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Solving Redundancy Allocation Problem with Repairable Components Using Genetic Algorithm and Simulation Method

M. Shahriari*

Faculty of Management, South Tehran Branch, Islamic Azad University, Tehran, Iran

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Abstract

Abstract: Reliability optimization problem has a wide application in engineering area. One of the most important problems in reliability is redundancy allocation problem (RAP). In this research, we worked on a RAP with repairable components and k-out-of-n sub-systems structure. The objective function was to maximize system reliability under cost and weight constraints. The aim was determining optimal components number of each subsystem, including the optimal number of repairmen allocated to each subsystem. Because this model belongs to Np. Hard problem, we used genetic algorithm (GA) for solving the presented model and response surface methodology (RSM) was used for tuning of algorithm parameters. Also for calculating the reliability of each subsystem (and system reliability) we used a simulation method. Finally, a numerical example was solved to test the algorithm performance.

Keywords: Redundancy Allocation Problem, k-out-of-n sub-systems, Common Cause Failures, Genetic Algorithm.

*. Shahriari.mr@gmail.com

1- Introduction

The aims of reliability systems are to find a better way for increasing the life cycle of components and systems. One of the most famous models in reliability is redundancy allocation problem (RAP) that tries to increase the system reliability by paralleling some components to each subsystem.

Fyffe et al. [8] were the first researchers who presented RAP. The object of their model was maximizing the system reliability under cost and weight constraints. They solved their model by dynamic programming. Nakagawa and Miyazaki [13] presented a nonlinear problem for reliability optimization model. Ha. C, and Kuo [10] formulated RAP with similar subsystems as a non-convex integer programming. Tillman et al. [16] reviewed 144 studies that made on reliability problems with different redundancy models and presented common reliability-redundancy method. In 1992, Chern [3] proved that RAP belongs to Np. Hard problem because of its calculation time. Many heuristic and meta-heuristic methods were used for solving this problem at that time. The heuristic methods that were presented by Sharma and Venkateswarn [15], Aggarwal [2], Aggarwal et al. [1], Gopal et al. [9] and Nakagawa and Nakashima [14] were very similar to each other. Nakagawa and Nakashima [14] compared the performance of NN, GAGI, and MSV heuristic algorithms used for solving general RAP. Ida et al. [12] and Yokota et al. [17] were the first researchers who solved RAP with series-parallel configuration and different failure modes with genetic algorithm (GA). Coit and Smith [7] presented the mathematical model of RAP for maximizing system reliability under uncertainty of components reliability and Coit, D.W. and Smith [4] solved this problem using GA. Coit, D.W., and Smith [5] made some changes on objective function of this model and solved the new model with GA.

In this study, we present a RAP with series-parallel configuration and k-out-of-n sub-systems. The components are repairable and each repairman is only able to repair the components of a specific subsystem. The objective function is to maximize system reliability and the system variables are number and type of each subsystem components and repairman. The paper is divided into five parts. The second part is problem definition. Third part deals with solving methodologies. A numerical example is presented in part four and the final part is conclusion and further studies.

2- Problem definition

In this paper, we present a single-objective RAP. The objective function of the problem is maximizing system reliability under cost and weight constraint. The subsystems are serially connected and the components in each subsystem are parallel. The configuration of subsystem is k-out-of-n. The components are repairable and each repairman can only work on the components of the allocated subsystem. The aim of model is determining the number and type of components in each subsystem, including the number of repairmen allocated to each subsystem. Because calculating the total repair time of the components in each subsystem in addition to the reliability of the system is very complex, we used a simulation method for calculating them.

2-1- Model assumptions

Some basic assumptions of model were as follows:

- The system is series-parallel,
- Components have constant failure rate (CFR),
- The number of the subsystem is deterministic,
- For each subsystem, different component types are available,
- The components are different based on cost and operation rate,
- The components are repairable,
- It is possible to allocate different component types for each subsystem.

2-2- Nomenclatures

$R(t)$:	System reliability at time t,
s:	Number of subsystems,
i:	Subsystems index, $i = 1,2,\ldots,s$,
k_i:	Minimum number of components in subsystem i,
v_i:	Maximum number of components in subsystem i,
j:	Component type index, $j = 1,2,3,4$
n_{ij}:	Number of component type j that allocated to subsystem i,
w_{ij}:	Weight of component type j that allocated to subsystem i,
W:	Upper bound of system weight,
c_{ij1}:	Price of component type j that allocated to subsystem i,
c_{i2}:	Price of recruitment each repairman for subsystem i,
c_{i3}:	The time dependent repair cost of components that allocated to subsystem i,
C:	Upper bound of system cost,
m_i:	Number of repairman recruitment for repairing components of subsystem i,

t_i : Total repairing time of component in subsystem i ,

t : Mission time of system.

2-3- Mathematical model

The mathematical model is as follows:

$$Max \qquad R(t) = \prod_{i=1}^{s} R_i(t) \tag{1}$$

$$S.t: \tag{2}$$

$$k_i \leq \sum_{i=1}^{4} n_{ij} \leq v_i \tag{3}$$

$$\sum_{i=1}^{s} \sum_{j=1}^{4} w_{ij}.n_{ij} \leq W \tag{4}$$

$$\sum_{i=1}^{s} \sum_{j=1}^{4} c_{ij1}.n_{ij} + \sum_{i=1}^{s} \left(c_{i2}.m_i + c_{i3}.t_i\right) \leq C \tag{5}$$

3- Solving method

We used simulation method for determining the total repair time of the components in each subsystem and calculating reliability of each subsystem as well as whole system, and a general GA for determining the optimal system parameters.

3-1- Genetic algorithm

Genetic algorithm (GA) is one of the most applicable algorithms for solving RAP. This algorithm was presented by Holland [11], in 1992. The pseudocode of this algorithm presented in Figure 1.

Procedure: Genetic Algorithm

Step 1: Set t:=0

Step 2: Generate initial population, p(t).

Step 3: Evaluate p(t) to create fitness values

Step 4: While (not termination coordination) do:

Step 5: Recombine p(t) to yield c(t), selecting from p(t) according to the fitness values.

Step 6: Evaluate c(t)

Step 7: Generate p(t+1) from p(t) and c(t)

Step 8: Set t:=t+1

Step 9: End.

Step 10: Stop

Fig 1. Pseudocode of GA

3-1-1. Chromosome

The chromosome used for illustrating the solution of the presented model is presented in Figure 2.

3-1-2. Algorithm operators

We used crossover, mutation operators for creating new generation and elitism to have good solutions in new generation.

3-1-3. Crossover operator

For crossover operator, we select two parents and then generate an integer number between 1 and s. Next, we change the first part of the first parent with the second part of second parent and the second part of the first parent with the first part of second parent as shown in Figure 3.

	Sub-system 1	2	...	s
Component type 1	n_{11}	n_{21}	...	n_{s1}
Component type 2	n_{12}	n_{22}	...	n_{s2}
Component type 3	n_{13}	n_{23}	...	n_{s3}
Component type 4	n_{14}	n_{24}	...	n_{s4}
Repairman	m_1	m_2	...	m_s

Figure 2. Chromosome of the problem

Integer Random Variable Between 1 and 14=8

Parent 1

Component type \ Sub-sysytem	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	2	2	1	0	0	0	2	1	2	0	0	1	1	0
2	0	2	1	2	2	0	2	1	2	2	0	0	0	4
3	1	0	1	1	0	1	1	2	0	2	2	0	0	0
4	0	0	0	1	2	1	0	0	0	2	2	3	1	0
m	1	1	0	1	1	0	2	1	1	2	1	1	0	1

Parent 2

Component type \ Sub-sysytem	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	2	0	2	1	0	2	4	0	1	2	2	0	2
2	1	1	2	2	1	2	0	2	0	1	4	0	1	2
3	0	1	2	0	0	2	0	0	0	2	0	2	2	1
4	0	2	0	2	2	0	2	0	4	0	0	0	1	0
m	0	1	1	2	1	1	1	2	1	1	2	1	1	1

Offspring 1

Component type \ Sub-sysytem	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	2	2	1	0	0	0	2	1	0	1	2	2	0	2
2	0	2	1	2	2	0	2	1	0	1	4	0	1	2
3	1	0	1	1	0	1	1	2	0	2	0	2	2	1
4	0	0	0	1	2	1	0	0	4	0	0	0	1	0
m	1	1	0	1	1	0	2	1	1	1	2	1	1	1

Offspring 2

Component type \ Sub-sysytem	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	2	0	2	1	0	2	4	2	0	0	1	1	0
2	1	1	2	2	1	2	0	2	2	2	0	0	0	4
3	0	1	2	0	0	2	0	0	0	2	2	0	0	0
4	0	2	0	2	2	0	2	0	0	2	2	3	1	0
m	0	1	1	2	1	1	1	2	1	2	1	1	0	1

Figure 3. Crossover operator

3-1-4. Mutation operator

For mutation operator, we select one parent and then generate an integer number between 1 and s. Then, we change the first part of the parent with the second part as shown in Figure 4.

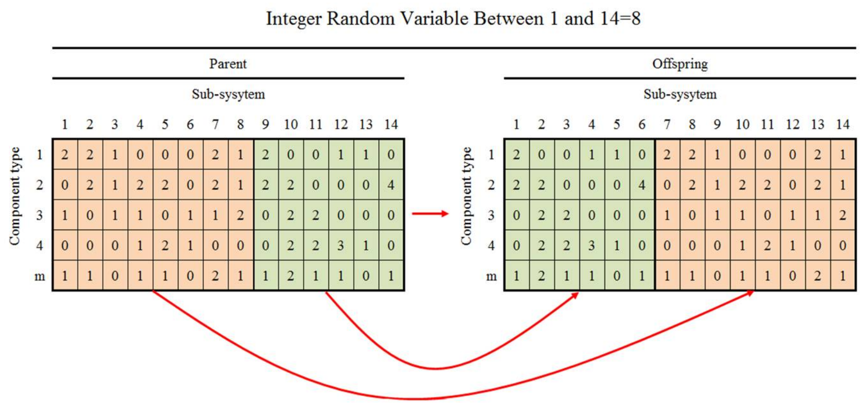

Figure 4. Mutation operator

3-1-5. Penalty function

For evaluation of each chromosome, we consider a fitness function, which considers the reliability of each chromosome. After generating initial chromosomes, crossover, and mutation operator, the new chromosomes may be infeasible. For avoiding infeasible selection of chromosomes, we consider a penalty function for fitness function. The penalty function presented in Equations (6) to (8) are as follows:

$$\text{Fitness Function} = \frac{R(t)}{PF_1 \times PF_2} \qquad (6)$$

$$PF_1 = MAX\left\{ \frac{\sum_{i=1}^{s} \sum_{j=1}^{4} w_{ij} n_{ij}}{W}, 1 \right\} \qquad (7)$$

$$PF_1 = MAX\left\{ \frac{\sum_{i=1}^{s} \sum_{j=1}^{4} c_{ij1}.n_{ij} + \sum_{i=1}^{s} (c_{i2}.m_i + c_{i3}.t_i)}{C}, 1 \right\} \qquad (8)$$

3-2- Simulation method

For calculating fitness function of each created chromosomes, we used a simulation method. The flowchart of the event is presented in Figures 4 and 5.

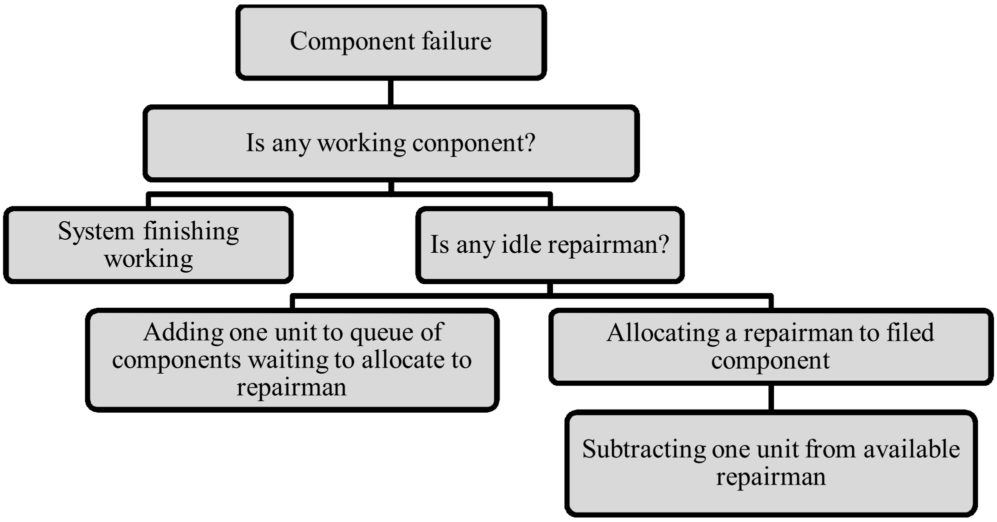

Fig 4. Component failure diagram of simulation

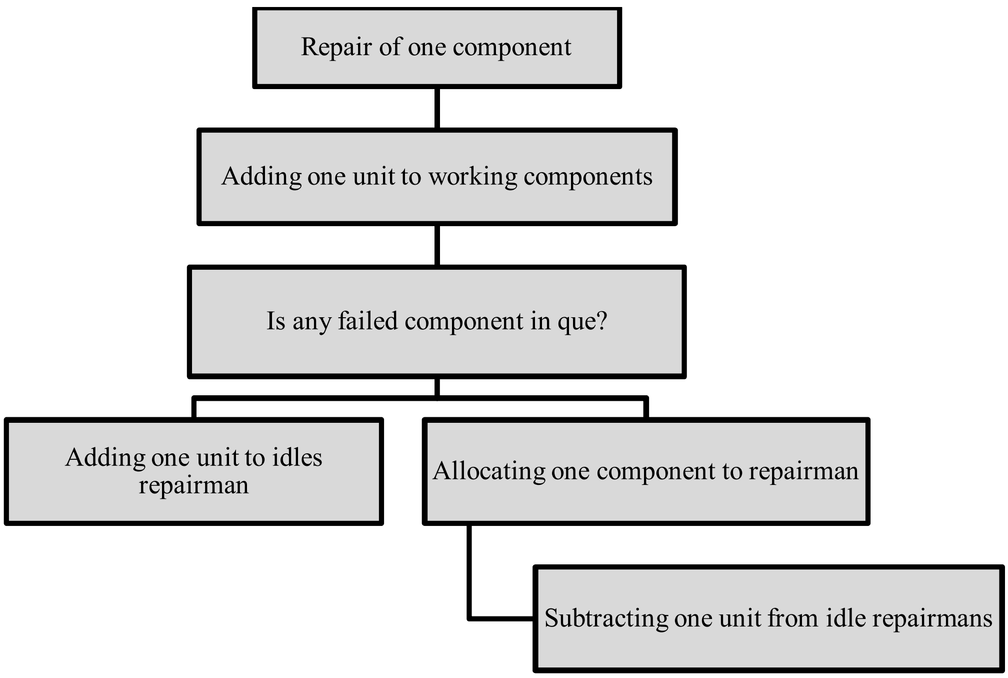

Figure 5. Component Repair diagram of simulation

3-3- Parameter tuning

We used response surface methodology (RSM), for algorithm parameter tuning. The algorithm parameters are population size ($nPop$), crossover operator probability (P_c), mutation operator probability (P_m), and algorithm iteration ($MaxIt$). The boundaries and optimum values of the algorithm parameters are presented in Table 1.

Table 1. The boundaries and optimum values of algorithm parameters

	Optimal value	**Upper value**	**Lower value**
nPop	50	100	100
P_c	0.4	0.7	0.63
P_m	0.1	0.3	0.1
MaxIt	100	200	200

4- Numerical example

For illustrating the performance of the presented algorithm, we solve a numerical example. This example has been already solved by Coit [7]. The parameters of examples are presented in Table 2. The other parameters are $(n_{\max}=6)$, $W=220$, $C=500$ and $t=100$. The values of c_{i2} and c_{i3} are calculated in Equations (09) and (10).

	Choice 1 (j=1)			Choice 2 (j=2)			Choice 3 (j=3)			Choice 4 (j=4)		
i	λ_{i1}	c_{i1}	w_{i1}	λ_{i2}	c_{i2}	w_{i2}	λ_{i3}	c_{i3}	w_{i3}	λ_{i4}	c_{i4}	w_{i4}
1	0.005320	1	3	0.000726	1	4	0.00499	2	2	0.008180	2	5
2	0.008180	2	8	0.000619	1	10	0.00431	1	9	-	-	-
3	0.013300	2	7	0.011000	3	5	0.01240	1	6	0.004660	4	4
4	0.007410	3	5	0.012400	4	6	0.00683	5	4	-	-	-
5	0.006190	2	4	0.004310	2	3	0.00818	3	5	-	-	-
6	0.004360	3	5	0.005670	3	4	0.00268	2	5	0.000408	2	4
7	0.010500	4	7	0.004660	4	8	0.00394	5	9	-	-	-
8	0.015000	3	4	0.001050	5	7	0.01050	6	6	-	-	-
9	0.002680	2	8	0.000101	3	9	0.000408	4	7	0.000943	3	8
10	0.014100	4	6	0.006830	4	5	0.001050	5	6	-	-	-
11	0.003940	3	5	0.003550	4	6	0.003140	5	6	-	-	-
12	0.002360	2	4	0.007690	3	5	0.013300	4	6	0.011000	5	7
13	0.002150	2	5	0.004360	3	5	0.006650	2	6	-	-	-
14	0.011000	4	6	0.008340	4	7	0.003550	5	6	0.004360	6	9

$$c_{i2} = 3.\sum_{j=1}^{4} c_{ij1} \quad ; \quad i = 1,3,6,9,12,14$$
$$c_{i2} = 4.\sum_{j=1}^{3} c_{ij1} \quad ; \quad i = 2,4,5,7,8,10,11,13 \tag{9}$$

$$c_{i3} = \frac{c_{i2}}{10} \quad ; \quad i = 1,2,\ldots,14 \tag{10}$$

We solved the presented example with GA using simulation methods and the optimal solution for this example with 20 times running the algorithm is presented in Figure 6. The total cost and weight of the problem are $C = 499.9375$ and $W = 219$ and the optimal system reliability is $R(100) = 0.9963$.

5- Conclusion and further studies

In this paper we presented a RAP with series-parallel configuration and k-out-of-n subsystems. The components are repairable and each repairman is only able to repair the components of specific subsystem. The objective function is maximizing system reliability and the system variables are number and type of each subsystem components and repairman. We used simulation method for calculating system reliability and GA for solving the numerical example. As we expected, the reliability of the system is greater than the reliability of the system with non-repairable components.

For further studies different assumptions can be considered. For example, complex configuration can be considered for subsystems. Also different meta-heuristic algorithms can be used for solving the presented model to compare their results with the results of this algorithm.

Sub-sysytem

Component type	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	2	1	1	0	0	3	1	2	0	0	1	0	2
2	0	2	1	2	2	0	0	0	1	0	2	1	1	0
3	1	0	0	0	0	0	0	2	0	2	0	0	2	0
4	2	0	0	0	0	2	0	0	0	0	0	1	0	0
m	1	1	0	1	1	1	1	1	1	2	1	1	2	1

Figure 6. The optimal chromosome of problem

References

[1] Aggarwal, K. K, Gupta, J. S. and Misra, K. B., “A New Heuristic Criterion for Solving a Redundancy Optimization Problem”, IEEE Transactions on Reliability 1975; Vol. 24, pp. 86-87.

[2] Aggarwal, K. K., “Redundancy Optimization in General Systems”, IEEE Transaction on Reliability, 1976; Vol. 25, pp. 330-332.

[3] Chern, M. S., “On the Computational Complexity of Reliability Redundancy Allocation in a Series System”, Operation Research Letters 1992; Vol. 11, pp. 309-315.

[4] Coit, D. W. and Smith, A., “Considering Risk Profiles in Design Optimization for Series Parallel Systems”, Proceedings of the Reliability & Maintainability Symposium, Philadelphia, 1997.

[5] Coit, D. W. and Smith, A., “Redundancy Allocation to Maximize a Lower Percentile of the System Time-to-Failure Distribution”, IEEE Transactions on Reliability 1998; Vol. 47, pp. 79-87.

[6] Coit, D. W. and Smith, A., “Stochastic Formulations of the Redundancy Allocation Problem”, Proceedings of the Fifth Industrial Engineering Research Conference, Minneapolis, 1996.

[7] Coit, D. W., “Maximization of System Reliability with a Choice of Redundancy Strategies”, IIE Transactions 2003; Vol. 35, No. 6, pp. 535-544.

[8] Fyffe, D. E., Hines, W.W. and Lee, N.K., “System Reliability Allocation and Computational Algorithm”, IEEE Transactions on Reliability 1968; Vol. 17, pp.64-69,

[9] Gopal, K., Aggarwal, K.K., and Gupta, J. S., “An Improved Algorithm for Reliability Optimization”, IEEE Transactions on Reliability 1978; Vol. 27, pp. 325–328.

[10] Ha, C. and Kuo, W., “Reliability Redundancy Allocation: An Improved Realization for Non-convex Nonlinear Programming Problems”, European Journal of Operational Research 2006; Vol. 171, pp. 24-38.

[11] Holland, J., 1992, “Adaptation Natural and Artificial ststems”, University of Michigan

[12] Ida, K., Gen, M. and Yokota, T., “System Reliability Optimization with Several Failure Modes by Genetic Algorithm”, Proceeding of the 16th International Conference on Computers and Industrial Engineering, Ashikaga of Japan, 1994.82.

[13] Nakagawa, Y. and Miyazaki, S., Surrogate Constraints Algorithm for

Reliability Optimization Problems with Tow Constraints, IEEE Transaction on Reliability 1981; Vol.30, pp. 175-180.

[14] Nakagawa, Y. and Nakashima, K., "A Heuristic Method for Determining Optimal Reliability Allocation", IEEE Transaction on Reliability 1977; Vol. 26, pp. 156-161.

[15] Sharma, J. and Venkateswarn, K.V., "A Direct Method for Maximizing the System Reliability", IEEE Transactions on Reliability 1971; Vol. 20, pp. 256-259.

[16] Tillman, F. A., Hwang, C.L. and Kuo, W., "Determining Component Reliability and Redundancy for Optimum System Reliability", IEEE Transactions on Reliability, 1977; Vol. 26, pp. 162-165.

[17] Yokota, T., Gen, M. and Ida, K., "System Reliability of Optimization Problems with Several Failure Modes by Genetic Algorithm", Japanese Journal of Fuzzy Theory and systems 1995; Vol. 7, pp. 117-1.