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RESEARCH

# Low and high frequency instabilities in an explosion-generated-plasma and possibility of wave triplet

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**Abstract** An explosion-generated-plasma is explored for low and high frequency instabilities by taking into account the drift of all the plasma species together with the dust particles which are charged. The possibility of wave triplet is also discussed based on the solution of dispersion equation and synchronism conditions. High frequency instability (HFI) and low frequency instability (LFI) are found to occur in this system. LFI grows faster with the higher concentration of dust particles, whereas its growth rate goes down if the mass of the dust is higher. The ion and electron temperatures affect its growth in opposite manner and the electron temperature causes this instability to grow. In addition to the instabilities, a simple wave is also observed to propagate, whose velocity is larger for larger wave number, smaller mass of the dust and higher ion temperature.

**Keywords** Dust particles · Explosion-generated-plasma · Dispersion equation · Low frequency instability · High frequency instability

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## Introduction

An electromagnetic interference on electronic systems due to high-power microwaves (HPMs) introduces noise or signals into the electronic systems. This could cause a temporary system malfunction and component degradation; even a permanent physical damage is also possible at high levels of irradiation. The HPM sources have been under investigation for several years as potential weapons for a variety of combat, sabotage, and terrorist applications [1–3]. The pulse released by an electromagnetic weapon lasts for an extremely short time, i.e. around 100 picoseconds. Hence, the absorption of this blast of high energy by anything capable of conducting electricity (including nerves and neurons) overwhelms the recipient. As a result, the computers used in data processing systems, communications systems, satellites, industrial controls, displays, military systems, radar, HF, VHF, UHF, and television equipment are all susceptible to the electromagnetic pulses (EMPs).

If we think about the explosion-generated-plasma (EGP), we find that the beams of ions and electrons are a source of free energy which can be transferred to waves. So instabilities can evolve in the nonlinear systems. If conditions are favourable, the resonant interaction of the waves in plasma can lead to nonlinear instabilities, in which all the waves grow faster than exponentially and attain enormously large amplitudes in a finite time or after a finite distance, depending on whether temporal or spatial growth is considered. These instabilities are referred to as explosive instabilities. Such instabilities could be of considerable practical interest, as these seem to offer a mechanism for rapid dissipation of coherent wave energy into thermal motion, and hence may be effective for plasma heating [4–7]. A consistent theory of explosive instability shows that in the three-wave approximation amplitudes of all the waves tend to infinity over a finite time called explosion time. The simplest wave coupling process that can exhibit explosive character is the coupling of three-waves with fixed phases [5, 8, 9]. For a wave triplet, all the three waves grow simultaneously. This phenomenon was first described by Cairns [10] using the kinetic equation which takes into account the interactions of waves with random phases and different signs of energy. Fainshtein and Chernova [11] have investigated the high power electromagnetic radiation from the development of explosive and high-frequency instabilities in a system consisting of a relativistic ion beam and a nonisothermal plasma. Based on an asymptotic method they derived and analyzed truncated equations for the complex mode amplitudes, and showed that the explosion is stabilized by a nonlinear frequency shift, while the high-frequency instability is analogous to the decay of low-frequency modes.

This can be seen that in most of the investigations the researchers have used the Boltzmann distribution of the electrons keeping in mind their mass to be negligible. Moreover, the electrons are employed to make background only. However, we consider the finite mass of the electrons and also take into account the dust particles which are always present in most of such plasma and whose charge may fluctuate due to currents flowing into the dust [12, 13]. In view of a strong nonisothermal plasma, we consider the temperature of electrons ($T_e$) to be much higher than that of the ions ($T_i$), i.e. $T_e \gg T_i$ [14–17]. For the sake of generality, we consider initial drift of all the plasma species and derive the dispersion equation for this system and solve it numerically for investigating the instabilities.

## Basic model

The plasma is taken to compose electrons, singly charged ions and negatively charged dust grains of uniform mass and charge. Hence, the quasineutrality condition reads $n_{i0} = n_{e0} - \alpha Z_d n_{d0}$, where $n_{i0}$, $n_{e0}$ and $n_{d0}$ are the unperturbed number density of the ions, electrons and dust grains, respectively, and $Z_d$ is the magnitude of the charge on the dust. The parameter $\alpha$ represents the nature of charge on the dust, and it is positive (negative) for the positively (negatively) charged dust grains.

### Basic equations and dispersion relation

If $n_i(n_e)$ is the density of ions (electrons), $M(m)$ is the mass of ion (electron) and $\vec{\upsilon}_i(\vec{u}_e)$ is the ion (electron) fluid velocity along with their unperturbed values as $\upsilon_0$ and $u_0$ in the $x$-direction, then one-dimensional continuity equation and momentum equation for the ion, electron and dust fluids can be written as

$$(n_i)_t + (n_i \upsilon_i)_x = 0 \tag{1}$$

$$(n_e)_t + (n_e u_e)_x = 0 \tag{2}$$

$$(n_d)_t + (n_d \upsilon_d)_x = 0 \tag{3}$$

$$(u_e)_t + u_e (u_e)_x + \frac{M}{m n_e}(n_e)_x - \frac{M}{m}\phi_x = 0 \tag{4}$$

$$(\upsilon_i)_t + \upsilon_i (\upsilon_i)_x + \frac{\sigma}{n_i}(n_i)_x + \phi_x = 0 \tag{5}$$

$$(\upsilon_d)_t + \upsilon_d (\upsilon_d)_x + \frac{\alpha M}{m_d}\phi_x = 0 \tag{6}$$

The system of equations can be closed with the following Poisson's equation.

$$\phi_{xx} - n_e + n_i + n_d Z_d \alpha = 0. \tag{7}$$

In the above equations, the subscripts $x$ and $t$ denote the respective differentiation. The densities are normalized by a background density $n_0$, potential $\phi$ by $T_e/e$, time $t$ by the inverse of frequency $\omega_{pi} = \sqrt{e^2 n_0/\varepsilon_0 M}$, velocities $\upsilon_i, u_i, \upsilon_d$ by the ion acoustic speed $C_s = \sqrt{T_e/M}$ and length $x$ by the Debye length $\lambda_{De} = \sqrt{\varepsilon_0 T_e / e^2 n_0}$.

The solution of the above equations is obtained for the variation of perturbed quantities as $\psi_1 \sim \exp(i\omega t - ikx)$ together with $\psi_1 \equiv n_{i1}, n_{e1}, n_{d1}, \vec{\upsilon}_{i1}, \vec{u}_{e1}, \phi_1, \vec{\upsilon}_{d1}$. Here $\omega$ is the frequency of oscillations and $k$ is the wave number. Hence, the following expressions for the perturbed densities are obtained from the basic fluid equations.

$$n_{i1} = \frac{k^2 n_{i0}\phi_1}{(\omega - k\upsilon_0)^2 - \sigma k^2} \tag{8}$$

$$n_{e1} = \frac{k^2 \phi_1}{\frac{m}{M}(\omega - k u_0)^2 - k^2} \tag{9}$$

$$n_{d1} = \frac{M}{m_d}\frac{k^2 \alpha Z_d n_{d0}\phi_1}{(\omega - k\upsilon_{d0})^2} \tag{10}$$

The use of these expressions in the Poisson's Eq. (7) yields

$$\frac{k^2\phi_1}{\frac{m}{M}(\omega - k u_0)^2 - k^2} - \frac{k^2 n_{i0}\phi_1}{(\omega - k\upsilon_0)^2 - \sigma k^2} - \frac{k^2\alpha^2 Z_d^2 n_{d0} M \phi_1}{m_d(\omega - k\upsilon_{d0})^2} + k^2\phi_1 = 0 \tag{11}$$

After simplifying the above equation, we get the following dispersion equation

$$\frac{m m_d}{M^2}\omega^6 - b_0\omega^5 + \omega^4 b_1 + \omega^3 b_2 + \omega^2 b_3 + \omega b_4 + a_1 a_2 a_3 + a_2 a_4 + a_1 a_4 (n_{i0} - a_2) = 0 \tag{12}$$

Here

$$b_0 = \frac{m_d}{M}\left(a_{11} + \frac{2m}{M}k\upsilon_{d0}\right), b_1 = a_{15} + \frac{m_d}{M}\left(1 - \frac{n_{i0}m}{M}\right) - a_3\frac{m}{M}, b_2 = a_3a_{11} - a_{16} + n_{i0}a_8 - a_5,$$

$$b_3 = a_3a_{12} - a_{17} + a_6 + n_{i0}a_9, b_4 = a_{18} - a_7 + a_3a_{13} + n_{i0}a_{10}, a_1 = k^2\left(1 - \frac{m}{M}u_0^2\right),$$

$$a_2 = k^2(\upsilon_0^2 - \sigma), a_3 = \alpha^2 Z_d^2 n_{d0}, a_4 = \frac{m_d}{M}k^2\upsilon_{d0}^2$$

together with

$$a_5 = \frac{m_d}{M}k(\upsilon_0 + \upsilon_{d0}), a_6 = \frac{m_d}{M}k^2\left[\upsilon_0(4\upsilon_{d0} + \upsilon_0) + \upsilon_{d0}^2\right],$$

$$a_7 = 2\frac{m_d}{M}k^3\upsilon_0\upsilon_{d0}(\upsilon_{d0} + \upsilon_0),$$

$$a_8 = \frac{2mm_d}{M^2}k(u_0 + \upsilon_{d0}), a_9 = \frac{m_d}{M}\left[a_1 - \frac{m}{M}\upsilon_{d0}k^2(4u_0 + \upsilon_{d0})\right],$$

$$a_{10} = \frac{2m_d}{M}k\upsilon_{d0}\left[\frac{k^2u_0m}{M}\upsilon_{d0} - a_1\right],$$

$$a_{11} = \frac{2m}{M^2}k(u_0 + \upsilon_0), a_{12} = a_1 - \frac{m}{M}\upsilon_0k^2(4u_0 + \upsilon_0),$$

$$a_{13} = 2k\upsilon_0\left(\frac{m}{M}u_0k^2\upsilon_0 - a_1\right),$$

$$a_{15} = \frac{m_d}{M}\left[k\upsilon_{d0}\left(\frac{m}{M}k\upsilon_{d0} + 2a_{11}\right) - a_{12}\right],$$

$$a_{16} = \frac{m_d}{M}[a_{13} + k\upsilon_{d0}(k\upsilon_{d0}a_{11} - 2a_{12})],$$

$$a_{17} = \frac{m_d}{M}k\left[a_1k\upsilon_0^2 - 2a_{13}\upsilon_{d0} + k\upsilon_{d0}^2a_{12}\right],$$

$$a_{18} = \frac{m_d}{M}\upsilon_{d0}k^2\left[2a_1k\upsilon_0^2 - \upsilon_{d0}a_{13}\right].$$

## Results and discussion

The dispersion Eq. (12) carries a term $\alpha^2$, it means the same results are expected for the cases of positively charged dust grains and negatively charged dust grains. We numerically solve Eq. (12) by giving typical values to various parameters in view of the plasma generated in an explosion. Hence, $k = 0.5$, $\upsilon_{d0} = 0.002$, $n_{i0} = 1.1$, $M = 23.38 \times 10^{-27}$ kg, $m_d = 10^{-20}$ kg, $u_0 = 0.4$, $\upsilon_0 = 0.04$, $n_{nd0} = 0.001$, $Z_d = 100$, $T_i = 1$ eV and $T_e = 10$ eV [18–22]. Our numerical calculations show that two types of instabilities occur in the plasma if we consider the role of dust mass, which is present in the said plasma. The growth rates of these instabilities are found to be influenced by density and mass of the dust grains. For the parameters within this range, we observe that there are two roots that satisfy the condition for unstable growth of the disturbances. However, in some cases more roots are found, but their growth is too low, showing that these roots are very small perturbations in the system. We solve Eq. (12) for the complex root of $\omega(\equiv \omega_R - i\gamma)$ and plot the normalized growth rate $\gamma$ in Figs. 1, 2, 3, 4. Since the normalization is

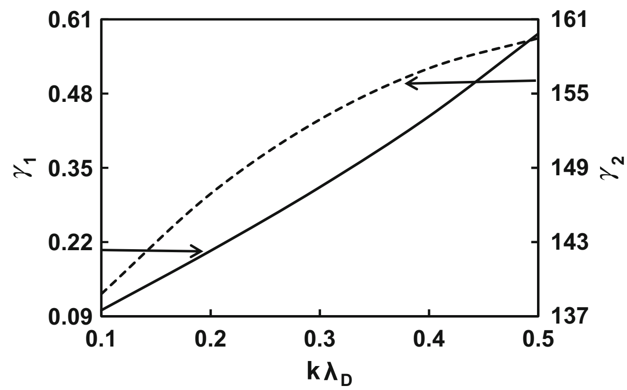


**Fig. 1** Variation of growth rates with normalized wave number, when $\upsilon_{d0} = 0.002$, $n_{i0} = 1.1$, $M = 23.38 \times 10^{-27}$ kg, $m_d = 10^{-20}$ kg, $u_0 = 0.4$, $\upsilon_0 = 0.04$, and $n_{nd0} = 0.001$, $Z_d = 100$, $T_i = 1$ eV and $T_e = 10$ eV

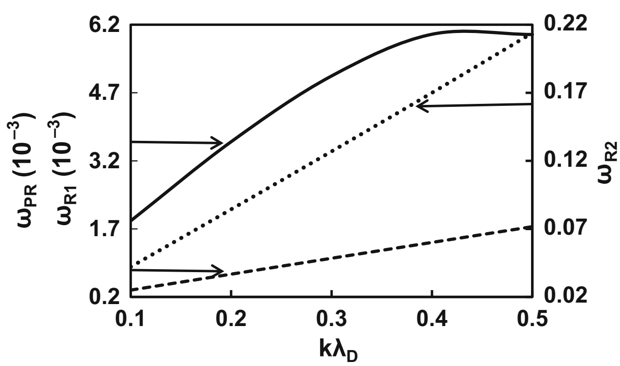


**Fig. 2** Variation of propagating and real frequencies with normalized wave number (*dashed line*) when the other parameters are the same as in Fig. 1. The *solid line* (*dotted line*) corresponds to $\omega_{R1}(\omega_{R2})$

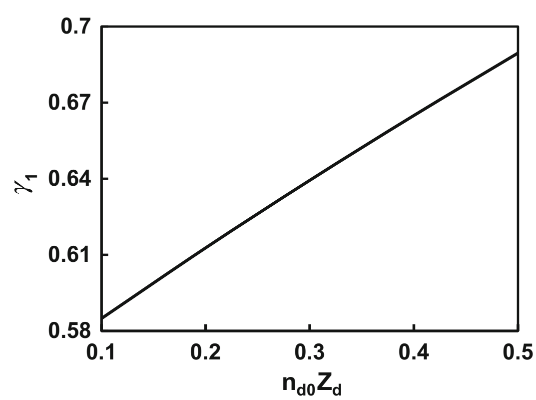


**Fig. 3** Variation of growth rate $\gamma_1$ with dust density, when $k = 0.5$ and the other parameters are the same as in Fig. 1

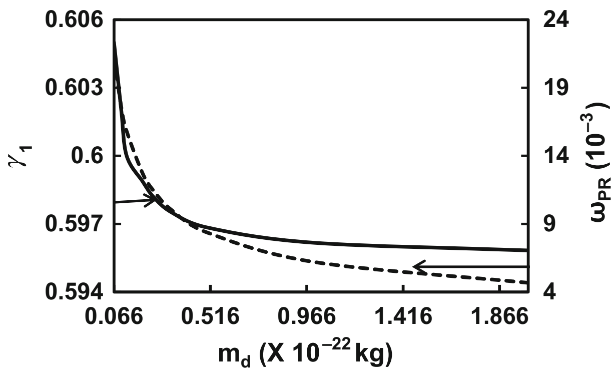


**Fig. 4** Variation of growth rate $\gamma_1$ and propagating frequency with dust mass, when $k = 0.5$ and the other parameters are the same as in Fig. 1

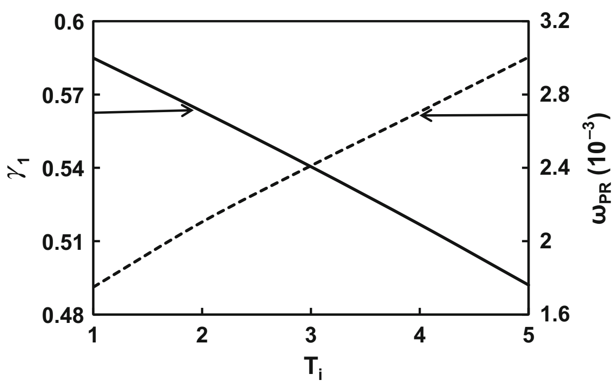


**Fig. 5** Dependence of growth rate $\gamma_1$ on ion temperature, when $k = 0.5$ and the other parameters are the same as in Fig. 1

done with respect to ion plasma frequency, the growth rate $\gamma < 1$ is called to correspond to lower growth rate instability (LFI) whereas the growth rate $\gamma > 1$ to higher growth rate instability (HFI).

Figure 1 shows the variation of growth rates of the instabilities with wave number. Here $\gamma_2$ corresponds to the higher growth rate instability (called HFI) and $\gamma_1$ corresponds to the lower growth rate instability (LFI). Both the growth rates $\gamma_1$ (solid line) and $\gamma_2$ (dashed line) behave similarly and these go higher for the longer values of $k$.

Figure 2 shows that the higher rate instability is a constant velocity wave. However, the lower growth rate instability (the graph marked with $\omega_{R2}$) is found to show wave number dependence behaviour. This becomes a constant frequency wave at the larger value of wave number. In addition to the two types of instabilities, this Figure shows that there exists a propagating mode also, whose velocity remains a constant.

Lower growth rate instability (LFI) is found to be influenced by the presence of dust grains. For example, Fig. 3 show that the growth rate $\gamma_1$ is increased when the dust grains in larger number are present. This is due to the

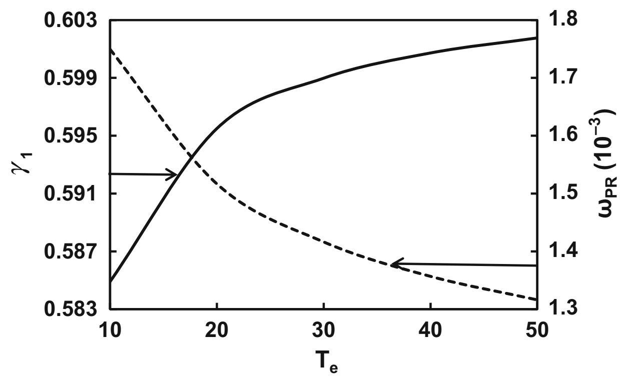


**Fig. 6** Weak dependence of growth rate $\gamma_1$ and frequency of propagating mode on electron temperature, when $k = 0.5$ and the other parameters are the same as in Fig. 1

negative charge on the dust, which provides larger restoring force to the oscillations of the species because of which the growth is enhanced. However, the mass of dust particles shows opposite effect on the growth rate $\gamma_1$, as expected. Similar effect of dust particles mass is observed on the frequency and hence, on the speed of propagating mode (please see the graph marked with $\omega_{PR}$ in Fig. 4).

The effect of ion and electron temperatures on the growth rate $\gamma_1$ is shown in Figs. 5 and 6, respectively. The ion temperature is found to reduce the growth of lower growth rate instability (LFI), whereas the electron temperature enhances the growth rate $\gamma_1$. However, the growth tries to saturate at larger value of electron temperature. Similar effect of electron temperature on the growth rate of dissipative instability has been observed by Keidar and Beilis [23]. The higher growth rate for the case of higher electron temperature can be explained based on the probability of the collisions. On the other hand, the propagating mode (marked with $\omega_{PR}$) behaves oppositely with the ion and electron temperatures. The frequency $\omega_{PR}$ and hence, the speed of this wave is found to reduce with the electron temperature and to enhance with the ion temperature (Fig. 6).

## Wave triplet analysis

Now we discuss the possibility of wave triplet in EGP having dust grains. For this, we reproduce Eq. (12) in the following form

$$\omega^3\left[\frac{mm_d}{M^2}\omega^3 - a_{14}\omega^2 + \omega b_1 + b_2\right] + \omega^2 b_3 + \omega b_4 + \{a_1a_2a_3 + a_2a_4 + a_1a_4(n_{i0} - a_2)\} = 0 \quad (13)$$

This equation shall yield three waves (triplet) if the coefficients $b_3$ and $b_4$ together with the term in curly bracket vanishes. It means

$$a_{12}a_3 - a_{17} + a_6 + n_{i0}a_9 = 0 \quad (14)$$

$$a_{18} - a_7 + a_3a_{13} + n_{i0}a_{10} = 0 \quad (15)$$

$$a_1a_2a_3 + a_2a_4 + a_1a_4(n_{i0} - a_2) = 0. \quad (16)$$

By solving these equations, we can get the following expression for the drift velocity of the dust grains.

$$\upsilon_{d0} = \frac{\frac{M}{m_d}a_3a_{13}\left[1 - \frac{m}{M}n_{i0} - \frac{a_{12}}{a_1a_2}(a_2 + n_{i0}a_1)\right] - (a_2 - a_1a_2 + n_{i0}a_1)\left(2\frac{m}{M}n_{i0}ku_0 - a_{13} - 2k\upsilon_0\right)}{2k(a_2 - a_1a_2 + n_{i0}a_1) - 2k\left(2\frac{m}{M}n_{i0}ku_0 - a_{13} - 2k\upsilon_0\right)^2} \quad (17)$$

This expression shows the dependence of the drift velocity on the densities, masses, temperatures and initial velocities of ions and electrons in addition to the mass of dust grains and wave number $k$.

Along with the use of this expression, Eq. (13) yields $\frac{mm_d}{M^2}\omega^3 - a_{14}\omega^2 + \omega b_1 + b_2 = 0$, which can be reformulated as

$$\omega^3 + a\omega^2 + b\omega + d = 0 \quad (18)$$

Here

$$a = -\frac{M^2a_0}{mm_d}, b = \frac{M^2b_1}{mm_d} \text{and} \quad d = \frac{M^2b_2}{mm_d}.$$

Every cubic equation with real coefficients has at least one solution $\omega$ among the real numbers. This is a consequence of the intermediate value theorem. We can distinguish several possible cases using the discriminant

$$\Delta = 18abd - 4a^3d + a^2b^2 - 4b^3 - 27d^2 \quad (19)$$

Since we are interested in propagating waves, three distinct real roots of Eq. (14), $\Delta$ should be positive.

Further, we assume these roots as $\alpha$, $\beta$ and $\gamma$. Hence

$$\omega^3 + a\omega^2 + b\omega + d = (\omega - \alpha)(\omega - \beta)(\omega - \gamma) \quad (20)$$

The roots of the above equation would satisfy

$$\alpha + \beta + \gamma = -a \quad (21)$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = b \quad (22)$$

$$\alpha\beta\gamma = -d \quad (23)$$

For the occurrence of triplets, the roots $\alpha$, $\beta$ and $\gamma$ should additionally meet the following condition

$$\alpha + \beta = \gamma. \quad (24)$$

Finally, we obtain

$$\alpha = \omega_1 \equiv -\frac{a^2 - \sqrt{a^4 - 32da}}{4a},$$

$\beta = \omega_2 \equiv -\frac{a^2 + \sqrt{a^4 - 32da}}{4a}$. The following condition is also required to be satisfied

$$a^3 + 8d - 4ab = 0.$$

The coefficients a, b and d show their dependence on the wave number k through the drift velocity $\upsilon_{d0}$ and other coefficients such as $a_0$, $b_1$ and $b_2$. Hence, it would be possible to achieve conditions for synchronism, i.e. $k_3 = k_1 + k_2$ and $\omega_1 = \omega_2 + \omega_3$ (which we have written as $\gamma = \alpha + \beta$) in the present explosion-generated-plasma. It means this plasma would support the excitation of wave triplet. This is also supported by Fig. 7, which is plotted based on the dispersion Eq. (12) and shows that a wave with negative energy is possible and the conditions for synchronism can be fulfilled [4, 15, 16, 23].

## Conclusions

In an EGP, dispersion relation was derived and solved numerically to investigate the evolved instabilities under the effect of initial drifts of ions, electrons and charged dust particles. Two types of instabilities, naming LFI and HFI, were found in the said plasma in addition to a constant velocity propagating mode. The phase velocity of the mode and the growth rate of the LFI were observed to be reduced

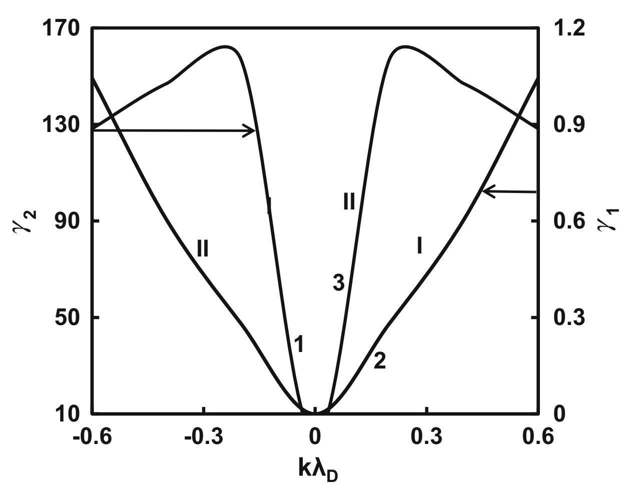


**Fig. 7** Dispersion curves corresponding to Eq. (12)

for higher mass of the dust particles. Only the LFI showed the dependence on the ion and electron temperatures. These results are based on the fixed charge on the dust grains. However, we can estimate the consequences of dust charge fluctuations based on the nonlinear property of the plasma. This has been seen that the system becomes more nonlinear when the charge on the dust grains fluctuates. Since nonlinearity generally enhances the wave amplitude [12], higher amplitude instabilities are expected in the EGP if the charge on the dust grain is observed to fluctuate.

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