bio-MCDEA model provided by Ghasemi et al. (2014) as follows:

Model 3: Bi-objective multiple criteria data envelopment analysis (BiO-MCDEA) model

$$
\begin{aligned}
\min\ & h = \left(w_2\, \mathrm{M} + \mathrm{w}_3 \textstyle\sum_{j=1}^{n} d_j\right) \\
s.t.\ & \textstyle\sum_{i=1}^{m} v_i x_{i0} = 1, \qquad (3) \\
& \textstyle\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \\
& j = 1, \dots, n, \\
& M - d_j \geq 0,\ j = 1, \dots, n, \\
& u_r \geq \varepsilon,\ \ r = 1, \dots, s, \\
& v_i \geq \varepsilon,\ \ i = 1, \dots, m,\ \ d_j \geq 0, \\
& j = 1, \dots, n.
\end{aligned}
$$

In multiple-objective linear programming (MOLP) and multiple-criteria decision-making (MCDM) models, the set of a problem's optimal solutions are efficient or non-dominated solutions that optimize all objective functions simultaneously.

The aim is to provide a model with an objective function in order to obtain the efficiency scores and efficient solutions.
We use the weighted sum method to solve the MCDEA model; to this end, we turn the problem into a single-objective problem. In this regard, we assume: min $w_1 d_o + w_2 M + w_3 \sum_{j=1}^{n} d_j$. The weight vectors (w_1, w_2, w_3) are selected based on the importance of each objective function according to the decision-maker's preferences. If we choose the weight vector $W = (0,1,1)$ to solve Model (1), the resulting model would be called BIO-MCDEA. w_2 and w_3 represent the significance of functions M and $\sum d_j$, respectively, as selected by the decision-maker; therefore, any changes in them would lead to changes in the efficiency scores. Note that Model (3) is initially a bio-objective problem presented as min $\{M, \sum_{j=1}^{n} d_j\}$, which we solve using the adaptive weighted sum method; in this regard, we put min $\left(w_2 M + w_3 \sum_{j=1}^{n} d_j\right)$ and turn Model (3) into a single-objective problem. Ghasemi et al. (2014) considered the weights equal in solving the same problem. In this model, since the importance of the second objective function is to further increase the discrimination power, greater weights are assumed for the second objective function. The difference between bio-MCDEA and MCDEA models is in the presence or absence of objective d_0; because when $\sum_{j=1}^{n} d_j$ is minimized in the bio-MCDEA model, d_0 is minimized as well. Now we present new DEA-R-I based models which can overcome the following problems existing in previous models:

1. Improving the discrimination power
2. Eliminating pseudo-inefficiency
3. Overcoming efficiency underestimation.

The following DEA-R-I model was presented based on the ratio of inputs to outputs for analyzing the efficiency of decision-making units and obtaining the efficiency scores (Wei et al., 2011c).

Model 4: An input-oriented ratio-based data envelopment analysis (DEA-R-I) model.

$$\theta_o^* = max \quad \theta_o$$
$$s.t. \quad \sum_{i=1}^{m} \sum_{r=1}^{s} w_{ir} \frac{\left(x_{ij}/y_{rj}\right)}{\left(x_{io}/y_{ro}\right)} \geq \theta_o \;, \quad j = 1, \dots, n, \qquad (4)$$
$$\sum_{i=1}^{m} \sum_{r=1}^{s} w_{ir} = 1, \quad w_{ir} \geq \epsilon, \quad \theta_o \geq 0.$$

In this model, w_{ir} is the weight corresponding to the ratio of the i-th input variable to the r-th output variable.
There are two types of inefficiency in conventional DEA models:
1- Weak efficiency; 2- Pseudo-inefficiency.
If our model detects inefficient units as efficient ones, we call them weakly efficient; we use two-phase models (see: Cooper et al., 2007) or SBM model (see: Tone, 2001) to resolve the issue of weak efficiency. Pseudo-inefficiency occurs when efficient units are wrongly identified as inefficient; this happens as a result of the natural weight limitations in conventional DEA models such as CCR and BCC models. DEA-R-I models are used to deal with the concept of pseudo-inefficiency. These models provide a bigger solution space for weight selection, which causes the input-oriented efficiency scores calculated by them to be greater than or equal to the scores obtained from conventional DEA models. The inherent weight limitations in conventional DEA models results in an inaccurate calculation of efficiency scores and therefore, some units are mistaken as being inefficient; in this regard, the decision-maker would then be forced to make changes in the inputs and outputs of these units to make them efficient, which could burden the DM with very high expenses. Therefore, it is imperative to use models such as DEA-R-I, which can provide us with accurate efficiency scores. The fact that the

efficiency scores obtained from input-oriented DEA-R-I models are greater than or equal to their corresponding values in input-oriented CCR models doesn't reveal the weakness of these models; on the other hand, these models prevent the issues of efficiency underestimation and discrimination power and poor weight dispersion.

The issue of efficiency underestimation is another important subject in data envelopment analysis; this problem happens when the efficiency of units is inaccurately measured by conventional models such as CCR and BCC; in such cases, the efficiency scores calculated by conventional models are lower than their actual values based on the level of inputs and outputs. We can use the DEA-R-I models in studies with such outcomes. These models consider a weight for each inputs/outputs ratio and involve all ratios in the efficiency measurement; whereas, the conventional models don't involve the input and output components which have a corresponding weight of zero in their calculations and therefore, the efficiency scores aren't calculated correctly and the efficiency underestimation issue occurs. We must note that if the efficiency scores obtained from CCR and DEA-R-I models were the same, efficiency underestimation wouldn't occur; on the other hand, if the non-zero weights related to outputs were not the same, the efficiency scores of CCR and DEA-R-I models will not be equal; in this relation, the efficiency scores obtained from the DEA-R-I models will be greater than or equal to their corresponding values in the input-oriented CCR models and efficiency underestimation will not occur; now, we arrive at the following two assumptions (see Wei et al., 2011b):

H1. When DEA-R-I weights are concentrated on one output, the CCR-I efficiency, and DEA-R-I efficiency, are the same.

H2. When DEA-R-I weights are not concentrated on multiple outputs, DEA-R-I efficiency and CCR-I efficiency are not the same, with the exception of, every output of the referenced DMU is the same times of corresponding output of targeted DMU.

According to the abovementioned reasons, input-oriented DEA-R models are more capable to deal with efficiency underestimation and discrimination power and poor weight dispersion problems; therefore, the higher efficiency scores of these models compared to CCR models is not proof of weakness, but strength.

As mentioned by Wei et al. (2011b), the efficiency and super-efficiency values obtained from the abovementioned models are greater than or equal to the corresponding values in CCR-I models; therefore, these models prevent the problems of efficiency underestimation and pseudo-inefficiency and are preferred to traditional DEA models.

To determine the value of ϵ in Model (4), we can solve the following model and arrive at the maximum value of ϵ.

$$\begin{aligned} \max \quad & \epsilon \\ s.t. \quad & \sum_{i=1}^{m}\sum_{r=1}^{s} \overline{w}_{ir} \frac{\left({x_{ij}}/{y_{rj}}\right)}{\left({xio}/{y_{ro}}\right)} \geq \theta \quad , \\ & j = 1, \dots, n, \qquad (5) \\ & \sum_{i=1}^{m}\sum_{r=1}^{s} \overline{w}_{ir} = 1, \quad \overline{w}_{ir} \geq \epsilon, \quad \theta \geq 0. \end{aligned}$$

Theorem 1: Model (4) is feasible for every $\epsilon \leq \epsilon^*$ and infeasible for every $\epsilon^* \leq \epsilon$.

Proof: Assume that $(\overline{w}_{ir}{}^*,\ \theta^*, \epsilon^*)$, $i = 1, \dots, m,\ r = 1, \dots, s,$ is the optimal solution for the Model (5). Then $(\overline{w}_{ir}{}^*$, $\theta^*, \epsilon),\ i = 1, \dots, m,\ r = 1, \dots, s,\ \ \epsilon \leq \epsilon^*$ is the feasible solution for the Model (5), Since Model (5) and Model (4) have the same feasible region, then $(\overline{w}_{ir}{}^*$, $\theta^*, \epsilon),\ i = 1, \dots, m,\ r = 1, \dots, s,\ \ \epsilon \leq \epsilon^*$ is the feasible solution for the Model (4). We must note that Model (5) is always feasible, it is sufficient to introduce a feasible solution for the model. Without loss of generality, suppose $\theta = 0$, $\bar{w}_{ir} = \frac{1}{m*s}$, $i = 1, \dots, m$, $r = 1, \dots, s$. It is easy to verify that is a feasible solution for Model (5).

Now, suppose that $(\bar{w}_{ir}{}', \theta', \epsilon)$, $i = 1, \dots, m$, $r = 1, \dots, s$, $\epsilon > \epsilon^*$ is a feasible solution to Model (4). Since the feasible regions of Models (4) and (5) are the same, $(\bar{w}_{ir}{}', \theta', \epsilon)$, $i = 1, \dots, m$, $r = 1, \dots, s$, $\epsilon > \epsilon^*$ is therefore the feasible solution to model (5) with a larger objective function value than $(\bar{w}_{ir}{}', \theta', \epsilon^*)$, which is in contradiction with the optimality of ϵ^*. As a result, Model (4) is infeasible for every $\epsilon^* \leq \epsilon$. ■

Theorem 2: $0 \leq \theta_o^* \leq 1$.

Proof: Since the first constraint of Model (4) applies to every $j = 1, \dots, n$, if we let $j = 0$, we will have: $\sum_{i=1}^{m} \sum_{r=1}^{s} w_{ir} \geq \theta_o^*$, however, according to the second constraint $\sum_{i=1}^{m} \sum_{r=1}^{s} w_{ir} = 1$, therefore $0 \leq \theta_o^* \leq 1$. ■

3. MCDEA models based on DEA-R-I models

In this section, the models in the previous section are developed based on DEA-R-I models.

First, consider the MCDEA model proposed by Li and Reeves (1999) in the previous section. Based on models of DEA-R-I, the multi-criteria model for evaluating efficiency is presented as follows:

Model 6: An input-oriented ratio-based multiple criteria data envelopment analysis model (MCDEA-I-R)

$$
\begin{aligned}
\max \quad & \theta_o \\
\min \quad & M \\
\min \quad & \sum_{j=1}^{n} d_j \\
s.t. \quad & \sum_{i=1}^{m} \sum_{r=1}^{s} w_{ir} \frac{\left({x_{ij}}/{y_{rj}}\right)}{\left({xio}/{y_{ro}}\right)} - d_j = \theta_o, \quad j = 1, \dots, n, \qquad (6) \\
& \sum_{i=1}^{m} \sum_{r=1}^{s} w_{ir} = 1, \qquad w_{ir} \geq \varepsilon, \\
& \theta_o \geq 0, \quad i = 1, \dots, m \quad, r = 1, \dots, s, \\
& M - d_j \geq 0, \ j = 1, \dots, n.
\end{aligned}
$$

In the model, θ_o is the efficiency value and d_j is the distance between the efficiency score of the DMU under evaluation and the weighted sum of the input to output ratio of all DMUs to that of the DMU under evaluation. M represents the maximum deviation. In order to solve the problem above, we can use the MOLP problem solving methods such as the weighted sum of objective functions. By selecting the proper weight vector, we can calculate the efficiency values obtained from the model for the unit under evaluation as $h_o = 1 - d_o$; furthermore, by increasing the weight corresponding to the objective function θ_o, we can increase the efficiency values and therefore, prevent pseudo-inefficiency and efficiency underestimation.

Theorem 3: Model (6) is feasible for every $\epsilon^* \leq \epsilon$ and infeasible for every $\epsilon \leq \epsilon^*$.

Proof: Assume that $(\bar{w}_{ir}, \bar{\theta}_o, \epsilon^*)$, $i = 1, \dots, m$, $r = 1, \dots, s$, is the optimal solution for the Model (5). Also, let $\bar{d}_j = \sum_{i=1}^{m} \sum_{r=1}^{s} \bar{w}_{ir} \frac{\left({x_{ij}}/{y_{rj}}\right)}{\left({xio}/{y_{ro}}\right)} - \bar{\theta}_o$, j=1,…n,

$$\bar{M} = \max_{1 \leq j \leq n} \left\{ \sum_{i=1}^{m} \sum_{r=1}^{s} \bar{w}_{ir} \frac{\left({x_{ij}}/{y_{rj}}\right)}{\left({xio}/{y_{ro}}\right)} - \bar{\theta}_o \right\}.$$

It is clear than $\left(\bar{M}, \bar{w}_{ir}, \bar{d}_j, \bar{\theta}_o, \epsilon^*\right)$, $i = 1, \dots, m$, $r = 1, \dots, s$, $j = 1, \dots, n$, be a feasible solution for Model (6). Hence $\left(\bar{M}, \bar{w}_{ir}, \bar{d}_j, \bar{\theta}_o, \epsilon\right)$, $i = 1, \dots, m$, $r = 1, \dots, s$, $j = 1, \dots, n$, $\epsilon \leq \epsilon^*$ is as a feasible solution for the Model (6).

Now on the contrary, suppose that $\left(\bar{M}, \bar{w}_{ir}, \bar{d}_j, \bar{\theta}_o, \epsilon\right)$, $i = 1, \dots, m$, $r = 1, \dots, s$, $j = 1, \dots, n$, $\epsilon > \epsilon^*$ is a feasible solution for Model (6). Hence $(\bar{w}_{ir}, \bar{\theta}_o, \epsilon)$, $i = 1, \dots, m$, $r = 1, \dots, s$, $\epsilon > \epsilon^*$ is a feasible solution Model (5) with a larger objective function value than

$(\bar{w}_{ir}, \bar{\theta}_o, \epsilon^*)$, $i = 1, \ldots, m$, $r = 1, \ldots, s$, which contradicts the optimality condition. As a result, Model (6) is infeasible for $\epsilon > \epsilon^*$. ■

Theorem 4: The optimal value of θ_o in Model (6) is always equal to or greater than unity.

Proof: In order to show that $0 \le \theta_o^* \le 1$, we let $j = o$ in the first equation of Model (6); therefore, we arrive at $\sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} - d_o = \theta_o$, however, according to the second constraint of Model (6) : $\sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} = 1$, $\theta_o \ge 0$, $d_o \ge 0$ therefore $1 - d_o = \theta_o$. Hence $0 \le 1 - d_o = \theta_o \le 1$. By attention to θ_o is an arbitrary feasible solution of Model (6) then $0 \le \theta_o^* \le 1$. ■

Theorem 5: The objective function value of Model (6) is bounded.

Proof: We prove that Model (6) is always feasible in Theorem (3). In accordance with the adaptive weighted sum method for solving Model (6), we put $\min -w_1\,\theta_o + w_2 M + w_3 \sum_{j=1}^{n} d_j$. By attention to Theorem (4), $0 \le \theta_o \le 1$. The resulting objective function is non-negative on account of a non-negative linear combination of $-\theta_o$ and the other two positive terms. Due to the fact that Model (6) is a min-type problem the proof completes. ■

As mentioned previously, one of the methods of solving MCDM problems is to utilize goal programming. In the previous section, the model provided by Bal et al. (2008, 2010) was based on goal programming. In this section, we attempt to extend that model based on the models of DEA-R-I. Our goal programming model is presented based on the ratio of inputs to outputs as follows:

Model 7: An input-oriented ratio-based goal programming data envelopment analysis model (GPDEA-I-R)

$$\min \quad \left\{ d_1^- + d_1^+ + \sum_{j=1}^{n} d_{3j}^- + \sum_{j=1}^{n} d_j + \sum_{j=1}^{n} d_{2j}^- + \sum_{j=1}^{n} d_{2j}^+ \right\}$$

$$\begin{aligned}
s.t.\quad & \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} \frac{\left({x_{ij}}/{y_{rj}} \right)}{\left({xio}/{y_{ro}} \right)} - d_j + d_{2j}^- - d_{2j}^+ = \theta_o, \quad j = 1, \ldots, n, \\
& \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} + d_1^- - d_1^+ = 1, \\
& M - d_j + d_{3j}^- - d_{3j}^+ = 0, \quad j = 1, \ldots, n, \\
& w_{ir} \ge \varepsilon, \quad \theta_o \ge 0, \quad i = 1, \ldots, m, \quad r = 1, \ldots, s, \\
& d_j \ge 0, \quad j = 1, \ldots, n, \quad d_1^-, d_1^+ \ge 0, \quad d_{2j}^-, d_{2j}^+ \ge 0, \\
& d_{3j}^-, d_{3j}^+ \ge 0, \quad j = 1, \ldots, n.
\end{aligned} \tag{7}$$

In this model, d_1^-, d_1^+ are wanted and unwanted deviations for the goal the weight restriction equal to unity, d_{2j}^-, d_{2j}^+ $(j = 1, \ldots, n)$ are wanted and unwanted deviations for the goal the efficiency score of the DMU under evaluation and distance between d_j with the weighted sum of the ratio of inputs to outputs of all DMUs to that of the DMU under evaluation. Whereas d_{3j}^+ and d_{3j}^- $(j = 1, \ldots, n)$ are the wanted and unwanted deviation variables for the goal $M - d_j \ge 0$. The objective is to minimize deviations for holding the main constraints.

Model 8: Alternative an input-oriented ratio-based goal programming data envelopment analysis model (AGPDEA - I-R)

$$\min \quad \left\{ d_1^- + d_1^+ + \sum_{j=1}^{n} d_{3j}^- + \sum_{j=1}^{n} d_j \right\}$$

$$\begin{aligned}
s.t.\quad & \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} \frac{\left({x_{ij}}/{y_{rj}} \right)}{\left({xio}/{y_{ro}} \right)} - d_j = \theta_o, \quad j = 1, \ldots, n, \\
& \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} + d_1^- - d_1^+ = 1, \\
& M - d_j + d_{3j}^- - d_{3j}^+ = 0, \quad j = 1, \ldots, n, \\
& w_{ir} \ge \varepsilon, \quad \theta_o \ge 0, \quad i = 1, \ldots, m, \quad r = 1, \ldots, s, \\
& d_j \ge 0, \quad j = 1, \ldots, n, \quad d_1^-, d_1^+ \ge 0, \\
& d_{3j}^-, d_{3j}^+ \ge 0, \quad j = 1, \ldots, n.
\end{aligned} \tag{8}$$

As discussed in the second section, Ghasemi et al. (2014) provided the bio-MCDEA model in order to deal with the topic of the discrimination power in DEA. Their bi-objective model provided the efficiency value. Meanwhile, when there were no weights restrictions for input and output weights, the problem of GPDEA and MCDEA models was not solved and some weights of input and output components became zero. In this section, we develop a Bio-MCDEA model based on DEA-R-I models. Therefore, the following Bio-MCDEA-R -I model is proposed based on the ratio of inputs to outputs, aiming to overcome the problems of discrimination power, efficiency underestimation, and pseudo-inefficiency.

Model 9: An input-oriented ratio-based bi-objective MCDEA (Bio-MCDEA) data envelopment analysis model (Bio-MCDEA-R -I)

$$\min \quad h = \left(w_2\,\mathrm{M} + \mathrm{w}_3 \textstyle\sum_{j=1}^{n} d_j\right)$$

$$s.t. \quad \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir}\frac{\left({}^{x_{ij}}/_{y_{rj}}\right)}{\left({}^{xio}/_{y_{ro}}\right)} - d_j = \theta_o\,, \quad j = 1,\dots,n, \tag{9}$$

$$\sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} = 1, \quad w_{ir} \geq \varepsilon, \quad \theta_o \geq 0.$$

$$M - d_j \geq 0,\ j = 1,\dots,n.$$

In this model, $d_j,\ j = 1,\dots,n,$ is the distance between the efficiency score of the DMU under evaluation and the weighted sum of the ratio of inputs to outputs of all DMUs to that of the DMU under evaluation and θ_o is the efficiency value which is maximized with minimizing d_js.

Theorem 6: Model (9) is feasible for every $\epsilon^* \leq \epsilon$ and infeasible for every $\epsilon \leq \epsilon^*$.

Proof: Since Models (6) and (9) have the same feasible regions, the proof is similar to our proof for Theorem (3). ■

Theorem 7: The optimal value of θ_o in Model (9) is always equal to or greater than unity.

Proof: In order to show that $0 \leq \theta_o^* \leq 1$, we let j = o in the first equation of Model (9); therefore, we arrive at $\sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} - d_j = \theta_o$, however, according to the second constraint of Model (9): $\sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} = 1$, $\theta_o \geq 0$, $d_o \geq 0$ therefore $1 - d_o = \theta_o$. Hence $0 \leq 1 - d_o = \theta_o \leq 1$. By attention to θ_o is an arbitrary feasible solution of Model (9) then $0 \leq \theta_o^* \leq 1$. ■

Theorem 8: The objective function value of Model (9) is bounded.

Proof: We prove that Model (9) is always feasible in Theorem (6). The objective function of Model (9) is a non-negative linear combination of positive functions. Due to the fact that Model (9) is a min-type problem the proof completes. ■

In the above model, in order to increase efficiency and prevent pseudo-inefficiency and efficiency underestimation problems, the weight of θ_o can be increased. In the numerical example section, the sensitivity analysis of solutions to the above models to the change of weights in the objective function is provided.

4. Comparison of presented models versus previous ones

In this section, we engage in a comparison of previous models with our presented models based on DEA-R-I through numerical examples. In this section, aiming to discuss to deal with two inter-related problems: weak discriminating power and unrealistic weight distribution. We engage in a comparison of results published by Ghasemi et al. (2014) in order to evaluate efficiency using the MCDEA models, as well as the models presented in this study. To this aim, we made use of the data set used by Ghasemi et al. (2014). A set of hypothetical data produced randomly from a uniform distribution function for 10 decision-making units with 4 inputs and 4 outputs are considered (Ghasemi et al., 2014). The data are presented in Table 1.

We select the appropriate value of ϵ for solving Models (4) , (6), (9) according to the Theorems (1), (3) and (6). After solving Model (5), we obtain $\epsilon^* = 0.0625$ for all DMUs. As regards $\sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} = 1, \quad w_{ir} \geq \epsilon$, then $\epsilon \leq \frac{1}{8} = 0.125$.

As respects Ghasemi et al. (2014) used of $\epsilon = 0.001$ for solving their models, so we use of $\epsilon = 0.001$ for solving our models.

We first consider the results from the GPDEA-CCR model. As the results in Table 2 demonstrate, the efficiency values provided by Bal et al. (2008, 2010) are incorrect and given the zero input and output multipliers of unit 1, its efficiency is falsely determined as 0.988, while the true value equals to zero. The same argument stands for unit 5. The efficiency values provided by Bal et al. (2008, 2010) for other units, as well as the true values, are presented in Table 2.

Table 2 presents the results obtained from the GPDEA-CCR model. Since all optimal weights related to input and output components corresponding with units one and three equal to zero, the efficiency scores are inaccurately presented by Bal et al. (2008, 2010) as 0.968 and 0.95, respectively. If we calculate the efficiency score based on the concept of economic efficiency, meaning the ratio of a weighted set of outputs to a weighted set of inputs, we will arrive at the accurate efficiency scores, as presented in the second column of Table 2. Therefore, considering that the optimal input and output weights related to units one and three become zero, the accurate efficiency score for these units would be zero.

Table 1. Example 1 dataset

DMU	Outputs				Inputs			
	y_1	y_2	y_3	y_4	x_1	x_2	x_3	x_4
1	47	93	54	65	32	50	82	46
2	88	56	92	80	61	56	68	37
3	94	65	80	80	42	58	45	34
4	50	53	93	97	73	39	88	81
5	47	42	70	52	45	38	68	41
6	86	45	100	47	86	62	44	32
7	83	91	62	74	38	74	71	74
8	79	60	72	98	61	54	70	62
9	85	68	51	41	84	52	38	47
10	78	95	70	92	87	47	31	52

Table 2. GPDEA- CCR results based on Example 1 dataset

DMU	Output weights				Input weights				Efficiency (true values)	Efficiency provided by Bal et al. (2010)
	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4		
1	0	0	0	0	0	0	0	0	0	0.968
2	0.00317	0.00434	0.00464	0	0.00403	0.01347	0	0	0.948	0.951
3	0.00333	0.00456	0.00488	0	0.00424	0.01417	0	0	1	1
4	0	0.00488	0.00797	0	0.00336	0.01182	0.00059	0.00298	1	1
5	0	0	0	0	0	0	0	0	0	0.950
6	0.00268	0.00367	0.00392	0	0.00341	0.01140	0	0	0.788	0.794
7	0.00070	0.00371	0.00564	0	0.00245	0.00990	0	0.00235	0.745	0.779
8	0.00084	0.00446	0.00679	0	0.00295	0.01193	0	0.00283	0.823	0.843
9	0.00305	0.00417	0.00446	0	0.00388	0.01297	0	0	0.771	0.767
10	0.00322	0.00441	0.00471	0	0.00409	0.01370	0	0	1	1

Now, we examine the results of the GPDEA- R-I model, as presented in Table 3. As can be observed, the last column of the table shows the efficiency values and all units are efficient, except units 5, 6 and 7. The main reason is not meeting the main condition of DEA in the evaluation of efficiency; in this regard, the number of DMUs should be 3 times as many as the sum of inputs and outputs. Accordingly, as there are 4 inputs and 4 outputs here, there should be 24 decision-making units, while the number of units under analysis is 10. Comparing Table 3 with Table 2, it can be observed that in both models, units 3, 4 and 10 are efficient and the efficiency values of the GPDEA-R-I model are greater than or equal to the corresponding values in the GPDEA-CCR model for all units. Considering the zero values of certain weights in GPDEA-CCR models and the existing problem of discrimination power, in the corresponding GPDEA-R-I models, the weight average corresponding to the ratio of each input to output component is non-zero; i.e., $\frac{\sum_{i=1}^{m} w_{ir}}{m} \neq 0$, $\frac{\sum_{r=1}^{s} w_{ir}}{s} \neq 0$. All weights are provided in Table 3 below. The GPDEA-R-I model prevents the problems of efficiency underestimation and pseudo-inefficiency. As can be seen, the efficiency values are greater than their corresponding values in the GPDEA-CCR model.

Figure 1 is a point chart for comparison of efficiency scores between GPDEA-R-CCR and GPDEA-CCR models. Now we analyze the results in MCDEA models under assumption of constant returns to scale technology. As can be seen in the Figure 1, all efficiency scores obtained from the GPDEA-R-CCR model are greater than or equal to their corresponding scores in the GPDEA- CCR models. This demonstrates that in comparison with the GPDEA-CCR models, GPDEA-R-CCR models can prevent the efficiency underestimation and discrimination power and poor weight dispersion issues and increases the power of discrimination.

Table 3. GPDEA-R-CCR results based on Example 1 dataset

DMU	weights																
	w_{11}	w_{12}	w_{13}	w_{14}	w_{21}	w_{22}	w_{23}	w_{24}	w_{31}	w_{32}	w_{33}	w_{34}	w_{41}	w_{42}	w_{43}	w_{44}	Efficiency
1	0.001	0.001	0.114	0.001	0.537	0.087	0.125	0.001	0.001	0.105	0.001	0.001	0.001	0.022	0.001	0.001	1.000
2	0.001	0.001	0.127	0.001	0.275	0.421	0.001	0.001	0.001	0.090	0.076	0.001	0.001	0.001	0.001	0.001	1.000
3	0.022	0.001	0.071	0.001	0.391	0.504	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
4	0.001	0.001	0.153	0.001	0.395	0.122	0.055	0.001	0.001	0.194	0.001	0.001	0.001	0.071	0.001	0.001	1.000
5	0.001	0.001	0.116	0.001	0.154	0.232	0.071	0.001	0.001	0.259	0.158	0.001	0.001	0.001	0.001	0.001	0.965
6	0.034	0.001	0.084	0.001	0.321	0.549	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.788
7	0.032	0.001	0.001	0.001	0.156	0.132	0.510	0.001	0.001	0.096	0.001	0.001	0.001	0.063	0.001	0.001	0.875
8	0.001	0.001	0.159	0.001	0.143	0.285	0.038	0.001	0.001	0.060	0.227	0.001	0.001	0.078	0.001	0.001	1.000
9	0.001	0.001	0.513	0.001	0.236	0.137	0.070	0.001	0.001	0.033	0.001	0.001	0.001	0.001	0.001	0.001	1.000
10	0.042	0.001	0.451	0.001	0.321	0.157	0.019	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000

Now we analyze the results of MCDEA models with constant returns to scale technology assumption. Table 4 presents the results of Model (1). In order to solve the model, we engage in a minimization of sum of deviations, i.e., $\min \sum_{j=1}^{n} d_j$ or minsum, as mentioned by Ghasemi et al. (2014). The weight of the fourth output equals to zero in all units. Moreover, the weights corresponding to the third input are non-zero values for unit 4 only, and zero for the other units. This is caused by the low capability of the MCDEA model; for instance, its lower discrimination power and the fact that it ignores the significance of third and fourth inputs and the fourth output in the calculation of efficiency. The weights corresponding to the fourth input are zero in units 2, 3, 6, 7 and 8, which leads to the imprecise calculation of efficiency values and thus, occurrence of pseudo-inefficiency.

Fig 1. Compare efficiency scores obtained from the models of GPDEA-R-CCR and GPDEA-CCR

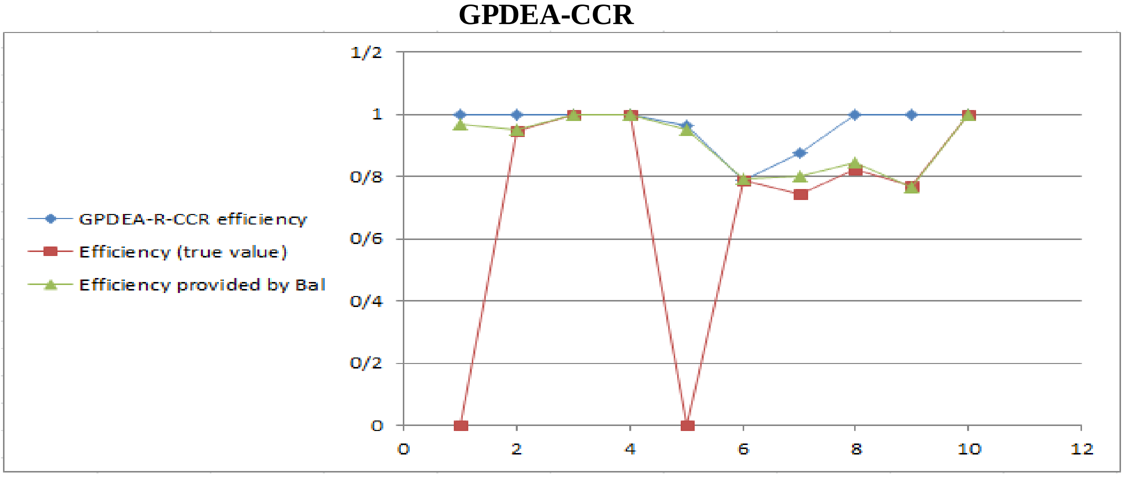


Table 4. Minsum DEA- CCR results based on Example 1 dataset

DMU	Output weights				Input weights				Efficiency
	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4	
1	0.00102	0.00543	0.00827	0	0.00359	0.01453	0	0.00345	1
2	0.00317	0.00434	0.00464	0	0.00403	0.01347	0	0	0.948
3	0.00333	0.00456	0.00488	0	0.00424	0.01417	0	0	1
4	0	0.00488	0.00797	0	0.00336	0.01182	0.00059	0.00298	1
5	0.00119	0.00636	0.00967	0	0.00420	0.01699	0	0.00403	1
6	0.00268	0.00367	0.00392	0	0.00341	0.01140	0	0	0.788
7	0.00070	0.00371	0.00564	0	0.00245	0.00990	0	0.00235	0.745
8	0.00084	0.00446	0.00679	0	0.00295	0.01193	0	0.00283	0.823
9	0.00305	0.00417	0.00446	0	0.00388	0.01297	0	0	0.771
10	0.00322	0.00441	0.00471	0	0.00409	0.01370	0	0	1

Now, we consider the results of Model (6). As can be observed from Tables 5, 6, 7, 8 and 9, the adaptive weighted sum method can be used to solve Model (6). In Table 5, weights are considered as the vector (0. 5, 0.45, 0.05) and the last column of Table 5 indicates that units 1, 3, 4 and 10 are efficient, and the others inefficient. Other columns show the weights corresponding to the ratios of all input to output components. Comparing the efficiency values obtained from MCDEA and MCDEA-R-I models, it can be observed that all values in the MCDEA-R-I models are greater than or equal to the corresponding values in the MCDEA models. Only in unit 5 is the MCDEA efficiency value greater than the MCDEA–R-I efficiency value. The value obtained from MCDEA is equal to 1 and the corresponding value in the MCDEA-R-I model equals to 0.986, which is due to the decrease in the weight of θ_o in the objective function, which here is w_{θ_o}=0.5. By increasing the corresponding weight of θ_o in the objective function, the efficiency values can be increased in the MCDEA-R-I model, so they would be greater than their corresponding values in MCDEA. As can be seen in Table 5, the average weighted sum corresponding to each input to output ratio equals to a non-zero value.

In the presented numerical example, we use the adaptive weighted sum method to solve the MOLP problems; in this regard, the corresponding weights of the objective functions are selected based on the manager or decision-maker's preferences. For sensitivity analysis and comparison of results obtained from the presented models with the results of previous models, we use a normal weight vector based on the importance of each objective function to solve the models.

Table 5. MCDEA-R-I results based on Example 1 dataset. Weight vector = (0.5, 0.45, 0.05)

DMU	weights																
	w_{11}	w_{12}	w_{13}	w_{14}	w_{21}	w_{22}	w_{23}	w_{24}	w_{31}	w_{32}	w_{33}	w_{34}	w_{41}	w_{42}	w_{43}	w_{44}	Efficiency
1	0.151	0.001	0.061	0.001	0.001	0.263	0.513	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
2	0.189	0.001	0.001	0.001	0.052	0.380	0.368	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.958
3	0.126	0.001	0.001	0.001	0.054	0.352	0.456	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
4	0.001	0.001	0.172	0.001	0.433	0.245	0.138	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
5	0.259	0.001	0.001	0.001	0.002	0.327	0.400	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.986
6	0.001	0.001	0.149	0.001	0.301	0.366	0.171	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.795
7	0.001	0.001	0.104	0.001	0.419	0.218	0.247	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.777
8	0.195	0.001	0.001	0.001	0.055	0.316	0.422	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.892
9	0.183	0.001	0.001	0.001	0.345	0.197	0.264	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.855
10	0.319	0.001	0.001	0.001	0.050	0.194	0.424	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000

Analyzing the results from the MCDEA-R-I model with the weight vector (0.7, 0.25, 0.05) for each objective, as explained in Table 6, it can be witnessed that units 1, 3, 4, 6 and 10 are efficient, and the others inefficient. In all units except unit 5, all efficiency values are greater than or equal to the corresponding values in Table 4, and for all units, the weight averages of the ratio of input to output components are non-zero values in all components, $\frac{\sum_{i=1}^{m} w_{ir}}{m} \neq 0$, $\frac{\sum_{r=1}^{s} w_{ir}}{s} \neq 0$. For instance, the first input component of $\frac{\sum_{r=1}^{s} w_{1r}}{s} \neq 0$ is non-zero value, which shows that all input and output components are involved in the calculation of efficiency. For example, the efficiency values of units 8 and 9 are 0.897 and 0.931, respectively, which are greater than their corresponding values in the MCDEA model, meaning 0.823 and 0.771, as presented in Table 4. The MCDEA-R-I model prevents the problem of pseudo-inefficiency; for instance, by solving the DEA-R-I model unit 6 is determined efficient (Table 6), while its efficiency score is equal to 0.788 in Table 4 relating MCDEA, which shows that this unit is MCDEA inefficient. The average efficiency of all units in the MCDEA-R-I model (Table 6) is greater than the corresponding value in MCDEA (Table 4). It should be noted that the DEA-R-I models have been considered input-oriented and the production technology is constant returns to scale.

Table 6. MCDEA-R-I results based on Example 1 dataset. Weight vector = (0.7, 0.25, 0.05)

DMU	weights																
	w_{11}	w_{12}	w_{13}	w_{14}	w_{21}	w_{22}	w_{23}	w_{24}	w_{31}	w_{32}	w_{33}	w_{34}	w_{41}	w_{42}	w_{43}	w_{44}	Efficiency
1	0.151	0.001	0.061	0.001	0.001	0.263	0.513	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
2	0.183	0.001	0.003	0.001	0.001	0.357	0.445	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.978
3	0.126	0.001	0.001	0.001	0.054	0.352	0.456	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
4	0.402	0.001	0.001	0.001	0.006	0.269	0.312	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
5	0.259	0.001	0.001	0.001	0.001	0.327	0.400	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.986
6	0.001	0.001	0.123	0.001	0.001	0.001	0.646	0.001	0.001	0.169	0.001	0.001	0.018	0.033	0.001	0.001	1.000
7	0.001	0.001	0.192	0.001	0.343	0.453	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.907
8	0.189	0.001	0.001	0.001	0.001	0.293	0.504	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.897
9	0.196	0.001	0.001	0.001	0.560	0.231	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.931
10	0.319	0.001	0.001	0.001	0.050	0.194	0.424	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000

Figure 2 is a point chart comparing the efficiency scores of Minsum DEA- CCR and MCDEA-R-I models. Since both models are multi-objective programming problems, we use the adaptive weighted sum method to solve them. If we use the weight vector $\mathrm{W} = (1,0,0)$ to solve model (1), the resulting single-objective model, which we call Minsum, will minimize the set of deviations. To solve model (6), we first select the weights $W_1 = (0.5, 0.45, 0.05)$ and $W_2 = (0.7, 0.25, 0.05)$; results are presented in Tables 5 and 6, respectively. Figure 2 compares the efficiency scores obtained from the Minsum model and the MCDM-R-I model for the weights W_1 and W_2. As can be witnessed, the scores obtained from model (6) are greater than or equal to their similar scores in Model (1) for all units; this proves that MCDM-R-I models can prevent the efficiency underestimation and discrimination power and poor weight dispersion issues better than Minsum models and increase the power of discrimination.

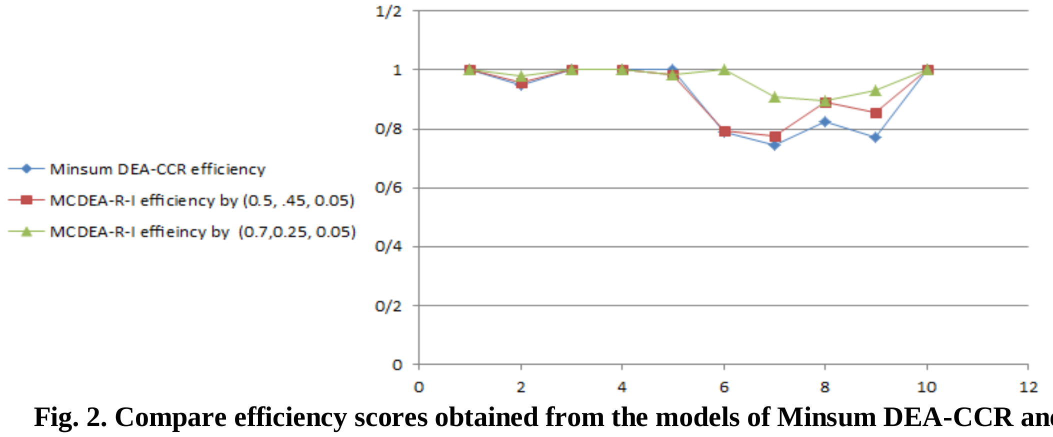


Fig. 2. Compare efficiency scores obtained from the models of Minsum DEA-CCR and MCDM-R-I

Now, to analyze the sensitivity of the MCDEA-R-I model to weight vector changes in the objective function, we consider the weight vector (0.8, 0.1, 0.1) for the objective function of the MCDEA-R-I model. According to the obtained results (Table 7), units 1, 2, 3, 4, 6 and 10 are efficient and the other units are inefficient, and similarly, the efficiency values of the MCDEA-R-I model are greater than or equal to the corresponding values in MCDEA (Table 5); however, this result is not true for unit 5. For instance, the efficiency scores of units 8 and 9 are 0.897 and 0.931 in MCDEA-R-I, respectively, while the corresponding values obtained from the MCDEA model (Table 4) equal to 0.823 and 0.771, which shows that the efficiency underestimation problem is resolved in DEA-R-I models. Also, we can observe that unit 6 is efficient in the MCDEA-R-I model (Table 7), while being inefficient in MCDEA (Table 4) with the efficiency score of 0.778. This reveals the existence of pseudo-inefficiency in the MCDEA model, while the MCDEA-R-I model prevents it.

Comparing the results from MCDEA-R-I and MCDEA models and using the weight vector W = (0.6, 0.4, 0) for the MCDEA-R-I model, the results indicate that the efficiency values of the MCDEA-R-I model are greater than or equal to those of the MCDEA model (see Tables 4 and 8). For instance, the efficiency scores for units 8 and 9 are 0. 823 and 0.771 in the MCDEA model, respectively, while in the MCDEA-R-I model (Table 8), their values equal to 0.888 and 0.93, respectively. The weight averages obtained from the MCDEA-R-I model are non-zero values for every input and output component; i.e., all input and output components are involved in the calculation of efficiency.

Table 7. MCDEA-R-I results based on Example 1 dataset. Weight vector = (0.8, 0.1, 0.1)

DMU	weights																
	w_{11}	w_{12}	w_{13}	w_{14}	w_{21}	w_{22}	w_{23}	w_{24}	w_{31}	w_{32}	w_{33}	w_{34}	w_{41}	w_{42}	w_{43}	w_{44}	Efficiency
1	0.139	0.001	0.069	0.001	0.001	0.260	0.519	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
2	0.157	0.001	0.001	0.001	0.001	0.313	0.497	0.001	0.001	0.001	0.001	0.001	0.020	0.001	0.001	0.001	1.000
3	0.119	0.001	0.003	0.001	0.001	0.322	0.544	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
4	0.404	0.001	0.001	0.001	0.001	0.271	0.313	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
5	0.259	0.001	0.001	0.001	0.001	0.327	0.400	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.986
6	0.001	0.001	0.123	0.001	0.001	0.001	0.646	0.001	0.001	0.169	0.001	0.001	0.018	0.033	0.001	0.001	1.000
7	0.001	0.001	0.192	0.001	0.343	0.453	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.907
8	0.189	0.001	0.001	0.001	0.001	0.293	0.504	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.897
9	0.196	0.001	0.001	0.001	0.560	0.231	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.931
10	0.297	0.001	0.009	0.001	0.001	0.177	0.504	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000

Table 8. MCDEA-R-I results based on Example 1 dataset. Weight vector = (0.6, 0.4, 0.0)

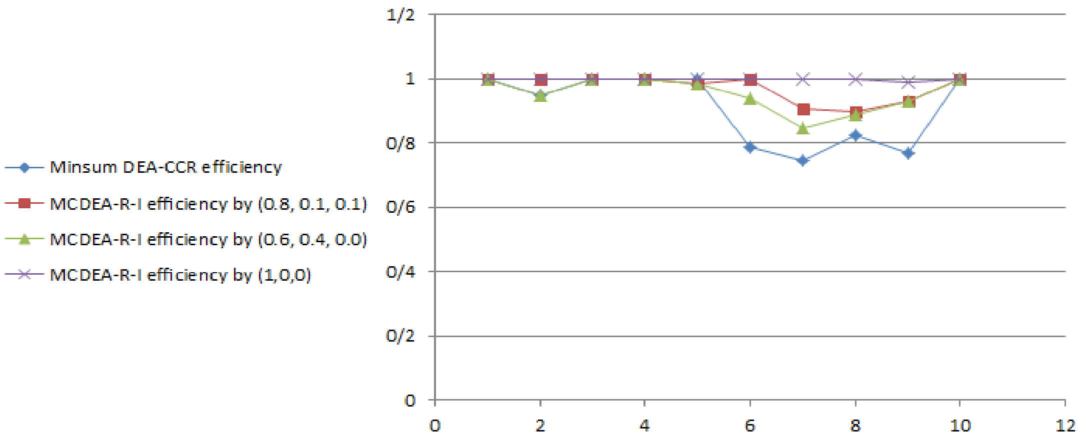
DMU	weights																
	w_{11}	w_{12}	w_{13}	w_{14}	w_{21}	w_{22}	w_{23}	w_{24}	w_{31}	w_{32}	w_{33}	w_{34}	w_{41}	w_{42}	w_{43}	w_{44}	Efficiency
1	0.139	0.001	0.069	0.001	0.001	0.260	0.519	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
2	0.157	0.001	0.001	0.001	0.001	0.313	0.497	0.001	0.001	0.001	0.001	0.001	0.020	0.001	0.001	0.001	1.000
3	0.119	0.001	0.003	0.001	0.001	0.322	0.544	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
4	0.404	0.001	0.001	0.001	0.001	0.271	0.313	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
5	0.259	0.001	0.001	0.001	0.001	0.327	0.400	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.986
6	0.001	0.001	0.123	0.001	0.001	0.001	0.646	0.001	0.001	0.169	0.001	0.001	0.018	0.033	0.001	0.001	1.000
7	0.001	0.001	0.192	0.001	0.343	0.453	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.907
8	0.189	0.001	0.001	0.001	0.001	0.293	0.504	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.897
9	0.196	0.001	0.001	0.001	0.560	0.231	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.931
10	0.297	0.001	0.009	0.001	0.001	0.177	0.504	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000

In the following, we will solve Model (5) with the weight vector (1, 0, 0) in order to analyze the sensitivity of the MCDEA-R-I model to changes in the weight vector (see Table 9 below). In this case, in order to maximize the significance of θ_o in the model, its weight is considered equal to one and the weights of the other two objective functions are assumed zero. As shown in Table 9, all units are efficient except unit 9, and with an increase in the weight of θ_o in the objective function, the problem of pseudo-inefficiency is completely solved. Even Unit 5 is identified as efficient in that case, and its efficiency would equal to one. For all rows and columns in Table 9 corresponding to the weights of the input to output ratios, the weight average for each row and column is a non-zero value. As an instance, the ratio of the first input to any other output is a non-zero value. However, due to the weights restriction $\sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir} = 1$, there can't be a zero value for all the weights and the DEA-R-I model would naturally impose a weight restriction on the problem. Figure 3 is a point chart demonstrating the efficiency scores of Minsum DEA- CCR and MCDEA-R-I models for weights $W_3 = (0.8, 0.1, 0.1)$, $W_4 = (0.6, 0.4, 0)$ and $W_5 = (1, 0, 0)$; results are presented in Tables 4, 7, 8 and 9, respectively.

Table 9. MCDEA-R-I results based on Example 1 dataset. Weight vector = (1, 0, 0)

DMU	weights																
	w_{11}	w_{12}	w_{13}	w_{14}	w_{21}	w_{22}	w_{23}	w_{24}	w_{31}	w_{32}	w_{33}	w_{34}	w_{41}	w_{42}	w_{43}	w_{44}	Efficiency
1	0.001	0.985	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
2	0.001	0.275	0.001	0.001	0.001	0.001	0.647	0.001	0.001	0.001	0.001	0.001	0.065	0.001	0.001	0.001	1.000
3	0.001	0.162	0.001	0.001	0.001	0.001	0.771	0.001	0.001	0.001	0.001	0.001	0.055	0.001	0.001	0.001	1.000
4	0.001	0.562	0.001	0.001	0.001	0.001	0.424	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
5	0.001	0.434	0.001	0.001	0.001	0.001	0.552	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
6	0.001	0.001	0.001	0.001	0.001	0.001	0.895	0.001	0.001	0.001	0.011	0.001	0.081	0.001	0.001	0.001	1.000
7	0.001	0.424	0.001	0.001	0.001	0.001	0.404	0.001	0.159	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
8	0.001	0.276	0.001	0.174	0.001	0.001	0.019	0.477	0.001	0.001	0.044	0.001	0.001	0.001	0.001	0.001	1.000
9	0.001	0.001	0.001	0.001	0.946	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.040	0.001	0.001	0.001	0.989
10	0.001	0.001	0.001	0.001	0.935	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.051	0.001	0.001	0.001	1.000

Fig. 3. Compare efficiency scores obtained from the models of Minsum DEA-CCR and MCDM-R-I



Ghasemi et al. (2014) presented the Bio-MCDEA model to overcome the lack of discrimination power in DEA. Table 1 presents the results from application of this model to the data. As can be seen, this model could not fully resolve the problem of low discrimination power in GPDEA and MCDEA models and if the weight restriction is removed, some weights would still have zero values. For example, the weight of the fourth output equals to 0.001 in all units and becomes zero when removing the weight restriction. For the weight vector of the third input, the weight corresponding to unit 4 is the only one with a non-zero value, i.e., the problem of discrimination power still remains.

Table 10. BiO-MCDEA model results based on Example 1 dataset (ϵ= 0.0001)

DMU	Output weights				Input weights				Efficiency	Super Efficiency	Rank
	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4			
1	0.00420	0.00481	0.00573	0.00010	0.00453	0.01678	0.00010	0.00016	0.961	0.961	4
2	0.00290	0.00435	0.00480	0.00010	0.00404	0.01324	0.00010	0.0014	0.948	0.948	5
3	0.00358	0.00408	0.00488	0.00010	0.00386	0.01429	0.00010	0.00013	1	1.210	2
4	0.00010	0.00486	0.00782	0.00010	0.00344	0.01191	0.00010	0.00288	1	1.079	3
5	0.00420	0.00624	0.00690	0.00010	0.00576	0.01906	0.00010	0.00024	0.947	0.947	6
6	0.00245	0.00369	0.00408	0.00010	0.00344	0.01123	0.00010	0.00011	0.789	0.789	8
7	0.00116	0.00373	0.00522	0.00010	0.00283	0.01031	0.00010	0.00165	0.767	0.767	9
8	0.00147	0.00445	0.00617	0.00010	0.00339	0.01237	0.00010	0.00190	0.837	0.837	7
9	0.00279	0.00418	0.00463	0.00010	0.00389	0.01275	0.00010	0.00014	0.761	0.761	10
10	0.00294	0.00441	0.00488	0.00010	0.00410	0.01346	0.00010	0.00015	1	1.419	1

In the following, we engage in an analysis of results obtained from the Bio-MCDEA-R model. In this regard, the weight vector $(w_1, w_2) = (0.5, 0.5)$ is used to solve the Bio-MCDEA-R model. Results from the Bio-MCDEA-R model are presented in Table 11. The last column of the table provides the efficiency values. As can be observed, units 1, 3, 4 and 10 are efficient and the others inefficient. All efficiency values in the Bio-MCDEA-R model are greater than or equal to the corresponding values in the Bio-MCDEA model. As can be seen, the Bio-MCDEA-R model prevents the pseudo-inefficiency problem. For instance, the Bio-MCDEA-R model considers unit 1 efficient, while it is inefficient in the Bio-MCDEA model.

Table 11. Bio-MCDEA-R results based on Example 1 dataset. Weight vector (0.5, 0.5)

DMU	weights																
	w_{11}	w_{12}	w_{13}	w_{14}	w_{21}	w_{22}	w_{23}	w_{24}	w_{31}	w_{32}	w_{33}	w_{34}	w_{41}	w_{42}	w_{43}	w_{44}	Efficiency
1	0.124	0.001	0.041	0.001	0.001	0.210	0.450	0.001	0.001	0.001	0.001	0.001	0.001	0.164	0.001	0.001	1.000
2	0.001	0.001	0.140	0.001	0.365	0.333	0.150	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.950
3	0.001	0.001	0.116	0.001	0.371	0.312	0.189	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
4	0.001	0.001	0.172	0.001	0.433	0.245	0.138	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
5	0.258	0.001	0.001	0.001	0.068	0.338	0.325	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.946
6	0.001	0.001	0.150	0.001	0.336	0.379	0.124	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.777
7	0.001	0.001	0.104	0.001	0.419	0.218	0.247	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.777
8	0.001	0.001	0.167	0.001	0.370	0.278	0.173	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.888
9	0.183	0.001	0.001	0.001	0.345	0.197	0.264	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.855
10	0.001	0.001	0.278	0.001	0.366	0.172	0.172	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000

Now, in order to analyze the sensitivity of the Bio-MCDEA-R model to changes in the weight vector of the objective function, the weight vector $(w_1, w_2) = (0.2, 0.8)$ is applied. Results indicate the efficiency of units 1, 3, 4 and 10, and the inefficiency of other units (see Table 12). As can be seen in the last column, the values in Table 12 are greater than or equal to those of the Bio-MCDEA model, and similar to Table 10's results, unit 1 is efficient in the Bio-MCDEA- R model, though the Bio-MCDEA model identifies it as inefficient. Now, the weight vector $(w_1, w_2) = (0.4, 0.6)$ is used to analyze the Bio-MCDEA-R model's sensitivity to changes in the weight vector of the objective function (See Table 13).

Table 12. Bio-MCDEA-R results based on Example 1 dataset. Weight vector= (0.2, 0.8)

DMU	weights																
	w_{11}	w_{12}	w_{13}	w_{14}	w_{21}	w_{22}	w_{23}	w_{24}	w_{31}	w_{32}	w_{33}	w_{34}	w_{41}	w_{42}	w_{43}	w_{44}	Efficiency
1	0.124	0.001	0.041	0.001	0.001	0.210	0.450	0.001	0.001	0.001	0.001	0.001	0.001	0.164	0.001	0.001	1.000
2	0.157	0.001	0.001	0.001	0.001	0.313	0.497	0.001	0.001	0.001	0.001	0.001	0.020	0.001	0.001	0.001	1.000
3	0.001	0.001	0.116	0.001	0.371	0.312	0.189	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
4	0.001	0.001	0.172	0.001	0.433	0.245	0.138	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
5	0.082	0.001	0.083	0.001	0.001	0.339	0.382	0.001	0.001	0.001	0.001	0.001	0.103	0.001	0.001	0.001	1.000
6	0.001	0.001	0.144	0.001	0.001	0.001	0.642	0.001	0.042	0.160	0.001	0.001	0.001	0.001	0.001	0.001	1.000
7	0.204	0.001	0.023	0.001	0.001	0.645	0.001	0.001	0.116	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
8	0.135	0.001	0.001	0.001	0.431	0.001	0.173	0.249	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.970
9	0.196	0.001	0.001	0.001	0.560	0.231	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.931
10	0.001	0.001	0.278	0.001	0.366	0.172	0.172	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000

Table 13. Bio-DEA-R-CCR results based on Example 1 dataset. Weight vector= (0.4, 0.6)

DMU	weights																
	w_{11}	w_{12}	w_{13}	w_{14}	w_{21}	w_{22}	w_{23}	w_{24}	w_{31}	w_{32}	w_{33}	w_{34}	w_{41}	w_{42}	w_{43}	w_{44}	Efficiency
1	0.124	0.001	0.041	0.001	0.001	0.210	0.450	0.001	0.001	0.001	0.001	0.001	0.001	0.164	0.001	0.001	1.000
2	0.001	0.001	0.140	0.001	0.365	0.333	0.150	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.950
3	0.001	0.001	0.116	0.001	0.371	0.312	0.189	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
4	0.001	0.001	0.172	0.001	0.433	0.245	0.138	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000
5	0.259	0.001	0.001	0.001	0.002	0.327	0.400	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.986
6	0.001	0.001	0.170	0.001	0.001	0.061	0.543	0.001	0.001	0.213	0.001	0.001	0.001	0.001	0.001	0.001	0.939
7	0.001	0.001	0.169	0.001	0.173	0.364	0.263	0.001	0.001	0.019	0.001	0.001	0.001	0.001	0.001	0.001	0.849
8	0.001	0.001	0.167	0.001	0.370	0.278	0.173	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.888
9	0.184	0.001	0.001	0.001	0.528	0.246	0.001	0.001	0.001	0.001	0.001	0.001	0.029	0.001	0.001	0.001	0.930
10	0.001	0.001	0.278	0.001	0.366	0.172	0.172	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	1.000

Fig. 4. Compare efficiency scores obtained from the models of Bio-MCDEA and Bio-MCDEA –R

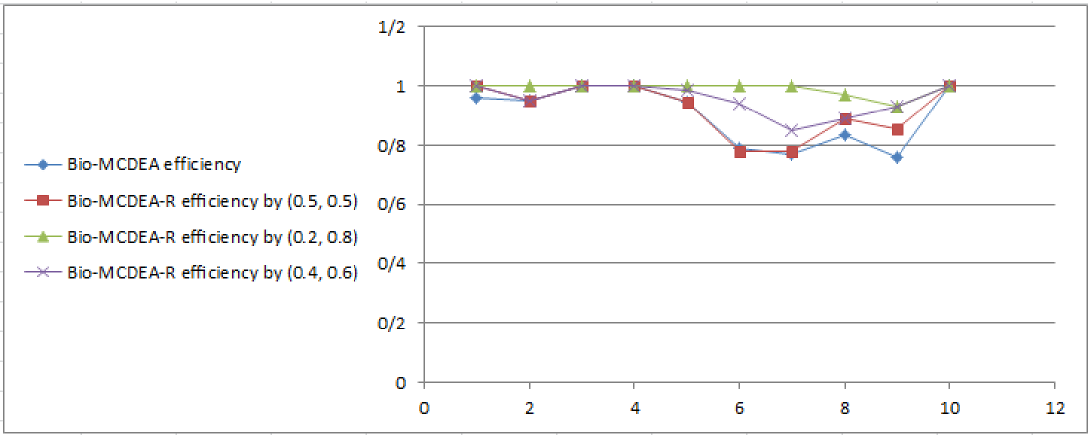


Figure 4 is a point chart comparing the efficiency scores of Bio-MCDEA and Bio-MCDEA-R models. In order to solve the Bio-MCDEA model, we use the weight vectors $W = (0.5,0.5)$, and the weight vectors $W = (0.5,0.5)$, $W = (0.2,0.8)$, and $W = (0.4,0.6)$ are used to solve the Bio-MCDEA-R model. Results are provided in Tables 10, 11, 12 and 13, respectively. As can be seen, all values obtained from model (9) are greater than or equal to their corresponding values in Model (3), which reveals that Model (9) can prevent the efficiency underestimation and lack of discrimination power and poor weight dispersion in Model (3) and cause the Bio-MCDEA-R model to have a higher weight discrimination power comparing to Bio-MCDEA models.

5. Conclusion

This article deals with three important issues, namely the discrimination power of weights, underestimation of efficiency and pseudo-inefficiency. In this regard, we engaged in a comparison of results from the GPDEA models provided by Bal et al. [25, 26], the MCDEA model proposed by Li and Reeves [23] and the Bio-MCDEA presented by Ghasemi et al. [24]. As discussed in the previous sections, the main drawback of all these models is low discrimination power and the issue that some input and output components are not involved in the evaluation of efficiency when weights become zero. In order to overcome said problems, we made use of DEA-R-I models since they enjoy the following properties: Firstly, the efficiency values obtained from these input-oriented models under constant returns to scale assumption are greater than or equal to the corresponding efficiency values in CCR models. Secondly, CCR models falsly introduce some efficient units as inefficient, due to an inherent weight restriction and the zero weights corresponding to certain input and output components, while this does not occur in DEA-R-I models. Thirdly, since in the optimal solutions obtained from CCR models, some weights corresponding to certain input and output components equal to zero, the efficiency values are calculated inaccurately for some units (less than the true values), which is referred to as efficiency underestimation. This issue has been resolved, to some extent, in DEA-R-I based models. In this paper, the models presented by Ghasemi et al. [24], including GPDEA, MCDEA, and Bio-MCDEA, all of which exhibit a multi-objective programming framework, were developed based on the DEA-R-I models and three new models were introduced. Through numerical examples, we demonstrated the advantages of these models over previous ones. In this regard, we showed that the three aims of increasing discrimination power, reducing efficiency underestimation and eliminating pseudo-inefficiency can be achieved by choosing appropriate weights for the objective functions. The models presented in this paper are based on DEA-R and consider a ratio of inputs to outputs in their calculations. In the first model, there is a weight restriction as a constraint, which prevents the weight average from equaling to zero. One of the features of the proposed models is that they are always feasible and when there is a higher number of units under evaluation compared to the number of inputs and outputs, the accuracy of results would increase. For future studies, the above-mentioned models can be extended to variable returns to scale and other MOLP problem solving methods, including interactive methods such as STEM and ZW, can be used to solve the models. We also suggest development of the above-mentioned models for imprecise data.

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