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An Application of Discounted Residual Income for Capital Assets Pricing by Method Curve Fitting with Sinusoidal Functions

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ABSTRACT:
The basic model for valuation of firm is the Dividend Discount Model (DDM). When investors buy stocks, they expect to receive two types of cash flow: dividend in the period during which the stock is owned, and the expected sales price at the end of the period. In the extreme example, the investor keeps the stock until the company is liquidated; in such a case, the liquidating dividend becomes the sales price. The residual income valuation model is an alternative model to the divided discounted model or valuation based on multiples in determining a value of a company. It decomposes the company value of a company into two imaginative parts: (1) the real assets of the company. It is assumed that these assets are leased to the company at a certain rate of return. (2) The present value of the future "Residual Incomes". Residual income refers to the income part which is achieved above the expected return on the real assets (the previously mentioned first part). Thus we examined the effects of Discounted Residual Income (DRI) on stock price of companies listed in Tehran Security Exchange (TSE) by method curve fitting with sinusoidal functions. Our sample includes 1920 firm- year of companies listed in TSE during the period 2005-2010.The results show that there is a significant relationship between discounted residual income and firm's value. Also the result shows that if we use method curve fitting with sinusoidal functions for fitting model, we can regress the best model and the explanatory power of model will increase.

Keywords: *Residual income, Valuation model, Sinusoidal functions, Stock price, Discounted models*

INTRODUCTION

The purposes of the corporations are different as viewed by different people. Some thinkers believe that the main goal of a corporation should be the maximization of social welfare. On the contrary others, believe the main goal of a corporation should be the maximization of stock prices since it can fulfill the social welfare. On the other hand, according to SFAS 2, one of the qualitative characteristics of accounting information is having the predictive value, and also accounting information should have the ability to increase decision makers' anticipation power of event's results. Stock price is an item that people are very eager to anticipate, because stock price is important for internal users (i.e. financial and executive managers) and external users (i.e. creditors and investors). In order to provide a model for anticipating stock prices, factors that affecting the stock prices must be

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determined. This paper examines a valuation model that is useful to investors in assessing firm value. Thus, the importance of this research is related to the fact that investors' intention in investing in companies' stock is getting rational returns including dividends and capital gains. Therefore, having a criterion in deciding when to buy and when to sell shares depend on the future stock price changes. The present research has been tried to assess a model among the models of stock valuation, and to determine the effects of explanatory variables on stock price according to economic environment in Iran.

The Relationship between the Discounted Residual Income and Firm Value

The basic model for valuation of firm is the Dividend Discount Model (DDM) (Miller and Modigliani, 1961). When investors buy stocks, they expect to receive two types of cash flow: dividend in the period during which the stock is owned, and the expected sales price at the end of the period. In the extreme example, the investor keeps the stock until the company is liquidated; in such a case, the liquidating dividend becomes the sales price. Under the assumption of an infinite time horizon, the DDM can be expressed as (Plenborg, 2002):

$$P_0=\sum_{t=1}^{\infty}\frac{\mathrm{DIV_t}}{(1+\mathrm{K_e})^{\mathrm{t}}} \qquad (1)$$

Where **P** is the firm value, **div** the dividends, and $\mathbf{k_e}$ the cost of capital. The estimated market value of a firm's equity should be unaffected by the valuation approach applied, so it is important to ensure that the valuation approaches are conceptually equivalent to one another. Since the DDM is the theoretically correct model, it may be surprising to some that considerable effort and resources are employed to develop alternative valuation approaches. One reason is that under the DDM, dividends are treated as the distribution rather than the creation of wealth. Penman (1992) describes it as the dividend conundrum 'price is based on future dividends but observed dividends do not tell us anything about price'. Ideally, the valuation approach chosen incorporates those variables that show the creation of wealth rather than the distribution of wealth. Among other things, this will ease the interpretation of firm value estimates for both financial analysts and end-users (investors). The residual income (RI) approach was introduced by Edwards and Bell (1961) and subsequently further developed by Peasnell (1982) and Ohlson (1995). It is derived from the DDM. RI is a variation of the better-known economic value added (EVA) approach (Stewart, 1991); it measures firm value from an equity-holder's perspective rather than from a lender's and an equity-holder's perspective (EVA approach) (Plenborg, 2002). The residual income can be expressed as:

$$RI_t = \mathrm{NI_t} - (\mathrm{k_e}.\mathrm{BV_{t-1}}) \qquad (2)$$

Where **RI** is the residual income, **NI** the net income, $\mathbf{k_e}$ the cost of capital and **BV** the book value of equity. SO the discounted residual income model can be expressed as:

$$P_0 = BV_0 + \sum_{t=1}^{\infty}\left(\frac{\mathrm{NI_t}-(\mathrm{k_e}.\mathrm{BV_{t-1}})}{(1+\mathrm{k_e})^{\mathrm{t}}}\right) \qquad (3)$$

Where **NI** is the net income, **BV** the book value of equity, and $\mathbf{k_e}$ the cost of capital.

Perek and Perek (2012) incorporated an empirical approach to compare the outcomes of two different methods: residual income and discounted cash flow valuation models. The aim of their study was to test whether these methods result in different values and to contribute to the understanding of why these two valuation techniques, although similar in theory, may generate different results when applied to real life companies. Their study shows that the residual income model results in lower company valuation compared with the discounted cash flow model. This finding may be due to certain specific characteristics of Turkish companies.

Fourier Series

Just before 1800, the French mathematician/ physicist/engineer Jean Baptiste Joseph Fourier made an astonishing discovery. As a result of his investigations into the partial differential equations modeling vibration and heat propagation in bodies, Fourier was led to claim that "every" function could be represented by an infinite series of elementary trigonometric functions sinuses and cosines.

The study of Fourier series is a branch of Fourier analysis. Prior to Fourier's work, no solution to the heat equation was known in the general case, although particular solutions were known if the heat source behaved in a simple way, in particular, if the heat source was a sine or cosine wave. These simple solutions are now sometimes called Eigen solutions. Fourier's idea was to model a complicated heat source as a superposition (or linear combination) of simple sine and cosine waves, and to write the solution as a superposition of the corresponding Eigen solutions. This superposition or linear combination is called the Fourier series (Fourier, 2003).

Although the original motivation was to solve the heat equation, it later became obvious that the same techniques could be applied to a wide array of mathematical and physical problems, and especially those involving linear differential equations with constant coefficients, for which the Eigen solutions are sinusoids. The Fourier series has many such applications in electrical engineering, vibration analysis, acoustics, optics, signal processing, image processing, quantum mechanics, econometrics, thin-walled shell theory, etc.

Fourier Series Expanded in Time (t) with Period (T)

In mathematics, a Fourier series decomposes periodic functions or periodic signals into the sum of a (possibly infinite) set of simple oscillating functions, namely sinuses and cosines (or complex exponentials). The study of Fourier series is a branch of Fourier analysis.

The Fourier series is named in honor of Joseph Fourier, who made important contributions to the study of trigonometric series. Prior to Fourier's work, no solution to the heat equation was known in the general case, although particular solutions were known if the heat source behaved in a simple way, in particular, if the heat source was a sign or cosine wave. These simple solutions are now sometimes called Eigen solutions. Fourier's idea was to model a complicated heat source as a superposition (or linear combination) of simple sine and cosine waves, and to write the solution as a superposition of the corresponding Eigen solutions. This superposition or linear combination is called the Fourier series.

From a modern point of view, Fourier's results are somewhat informal, due to the lack of a precise notion of function and integral in the early nineteenth century.

Although the original motivation was to solve the heat equation, it later became obvious that the same techniques could be applied to a wide array of mathematical and physical problems, and especially those involving linear differential equations with constant coefficients, for which the Eigen solutions are sinusoids. The Fourier series has many such applications in electrical engineering, vibration analysis, acoustics, optics, signal processing, image processing, quantum mechanics, and econometrics.

Let the function $f(t)$ be periodic with period $T = 2L$ where

$$\omega_{0}=\frac{2\pi}{T} \quad (4)$$

Hence the Fourier series is

$$f(t) = \frac{a_0}{2} + \sum_m a_m cos(m\omega t) + \sum_m b_m sin(m\omega t) \quad (5)$$

Where

$$a_0 = \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} f(t)dt \quad (6)$$

$$a_m = \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} f(t)\cos(n\omega t)dt \quad (7)$$

$$b_m = \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} f(t)\sin(n\omega t)dt \quad (8)$$

(Note: half the range of integration = π/ω)

Curve Fitting with Sinusoidal Functions

Curve fitting is the process of constructing a curve, or mathematical function that has the best fit to a series of data points, possibly subject to constraints. Curve fitting can involve either interpolation, where an exact fit to the data is required, or smoothing, in which a "smooth" function is constructed that approximately fits the data. A related topic is regression analysis, which focuses more on questions of statistical inference such as how much uncertainty is present in a curve that is fit to data observed with random errors. Fitted curves can be used as an aid for data visualization, to infer values of a function where no data are available, and to summarize the relationships among two or more variables. Extrapolation refers to the use of a fitted curve beyond the range of the observed data, and is subject to a greater degree of uncertainty since it may reflect the method used to construct the curve as much as it reflects the observed data (Coope, 1999).

The Gauss–Newton algorithm is a method used to solve non-linear least squares problems. It can be seen as a modification of Newton's method for finding a minimum of a function. Unlike Newton's method, the Gauss–Newton algorithm can only be used to minimize a sum of squared function values, but it has the advantage that second derivatives, which can be challenging to compute, are not required.

Non-linear least squares problems arise for instance in non-linear regression, where parameters in a model are sought such that the model is in good agreement with available observations.

Least-squares fit of a sinusoidal function is to determine coefficient values that minimize:

$$s_r = \sum_{i=1}^{N}\{y_i - [A_0 + A_1\cos(\omega t_i) + B_1\sin(\omega t_i)]\}^2 \quad (9)$$

$$\begin{bmatrix} N & \sum\cos(\omega_0 t) & \sum\sin(\omega_0 t) \\ \sum\cos(\omega_0 t) & \sum\cos^2(\omega_0 t) & \sum\cos(\omega_0 t).\sin(\omega_0 t) \\ \sum\sin(\omega_0 t) & \sum\cos(\omega_0 t).\sin(\omega_0 t) & \sum\sin^2(\omega_0 t) \end{bmatrix} \times \begin{Bmatrix} A_0 \\ A_1 \\ B_1 \end{Bmatrix} = \begin{Bmatrix} \sum y \\ \sum y\cos(\omega_0 t) \\ \sum y\sin(\omega_0 t) \end{Bmatrix} \quad (10)$$

For system

$$\int_0^T \cos(\omega_0 t)\,dt = -\frac{1}{\omega_0}\sin(\omega_0 t)\begin{cases} T \\ 0 \end{cases} = 0 \quad (11)$$

Where

$$\begin{bmatrix} N & 0 & 0 \\ 0 & N/2 & 0 \\ 0 & 0 & N/2 \end{bmatrix} \times \begin{Bmatrix} A_0 \\ A_1 \\ B_1 \end{Bmatrix} = \begin{Bmatrix} \sum y \\ \sum y\cos(\omega_0 t) \\ \sum y\sin(\omega_0 t) \end{Bmatrix} \quad (12)$$

$$A_0 = \frac{\sum y}{N} \quad (13)$$

$$A_1 = \frac{2}{N}\sum y\cos(\omega t) \quad (14)$$

$$A_2 = \frac{2}{N}\sum y\sin(\omega t) \quad (15)$$

RESEARCH METHOD

Research Sample

In this study, research sample is selected by considering all companies listed on Tehran stock exchange from 2005-2010.Next step in the sampling process was to put aside those companies that their yearend was not 21th of Mach (Iranian New year Colander). Then among the remained companies those were chosen that had continuing operation during the period of 2005 to 2010. Having set these limitations the remaining companies that were 1920 firm- year were considered as research sample. Required data about these companies in stock exchange extracted from available data using Tadbir Pardaz and Rahavarde Novin softwares.

Method of Testing the Research Hypothesis

The residual income valuation formula (Model 3) is very similar to a multistage dividend discount model (Model 1), substituting future dividend payments for future residual earnings. Using the same basic principles as a dividend discount model to calculate future residual earnings, we can derive an intrinsic value for a firm's stock. In contrast to the discounted cash flows approach (DCF approach) which uses the weighted average cost of capital for the discount rate, the appropriate rate for the residual income strategy is the cost of equity. So we state the research hypothesis as:

H0: There are a significant relationship between discounted residual income and stock price.

In order to test the research's hypothesis we regress a regression of discounted model for evaluating stock. It means we test the regression of model 16.

$$P_{0i} = a_0 + a_1 DRI_i + f(i) \quad (16)$$

Where:

P_{0i} = Present Value of stock of firm i

$$DRI_i = DiscountedResidualincome = BV_{0i} + \sum_{t=1}^{5} \left(\frac{NI_{i,t} - (k_{ei}.BV_{i,t-1})}{(1+k_{ei})^t} \right) \quad (17)$$

F (i) = A function for model's errors = Fourier series Expanded In Firm i by sinusoidal function that it has been explained in model 5.

It is notable that we discounted residual income for five years (2006-2010) and we regress model for year 2005. Thus we used cross section regression for testing research hypothesis.

RESULTS AND DISCUSSION

We used STATA software statistic for determining descriptive statistics. Some descriptive statistics have been presented in table 1. Table 1 shows the mean, standard deviation, minimum and maximum data. Also table 1 shows Pearson correlation coefficient between variables of research.

The table 1 shows that there is a positive correlation between explanatory variables of research and stock price. Also table 1 shows that there is a significant correlation between Fourier Seri (F(i)) and stock price but it's necessary to regress model 16 for determining the effects of explanatory variables on stock price.

We used MATLAB software for this purpose and the results have been presented in table 2.

Table 2 shows that the adjusted R-squared of model is equal to 0.9985. This means that we can explain stock price changes by approximately 99.85 percent of the explanatory variables in model 16.

Table 1: Descriptive statistics

Panel A: Summary Statistics

Variable	Mean	Std. Dev.	Min	Max
Stock Price (P_0)	182159.9	139666	17010	511200
Discounted Residual income (DRI)	74065.16	150559.4	-501499	553825
Fourier Seri (f(i))	108094.7	176430.3	-145025	566379

Panel B: Pearson Correlation Coefficient

		P_0	DRI	F(i)
	P_0	1		
	DRI	0.2627	1	
	F(i)	0.5675	-0.0645	1

Table 2: Evaluating value of stock based on method curve fitting with sinusoidal functions

Variable	Coefficient	Std. Err	T-Test (P-Value)
DRI	0.2436701	0.1206915	2.02 (0.04)

General Model Fourier8:

f(x) =a0 + a1*cos(x*w) + b1*sin(x*w) +a2*cos(2*x*w) + b2*sin(2*x*w) + a3*cos(3*x*w) + b3*sin(3*x*w) +a4*cos(4*x*w) + b4*sin(4*x*w) + a5*cos(5*x*w) + b5*sin(5*x*w) +a6*cos(6*x*w) + b6*sin(6*x*w) + a7*cos(7*x*w) + b7*sin(7*x*w) +a8*cos(8*x*w) + b8*sin(8*x*w)

Coefficients (with 95% confidence bounds):

a0 = -3.646e+005 (-1.887e+006, 1.158e+006)

a1 = -7.689e+005 (-2.551e+006, 1.013e+006)

b1 = 4.784e+005 (-1.954e+006, 2.911e+006)

a2 = -5.443e+004 (-9.764e+005, 8.675e+005)

b2 = 8.347e+005 (-1.699e+006, 3.368e+006)

a3 = 4.899e+005 (-1.629e+006, 2.609e+006)

b3 = 4.552e+005 (-1.023e+005, 1.013e+006)

a4 = 4.897e+005 (-6.817e+005, 1.661e+006)

b4 = -6.366e+004 (-1.185e+006, 1.058e+006)

a5 = 1.621e+005 (-1.134e+005, 4.376e+005)

b5 = -2.711e+005 (-1.293e+006, 7.51e+005)

a6 = -7.171e+004 (-6.267e+005, 4.833e+005)

b6 = -1.543e+005 (-3.212e+005, 1.267e+004)

a7 = -7.465e+004 (-2.451e+005, 9.577e+004)

b7 = -1.29e+004 (-2.114e+005, 1.856e+005)

a8 = -1.372e+004 (-6.02e+004, 3.277e+004)

b8 = 1.953e+004 (-4.433e+004, 8.339e+004)

w = 0.08063 (0.06841, 0.09285)

Goodness of fit:

SSE: 1.814e+009

R-square: 0.999

Adjusted R-square: 0.9985

RMSE: 6820

CONCLUSION

For the purpose of equity valuation, it is important to assess the true fundamental economic strengths of a firm. Over the past decade, the Residual Income Model (RIM) has become widely accepted as a theoretical framework for equity valuation based on fundamental information from accounting data. Successful applications of the RIM are desirable to contribute a fundamental perspective to pricing decisions (Huang and Song 2005). In this study we examined the effects of discounted residual income on stock price (firm's value) based on method curve fitting with sinusoidal functions. The results of research show that the research hypothesis is acceptable and there is a significant relationship between them. This result consists with result of Plenborg's study (2002). Also result shows the explanatory power of discounted residual income in Tehran Security exchange is so high. Therefore the results show Discounted Residual Income Model

(DRIM) is a suitable model for capital assets pricing in Tehran Security Exchange, because there is a significant relationship between discounted residual income and stock price.

When investors want to decide about buying or selling share or evaluating price of a stock, it's better to notice to residual income of firm. Also we suggest to future researchers to do our research's method for forecasting stock price.

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