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Research Article

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Target Setting in the Process of Merging and Restructuring of Decision-Making Units using Multiple Objective Linear Programming

J. Gerami*

Department of mathematics, Shiraz Branch, Islamic Azad University, Shiraz, Iran

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Abstract

This paper presents a novel approach to achieving the goals of data envelopment analysis in the process of reconstruction and integration of decision-making units by using multiple objective linear programming. In this regard, first, we review inverse data envelopment analysis models for data reconstruction and integration. We present a model with multi-objective linear programming structure in inverse data envelopment analysis. The superiority of the model to prioritize inputs and outputs in the input and output data integration process. Finally, we used of bank data for showing the validity of proposed approach and we proposed results.

Keywords: Inverse data envelopment analysis, merging, restructuring, Banking industry, multiple objective linear programming.

1. Introduction

One way to improve corporate performance is to adjust the level of inputs and outputs according to the level of inputs and outputs available [1], [2]. Therefore, it is important to build new decision making units that can be created. In this regard, we can use inverse data envelopment analysis to increase the environmental and operational efficiency of companies. In this regard, according to the manager, we can determine the expected efficiency and

*. Corresponding author: Email: geramijavad@gmail.com

provide new decision-making units with a stabilization or creation and division scenario. In this context, by predefining expected efficiency, we can achieve the desired level of new inputs and outputs by stabilizing the efficiency score. [3], [4], [5], [6], [7], [8], [9], [10], [11].

One way to create and build new units with the expected efficiency score is to use inverse data envelopment analysis. See the following articles for further studies on inverse data envelopment analysis [12], [13], [14], [15].

In the following Gattoufi et al. [11], Provides a new metallurgy to detect the amount of reduction required in inputs or increments in outputs to stabilize the target efficiency level value, and by integrating input and output levels, two decision-making units have a combined decision-making unit with the expected efficiency score.

Amin et al. [16], [17] presented a general model for corporate restructuring by using inverse data envelopment analysis.

One of the issues in merging and restructuring decision-making units to obtain new units with predetermined efficiency scores is prioritizing inputs and outputs. Previous models presented did not consider prioritization. The disadvantage of the approach presented in this paper is compared to the previous approach, prioritizing inputs and outputs based on the manager's point of view using multiple objective linear programming models.

This article is organized as follows. The second section, presents the initial models of reconstruction and integration of decision-making units using inverse data envelopment analysis. In the third section, we present a general model with a multiple objective structure to obtain decision units with a specific efficiency score. In the fourth part of the article, we illustrate proposed approach to a numerical example and finally present the results.

2. Reconstruction and integration using inverse data envelopment analysis

Consider n decision-making unit that each uses m input to product s output. Suppose that x_{ij} ,y_{rj} represent i-th input and r-th output of the decision-making unit j respectively. In this section, we perform the reconstruction and integration of the decision-making unit using two decision makers. Make a reconstruction with two decision makers DMU_k, DMU_l. Suppose, we call the new unit DMU_M.

For the input oriented model, suppose merge the inputs of the two decision units and create a new one. Gattoufi et al. [11] introduced the Generalized Inverse DEA (InvDEA) model in this regard. For the input oriented model, suppose we merge the inputs of the two decision units and create a new input. Consider the efficiency of the new unit for a certain amount. In the process of reconstruction and merging companies, we may create the same units with new inputs and outputs or create new units based on previous units. Consider the efficiency of the new unit for a certain amount. Suppose α_{ik}, α_{il} are the i input levels of units DMU_k, DMU_l.

Gattoufi et al. [11] presented the input oriented model as follows.

$$\begin{aligned}
\min \quad & \textstyle\sum_{i=1}^{m}(\alpha_{ik}+\alpha_{il}) \\
\text{s.t.} \quad & \textstyle\sum_{j\in F} x_{ij}\lambda_j + \lambda_M(\alpha_{ik}+\alpha_{il}) - \\
& (\alpha_{ik}+\alpha_{il})\,\bar{\theta} \le 0 \quad i=1,\dots,m, \\
& \textstyle\sum_{j\in F} y_{rj}\lambda_j + \lambda_M(y_{rk}+y_{rl}) \ge \\
& (y_{rk}+y_{rl}) \quad r=1,\dots,s, \\
& \textstyle\sum_{j\in F}\lambda_j = 1, \qquad\qquad (1) \\
& 0 \le \alpha_{ij} \le x_{ij} \quad j=k,l, \\
& i=1,\dots,m,\ \lambda_j \ge 0\ \forall\, j\in F.
\end{aligned}$$

In the above model λ_j show intensity vector, $\bar{\theta}$ is the target efficiency score of DMU_M F show indices are related to other decision units except l and k. In the process of rebuilding and merging companies, we may create the same units with new inputs and outputs or create new units based on previous units. Model (1) is a nonlinear model because the term $\lambda_M(\alpha_{ik}+\alpha_{il})$

makes the model nonlinear and an easy way to convert model (1) to a linear model is to remove the term $\lambda_M(\alpha_{ik}+\alpha_{il})$ from set of model constraints [11]. Then model (1) will convert to model (2).
Note that model (1) is a nonlinear model.

$$\begin{aligned} \min\ & \sum_{i=1}^{m}(\alpha_{ik}+\alpha_{il}) \\ \text{s.t.}\ & \sum_{j\in F} x_{ij}\lambda_j - (\alpha_{ik}+\alpha_{il})\,\bar{\theta} \le 0, \\ & i=1,\dots,m, \\ & \sum_{j\in F} y_{rj}\lambda_j \ge (y_{rk}+y_{rl}), \\ & r=1,\dots,s, \\ & \sum_{j\in F}\lambda_j = 1, \qquad (2) \\ & 0\le \alpha_{ij}\le x_{ij}, \\ & j=k,l,\ i=1,\dots,m, \\ & \lambda_j \ge 0\ \forall\, j\in F. \end{aligned}$$

Gattoufi et al. [11] presented the output oriented model as follows.

$$\begin{aligned} \max\ & \sum_{r=1}^{s}(\beta_{rk}+\beta_{rl}) \\ \text{s.t.}\ & \sum_{j\in F} x_{ij}\lambda_j + \lambda_M(x_{ik}+x_{il}) \le \\ & (x_{ik}+x_{il}),\quad i=1,\dots,m, \\ & \sum_{j\in F} y_{rj}\lambda_j + \lambda_M(\beta_{rk}+\beta_{rl}) - \\ & \bar{h}(\beta_{rk}+\beta_{rl}) \ge 0\quad r=1,\dots,s \\ & \sum_{j\in F}\lambda_j = 1, \qquad (3) \\ & 0\le y_{rj}\le \beta_{rj}, \\ & j=k,l,\ i=1,\dots,m \\ & \lambda_j \ge 0\ \forall\, j\in F. \end{aligned}$$

It should be noted that the variable β_{rk} corresponds to the output of the newly created unit namely DMU_M. The above model presents the additional level of the newly created unit must have compared to the units used to create the new unit to achieve a predetermined level of efficiency. We denote this efficiency by $\bar{h}$. Similar to the argument used in the input oriented model, we can convert model (3) to the following linear model.

$$\begin{aligned} \max\ & \sum_{r=1}^{s}(\beta_{rk}+\beta_{rl}) \\ \text{s.t.}\ & \sum_{j\in F} x_{ij}\lambda_j \le (x_{ik}+x_{il}) \\ & i=1,\dots,m, \\ & \sum_{j\in F} y_{rj}\lambda_j - \bar{h}(\beta_{rk}+\beta_{rl}) \ge 0, \\ & r=1,\dots,s, \\ & \sum_{j\in F}\lambda_j = 1, \qquad (4) \\ & 0\le y_{rj}\le \beta_{rj},\ \ j=k,l, \\ & r=1,\dots,m \\ & \lambda_j \ge 0, \forall\, j\in F. \end{aligned}$$

If the new unit is outside of the productivity possible set, then models (3) and (4) will be infeasible. In the next section, we present models for achieving goals in the process of integrating and creating new units based on multi-objective models.

3. Targeting using multiple objective linear programming

In this section, we develop a new model based on multiple objective linear programming models. At first present the input oriented model as follows.

$$\begin{aligned} \max\ & \sum_{i=1}^{m} -(\alpha_{ik}+\alpha_{il}) \\ \max\ & \sum_{i=1}^{m}(d_{ik}+d_{il}) \\ \max\ & d_{min} \\ \text{s.t.}\ & \sum_{j\in F} x_{ij}\lambda_j - (\alpha_{ik}+\alpha_{il})\,\bar{\theta} \le 0, \\ & i=1,\dots,m, \\ & \sum_{j\in F} y_{rj}\lambda_j \ge (y_{rk}+y_{rl}), \\ & r=1,\dots,s, \\ & \sum_{j\in F}\lambda_i = 1, \qquad (5) \\ & x_{ij}-\alpha_{ij}-d_{ij} = 0\ \ j=k,l, \\ & i=1,\dots,m, \\ & \lambda_j \ge 0\ \forall\, j\in F. \end{aligned}$$

In the above model λ_j show intensity vector, $\bar{\theta}$ is the target efficiency score of DMU_M. For new unit namely DMU_M, d_{ij} is the value of the deviation as to constraint $x_{ij}-\alpha_{ij}$. d_{min} shows the least deviation. The purpose of the above model is to minimize the level of new unit input namely $\alpha_{ik}+\alpha_{il}$. If the amounts of $d_{il}+d_{ik}$ maximized then $\alpha_{ik}+\alpha_{il}$ minimized. If the amounts of $d_{il}+d_{ik}$ maximized then $\sum_{i=1}^{m}(d_{il}+d_{ik})$ maximized. The above model is a model with MOLP structure in the input form.
Now, we proposed input oriented model for reconstruction the reconstruction

problem in the multiple objective structure as following.

$$\begin{aligned} \max\ & \sum_{r=1}^{s}(\beta_{rk}+\beta_{rl}) \\ \max\ & \sum_{r=1}^{s}(c_{rk}+c_{rl}) \\ \max\ & C_{min} \\ \text{s.t.}\ & \sum_{j\in F} x_{ij}\lambda_j \le (x_{ik}+x_{il}), \\ & i=1,\dots,m, \\ & \sum_{j\in F} y_{rj}\lambda_j - \bar{h}(\beta_{rk}+\beta_{rl}) \ge 0 \\ & r=1,\dots,s, \qquad (6) \\ & \sum_{j\in F}\lambda_j = 1, \\ & y_{rj} \le \beta_{rj} - y_{rj} - c_{rk} = 0, \\ & j=k,l, \\ & r=1,\dots,s,\quad \lambda_j \ge 0\ \forall\, \text{j} \in \text{F}. \end{aligned}$$

By maximizing c_{rj}, we can maximize β_{rj}. Since c_{min} is the minimum of the above values. By maximizing c_{min} all values are maximized.

4. Application

In this section, we use the introduced data by Gattoufi et al. [11], in order to show the validity of the proposed approach. Data include banks operating in the countries that are member of GCC. These countries are Kuwait, Oman, Saudi Arabia. The data for these 42 banks include two inputs and two outputs.

Inputs include:

1) Interest expenses
2) Non-interest expenses

Outputs include:

1) Interest income
2) Non-interest income

The data are listed in Table (1).

Table 1. The set of bank data.

Bank	Interest expenses	Non-interest expenses	Interest incomes	Non-interest incomes	Technical efficiency scores under VRS
B01	3956.796	1894.426	9001.004	8701.497	1
B02	481.239	319.976	974.854	597.726	0.677
B03	305.2	138.6	479.8	252.2	0.64
B04	4710.68	3996.259	12,920.34	6060.768	0.893
B05	1.018	1.282	3.054	0.377	1
B06	954.437	1208.703	1991.004	7278.097	1
B07	3.965	5.082	13.359	3.003	0.829
B08	14.63	16.863	44.659	14.938	0.738
B09	11.771	6.579	22.952	15.134	0.727
B10	364.92	244.75	923.51	1942.935	1
B11	4897.442	2787.181	11,294.61	9363.232	0.939
B12	14.665	8.973	28.124	10.971	0.67
B13	6.077	14.249	26.994	10.207	0.97
B14	397.627	371.535	894.845	1902.878	0.813
B15	661.12	830.166	2325.128	1748.531	0.953
B16	12.125	7.346	33.573	19.53	0.96
B17	1222.026	1049.479	2959.509	2651.546	0.785
B18	931.172	838.346	2460.798	2765.485	0.866
B19	4070.351	2845.498	8377.368	7726.906	0.77
B20	3721.233	858.463	6953.701	2779.716	1
B21	16.137	7.08	40.771	22.126	1
B22	150.706	132.504	538.754	129.956	1
B23	3857.94	2894.374	7439.526	10,239.09	0.91
B24	7994.808	2286.908	14,156.19	11,261.82	1
B25	9.689	6.975	22.432	6.032	0.756
B26	3292.736	1953.592	7041.164	3323.973	0.826
B27	402.772	321.189	906.237	775.778	0.678
B28	32.835	21.536	97.679	26.551	0.98
B29	6.737	7.854	18.402	4.504	0.69
B30	531.395	922.04	1672.093	1185.165	0.815
B31	152.51	190.361	685.374	769.898	1

B32	1.925	4.581	9.163	5.274	1
B33	4.889	6.737	17.402	5.082	0.84
B34	3233.619	2527.414	7959.733	4684.616	0.84
B35	5169.71	5405.975	15,189.61	9830.137	0.871
B36	6802.566	5608.863	19,958.04	15,716.89	1
B37	3111.952	2126.013	6895.572	4869.316	0.811
B38	3600.983	1319.711	6547.924	5116.082	0.876
B39	7781.754	8486.425	27,514.03	14,335.68	1
B40	4488.666	4531.419	12,157.91	12,380.68	1
B41	3188.736	1106.154	5727.009	6194.46	1

Suppose we consider Banks B_{02} and B_{03} as restructuring units and want to obtain a new unit with a level of efficiency one. Let us assume the merged bank is interested in being fully efficient. Moreover, assume the merged bank would like to give its efficiency to interest expenses of B_{02}, conformed by interest expenses of B_{03}, then non-interest expenses of B_{02} and finally non-interest expenses of B_{03}. We first solve the input oriented model and then the output oriented model. The results are in Tables (2) and (3).
As can be seen from the Tables (2) and (3), total inputs and outputs decreased and increased, respectively. An overall improvement in input and output levels was achieved. Given the input and output levels of the new unit, we therefore created a unit with a single efficiency level. The results can be interpreted similarly for the output orientation model.

5. Conclusion

One of the important issues in data envelopment analysis is the integration and reconstruction of decision making units. In this regard, this paper coverage two models of data envelopment analysis with multiple objective linear programming structure that uses for reconstruction of the unit. We show that the above model is preferable to the previous models because these models obtain appropriate target scores than the previous models and obtain new unit with lower input and more outputs level. We can expand the presented models to other models of DEA such as non-radial models and DEA models in the presence inaccurate data.

Table 2. Target setting with the output oriented model.

Method	β^*_{12}	β^*_{13}	β^*_{22}	β^*_{23}
MCDEA model (5)	562.36	3329.45	446.82	432.5
InvDEA model (2)	409.24	2009.48	548.33	394.21

Table 3. Target setting with the input oriented model.

Method	α^*_{12}	α^*_{13}	α^*_{22}	α^*_{23}
MCDEA model (6)	63.42	296.6	275.7	129.2
InvDEA model (2)	371.27	319.98	0	108.26

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