Dependence of Default Probability and Recovery Rate in Structural Credit Risk Models: Empirical Evidence from Greece

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ABSTRACT:
The main idea of this paper is to study the dependence between the probability of default and the recovery rate on credit portfolio and to seek empirically this relationship. We examine the dependence between PD and RR by theoretical approach. For the empirically methodology, we use the bootstrapped quantile regression and the simultaneous quantile regression. These methods allow to determinate the dependence between the PD and RR. This study is elaborated for a sample composed by 17 banks in Greece. The period of study is of 7 years (2006-2012). The measurement of this dependence is determinate by using 7 indicators: the probability of default, the recovery rate, the number of defaults, the expected value of losses, the growth rate of GDP in Greece and three dummy variables who the exit of another firm of the Athens Exchange, the new firm is quoted in the Athens exchange and the failure of Greece is declared. We use in our study two dependent variables PD and RR. The descriptive, correlation and regression analysis results are presented by STATA 12.

INTRODUCTION

This literature review briefly summarizes the way credit risk models, which have developed during the last thirty years, treat Recovery Rate and, more specifically, their relationship with the Probability of Default of an obligor. These models can be divided into two main categories (Atman et al., 2002):
✓ Credit pricing models.
✓ Portfolio credit value-at-risk (VaR) models.
    Thus, credit pricing models can in turn be divided into three main approaches:
✓ "First generation" structural-form models.

✓ "Second generation" structural-form models.
✓ Reduced-form models.
These three different approaches, together with their basic assumptions advantages, drawbacks and empirical performance:
✓ First generation structural-form models: the Merton approach.
✓ Second generation structural-form models.
✓ Reduced-form models.
✓ Credit value-at-risk models.

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✓ Some latest contributions on the PD-RR relationship[1].

It has been noted that default probabilities and default rates and average recovery rates are negatively correlated (Altman et al., 2005). Then, both variables also seem to be driven by the same common factor that is persistent over time and clearly related to be the business cycle: in recessions or industry downturns, default rates are high and recovery rates are low.

Thus, the main idea for this study is to answer the question follows: *As the Probability of Default is depended to the Recovery Rate and conversely?*

Literature Review

The main idea of this paper is to describe and to determine the relationship of dependence between probability of default and the recovery rate. In the literature, this dependence is treated by many authors. Table 1 summarizes many works developed and studied the dependence between the PD and the RR.

Dependence between the Probability of Default and the Recovery Rate: Analytical Analysis

By analyzing the previous literature, we can conclude that the default probability was estimated according to several approaches. So, the default probability can be estimated by basing itself on historical series of default by measuring the risk by the rating or the score (Altman, 1968). Empirically, the measures of score call on to alternatives as the analysis in main component, the logistic regression and the Probit analysis.

Jonkhart (1979) deducted the default probability from the spreads of rates available on markets. The works of Jonkhart are carried out by Iben and Litterman (1989), Wu and Yu (1996), Altman (1988, 1989), Asquith et al. (1989), Rosenberg and Gleit (1994), Hand and Henely (1997) and Thomas (2000), who deducted the default probability from historical data on the bonds having been lacking by type of rating and by type of term.

Merton (1979), Black and Scholes, Black and Cox (1976), Geske (1977) and Lee (1981) deducted the default probability from the

volatility of assets. This method is intended for the highly-rated credits.

In our study of research we are going to base ourselves on the analysis developed by Merton (1974). So, Merton's model is based on the hypothesis which the company **k** has a certain quantity of debt with zero-coupon. The nominal value of this debt is F_k and it becomes due in the maturity date **T**. The Firm is declared default if, in the date **T**, the value of its assets is lower than its nominal value, is, if $V_k(T) < F_k$. The recovery rate spells then under the following shape:

$$R_k = \frac{F_k}{V_k(T)} \tag{1}$$

And the loss in the case of default is:

$$L_k^* = 1 - R_k = \frac{F_k - V_k(T)}{F_k} \tag{2}$$

By using the function of Heaviside (**Θ**), we can determinate the loss individual can be expressed by the following form:

$$L_k = \left(1 - \frac{V_k(T)}{F_k}\right)\Theta\left(1 - \frac{V_k(T)}{F_k}\right) \tag{3}$$

So, in the Merton's model, the losses value and the recovery rate are directly determined by the value of assets to the date of maturity.

Thus, the stochastic modeling of the market value of a company $V_k(T)$ allows to evaluate his credit risk. The probability density function (pdf) of markets values in the date of maturity $P_{V_k}(V_k(T))$. So, the default probability is given by:

$$P_{D,k} = \int_0^{F_k} P_{V_k}(V_k(T)) dV_k(T) \tag{4}$$

And the recovery rate will be calculated as follows:

$$\langle R_k \rangle = \frac{1}{P_{D,k}} \int_0^{F_k} \frac{V_k(T)}{F_k} P_{V_k}(V_k(T)) dV_k(T) \tag{5}$$

1- PD : the Probability of default, RR: The Recovery Rate.

Table 1: The treatment of the dependence between PD and RR

	Main models and related empirical results	Treatment of PD	Relationship between PD and RR
Credit Pricing models			
First generation structural-form models	Merton (1974), Black and Cox (1976), Geske (1977), Vasicek (1984), Crouhy and Galai (1994), Jones et al. (1984).	PD and RR are a function of the structural characteristics of the firm. RR is therefore an endogenous variable.	PD and RR are inversely related.
Second generation structural-form models	Kim et al. (1993), Nielsen et al. (1993), Hull and White (1995), Longstaff and Schwartz (1995).	RR is exogenous and independent from the firm's asset value.	RR is generally defined as a fixed ratio of the outstanding debt value and is therefore independent from PD.
Reduced-form models	Litterman and Iben (1991), Madan and Unal (1995), Jarrow and Turnbull (1995), Jarrow et al. (1997), Lando (1998), Duffie and Singleton (1999), Duffie (1998) and Duffee (1999).	Reduced-form models assume an exogenous RR that is either a constant or a stochastic variable independent from PD.	Reduced-form models introduce separate assumptions on the dynamic of PD and RR, which are modeled independently from the structural features of the firm.
Latest contributions on the PD-RR relationship	Frye (2000), Jarrow (2001), Carey and Gordy (2001), Altman and Brady (2002).	Both PD and RR are stochastic variables which depend on a common systematic risk factor (the state of the economy).	PD and RR are negatively correlated. In the "macroeconomic approach" this derives from the common dependence on one single systematic factor. In the "microeconomic approach" it derives from the supply and demand of defaulted securities.
Credit value at risk models			
CreditMetrics®	Gupton, Finger and Bhatia (1997)	Stochastic variable (beta distr.)	RR independent from PD
Credit Portfolio View®	Wilson (1997a and 1997b).	Stochastic variable	RR independent from PD
Credit Risk+®	Credit Suisse Financial Products (1997).	Constant	RR independent from PD
KMV Credit Manager®	McQuown (1997), Crosbie (1999).	Stochastic variable	RR independent from PD

Let us consider now a portfolio of credit **K**, where the market value of every company is correlated in one or in several variables. Under the condition of the realizations of variables, we obtain different values of $P_{D,k}$ and of $\langle R_k \rangle$. In fact, we can demonstrate a functional dependence between the probability of default and the recovery rate. This is in contrast with what is evoked by certain surrounding areas of modeling which suppose the existence of an independence of these qualities.

By supposing the existence of a process of underlying distribution of the value of the company, we can easily deduct all the results relative to the measure of the probability of default and the recovery rate.

So, we consider a homogeneous portfolio of size **K**. The nominal value of this portfolio is $F_k = F$ and the first market values are $V_k(0) = V_0$.

The evolution in the time of the market value of a single firm k is modeled by a stochastic differential equation of the following form:

$$\frac{dV_k}{V_k} = \mu dt + \sqrt{c}\sigma dW_m + \sqrt{(1-c)}\sigma dW_k \qquad (6)$$

This equation describes a process of correlated diffusion to a determinist term μdt and a correlated linearly diffusion. The parameters of this process are; the constant μ, the volatility σ and the correlation **c** between the firm return and the market return. The process of Wiener indicated by dW_k and dW_m, describes the idiosyncratic fluctuations and the market fluctuations respectively.

So, the evaluation of the prices of the options on the financial market is based on two parameters important to know the volatility and the fluctuations of assets (Gatfaoui, 2006; Giovanni et al., 2012). In this aligned, the fluctuations in the prices of assets are understandable by two different and independent risk factors which are the systematic factor and the idiosyncratic factor.

Therefore, the pricing of assets is improved there because the distortions of the price of the underlying are decomposed into two parts:

✓ A component of market volatility stemming from systematic fluctuations in the price of asset (dW_m).

✓ A component of idiosyncratic volatility stemming from specific fluctuations in the price of asset (dW_k).

For increment of discreet time $\Delta t = \dfrac{T}{N}$, where the time is divided on N stages, we arrive at the discreet formulation of the stochastic differential equation above. The market value of k firms in the maturity can be written in the following form:

$$V_k(T) = V_0 \prod_{t=1}^{N}\left(1 + \mu\Delta t + \sqrt{c}\sigma\eta_{m,t}\sqrt{\Delta t} + \sqrt{1-c}\sigma\varepsilon_{k,t}\sqrt{\Delta t}\right) \qquad (7)$$

With, $\eta_{m,t}$ and $\varepsilon_{k,t}$ are independent random variables and they follow a normal distribution law. We will try in what follows to determine market return X_m, the number of default $N_D(X_m)$ and the recovery rate $\langle R(X_m)\rangle$. On the market return $\langle R(X_m)\rangle$ which defines the average yield of all k firms over a period of time to maturity.

$$X_m = \frac{1}{K}\sum_{k=1}^{K}\left(\frac{V_k(T)}{V_0} - 1\right)$$
$$= \frac{1}{K}\sum_{k=1}^{K}\prod_{t=1}^{N}(1 + \mu\Delta t + \sqrt{c}\sigma\eta_{m,t}\sqrt{\Delta t} + \sqrt{1-c}\sigma\varepsilon_{k,t}\sqrt{\Delta t}) - 1 \qquad (8)$$

For $K \rightarrow \infty$, we can express the average on k as the value of the hope $\varepsilon_{k,t}$. For the independence of $\varepsilon_{k,t}$ for different k and t, we can write then:

$$X_m + 1 = \prod_{t=1}^{N}\left(1 + \mu\Delta t + \sqrt{c}\sigma\eta_{m,t}\sqrt{\Delta t} + \sqrt{1-c}\sigma\langle\varepsilon_{k,t}\rangle\sqrt{\Delta t}\right) \qquad (9)$$

With :

$$\langle\varepsilon_{k,t}\rangle = 0$$

Then, the expression of $X_m + 1$ will be simplified as follows:

$$X_m + 1 = \prod_{t=1}^{N}\left(1 + \mu\Delta t + \sqrt{c}\sigma\eta_{m,t}\sqrt{\Delta t}\right)$$
$$= exp\left(\sum_{t=1}^{N} ln(1 + \mu\Delta t + \sqrt{c}\sigma\eta_{m,t}\sqrt{\Delta t})\right)$$
$$\approx exp\left(\left(\mu - \frac{c\sigma^2}{2}\right)T + \sigma\sqrt{c\Delta t}\sum_{t=1}^{N}\eta_{m,t}\right) \qquad (10)$$

In this stage, we apply the logarithm to the function above. The random variable $\eta_{m,t}$ follows a standard normal distribution. Then, we obtain:

$$ln(X_m + 1) = \left(\mu - \frac{c\sigma^2}{2}\right)T + \sigma\sqrt{cT}\frac{1}{\sqrt{N}}\sum_{t=1}^{N}\eta_{m,t} \qquad (11)$$

Thus, the variable $ln(X_m + 1)$ is normally distributed with average $(\mu T - \frac{c\sigma^2 T}{2})$ and variance $c\sigma^2 T$. So, by basing itself on the normal logarithmic distribution, we can write the probability density function as follows:

$$p_{X_m}(X_m) = \frac{1}{(X_m+1)\sqrt{2\pi c\sigma^2 T}}exp\left(\frac{\left(ln(X_m+1)-\mu T+\frac{1}{2}c\sigma^2 T\right)^2}{2c\sigma^2 T}\right) \qquad (12)$$

For a single firm k we can write:

$$ln\frac{V_k(T)}{V_0} = \sum_{t=1}^{N}ln\left(1 + \mu\Delta t + \sqrt{c}\sigma\eta_{m,t}\sqrt{\Delta t} + \sqrt{1-c}\sigma\varepsilon_{k,t}\sqrt{\Delta t}\right)$$
$$\approx ln(X_m + 1)\left(\mu - \frac{c\sigma^2}{2}\right)T + \sigma\sqrt{cT}\frac{1}{\sqrt{N}}\sum_{t=1}^{N}\eta_{m,t} \qquad (13)$$

Thus, the market return X_m is considered a constant. Thereafter, all variables $V_k(T)$ are independent and the variable $ln\frac{V_k(T)}{V_0}$ is normally distributed and we have an average $ln(X_m + 1) - \frac{(1-c)\sigma^2}{2}T$ and a variance $(1-c)\sigma^2 T$.

Since, we considered a homogeneous portfolio; we omit the index k in follows. This allows a better rating and effective results. The probability density function for the market value of a firm is given by the following form:

$$P_v(V(T))$$
$$= \frac{1}{(X_m + 1)\sqrt{2\pi(1-c)\sigma^2 T}}exp\left(\frac{\left(ln\frac{V(T)}{V_0} - ln(X_m + 1) + \frac{1}{2}(1-c)\sigma^2 T\right)^2}{2(1-c)\sigma^2 T}\right) \qquad (14)$$

So, the individual probability of default is given by the following function:

$$P_D(X_m)$$
$$= \int_0^F p_v(V(T))dV(T)$$
$$= \Phi\left(\frac{\left(ln\frac{V(T)}{V_0} - ln(X_m + 1) + \frac{1}{2}(1-c)\sigma^2 T\right)^2}{2(1-c)\sigma^2 T}\right) \qquad (15)$$

Where, Φ denotes the cumulative standard normal distribution. The expected value of the loss of individual default $L^* = 1 - \frac{V(T)}{F}$ can be calculated as follows:

$$\langle L^*(X_m)\rangle$$
$$= \frac{1}{P_D(X_m)}\int_0^F \left(1 - \frac{V(T)}{F}\right)p_v(V(T))dV(T)$$
$$= \frac{1}{P_D(X_m)}\left[\Phi\left(\frac{\left(ln\frac{V(T)}{V_0} - ln(X_m + 1) + \frac{1}{2}(1-c)\sigma^2\right)}{\sqrt{(1-c)}\sigma^2 T}\right) \right.$$
$$- exp\left(ln(X_m + 1)\right.$$
$$\left.\left. - ln\frac{F}{V_0}\right)\Phi\left(\frac{\left(ln\frac{V(T)}{V_0} - ln(X_m + 1) + \frac{1}{2}(1-c)\sigma^2\right)}{\sqrt{(1-c)}\sigma^2 T}\right)\right] \qquad (16)$$

The expected recovery rate is:

$$\langle R(X_m)\rangle = 1 - \langle L^*(X_m)\rangle \qquad (17)$$

In the case of a homogeneous portfolio, the loss of a portfolio (average loss) is obtained by the following form:

$$\langle L(X_m)\rangle = P_D(X_m)\langle L^*(X_m)\rangle \qquad (18)$$

For clarity, we introduce the function:

$$A(X_m) = ln\frac{F}{V_0} - ln(X_m + 1) \qquad (19)$$

Where B is the composite parameter which is written as follows:

$$B = \sqrt{(1-c)\sigma^2 T} \tag{20}$$

However, the expressions of $P_D(X_m)$ and $\langle R(X_m) \rangle$ are simplified as follows:

$$P_D(X_m) = \Phi\left(\frac{A(X_m) + \frac{1}{2}B^2}{B}\right) \tag{21}$$

And,

$$\langle R(X_m) \rangle = \frac{e^{-A(X_m)\Phi\left(\frac{A(X_m)+\frac{1}{2}B^2}{B}\right)}}{\Phi\left(\frac{A(X_m)+\frac{1}{2}B^2}{B}\right)} \tag{22}$$

The relationship between the probability of default and the recovery rate does not depend on B only, but it is set by $A(X_m)$. Thus, the parameter B can be measured by the probability of default and the recovery rate. In addition, reversing the expression of $P_D(X_m)$, we can express **A** in terms of P_D:

$$A = B\Phi^{-1}(P_D) - \frac{1}{2}B^2 \tag{23}$$

To clarify the effect of the idiosyncratic fluctuations and the market fluctuations on the volatility, Schäfer and Koivusalo (2011) proposed a relationship of functional dependence for the probability of default and the recovery rate. The recovery rate is expressed by the following form:

$$\langle L(P_D) \rangle = \frac{1}{P_D} exp\left(-B\Phi^{-1}(P_D) - \frac{1}{2}B^2\right)\Phi(\Phi^{-1}(P_D) - B) \tag{24}$$

If should be noted that this functional relationship depends on a single parameter B.

We can see that for higher values of B lead to an overall decrease in the recovery rate. From the equation above $\langle R(P_D) \rangle$, we can obtain the functional relationship of the portfolio loss and default probabilities:

$$\langle L(P_D) \rangle = P_D - exp\left(-B\Phi^{-1}(P_D) - \frac{1}{2}B^2\right)\Phi(\Phi^{-1}(P_D) - B) \tag{25}$$

For a high value of $K(K \to \infty)$, the idiosyncratic is zero and the market return X_m is defined only by the realization of the term $\eta_{m,t}$. The number of default $N_D(X_m)$ measure the number of times that the inequality $V_k(T) < F_k$ is feasible. We can estimate the value of the probability of default as follows:

$$P_D(X_m) \approx \frac{N_D(X_m)}{K} \tag{26}$$

The loss of the portfolio is then obtained as the average of the individual losses:

$$\langle L(X_m) \rangle = \frac{1}{K}\sum_{k=1}^{K} L_k \tag{27}$$

We can deduce the following relationship:

$$\langle L(X_m) \rangle = P_D(X_m)(1 - \langle R(X_m) \rangle) \tag{28}$$

The recovery rate is expressed as follows:

$$\langle R(X_m) \rangle = 1 - \frac{\langle L(X_m) \rangle}{P_D(X_m)} \approx 1 - \frac{K\langle L(X_m) \rangle}{N_D(X_m)} \tag{29}$$

Several studies have shown that the number of default $N_D(X_m)$ is strictly non-zero. This is justified based on a large portfolio is measured by K. If we based on the evolution of a portfolio, we can obtain different values for the market return (X_m), the number of defaults $N_D(X_m)$, the probability of default $P_D(X_m)$ and the recovery rate $\langle R(X_m) \rangle$.

RESEARCH METHOD
Data and Empirical Model

In this section, we identified the sources of our data. We present the data itself and describe the regression model. Finally, we use to investigate the relation of dependence between the probability of default and the recovery rate.

Data

In this study we employed the indicator of 17 banks quoted in the Athens Exchange of covered the period of 2006-2012. The list of banks included in this study is provided in table 2. The balance sheet data is collected from Statistical Bulletin of The Athens Exchange. In this study, we will use STATA12 (Data Analysis and Statistical Software 12) for data manipulation and inferences. The regression analysis is used to identify the dependence between PD and RR. The descriptive statistics applies to find the mean, the maximum, the minimum and standard deviation, Skweenes and Kurtosis of those variables. The Pearson correlation tests applied to deal with the problems.

Econometric Methodology

In this paper, we used two econometric techniques to measure the dependence between the probability of default and recovery rate. Those techniques are the Bootstrapped Quantile Regression and the Simultaneous Quantile Regression.

Then, we used these two techniques because the parameter quantile regression provides an estimate of the change in a specific quantile of the response variable produced by a unit change in the predictor.

Empirical Model

In our paper, we consider two models who describe the dependence between the probability of default and the recovery rate. We estimated the probability of default in function of seven variables. All these variables will be explained in follows (Frye, 2000; Altman, 2001; Gordy, 2001; Altman et al., 2002; Altman et al., 2005; Bruche and Gonzalez-Aguado, 2008; Becker, 2013).

The probability of default is estimated by the model presented as follow:

(Equation 1)

$$PD_t = f(RR_t, ND_t, L_t, Dummy1_t, Dummy2_t, Dummy3_t)$$

When,

- ✓ PD_t: The probability of default at the moment t.
- ✓ RR_t: The recovery rate at the moment t.
- ✓ ND_t: The number of default at the moment t.
- ✓ L_t : The expected value of losses at the moment t.
- ✓ $Dummy1_t$: This variable indicates that a new firm is listed in the Athens Exchange at the moment t. This variable that takes the value 1 when a new firm is listed in the Athens Exchange and takes 0 in the opposite occur.
- ✓ $Dummy2_t$: This variable indicates that a new firm is going out of the Athens Exchange at the moment t. This variable that takes the value 1 when a new firm is going out of the Athens Exchange and takes 0 in the opposite occur.
- ✓ $Dummy3_t$: This variable indicates that a Greece declare his failure at the moment t. This variable that takes the value 1 after the date of failure and 0 before the date of failure.

The used data are daily and which are collected from the publication of the Athens Exchange.

RESULTS AND DISCUSSION

Within the framework of this paper, we are going to present a descriptive statistics analysis of the various variables used in all estimations. These variables are used to estimate the dependence between probability of default and the recovery rate. So, we used the Software STATA 12 to obtain empirical results.

First of all, the number of the observations is limited to 1748 observations concerning the two models. The table 3 summarizes all the descriptive statistics (mean, max, min, the standard deviation, the Skewness and the Kurtosis) relative to variables used in the different estimation of the variable PD.

Table 2: List of banks

Name of Bank	The study period
ALPHA BANK (KO)	02/01/2006 – 31/12/2012
ASPIS BANK (KO)	02/01/2006 – 30/06/2010
ATTICA BANK (KO)	02/01/2006 – 31/12/2012
BANK OF CYPRUS (CR)	02/01/2006 – 31/12/2012
BANK OF GREECE (CR)	02/01/2006 – 31/12/2012
ΕΓΝΑΤΙΑ BANK (KO)	02/01/2006 – 20/09/2007
ΕΓΝΑΤΙΑ BANK (ΠΟ)	02/01/2006 – 20/08/2007
EMPORIKI BANK (CR)	02/01/2006 – 29/04/2011
EUROBANK EFG (KO)	02/01/2006 – 31/12/2012
GENIKI BANK (CR)	02/01/2006 – 31/12/2012
MARFIN EGNATIA BANK (CR)	02/01/2008 – 31/03/2011
MARFIN FINANCIAL GROUP (KO)	02/01/2006 – 30/03/2007
MARFIN POPULAR BANK (KO)	02/01/2008 – 11/04/2012
NATIONAL BANK (CR)	02/01/2006 – 31/12/2012
PIRAEUS BANK (CR)	02/01/2006 – 31/12/2012
PROTON BANK S,A, (CR)	02/01/2006 – 31/12/2012
TT HELLENIC POSTBANK (CR)	02/01/2008 – 30/12/2011

Table 3: Descriptive statistics

Variable	Obs	Mean	Std Div	Min	Max	Skewness	Kurtosis
PD	1748	0.5007853	0.0845174	0.1545267	0.7606353	0.2988851	2.873225
RR	1748	0.7433242	0.0993225	0.0076826	1	-0.0383176	2.363317
ND	1748	6.653638	1.326654	-0.0109848	10.00634	0.1338035	2.517658
L	1748	-0.109574	0.6235335	0.4428743	0.4199632	-6.181387	104.6915
Dummy1	1748	0.0005721	0.0239182	0	1	41.7732	1746.001
Dummy2	1748	0.0040046	0.063173	0	1	15.70727	247.7183
Dummy3	1748	0.4302059	0.4952465	0	1	0.2819365	1.079488

First of all, we used two types of estimations: The Bootstrapped Quantile Regression and The Simultaneous Quantile Regression. The choice of the two techniques is justified by the objective of this study. Then, the purpose of this paper is to explain the relationship of dependence between the Probability of Default and the Recovery Rate. Those econometric techniques allow describing the dependence between tow variables based on their volatilities.

Figure 1 summarizes the evolution of the probability of default and the recovery rate for each year (2006-2012). In figure 2 we present the evolution of the PD and the RR for the entire period that spans 1748 days.
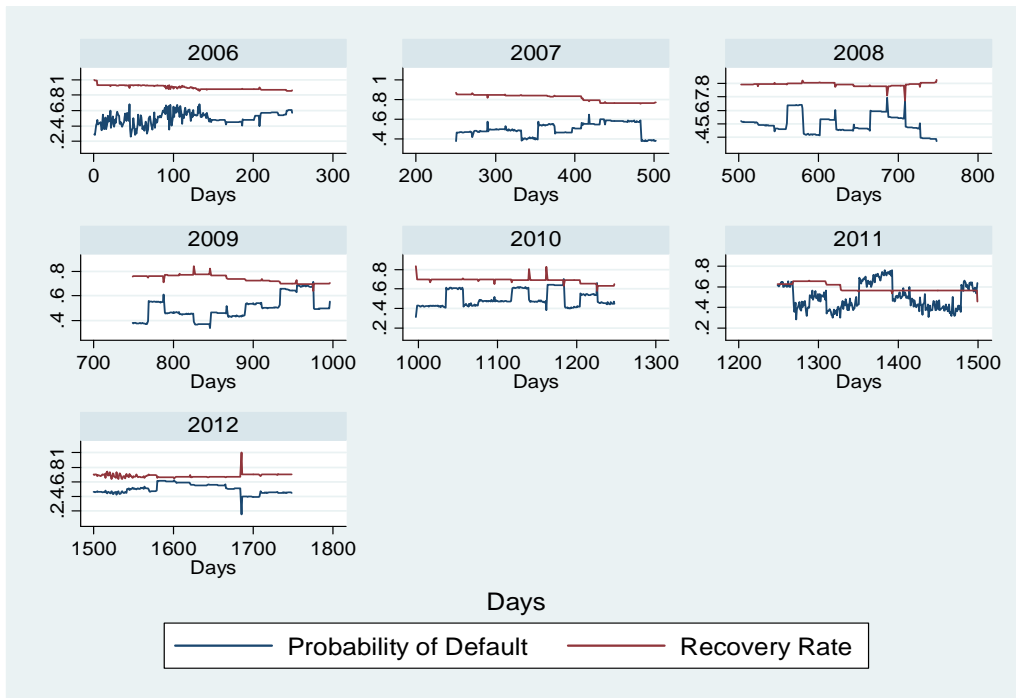
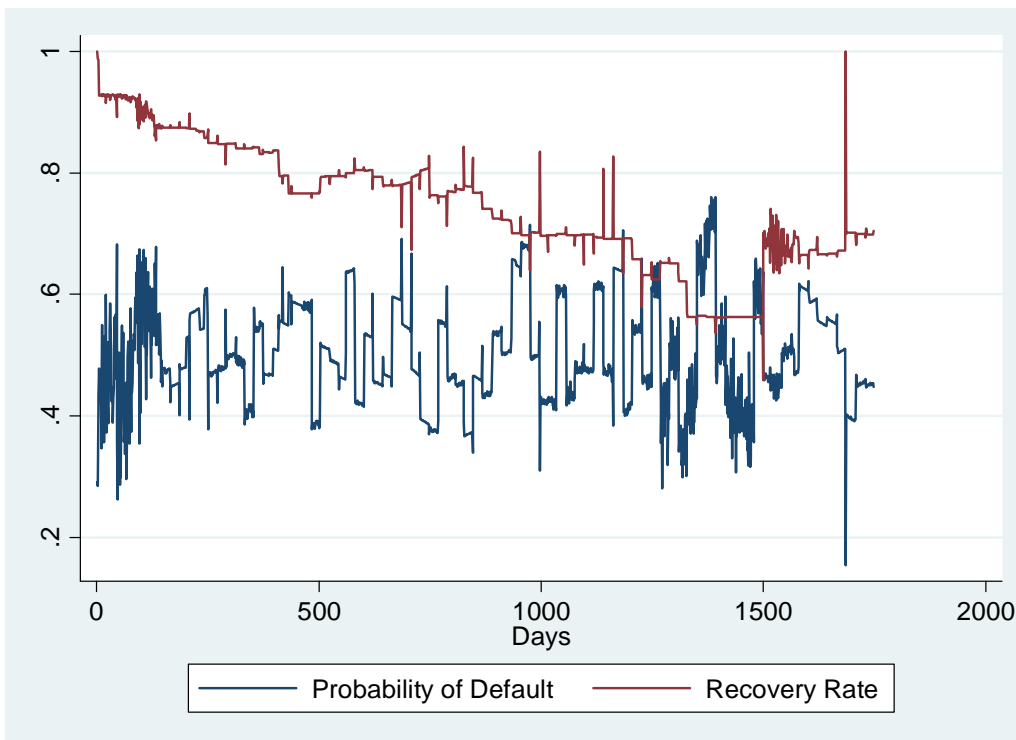**Figure 1: The evolution of the PD and the RR (by year)**

Figure 2: The evolution of the PD and the RR

In this paper, we made a test of the correlation between the various used variables. Table 4 summarizes the results relative to the correlation. So, the results show that all the coefficient of correlation of Pearson does not exceed the limit of tolerance of (0.7), so he does not cause problems during the estimation of the model who measure the PD.

We also conducted a test of the unit root time series. Thus, we used the Dickey-Fuller and Phillips-Perron test. According to the results presented in tables 5 and 6, we noticed that all the calculated values of the t-student or t-statistical values are below the critical thresholds of 1%, 5% and 10%. In this case, all the variables used in the model to be estimated are stationary.

Table 4: The matrix of correlation

	PD	RR	ND	L	Dummy1	Dummy2	Dummy3
PD	1.0000						
RR	-0.1116 (0.0000)*	1.0000					
ND	0.8495 (0.0000)*	0.1209 (0.0000)*	1.0000				
L	0.1059 (0.0000)*	0.1264 (0.0000)*	0.5142 (0.0000)*	1.0000			
Dummy1	0.0057 (0.8132)	0.0115 (0.6306)	0.0115 (0.6311)	0.0072 (0.7621)	1.0000		
Dummy2	0.0272 (0.2564)	-0.0330 (0.1680)	0.0343 (0.1520)	0.0269 (0.2611)	-0.0015 (0.9495)	1.0000	
Dummy3	00016 (0.9469)	-0.7990 (0.0000)*	-0.3087 (0.0000)*	-0.3878 (0.0000)*	-0.0208 (0.3850)	0.0181 (0.4498)	1.0000

Value significant in a threshold of: (*) 1%; (**) 5% et (***) 10%.

Table 5: The test of the unit root of Dickey-Fuller

Variables	t-statistic	Critical value at 1%	Critical value at 5%	Critical value at 10%	The hypothesis rejected
DP	-12.106	-3.430	-2.860	-2.570	H0: presence of unit root. So the variable is stationary
RR	-4.256	-3.430	-2.860	-2.570	H0: presence of unit root. So the variable is stationary
ND	-10.285	-3.430	-2.860	-2.570	H0: presence of unit root. So the variable is stationary
L	-13.770	-3.430	-2.860	-2.570	H0: presence of unit root. So the variable is stationary
Dummy1	-41.797	-3.430	-2.860	-2.570	H0: presence of unit root. So the variable is stationary
Dummy2	-41.942	-3.430	-2.860	-2.570	H0: presence of unit root. So the variable is stationary
Dummy3	-12.868	-3.430	-2.860	-2.570	H0: presence of unit root. So the variable is stationary

Table 6: The test of the unit root of Phillips-Perron

Variables	t-statistic	Critical value at 1%	Critical value at 5%	Critical value at 10%	The hypothesis rejected
DP	-233.223	-20.700	-14.100	-11.300	H0: presence of unit root. So the variable is stationary
RR	-297.631	-20.700	-14.100	-11.300	H0: presence of unit root. So the variable is stationary
ND	-153.469	-20.700	-14.100	-11.300	H0: presence of unit root. So the variable is stationary
L	-33.607	-20.700	-14.100	-11.300	H0: presence of unit root. So the variable is stationary
Dummy1	-1745.365	-20.700	-14.100	-11.300	H0: presence of unit root. So the variable is stationary
Dummy2	-1735.363	-20.700	-14.100	-11.300	H0: presence of unit root. So the variable is stationary
Dummy3	-1214.754	-20.700	-14.100	-11.300	H0: presence of unit root. So the variable is stationary

To pursue our analysis, we go, then, present the resultant of the estimation of the model of measure of the banking performance by using the Software STATA 12.We estimated the variables PD (tables 7 and 8). So, we estimated the variable PD by basing itself on 8 estimations for each of both variables and by using two econometric techniques.

In table 7 we used the Bootstrapped Quantile Regression. Then, we noticed that all the values of the statistical Pseudo-R² are almost equal to 0.80 in all estimates. So we can assume that the estimated model is characterized by a good linear adjustment.

In our model the probability of default will be estimated based on other explicative variables. The results of estimation are presented in table 7.

Table 7 summarizes all estimations relative to the model (PD), we notice that there are four significant variables with different thresholds. The first one, it is the variable RR, is statistically significant and negative in a 1% threshold in four estimations (1, 2, 3 and 5), in a 5% threshold in the sixth estimation and in a 10% threshold in the last estimations. In this frame, the variable RR has a negative impact on the probability of default of the banks of Greece. When we augmented the recovery rate, the banks have profit to supply an important exposure to failure. We can conclude that a high recovery rate allow to absorb losses incurred by banks in Greece.

The variable ND is statistically significant and positive in a 1% threshold in all estimations. The number of default of banks affects their probability of default. Thus, the high numbers of default reflect that the probability of default is high.

The variable L (the expected losses) is statistically significant and negative in a 5% threshold only in all estimation. This confirms the literature, because the high value of the expected losses allows the bank to minimize their probability of default in future.

For the three dummy variables used in our study, we conclude that only the dummy1 variable is significant in 5% threshold in the estimation 2 and 5. This variable affects negatively the probability of default. Then, the entry of a new bank in the Athens Exchange leads to the minimization of the probability of default of the existing banks in the financial market of Greece.

In table 8 we used the Simultaneous Quantile Regression. Then, we noticed that all the values of the statistical Pseudo-R² are equal to 0.80 in all estimates. So we can assume that the estimated model is characterized by a good linear adjustment.

In the model (PD) the probability of default will be estimated based on other explicative variables. The results of estimation are presented in table 7.

After using the second econometric techniques (Simultaneous Quantile Regression), we noticed that all the results have almost the same significance thresholds. So, for the two econometric techniques the impact of different variables remains the same.

Empirically, we conclude that the probability of default is inversely related to the recovery rate. The recovery rate is not constant; it decreases with increasing of the probability of default. This empirical result is confirmed by these figures follows. On these figures we presented the distribution of the probability of default and the recovery rate of all banks and by year. The dependence between PD and RR is justified by the correlation coefficients of Pearson who presented in table 4.

Table 7: Estimation by bootstrapped quantile regression

Dependent variable: PD							Period of estimation : 2006 – 2012	
Explicative variables	Estimation 1	Estimation 2	Estimation 3	Estimation 4	Estimation 5	Estimation 6	Estimation 7	Estimation 8
RR	-0.075498 (-4.35)*	-0.0753065 (-4.44)*	-0.0754981 (-4.38)*	-0.0364969 (-1.85)***	-0.075498 (-4.42)*	-0.0364969 (-1.75)**	-0.0364969 (-1.81)***	-0.0364969 (-1.82)***
ND	0.071905 (85.20)*	0.0719024 (86.20)*	0.0719049 (85.13)*	0.0727096 (98.80)*	0.071905 (81.03)*	0.0727096 (98.13)*	0.0727096 (97.62)*	0.0727096 (97.55)*
L	-0.0747226 (-11.79)*	-0.0747128 (-11.69)*	-0.0747225 (-11.70)*	-0.073463 (-9.98)*	-0.0747226 (-11.48)*	-0.073463 (-9.90)*	-0.073463 (-9.99)*	-0.073463 (-9.84)*
Dummy1		-0.0085701 (-2.18)**			-0.0085487 (-2.18)**	-0.0035373 (-1.28)		-0.0035373 (-1.29)
Dummy2			0.0006389 (0.06)		0.0006387 (0.06)		0.0042401 (0.31)	0.0042401 (0.31)
Dummy3				0.0121234 (1.45)		0.0121234 (1.35)	0.0121234 (1.42)	0.0121234 (1.41)
CONSTANT	0.070721 (4.70)*	0.0706089 (4.82)*	0.0707216 (4.79)*	0.0288906 (1.70)***	0.070721 (4.76)*	0.0288906 (1.59)	0.0288906 (1.66)***	0.0288906 (1.67)***
Number of obs	1748	1748	1748	1748	1748	1748	1748	1748
Pseudo R²	0.7997	0.7997	0.7997	0.8007	0.7997	0.8007	0.8007	0.8007

Value significant in a threshold of: (*) 1%; (**) 5% et (***) 10%.

Table 8: Estimation by simultaneous quantile regression

Dependent variable: PD							Period of estimation : 2006 – 2012	
Explicative variables	Estimation 1	Estimation 2	Estimation 3	Estimation 4	Estimation 5	Estimation 6	Estimation 7	Estimation 8
RR	-0.075498 (-4.41)*	-0.0753065 (-4.23)*	-0.0754981 (-4.20)*	-0.0365789 (-1.84)***	-0.075498 (-4.37)*	-0.0365789 (-1.57)	-0.0365868 (-1.91)***	-0.0365789 (-1.83)***
ND	0.071905 (83.62)*	0.0719024 (82.21)*	0.0719049 (84.06)*	0.0727134 (100.93)*	0.071905 (82.91)*	0.0727134 (46.53)*	0.0727117 (99.74)*	0.0727134 (96.00)*
L	-0.0747226 (-11.71)*	-0.0747128 (-11.56)*	-0.0747225 (-11.89)*	-0.0734714 (-10.03)*	-0.0747226 (-11.66)*	-0.0734714 (-10.01)*	-0.07347 (-10.25)*	-0.0734714 (-9.98)*
Dummy1		-0.0085701 (-2.20)**			-0.0085487 (-2.21)**	-0.0035433 (-0.80)		-0.0035433 (-1.27)
Dummy2			0.0006389 (0.05)		0.0006387 (0.05)		0.0042446 (0.30)	0.0042409 (0.31)
Dummy3				0.0121102 (1.37)		0.0121102 (1.42)	0.0121108 (1.42)	0.0121102 (1.41)
CONSTANT	0.070721 (4.75)*	0.0706089 (4.61)*	0.0707216 (4.79)*	0.0289338 (1.66)***	0.070721 (4.72)*	0.0289338 (1.61)	0.0289489 (1.73)***	0.0289338 (1.64)***
Number of obs	1748	1748	1748	1748	1748	1748	1748	1748
Pseudo R²	0.8021	0.8022	0.8031	0.8022	0.8022	0.8032	0.8032	0.8032

Value significant in a threshold of: (*) 1%; (**) 5% et (***) 10%.

The dependence between the Probability of Default and the Recovery rate of all banks is shows in figure 3, figure 4, figure 5, figure 6, figure 7, figure 8 and figure 9. All these figures shows the dependence between the Probability of Default and the Recovery rate of all banks by years (figures 3, 4, 5, 6, 7, 8 and 9).
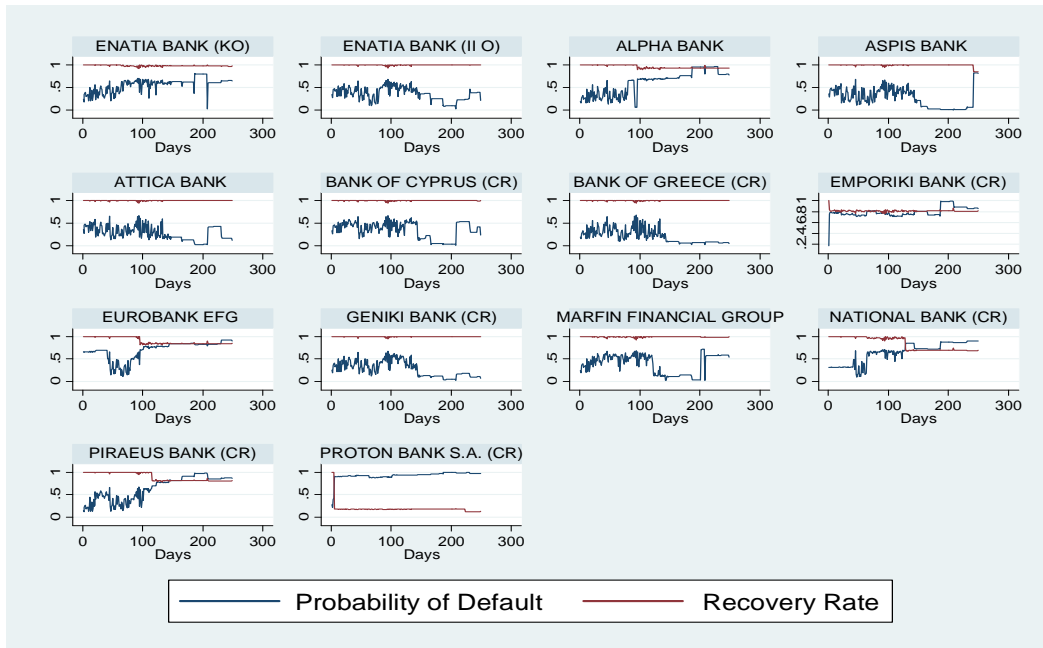
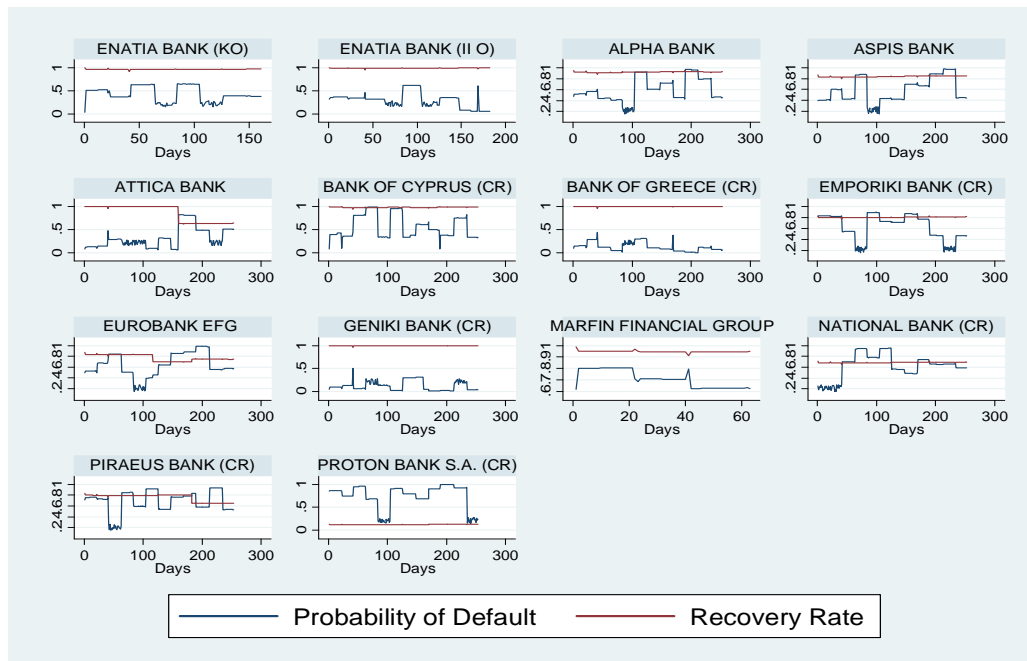**Figure 3: The evolution of the PD and the RR in 2006 (by banks)**

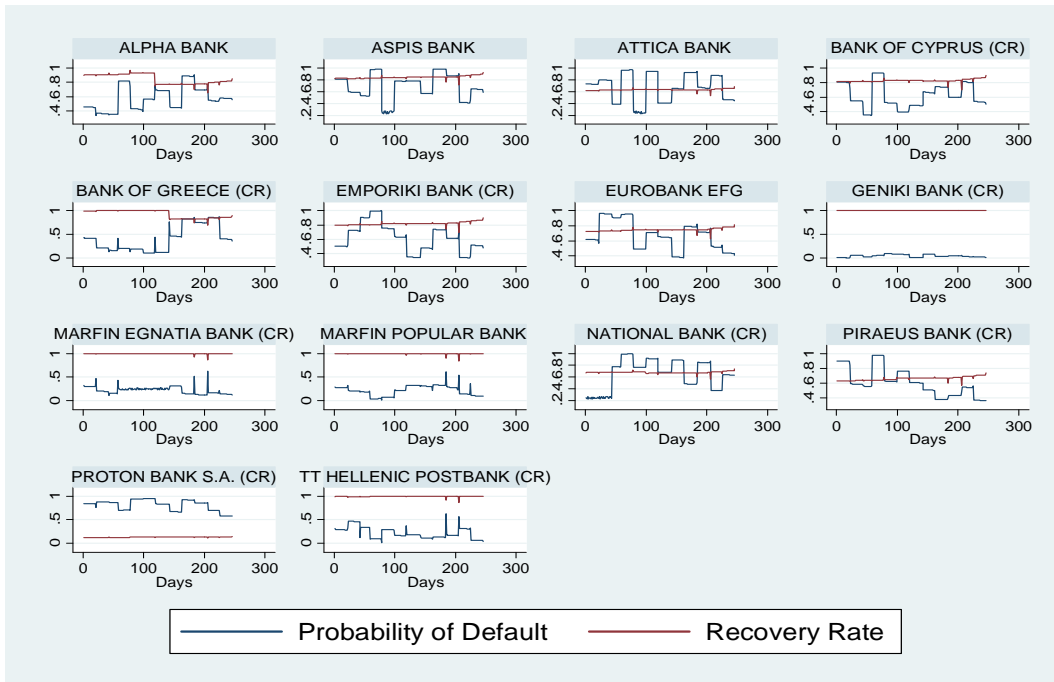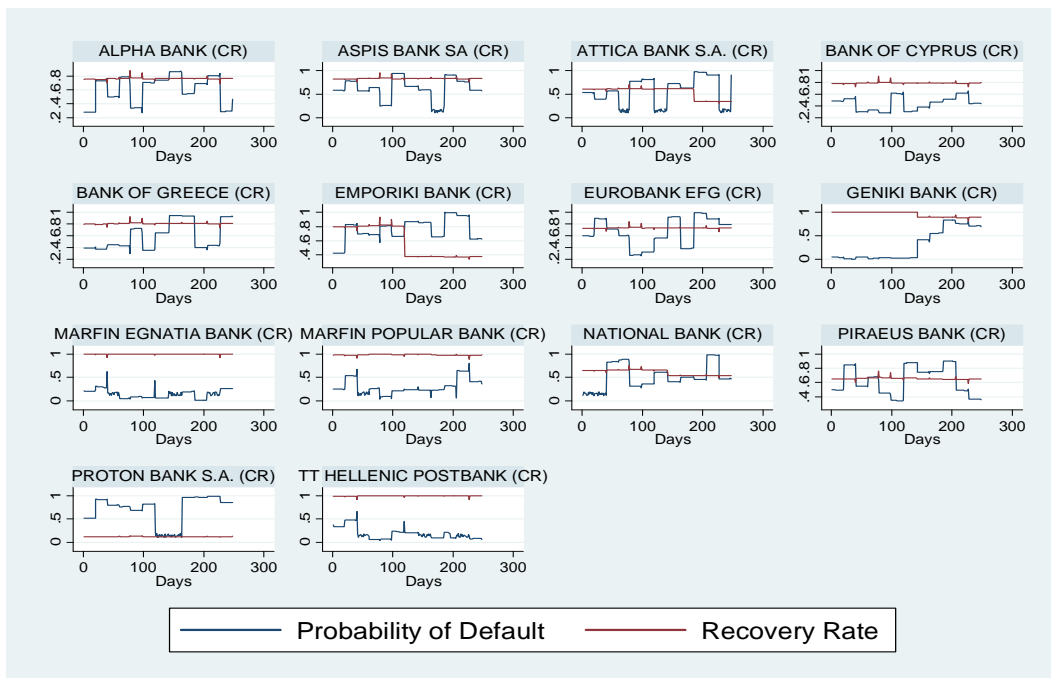**Figure 4: The evolution of the PD and the RR in 2007 (by banks)**

Figure 5: The evolution of the PD and the RR in 2008 (by banks)

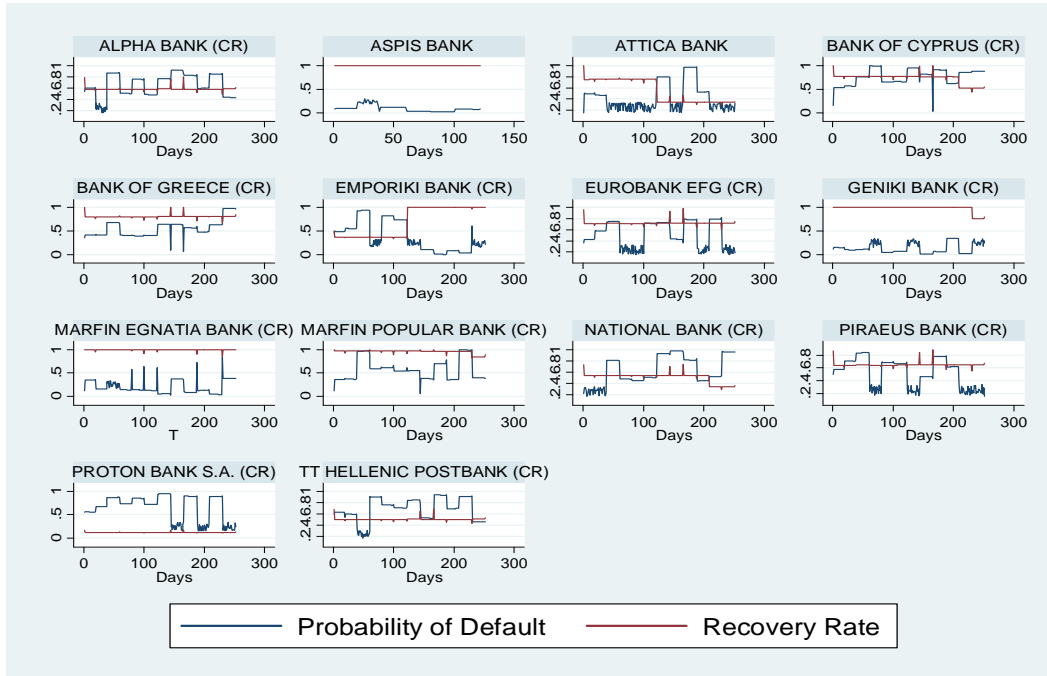
Figure 6: The evolution of the PD and the RR in 2009 (by banks)

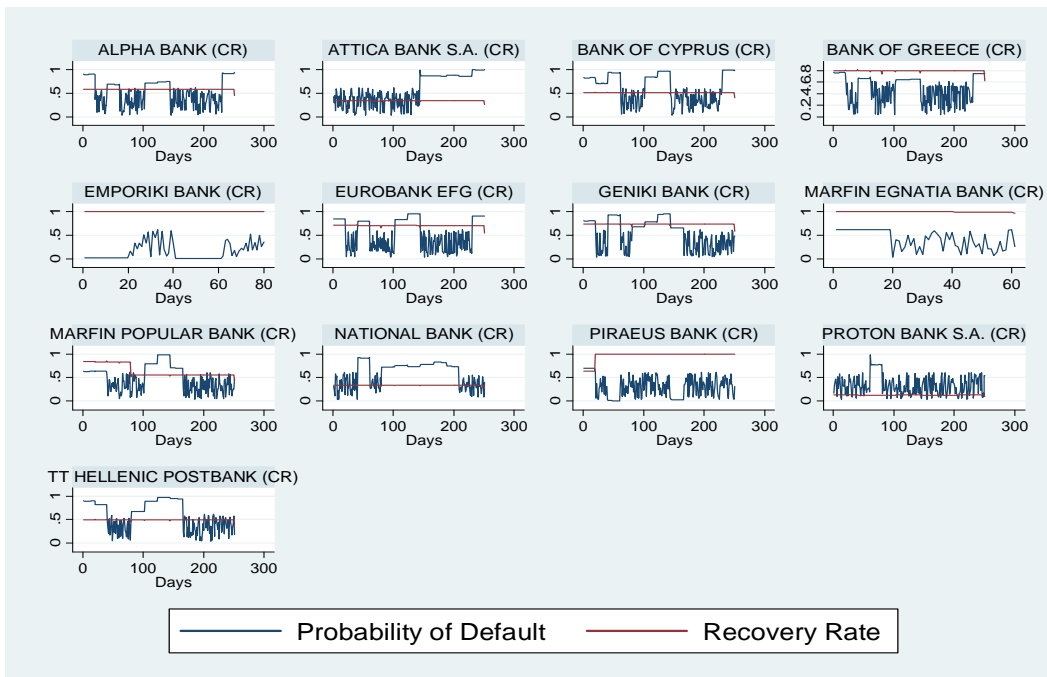**Figure 7: The evolution of the PD and the RR in 2010 (by banks)**

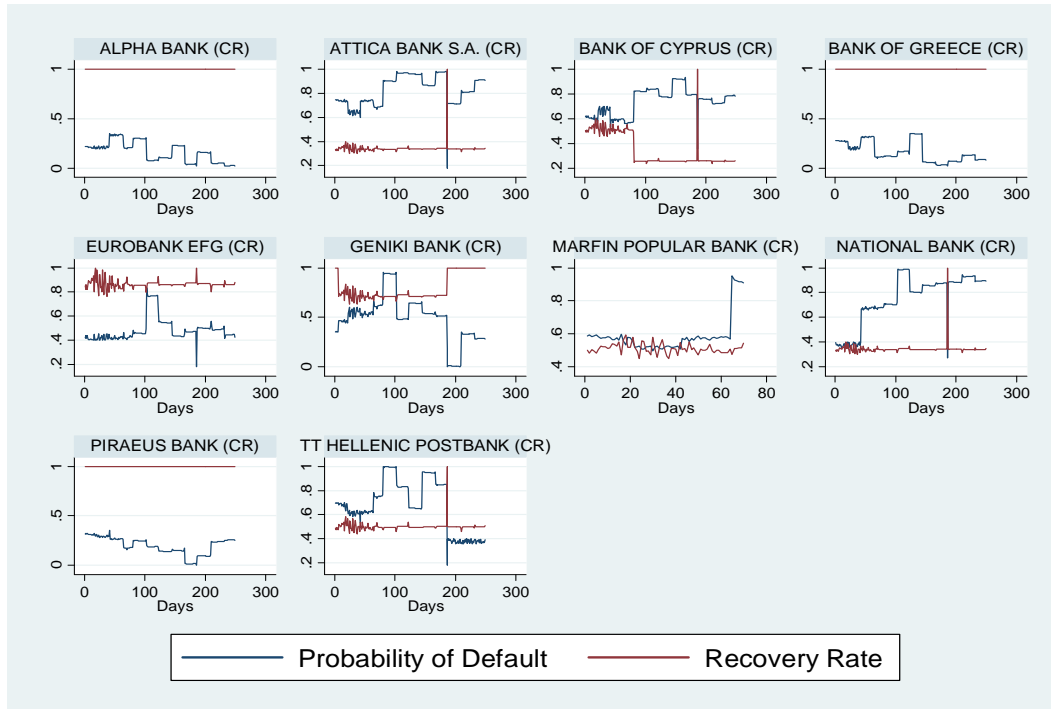**Figure 8: The evolution of the PD and the RR in 2011 (by banks)**

Figure 9: The evolution of the PD and the RR in 2012 (by banks)

Table 9: The VaR results

Confidence level	VaR
95%	0,3641361
99%	0,3641539
99.5%	0,3756432
99.9%	0,3778987

We also carry a Kruskal-Wallis equality-of-populations rank test. The Kruskal-Wallis one-way analysis of variance by ranks is a non-parametric method for testing whether samples originate from the same distribution. It is use for comparing more than two samples that are independent, or not related. The parametric equivalent of the Kruskal-Wallis test is the one-way analysis of variance (ANOVA). The results of this test can accept the hypothesis H0. ie the average of the different types of sample studied are not significantly different (Mean Rank = 6 < Index Kruskal-Wallis = 6,713).

We calculate also the Value-at-Risk on four confidence level chosen (table 9).

In recession periods, the number of defaulting banks or firms in generally rises. On top of this, the average amount recovered on the bonds of defaulting banks tends to decrease. Our paper purpose an econometric model in which

this joint time-variation in default rates and recovery rate distribution by a quantile regression, which given the importance if the volatilities.

CONCLUSION

The dependence between the probability of default and the recovery rate has a crucial influence on large credit portfolio losses. Thus, the probability of default and the recovery rate are often modeled independently in current credit risk models: KMV model, CreditMetrics model, Credit Risk+ and Credit Portfolio View.

In this paper we revisited the Merton model for different underlying processes with correlations. Then, we used the Bootstrapped Quantile Regression and the Simultaneous Quantile Regression to identify the dependence between the probability of default and the recovery rate.

Finally, we concluded that the probability of default and the recovery rate are inversely related. This result is confirmed by those table and figure presented in the fourth section.

Based on the results found in this study, the banks are obliged to maximize their recovery rate to reduce their probability of default.

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