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Synchronization of a Heart Delay Model with Using CPSO Algorithm in Presence of Unknown Parameters

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Abstract
Heart chaotic system and the ability of particle swarm optimization (PSO) method motivated us to benefit the method of chaotic particle swarm optimization (CPSO) to synchronize the heart three-oscillator model. It can be a suitable algorithm for strengthening the controller in presence of unknown parameters. In this paper we apply adaptive control (AC) on heart delay model, also examine the system stability by the Lyapunov stability theorem. Then we improve results with using CPSO algorithm and define an appropriate cost function. At the end of we implement the proposed approach on an example.

Keywords: Chaotic particle swarm optimization (CPSO), adaptive control (AC), heart three-oscillator model, Nonlinear oscillators, Synchronization.

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1. Introduction

Theory of synchronization is a recent research area that extensively investigated nowadays in many fields including mechanical systems, robotics, biological systems and etc. Synchronization of chaos is a phenomenon that may occur when two or more chaotic systems are coupled or one chaotic system drives the other. The problem of synchronization of chaotic systems in presence of environmental noise or the parameters of the uncertainty becomes more challenging [5, 7, 8, 11, 12, 21, 24]. The use of synchronization of chaos in secure communication is certainly the main reason for the success for such research fields. There are many control techniques to synchronize chaotic systems such as: linear control, AC and active control. AC technique is applied for synchronizing two different chaotic systems to design an appropriate controller. Generally, in this method the aim is to find a controller and also a law for updating the parameters, so that the state variables of slave and master systems are synchronized With the each other asymptotically, also the Lyapunov stability theorem is used to check system stability. Chung used a PID controller for the synchronization two exactly identical chaotic systems started from different initial conditions by applying the PSO algorithm [٣]. The method presented in [٣], is based on minimizing the summation of the norm of synchronization error in discrete model of the system via PSO algorithm [1, 16, 22, 23, 25]. Mathematical modelling of heart rhythm and synchronization of heart oscillators are the goal of many research efforts [13]. Cardiac conduction system can be considered as a network of self-stimulate elements. In the heart three-oscillator system, each oscillator is represented one of the heart natural important pacemaker: atrial sinus (AS) node and atrial ventricular (AV) and His-Purkinje complex (HP). Because these elements show oscillation behavior, they can be modeled as nonlinear oscillators. Proposed oscillator model is designed in order to reproduce time series of action potential of natural pacemaker's cardiac, Therefore heart system is presented by a system of differential equations. There are different methods for evaluating cardiac function by measuring certain signals [15]. As a case study the presented method is applied to nonlinear oscillators of heart system [13]. For modeling of cardiac pacemaker, the Vander Pol oscillator is offered which is considered as follows

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -d(x_1^{\,2} - 1)x_2 - x_1 + a\cos\omega t, \end{cases} \tag{1}$$

Where x_1, x_2 are Flow and voltage variables and a, d, ω are system parameters, that based on their amounts the heart dynamics may be chaotic or non chaotic.

There are numerous papers in areas AC [2, 9, 14, 18] and CPSO algorithm [1, 3, 16, 22, 23, 25] to synchronize, that we combine these two series on heart three-oscillator model.

In this paper, we propose a controller based on CPSO algorithm to synchronize two different chaotic systems in presence of unknown parameter. We synchronize heart three-oscillator model with AC in presence of unknown parameters and also checked system stability by the Lyapunov stability theorem. Then the parameters of controller adapt according to CPSO rule to minimize an appropriate cost function.

The article is organized as follows: In Section 2, we describe the CPSO algorithm and synchronization method, In Section 3, we present the heart mathematical model, In Section 4, we apply AC on heart delay model, then improve results with using CPSO algorithm, Section 5 contains implementation of the method and simulation and Section 6 is the conclusions.

2. CPSO algorithm and synchronization method

2.1. PSO algorithm

PSO algorithm which was proposed by Kennedy and Eberhart in 1995 is a relatively new optimization method for nonlinear system optimization. This algorithm can handle unknown parameters by online adapting the controller's gains to pursue its goal to optimize the cost function, since it is not significantly sensitive to the dynamic of model.

This technique involves simulating social behavior among particles that fly through a multidimensional search space. Particles would evaluate their positions or fitness levels with respect to the objective function in each of the iteration. In PSO algorithm the i^{th} particle x_i is defined as a potential solution in D-dimensional space, where

$$\boldsymbol{x}_i = (\boldsymbol{x}_{i_1}, \boldsymbol{x}_{i_2}, ..., \boldsymbol{x}_{i_D}).$$

Changing of the position of each particle from one to another iteration can be computed according to the distance between the current position and its previous best position and distance between the current position and the best particle among the neighborhood of particle in the population. Then, updating of the velocity and the particle position can be obtained by the following equations

$$v_i = w.v_{i-1} + c_1 \times r_1 \times (x_{i_{gbest}} - x_i) + c_1 \times r_2 \times (x_{i_{pbest}} - x_i) \quad (2)$$
$$x_{i+1} = x_i + v_i$$

Where w is the inertia weight and r_1, r_2 are two random values in the range [0, 1], $x_{i_{gbest}}$ is the best particle among the neighborhood of particle in the population and $x_{i_{pbest}}$ is the best historical position for particle x_i, also c_1, c_2 are the acceleration constants that represent the weighting of the stochastic acceleration terms that pull each particle x_i towards $x_{i_{gbest}}$ and $x_{i_{pbest}}$ positions. We use the Von Neumann or square topology in the canonical particle swarm than either of the traditional topologies. It is formed by arranging the population in a grid and connecting neighbors above, below, and to the right and left. The edges of the matrix are wrapped, **Figure 1**.

According to existing literature, c_1 and c_2 are often set to be 2. In order to reduce the number of iterations required to reach the optimal solution, a suitable selection of inertia weight (w) is introduced to provide a balance between global and local explorations. The inertia weight normally decreases linearly from 0.9 to 0.4 during the optimization process. The inertia weight can be set according to the equation (3),

$$w = w_{max} - \frac{w_{max} - w_{min}}{iter_{max}} \times iter, \quad (3)$$

Where $iter_{max}$ is the maximum number of iterations (generations), and $iter$ is the current number of iterations.

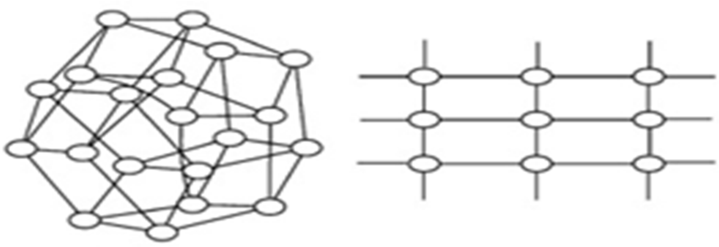

Figure 1. Von Neumann or square topology

2.2. CPSO algorithm

Because of premature convergence PSO algorithm, especially for search spaces with several local optima's, CPSO algorithm was developed based on the ergodicity of chaos optimization and the evolvement of PSO to improve the global convergence [25]. Moreover, in [16] and [25] is stated that chaos-based optimization algorithms can carry out overall searches at higher speeds than stochastic ergodic searches that depend on probabilities. We assume that chaotic maps are adopted to select the numerical values for the parameters of the particles velocity equation (2). Different numerical studies have been conducted for choosing the better chaotic maps. In order to enrich the searching behavior and to avoid being trapped into local optimum, the well-known Lorenz equations are employed for the hybrid PSO. That defined as follows

$$\begin{aligned} \dot{x} &= \delta (x - y) \\ \dot{y} &= \rho x - y - xz \\ \dot{z} &= -\beta z + xy \end{aligned} \tag{4}$$

Where

$$(x(0), y(0), z(0)) = (0,1,0)$$
$$(\delta, \rho, \beta) = (10, 28, 8/3)$$

2.3. The synchronization method

The synchronization of chaos is a process in which two or more identical or no identical chaotic systems with a distinct feature of motion set through a foreign force to achieve a set of common behaviors. The most common configuration for synchronous systems is considered as two subsystems which are coupled, one of them as slave system and the other as the master system. Purpose of synchronous is that slave system follows dynamics of master system. Chaotic systems according to the type and intensity of the coupling between the two systems are synchronized with each other via different methods, [4, 15]. For clarification, consider the following relationship between slave and master systems, respectively

$$\begin{aligned} \dot{u}(t) &= f_u(u, t) \\ \dot{v}(t) &= f_v(v, t) \end{aligned} \tag{5}$$

Where $u \in R^n, v \in R^n$ the system state variables, f_u and f_v are chaotic nonlinear functions. These two systems can be synchronized by applying an appropriate control signal and the special relationship between their trajectories. In this case we have

$$\lim_{t \to \infty} \left\| D_1(u(t)) - D_2(v(t)) \right\| \to 0 \tag{6}$$

Functions D_1 and D_2 determine the optimum relationship between state variables systems. In fact, they specify the type of synchronization.

3. Modeling of the heart

3.1. Record of the heart electrical activity

The walls of the heart muscle called myocardium, which consists of four cavities; such as right and left atrium (Upper part), and right and left ventricles (lower part) [13]. There are different methods for the assessment of cardiac function which is done by measuring certain signals including tape electrocardiogram (ECG) which is the recording of the waves of electrical activity of the heart by putting electrodes on the chest and around the heart. Each of the components on tape indicates the electrical activity of the heart cells. Stimulated sinus node causes electrical stimulation of the atria and creates a P wave that is the wave of atrial depolarization. After the wave of ventricular depolarization, the QRS complex(The combination of three waves Q , R and S) series arise and the subsequent wave creates T wave by ventricular repolarization. Sometimes another small wave after wave of T is seen that is called U wave. The reason of creation of the wave is repolarization papillary muscles in the heart. In distance of these waves there is not appropriate electrical activity and electro gram draws a horizontal straight line that is called the isoelectric line [10, 13].

3.2. Mathematical model of the heart

The idea of heartbeat system modeling with coupled nonlinear oscillators was explained in 1928 by Vander Pol and Vander Mark. Cardiac conduction system may be assumed as a self-stimulate pacemaker that is composed of two oscillator subsystems. The first subsystem is AS node that has the highest pulse (60-100 impulse per minute) between other oscillators of heart. The second subsystem consists of AV node (40-60 impulses per minute) [4].However, it was observed that the two oscillators for producing ECG signals are not very accurate. The reason is that the signal of first oscillator is related to the activation of AS node and right atrium and the signal of second oscillator is only related to the left ventricular depolarization. According to this hypothesis, it is possible to produce P curve, but QRS complex may not be produced, because this distance is mainly due to ventricular repolarization. These observations tempted us to incorporate a third oscillator, which represents the spread of a pulse through the heart that indicate HP in a physiological way [19, 26]. In order to create a general model, we assume that all oscillators should be coupled asymmetry. In addition, external stimulation is entered into the system with regard to the oscillator frequency. This developed model can be shown with a set of differential equations as follows

$$AS:\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = -d_1(x_1^2-1)x_2 - c_1x_1 + a_1\cos\omega t \\ \quad + R_{13}(x_1-x_3) + R_{15}(x_1-x_5)\end{cases} \quad (7)$$

$$AV:\begin{cases}\dot{x}_3 = x_4 \\ \dot{x}_4 = -d_2(x_3^2-1)x_4 - c_2x_3 + a_2\cos\omega t \\ \quad + R_{31}(x_3-x_1) + R_{35}(x_3-x_5)\end{cases}$$

$$HP:\begin{cases}\dot{x}_5 = x_6 \\ \dot{x}_6 = -d_3(x_5^2-1)x_6 - c_3x_5 + a_3\cos\omega t \\ \quad + R_{51}(x_5-x_1) + R_{53}(x_5-x_3)\end{cases}$$

Where pairs (x_1, x_2) , (x_3, x_4), and (x_5, x_6) respectively, show AS, AV and HP oscillators. Parameters of this model introduce in Table 1.

Because even small delays may Change the dynamics of the system, differential equations by incorporating time delay can cause drastic changes and creation of chaos in the system that has been described by the regular behavior, [19, 20, 26]. Accordingly proposed mathematical model can change to consider the aspects of delay in coupled terms. Thus the governing equations are changed as below, where $x_i = x_i(t)$, also $x_i^{\tau} = x_i(t-\tau)$ and τ is time delay

$$AS: \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -d_1(x_1^2 - 1)x_2 - c_1 x_1 + a_1 \cos\omega t \\ \quad + R_{13}(x_1 - x_3^{\tau_{13}}) + R_{15}(x_1 - x_5^{\tau_{15}}) \end{cases} \quad (8)$$

$$AV: \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -d_2(x_3^2 - 1)x_4 - c_2 x_3 + a_2 \cos\omega t \\ \quad + R_{31}(x_3 - x_1^{\tau_{31}}) + R_{35}(x_3 - x_5^{\tau_{35}}) \end{cases}$$

$$HP: \begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = -d_3(x_5^2 - 1)x_6 - c_3 x_5 + a_3 \cos\omega t \\ \quad + R_{51}(x_5 - x_1^{\tau_{51}}) + R_{53}(x_5 - x_3^{\tau_{53}}) \end{cases}$$

Table1. Parameters of three-oscillator system

Definition	Parameters	Value
AS frequency	c_1	1
AV frequency	c_2	2
HP frequency	c_3	1
AS voltage range	a_1	5
AV voltage range	a_2	6
HP voltage range	a_3	4
AS damping coefficient	d_1	-40
AV damping coefficient	d_2	6
HP damping coefficient	d_3	7
Frequency	ω	2.04
coupling coefficients between x_i and x_j	R_{ij}	R_{13}=1,R_{31}=2 R_{15}=1,R_{51}=2 R_{35}=1,R_{53}=3
AS membrane flow	x_1	0.02
AS membrane voltage	x_2	0.7
AV membrane flow	x_3	0
AV membrane voltage	x_4	0
HP membrane flow	x_5	0
HP membrane voltage	x_6	0

4. Designing proper control signal

4.1. Problem definition

Consider the following two systems defined by

$$\begin{aligned} \dot{x} &= f(x) \\ \dot{y} &= g(y) + u \end{aligned} \tag{9}$$

Where $f(x)$ and $g(y)$ are nonlinear chaotic functions in R^n. Here the first system is considered as master (drive) system and the other one as the slave (response) system. Let us suppose that the functions $f(x)$ and $g(y)$ are totally different nonlinear functions. Our goal is to design a control law for the slave system in such a way that both master and slave systems are synchronized. This means that the error state vector $e = y - x$ converges to zero as time converges to infinity [7]. During the synchronization of systems, it is possible that the system parameters be unknown. Therefore in this case for synchronization of systems, we use AC methods. By applying appropriate input controller to oscillators of model (8), we have

$$AS: \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -d_1(x_1^{\,2} - 1)x_2 - c_1 x_1 \\ \qquad + a_1 \cos \omega t + R_{13}(x_1 - x_3(t - \tau_{13})) \\ \qquad + R_{15}(x_1 - x_5(t - \tau_{15})) \end{cases}$$

$$AV: \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -d_2(x_3^{\,2} - 1)x_4 - c_2 x_3 \\ \qquad + a_2 \cos \omega t + R_{31}(x_3 - x_1(t - \tau_{31})) \\ \qquad + R_{35}(x_3 - x_5(t - \tau_{35})) + u_1 \end{cases} \tag{10}$$

$$HP: \begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = -d_3(x_5^{\,2} - 1)x_6 - c_3 x_5 \\ \qquad + a_3 \cos \omega t + R_{51}(x_5 - x_1(t - \tau_{51})) \\ \qquad + R_{53}(x_5 - x_3(t - \tau_{53})) + u_2 \end{cases}$$

4.2. AC based CPSO algorithm

In this method the controller is designed and is added to the slave system such that nonlinear parts of error dynamic between master and slave systems are eliminated. Also the Lyapunov stability theorem is used to check system stability. In the following we study synchronization of oscillators AS, AV and HP in case of time delay and that two of its parameters are unknown. Assume that all parameters are known and d_2, d_3 are unknown, **Figures 2-5**. We consider parameter error ($\tilde{d}_i$, $i = 2,3$) and the synchronization error $e_i(t)$,$i = 1,2,3,4$ as below [9, 18].

$$\tilde{d}_2 = d_2 - \hat{d}_2 \quad , \tilde{d}_3 = d_3 - \hat{d}_3$$

$$e_i(t) = x_i(t) - x_{i+2}(t - \tau_{i,i+2}) \ ,i = 1,2,3,4$$

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_3(t - \tau_{13})$$
$$\begin{aligned} \dot{e}_2 &= \dot{x}_2 - \dot{x}_4(t - \tau_{24}) = -d_1(x_1^{\,2} - 1)x_2 - c_1 x_1 \\ &+ a_1 \cos\omega t + R_{13}(x_1 - x_3(t - \tau_{13})) + R_{15}(x_1 - x_5(t - \tau_{15})) \\ &+ d_2(x_3^{\,2} - 1)x_4 + c_2 x_3 - a_2 \cos\omega t - R_{31}(x_3 - x_1(t - \tau_{31})) \\ &- R_{35}(x_3 - x_5(t - \tau_{35})) - u_1 \end{aligned}$$
$$\dot{e}_3 = \dot{x}_3 - \dot{x}_5$$

$$\begin{aligned} \dot{e}_4 &= \dot{x}_4 - \dot{x}_6 = -d_2(x_3^{\,2} - 1)x_4 - c_2 x_3 + a_2 \cos\omega t \\ &+ R_{31}(x_3 - x_1(t - \tau_{31})) + R_{35}(x_3 - x_5(t - \tau_{35})) + u_1 \\ &+ d_3(x_5^{\,2} - 1)x_6 + c_3 x_5 - a_3 \cos\omega t - R_{51}(x_5 - x_1(t - \tau_{51})) \\ &- R_{53}(x_5 - x_3(t - \tau_{53})) - u_2 \end{aligned}$$

Control signal with using estimation of parameters d_2 and d_3 are determined as

$$\begin{aligned} u_1 &= -d_1 x_1^2 x_2 + a_1 \cos \omega t \\ &+ \hat{d}_2 x_3^2 x_4 - c_1 x_3 - a_2 \cos \omega t \qquad (11) \\ &+ d_1 x_4 + c_2 x_5 - \hat{d}_2 x_2 + k_1 e_1 + k_2 e_2 + k_3 e_3 \\ u_2 &= -\hat{d}_2 x_3^2 x_4 + a_2 \cos \omega t \\ &+ \hat{d}_3 x_5^2 x_6 + c_3 x_5 - a_3 \cos \omega t \\ &+ \hat{d}_2 x_6 - c_2 x_5 - \hat{d}_3 x_4 + k_4 e_1 + k_5 e_3 \\ &+ k_6 e_4 + u_1 + k_7 \end{aligned}$$

To obtain the laws for updating parameters $\hat{d}_2$ and $\hat{d}_3$, the Lyapunov function method is used. Consider positive definite Lyapunov functions as

$$v_1 = \frac{e_2^2 + \tilde{d}_2^2}{2}$$

$$\begin{aligned} \dot{v}_1 &= \dot{e}_2 e_2 + \tilde{d}_2 \dot{\tilde{d}}_2 = [\tilde{d}_2 x_3^2 x_4 - \tilde{d}_2 x_4 \\ &+ (R_{13} + R_{31} + R_{15} - c_1 - k_1) e_1 \\ &+ (\hat{d}_2 + d_1 - k_2) e_2 \\ &+ (-R_{35} + R_{15} + c_2 - k_3) e_3] e_2 + \tilde{d}_2 \dot{\tilde{d}}_2 \end{aligned}$$

Thus by choosing

$$\dot{\tilde{d}}_2 = (x_2 - x_3^2 x_4) e_2$$

$$k_1 > R_{13} + R_{31} + R_{15} - c_1$$

$$k_2 > \hat{d}_2 + d_1$$

$$k_3 > -R_{35} + R_{15} + c_2$$

We have $\dot{v}_1 < 0$, also

$$v_2 = \frac{e_4^2 + \tilde{d}_2^2 + \tilde{d}_3^2}{2}$$

$$\begin{aligned} \dot{v}_2 &= \dot{e}_4 e_4 + \tilde{d}_2 \dot{\tilde{d}}_2 + \tilde{d}_3 \dot{\tilde{d}}_3 = [\tilde{d}_3 x_5^2 x_6 - \tilde{d}_3 x_4 \\ &+ (R_{51} - R_{31} - k_4) e_1 - \tilde{d}_2 x_3^2 x_4 + \tilde{d}_2 x_6 \\ &+ (\hat{d}_2 + \hat{d}_3 - k_6) e_4 + u_1 - k_7 \\ &+ (R_{35} + R_{53} + R_{51} - c_2 - k_2) e_3] e_4 + \tilde{d}_2 \dot{\tilde{d}}_2 + \tilde{d}_3 \dot{\tilde{d}}_3 \end{aligned}$$

Thus by choosing

$$\dot{\tilde{d}}_2 = (x_3^2 x_4 - x_6) e_4 \quad , \quad \tilde{d}_3 = (x_4 - x_5^2 x_6) e_4$$

$$k_2 > R_{35} + R_{53} + R_{51} - c_2 \ , \ k_4 > R_{51} - R_{31}$$

$$k_6 > \hat{d}_2 + \hat{d}_3$$

We have $\dot{v}_2 < 0$

then $\dot{v}_1$ and $\dot{v}_2$ are negative definite. Now, according to the Lyapunov stability theorem, we can conclude that systems $SA\text{-}AV\text{-}HP$ which have unknown parameters will be synchronized asymptotically with definite control function, and synchronization error approaches to zero asymptotically, **Figures 6- 9**.

To enhance the speed of synchronization, we optimize the unknown parameters in the controller with using CPSO algorithm. In our method the controller parameters ($\hat{d}_1, \hat{d}_2$) are adapted by applying CPSO algorithm which minimizes an appropriate cost function.

When the error of synchronization vanishes, the cost function reaches to its global minimum. To reach this aim, parameters of controller are considered as positions of the particles in CPSO algorithm. So the controller adapts with the CPSO algorithm in presence of unknown parameters which may exist in the system.

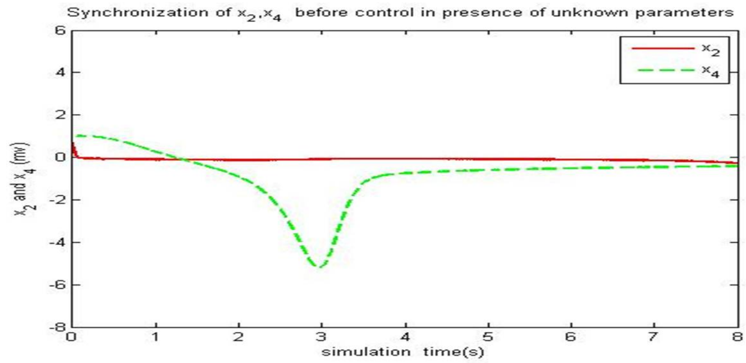

Figure 2

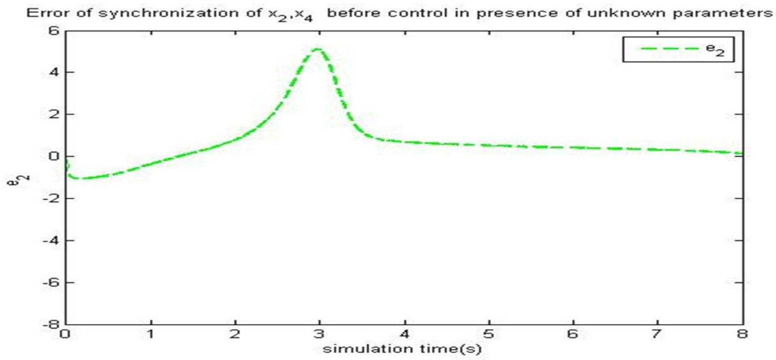

Figure 3

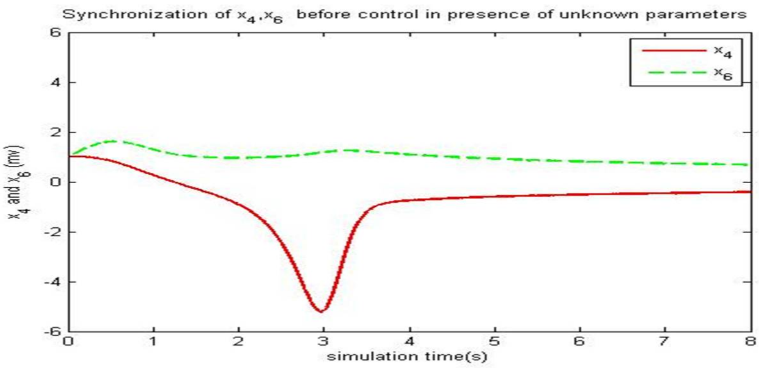

Figure 4

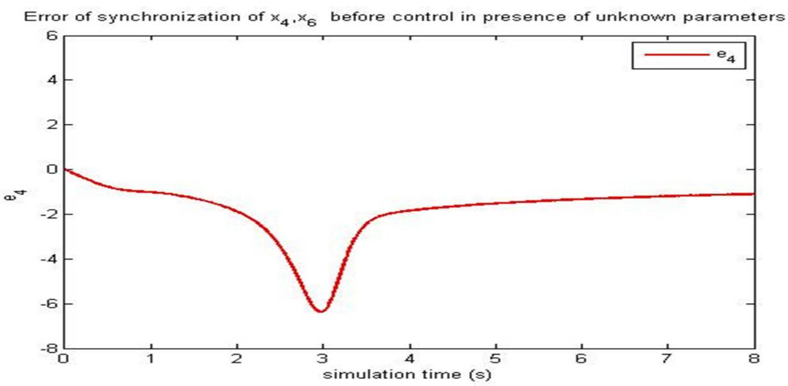

Figure 5

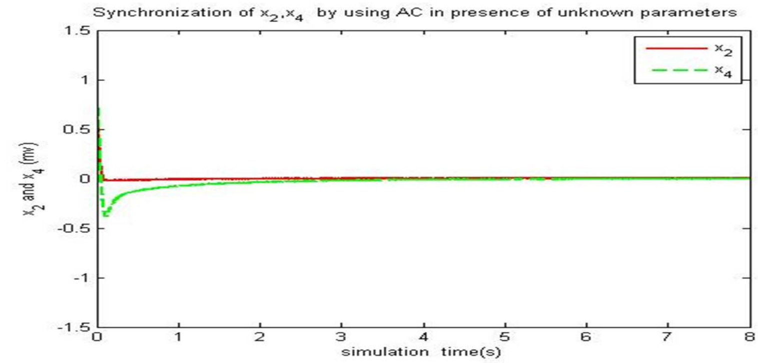

Figure 6

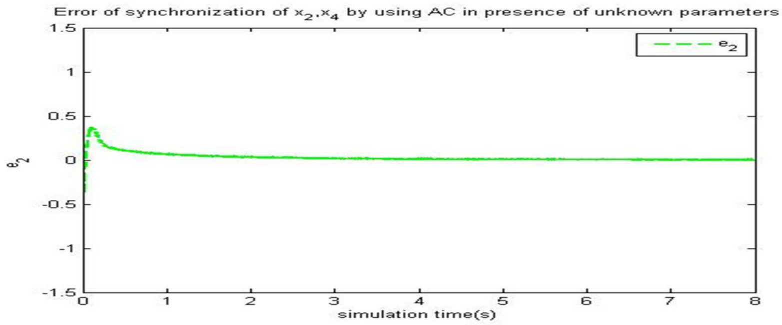

Figure 7

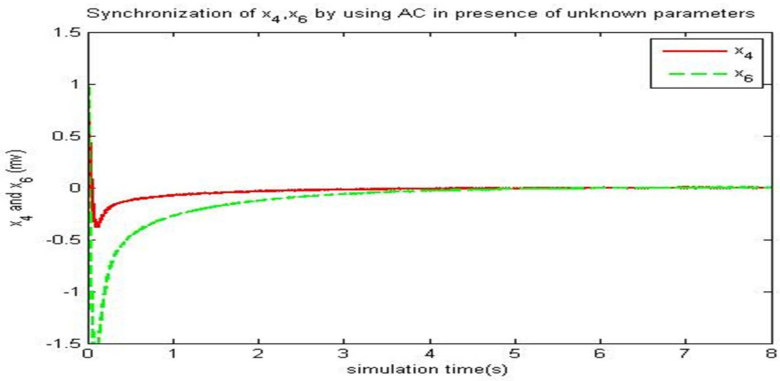

Figure 8

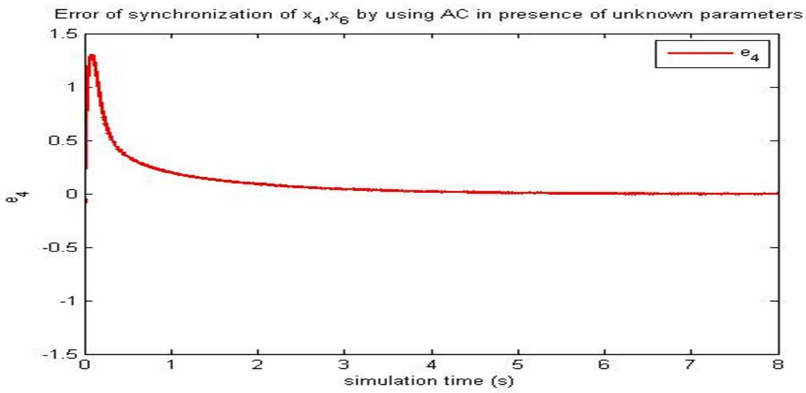

Figure 9

4.3 Cost function

Since the cost function affects on the controller performance, thus it must be defined carefully. Therefore we define the cost function as below

$$J=\int_0^T ((e_1)^2+(e_2)^2+(e_3)^2+(e_4)^2)dt \quad (12)$$

In which J is the continuous form of the cost function and T the simulation time. In many practical cases with the same initial condition, CPSO algorithm with this cost function must run several times to reach an appropriate controller. While the system is running, the position vector of the CPSO parameters is updated in ever iteration. First we define the following sequence then propose new cost function

$$\begin{aligned} &t_{i+1}=t_i+h, \quad t_1=0 \\ &0<h\le T \end{aligned} \quad (13)$$

Since we are looking for new issues, in this paper we assume that the small positive parameter h has to be selected randomly. Discussion about being variable h and its sensitivity analysis will be discussed in future work.

We propose the cost function as follows

$$J(i)=\int_{t_i}^{t_i+h} ((e_1)^2+(e_2)^2+(e_3)^2+(e_4)^2)dt \quad (14)$$

We assume h is near zero to preventchattering of response of system. Also, according to equation (13), t_i have been introduced with additional condition that for every i, $t_i \le T$. Index i indicates number of updating times of the CPSO algorithm or updating times of the parameters of controller. In addition, the cost function is calculated for each particle with the integral of summation of square of synchronization error by equation (14). Since it may easily get trapped in local optima, when the cost function does not change for a while (If during the implementation of the algorithm for several consecutive iterations (at least 30 iterations) the cost function remains constant), we reduce the parameter h.

The cost function must be defined such that its minimization in CPSO algorithm guarantees control of the system. To this aim, it can be considered as a positive semi-definite function of synchronization error, that we choose the square of the norm of error vector. While the system is running, it must be calculated online. This cost function is a function of time. It may lead to harsh changes in control signal and cause chattering in it. Especially, this case happens in the beginning when the error signal usually changes rapidly. To reduce this problem, the cost function is defined as the average of square of synchronization error vector norm in an appropriate time interval. If the time interval of integration does not change appropriately, this average may not adapt unfortunately. Unlike to beginning time, when the error signals approach to zero, this function can have inappropriate effects like chattering (especially in presence of time varying parameter uncertainties).

The most properties of proposed cost function are as follow

1. During algorithm running, it is calculated online.

2. The cost function is defined as the average of square of synchronization error vector norm in time interval $[t_i, t_{i+1}]$ This can extremely inhibit from chattering of the response of system. Also, near the crossover points it prevents from trapping in a local optima and it leads to failure of these algorithms since in these points the error signal is small.

3. In convergence, for preventing chattering of response of system, the integration interval becomes smaller until it reaches to some minimum length. The minimum interval length must be selected suitable, such that its average remains near zero and the controller could influence on behavior of the system, Thus, CPSO algorithm will be able to analyze the effectiveness of controller.

So, algorithm determines the parameters of controller as control law equation (11) to minimize the cost function defined in equation (14). The integrand of J defined in equation (14) is a positive semi-definite function, thus in order to minimize the integral it would approach to zero and it guarantees the stability of system. CPSO algorithm is applied for minimizing the cost function. While the system is running, the CPSO parameters update with calculated cost function. At the beginning of simulation, the algorithm updates rarely. Since CPSO algorithm adapts the controller in presence of unknown parameters, so uncertainties can not perturb the controller performance. The CPSO algorithm tends to minimize the cost function defined in (14) and it does not depend significantly on the parameters (k_i) of (11). Thus without any additional information, CPSO algorithm can defeat the uncertainty parameters.

5. Implementation of the method and simulation

5.1. Implementation of the method

As mentioned above, the parameters of the controller are considered as the positions of particles in swarm optimization algorithm. The adaptive method proposed as follow

Step 1: The initial positions and velocities of particles (parameters of controller) are determined randomly and changes.

Step 2: While system is running, the control signal is calculated and is applied to system by equation (11).

Step 3: The cost function is calculated for each particle by equation. (14).

Step 4: The best solution of each particle (P_i^k) and the swarm (X_g^k) are found and saved, when the time reaches the optimum point defined by equation (13), otherwise, the algorithm is followed by step 8.

Step 5: Checking of convergence of algorithm(the value of cost function convergence of zero).

Step 6: Update of the PSO algorithm via calculating the new positions and velocities of particles by equation (2).

Step 7: The interval of integration in cost function is reduced when the value of cost function does not change (Since we reduce the parameter h , equation (14)).

Step 8: Go to step 2.

5.2. Simulation

In this simulation, we solve a set of differential equations related to the master and slave systems with maximum step size 0.001. In performance of simulation, we used parameters defined in sub-section 4.1. The control signal is assumed to be saturated at u = ±60 because in real systems cannot be applied to an unbounded control law.

Case 1: **Systems with certain parameters**

In this case study, there is no uncertainty in the system and CPSO algorithm runs with the values of $pop = 30$(Population), $c_1 = c_2 = 2, w = 0.99$ where, pop represents the population size of swarm under consideration, c_1 and c_2 corresponding to the cognitive and the social parameters respectively and w is the inertia weight. Simulation results proposed in, **Figures 10-17**, that variation of cost function is versus time.

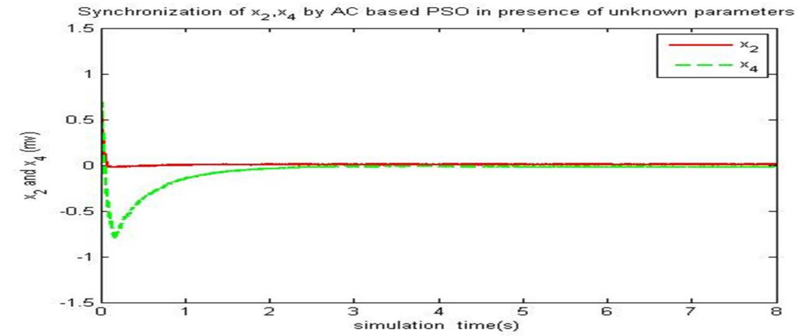

Figure 10

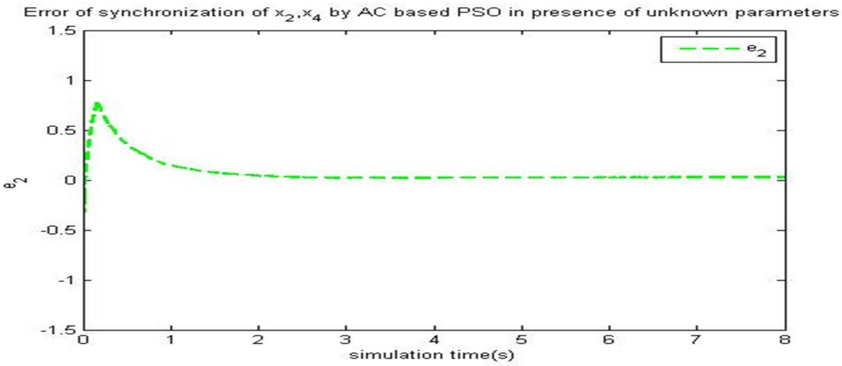

Figure 11

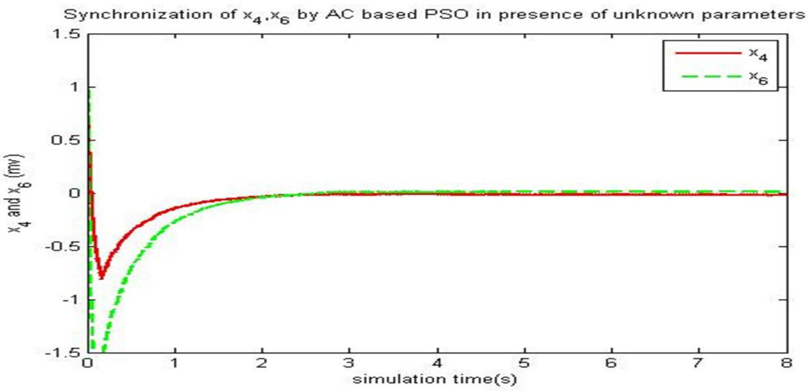

Figure 12

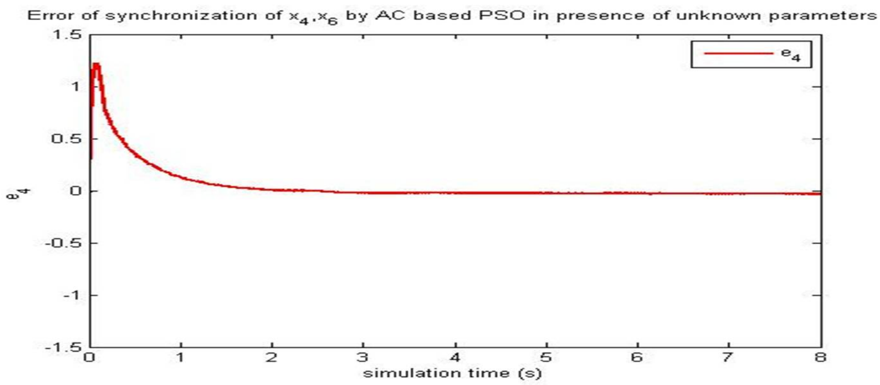

Figure 13

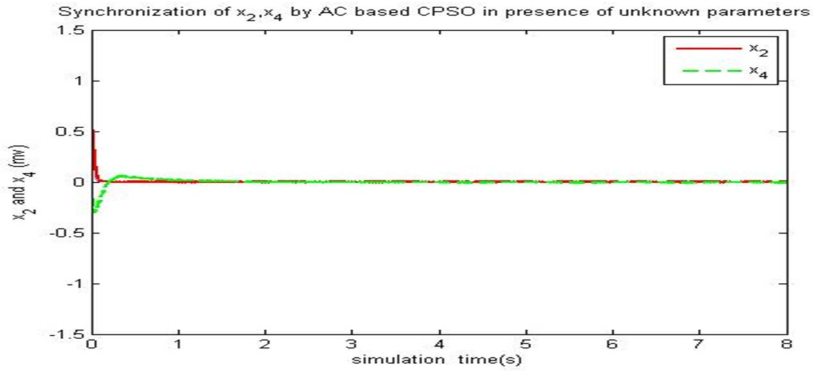

Figure 14

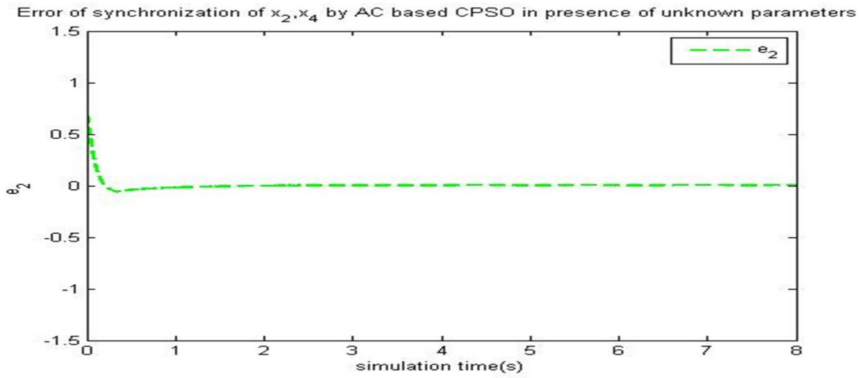

Figure 15

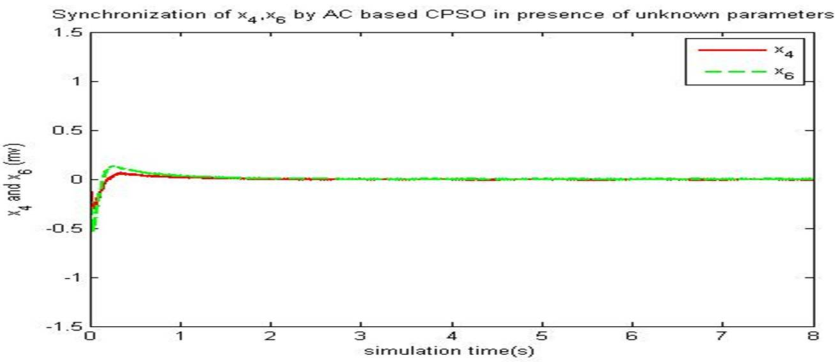

Figure 16

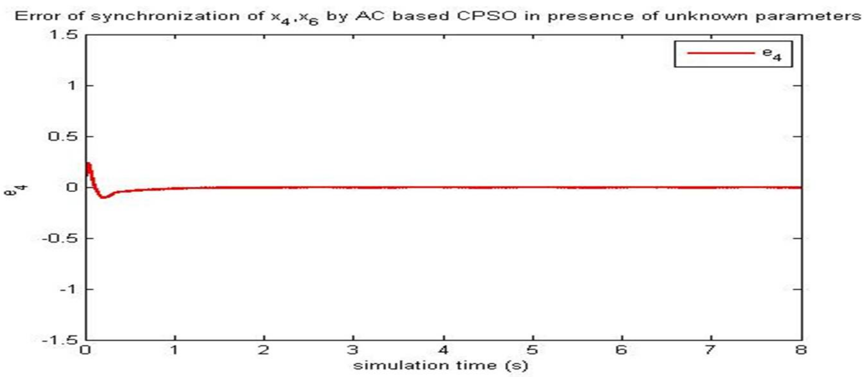

Figure 17

Case 2: Systems with uncertainty

In this case study, the parameters of CPSO algorithm run same as case 1. But this case study contains parameter uncertainty. For strengthening of controller in presence of parameters uncertainty, control law equation (11) is applied to a system which its real dynamic follows from equation (10) with following parameters, [14, 21]

$$R_{51} = 2 + 0.1 r_1(t) \quad R_{53} = 3 + 0.1 r_2(t)$$

$$d_1 = 5 + 0.1 r_3(t) \quad d_2 = 6 + 0.1 r_4(t)$$

$$d_3 = 7 + 0.1 r_4(t) \quad c_{11} = 1 + 0.1 r_6(t)$$

$$c_{12} = 2 + 0.1 r_5(t) \quad c_{13} = 1 + 0.1 r_6(t)$$

Where $r_i(t)$'s are normally distributed random functions. Simulation results are shown in **Figures 18- 21**. These results contain the variation of cost function versus time.

By choosing parameters as shown in **Table 1**, we designed controller to the model of three-oscillator system of the heart in case of time delay and in presence of unknown parameters by applying CPSO algorithm and AC.

Simulation results related to synchronization time delay problem with unknown parameters after over 50 runs are shown in **Tables 2,3**.

Case 1: With AC

Case 2: Using PSO algorithm on case1.

Case 3: Using CPSO algorithm on case1.

Case 4: Study uncertainty on case 2.

The results in **Tables 2, 3** show the advantages of the CPSO algorithm in compared to the adaptive and PSO methods. Also confirms its strength in the face of uncertainty.

Table 2. Synchronization time of x_2 and x_4 for time delay problem of the heart

Mean time				
	Case 1	Case 2	Case 3	Case 4
15	1.129	1.321	1.311	1.333
28	1.129	1320	1.309	1.328
40	1.129	1.318	1.306	1.325
50	1.129	1315	1.302	1.321
Best time				
	Case 1	Case 2	Case 3	Case 4
15	0.925	1.309	1.285	1.293
28	0.925	1.312	1.284	1.290
40	0.925	1308	1.286	1.288
50	0.925	1.306	1.283	1.286

Table 3. Synchronization time of x_4 and x_6 for time delay problem of the heart

Mean time				
	Case 1	Case 2	Case 3	Case 4
15	1.381	1.425	1.417	1.417
28	1.381	1.422	1.415	1.413
40	1.381	1.421	1.416	1.412
50	1.381	1.419	1.413	1.411
Best time				
	Case 1	Case 2	Case 3	Case 4
15	1.121	1.261	1.213	1.225
28	1.121	1.258	1.212	1.224
40	1.121	1.259	1.213	1.222
50	1.121	1.259	1.211	1.219

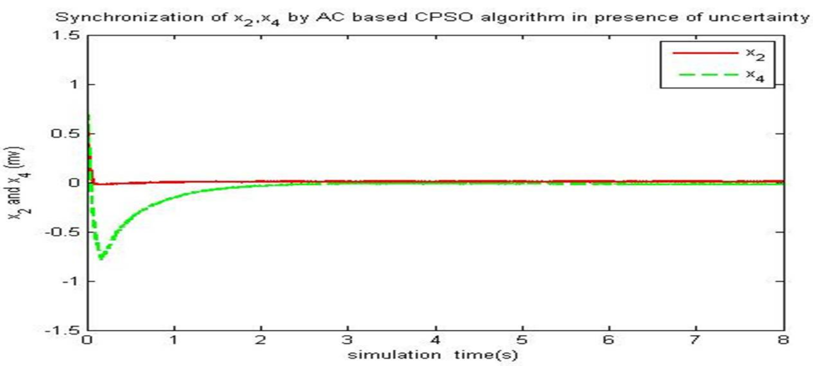

Figure 18

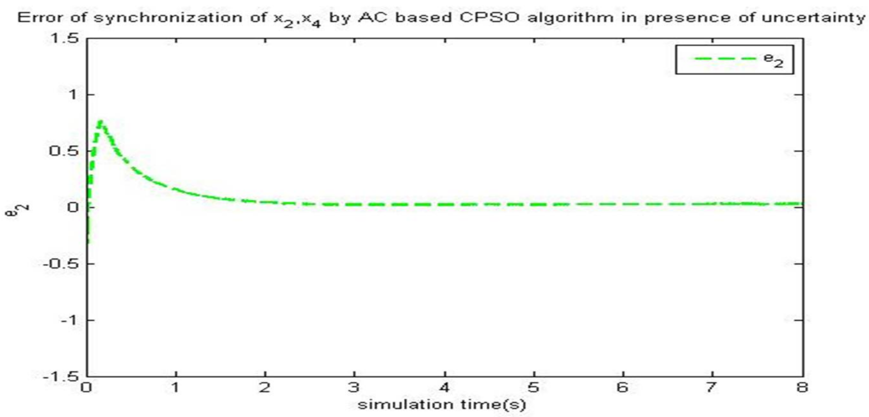

Figure 19

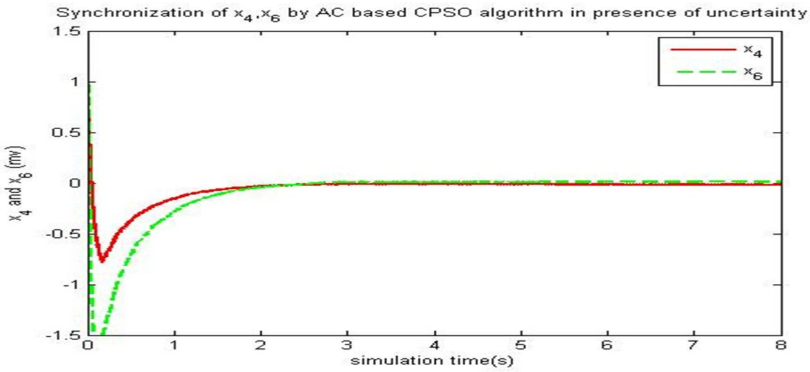

Figure 20

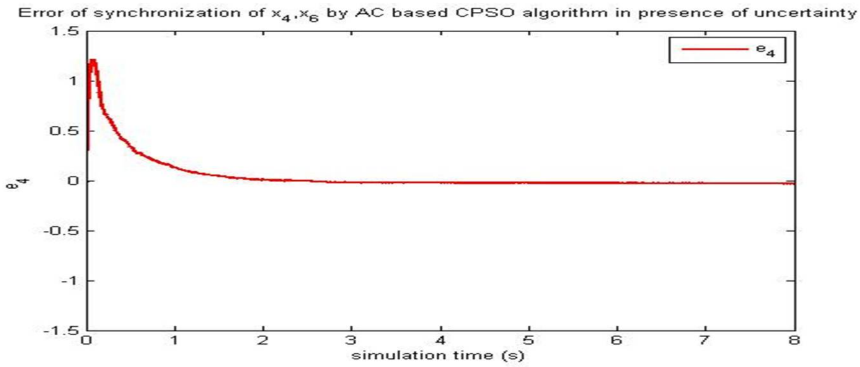

Figure 21

6. Conclusion

To synchronize the heart three-oscillator model in presence of parameter unknown, we proposed an AC based of CPSO algorithm. In this approach parameters of algorithm were adapted with CPSO to minimize tracking error. The most advantage of this controller is ability to eliminate tracking error without any information about uncertainty bounds. Because of strengthening of controller, the CPSO algorithm can defeat parameter uncertainties of the system even in presence of parameter unknown. Also we have shown that nonlinear oscillators can be used to model the heartbeat activity and dynamic rhythm of the heart is modeled in case of three-oscillator with delay. In this model each oscillator is represented one of the heart natural important pacemaker: AS, AV and HP.

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