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Research Article

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Calculation of non-radial efficiency of decision-making units with fuzzy data using GDEA model

A. Farshad , M. Rostamy-MalKhalifeh*
Islamic Azad University, Science and Research Branch, Tehran, Iran

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Abstract
All managers need to evaluate the units under their supervision. To evaluate the units, they must determine the evaluation indicators and then calculate the efficiency of each unit with the help of these indicators. In practice, many indicators may not be small, but have a qualitative value or their values are fuzzy in this article we want to calculate the units under evaluation that have the same inputs and outputs. To evaluate the performance of these units with fuzzy data, first for each unit, we calculate the nature of the output in such a way that according to the desired input, what can be the maximum value of the output because the number of outputs is more than one, so we have a multi-objective linear programming problem to be answered by solving the multi-objective linear programming problem We use flexible fuzzy. In the introduced model, we first change the fuzzy linear programming problem to a multi-objective linear programming problem with three objective functions using the alphabetical method arranged on triangular fuzzy numbers, so using this method is an optimal solution. Lexicography of the ML problem We find OP.

Keywords: Data envelopment analysis, Efficiency, Multi-objective linear programming problem. Fuzzy multi-objective linear programming problem

* Corresponding author: Email: Mohsen_rostamy@yahoo.com

1. Introduction

Data Envelopment Analysis (DEA) is an appropriate and practical approach for measuring the relative efficiency and ranking of a homogeneous decision unit: (DMUS) which is widely used in various fields such as health care, transportation, education. Power, energy, tourism, banking, insurance and finance. Assume the output and input data are clear and accurate. However, specific input and output data are not available in many real-world applications. Consequently, it is necessary to propose methods and models that can measure the performance of DMUs in the presence of vague and inaccurate data. [1] Data envelopment analysis is a mathematical planning approach used to evaluate the performance of decision units with similar activities and multiple inputs and outputs. For more studies about the different methods for measuring the efficiency of units in the DEA literature, see [2-7]. If there is an undesirable input or output between the inputs and outputs of the decision units, the amount of these inputs or outputs should be increased and decreased, respectively [8]. Data Envelopment Analysis (DEA) is used to measure the relative efficiency of a group of decision units with different inputs to generate different efficiencies. In order to discuss inaccurate data, the fuzzy concept is introduced [9]. Data envelopment analysis is a quantitative method used to measure the relative efficiency of organizational units with multiple inputs and outputs in order to achieve the relative efficiency of similar decision units [9]. Zadeh was a pioneering researcher in probabilistic planning who introduced the basic principles of possibility theory and then many studies have been done to expand the theory of possibility. He stated that probability theory is a mathematical theory for modeling certain types of uncertainty as an option for probability theory [1]. Since, in fuzzy linear programming (FLP) models, fuzzy coefficients can be The title of fuzzy variables and constraints are considered as fuzzy events, the possibility of fuzzy events can be defined using the theory of possibility. Therefore, to measure the chance of fuzzy events, the basic measures (Pos) and necessity (Nec) are introduced with optimistic and pessimistic views, respectively, then the measurement of validity (Cr) as the average amount of probability and necessity measures and. Finally, the general fuzzy criterion (GF), which is equivalent to a convex combination of feasibility and necessity criteria, is proposed [10]. Goa et al. Developed the feasibility approach in fuzzy data envelopment analysis models using the CCR model proposed by Charens et al. [11] They developed the fuzzy CCR model using Pos (Pos) and Necessity (Nec) measures. Have been created to deal with customer evaluation. Data coverage analysis (DEA) was proposed by Charnes et al. [11], developed using the Farrell idea [12] and then by others [10, 8, 13]. DEA is a method of measuring the efficiency of decision units (DMUS) with shared inputs and outputs. In real life, most data are uncertain and expressed in natural language, therefore, some researchers have conducted studies in this field and have used fuzzy set theory and fuzzy number comparison [13, 10, 9, 14].

Eun et al. [11] proposed a general model for DEA, called GDEA, which can integrate basic DEA models, especially CCR, BCC, and FDH models, into a radial solution in this model.

2- GDEA linear model

Eun et al. [15] presented a general model for DEA, called GDEA, that can solve basic DEA models, especially CCR, BCC, and FDH models in a single way. The GDEA model can measure performance in several Evaluate the basic model in specific cases, which is as follows:

$$
\begin{aligned}
& Max\,\triangle \\
& s.t.\triangle \le d_j + \alpha \begin{pmatrix} \sum_{i=1}^{s} u_r\left(y_{ro}-y_{rj}\right)+ \\ \sum_{i=1}^{m} v_i\left(-x_{io}+x_{ij}\right) \end{pmatrix}, \quad j=1,\dots,m \quad (1) \\
& \qquad \sum_{i=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1, \quad i=1,\dots,m, \quad r=1,\dots,s \\
& \qquad u_r, v_i \ge \varepsilon
\end{aligned}
$$

Where α is appropriately selected according to the problem data

$$d_j = Max_{ir}\left\{u_r\left(y_{ro}-y_{rj}\right), v_i\left(-x_{io}+x_{ij}\right)\right\}$$

3-The proposed method:

In the GDEA method, all units are evaluated and the weight found is proportional to all DMUs. Also in the GDEA model, the weight found optimizes the performance of all units. In this case, it is possible that one of the units has very bad conditions, and because this unit affects all units, in the proposed method, we change this model so that each of them maximizes the efficiency of unit j like itself. In fact, we consider it as a multi-objective linear programming (MOLP) problem, all of which must be maximized, and the efficiency of each of these units must be considered separately. The corresponding model for this mode is as follows:

$$
\begin{aligned}
& Max \quad \left\{\triangle_1,\dots,\triangle_n\right\} \qquad (2) \\
& s.t. \quad \triangle_j \le \tilde{d}_j + \alpha \begin{pmatrix} \sum_{i=1}^{s} u_r\left(y_{ro}-y_{rj}\right)+ \\ \sum_{i=1}^{m} v_i\left(-x_{io}+x_{ij}\right) \end{pmatrix}, \quad j=1,\dots,m \\
& \qquad \sum_{i=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1, \quad i=1,\dots,m, \quad r=1,\dots,s
\end{aligned}
$$

In the previous model we have n objective functions that should be maximized. There are various methods to solve this problem of multi-objective linear programming, in this part we use the weighted method. Note that weights can be at the manager's discretion, but in this article, we will determine the weights based on the importance of each DMU input and output. So that we specify each weight that we give to each of the target functions with Δ_j. We specify the weight of the target functions with Δ_j, which is defined as follows, and finally the model is as follows:

$$
\begin{aligned}
& Max \quad \sum_{j=1}^{n} \lambda_j \triangle_j \qquad (3) \\
& s.t. \quad \triangle_j \le d_j + \alpha \begin{pmatrix} \sum_{i=1}^{s} u_r\left(y_{ro}-y_{rj}\right)+ \\ \sum_{i=1}^{m} v_i\left(-x_{io}+x_{ij}\right) \end{pmatrix}, \quad j=1,\dots,n \\
& \qquad \sum_{i=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1, \quad i=1,\dots,m, \quad r=1,\dots,s \\
& \qquad u_r, v_i \ge \varepsilon \\
& \qquad d_j = Max_{ir}\left\{u_r\left(y_{ro}-y_{rj}\right), v_i\left(-x_{io}+x_{ij}\right)\right\} \\
& \qquad d_j = \frac{d'_j}{\sum_{j=1}^{n} d'_j} \; d'_j = Max_{ir}\left\{u_r\left(y_{ro}-y_{rj}\right), v_i\left(-x_{io}+x_{ij}\right)\right\}
\end{aligned}
$$

Given that in the real world many data are inaccurate, ie fuzzy, this paper assumes that the input and output of all units are triangular fuzzy numbers.

$$
\begin{aligned}
& Max \quad \sum_{j=1}^{n} \lambda_j \tilde{\triangle}_j \qquad (4) \\
& s.t. \quad \tilde{\triangle}_j \le \tilde{d}_j + \alpha \begin{pmatrix} \sum_{i=1}^{s} u_r\left(\tilde{y}_{ro}-\tilde{y}_{rj}\right)+ \\ \sum_{i=1}^{m} v_i\left(-\tilde{x}_{io}+\tilde{x}_{ij}\right) \end{pmatrix}, \quad j=1,\dots,n \\
& \qquad \sum_{i=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1, \quad i=1,\dots,m, \quad r=1,\dots,s \\
& \qquad u_r, v_i \ge \varepsilon \\
& \qquad d_j = Max_{ir}\left\{u_r\left(\tilde{y}_{ro}-\tilde{y}_{rj}\right), v_i\left(-\tilde{x}_{io}+\tilde{x}_{ij}\right)\right\}
\end{aligned}
$$

In the above model, λ_j are definite numbers, which are specified as follows: First, we specify $d_j^{'}$, which is specified according to the center of fuzzy numbers.

$$\lambda_j = \frac{d'_j}{\sum_{j=1}^{n} d'_j} \quad d'_j = Max_{ir}\left\{u_r\left(y^c{}_{ro} - y^c{}_{rj}\right), v_i\left(-x^c{}_{io} + x^c{}_{ij}\right)\right\}$$

To solve this problem, fuzzy linear programming, because the numbers are triangular, is represented by three real numbers as F = (l, c, u). The upper bound, denoted by u, is the maximum values that the fuzzy number F can take. The lower limit denoted by l is the minimum values that the fuzzy number F can take. The value of c is the most probable value of a fuzzy number.

In this method, we minimize the center of the triangular numbers. Then we maximize the difference between the upper and lower limits, and finally we minimize the sum of the upper and lower limits of the triangular numbers. According to this method, the model that is presented to solve it is as follows:

$$\begin{aligned}
& Min \sum_{j=1}^{n} \lambda_j \triangle_j^c \\
& Max\left(\sum_{j=1}^{n} \lambda_j^u \tilde{\triangle}_y^u\right) - \left(\sum_{j=1}^{n} \lambda_j^l \tilde{\triangle}_y^l\right) \\
& Min\left(\sum_{j=1}^{n} \lambda_j^u \tilde{\triangle}_y^u\right) + \left(\sum_{j=1}^{n} \lambda_j^l \tilde{\triangle}_y^l\right) \\
& s.t.\ d_j + \alpha \sum_{i=1}^{s} u_r^l\left(\tilde{y}_{ro}^L - \tilde{y}_{rj}^L\right) + \sum_{i=1}^{m} v_i^L\left(-\tilde{x}_{io}^L + \tilde{x}_{ij}^L\right) - \tilde{s}_r^L = \tilde{\triangle}_j^L \\
& \quad d_j + \alpha \sum_{i=1}^{s} u_r^c\left(\tilde{y}_{ro}^c - \tilde{y}_{rj}^c\right) + \sum_{i=1}^{m} v_i^c\left(-\tilde{x}_{io}^c + \tilde{x}_{ij}^c\right) - \tilde{s}_r^c = \tilde{\triangle}_j^c \\
& \quad d_j + \alpha \sum_{i=1}^{s} u_r^v\left(\tilde{y}_{ro}^v - \tilde{y}_{rj}^v\right) + \sum_{i=1}^{m} v_i^v\left(-\tilde{x}_{io}^v + \tilde{x}_{ij}^v\right) - \tilde{s}_r^v = \tilde{\triangle}_j^v \\
& \quad \sum_{i=1}^{s} u_r^l + \sum_{i=1}^{m} v_i^L = 1 \\
& \quad \sum_{i=1}^{s} u_r^c + \sum_{i=1}^{m} v_i^c = 1 \\
& \quad \sum_{i=1}^{s} u_r^v + \sum_{i=1}^{m} v_i^v = 1 \\
& \quad u_r^l, v_i^L, u_r^c, v_i^c, u_r^v, v_i^v \geq \varepsilon \\
& \quad i = 1,\dots,m \quad , \quad r = 1,\dots,s
\end{aligned}$$

In the above model, the d_js are specified as follows:

$$d_j^L = Max_{ir}\left\{u_r^L\left(\tilde{y}_{ro}^L - \tilde{y}_{rj}^L\right), v_i^L\left(-\tilde{x}_{io}^L + \tilde{x}_{ij}^L\right)\right\}$$

$$d_j^c = Max_{ir}\left\{u_r^c\left(\tilde{y}_{ro}^c - \tilde{y}_{rj}^c\right), v_i^c\left(-\tilde{x}_{io}^c + \tilde{x}_{ij}^c\right)\right\}$$

$$d_j^v = Max_{ir}\left\{u_r^v\left(\tilde{y}_{ro}^v - \tilde{y}_{rj}^v\right), v_i^v\left(-\tilde{x}_{io}^v + \tilde{x}_{ij}^v\right)\right\}$$

$$\lambda_j = \frac{d_j'^c}{\sum_{j=1}^{n} d_j'^c} \lambda_j = \frac{d_j'^L}{\sum_{j=1}^{n} d_j'^L} \lambda_j = \frac{d_j'^u}{\sum_{j=1}^{n} d_j'^u} \lambda_j$$

$$\tilde{d}_j'^L = Max_{ir}\left\{\left(\tilde{y}_{ro}^L - \tilde{y}_{rj}^L\right), \left(-\tilde{x}_{io}^L + \tilde{x}_{ij}^L\right)\right\}$$

$$\tilde{d}_j'^c = Max_{ir}\left\{\left(\tilde{y}_{ro}^c - \tilde{y}_{rj}^c\right), \left(-\tilde{x}_{io}^c + \tilde{x}_{ij}^c\right)\right\}$$

$$\tilde{d}_j'^v = Max_{ir}\left\{\left(\tilde{y}_{ro}^v - \tilde{y}_{rj}^v\right), \left(-\tilde{x}_{io}^v + \tilde{x}_{ij}^v\right)\right\}$$

4- conclusion:

In this paper, first the GDEA model is extended to the case where it can maximize the performance of each unit individually, then a non-radial model is presented and this model is extended with fuzzy data and a method for solving it is presented. It should be noted that in this study, given that in the real world, the existing data and conditions are not always definite and uncertainty may exist. To calculate the input and output indices, which were considered as triangular fuzzy numbers, a model was presented to generalize the data evaluation efficiency with this model in cases where there is uncertainty for the data.

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