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Gradient effects on dust lattice waves in paramagnetic dusty plasma crystals

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Abstract

Dust lattice modes are studied in a hexagonal two-dimensional lattice in plasma crystal, including paramagnetic dust particles. The gradients of magnetic fields, electric fields, and dust charge and also the interaction of dipole-dipole take into account. These gradients modify the levitation condition and affect the frequencies of dust lattice waves. The coupling between in-plane and out-of-plane modes gives rise to the *hybrid* mode, which is always an unstable mode. However, intersection of the in-plane mode with other modes does not result in mode-coupling instability.

Keyword: Dusty plasma, Crystal, Dust lattice, Mode coupling, Modulational instability

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Background

Dust lattice waves are produced by oscillations of regularly spaced charged microparticles suspended in a plasma crystal, which form as a result of strong mutual coulomb interaction [1,2]. Crystalline complex plasma structures have been observed in recent rf discharge experiments [3], in which the plasma sheath was embedded in an external magnetic field. Theoretical studies then followed for the investigation of conditions for magnetic-field-assisted crystal equilibria involving paramagnetic charged dust grains. The role of various forces acting on paramagnetic grains has been discussed by Yaroshenko et al. [4], where magnetic forces have been shown to prevail over the (weaker) electric polarization forces. Also, the effect of magnetic field in dusty plasma lattice has been studied by the group of Farokhi [5,6] recently.

Dust lattices support a variety of linear modes of which we single out: longitudinal [7] ($\sim x$, acoustic) and a transverse [8,9] ($\sim y$, shear) in-plane as well as a transverse (out-of-plane, inverse-optic) dust-lattice wave mode(s).

Recently, Yaroshenko et al. studied the vertical vibrations of a one-dimensional string of magnetized particles, taking into account the magnetic force associated with gradients of an external magnetic field, and they founded a new low-frequency oscillatory mode [10]. The influence of an inhomogeneous magnetic field, ion focusing effect, and equilibrium charge gradient on the propagation of dust lattice modes in a one-dimensional string by paramagnetic particles is considered in the study of Yaroshenko et al. [11], and they founded the modified dust lattice waves. Dust lattice waves in hexagonal dusty plasma crystal were studied before [12-14]. Linear bending mode in hexagonal dusty plasma crystal has been studied by Vladimirov [15].

A theoretical treatment of the *nonlinear* aspects of dust lattice modes in one-dimensional Yukawa crystals has been carried out in the study of Kourakis et al. [16], where the above aspects are incorporated in an exact nonlinear lattice model. Recently, Farokhi et al. have studied the nonlinear dust lattice modes in hexagonal dusty plasma crystals [17]. Mode coupling instability in hexagonal dusty plasma crystals has been studied recently [18-20].

In this paper, we consider a two-dimensional monolayer of microparticles forming the hexagonal-type two-dimensional crystal in the presence of an external electric field and investigate the propagation of dust lattice waves in this system theoretically, including effects relevant for the sheath region, namely, anisotropy of interactions caused by dipole-dipole interactions and the height-dependent charge variations.

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Vibrational modes in a hexagonal lattice of paramagnetic grains

In this section to describe the modes in dusty plasma with magnetized grains, we consider a hexagonal crystal, where the spherical dust grains have magnetic moment $\vec{m}$, parallel to the external magnetic field, according to Figure 1. The magnetic moment of a particle with radius a and magnetic permeability μ, in an external magnetic field B is shown in the following equation:

$$m = \frac{4\pi}{\mu_\circ}\frac{\mu - 1}{\mu + 2}a^3 B = \alpha B \tag{1}$$

The influence of a inhomogeneous magnetic field on a magnetized grains is as follows:

$$F_m = -\frac{\partial(-\vec{m}\cdot\vec{B})}{\partial z} = 2\alpha B(\partial B/\partial z) \approx 2\alpha B_0 B_0^{'} + 2\alpha\left(B_0 B_0^{''} + B_0^{'2}\right)z + \cdots, \tag{2}$$

where a series expansion used for B. The electric force, by using from the same series expansion for Q and E is expressed as follows:

$$F_E = QE = Q_0 E_0 + \left(Q_0 E_0^{'} + Q_0^{'} E_0\right)z + \cdots \tag{3}$$

The electrostatic and magnetic energy due to interaction between the origin (o'th) grain and its neighbors (i'th grains) of crystal can be written as follows:

$$U_{o,i} = \frac{Q_o Q_i}{4\pi\epsilon_o |r_{o,i}|}\exp\left(-\frac{|r_{o,i}|}{\lambda_D}\right) - \frac{\mu_\circ}{4\pi}\left[\frac{\vec{m}_o\cdot\vec{m}_i}{|r_{o,i}|^3} - \frac{3\left(\vec{m}_o\cdot\vec{r}_{o,i}\right)\left(\vec{m}_i\cdot\vec{r}_{o,i}\right)}{|r_{o,i}|^5}\right] \tag{4}$$

The particle interaction force acting on the o particle can be presented as follows:

$$\vec{F}_{o,i} = -\partial U_{o,i}/\partial\vec{r}_o, \tag{5}$$

The dipole interactions are also short ranged, so that we need only to consider the nearest neighbor particle interactions. The equation of motion for the origin particle in crystal is as follows:

$$\vec{F} = -\sum_i \nabla U_{o,i} + \vec{F}_E + \vec{F}_m - M\vec{g} \tag{6}$$

Using from Equations (2), (3), (4), (5), and (6), and considering only small oscillations (u, v and $z << d$) around the equilibrium position, it gives the component of linear equation of motion for origin particle:

$$\begin{aligned} M\ddot{u}_{m,n} + 2\nu M\dot{u}_{m,n} &= \left(\frac{Q_o^2}{4\pi\epsilon_o d^3}e^{-\kappa}\left(2 + 2\kappa + \kappa^2\right) - \frac{3\mu_o m_o^2}{\pi d^5}\right)\left(u_{m+1,n} + u_{m-1,n} - 2u_{m,n}\right) \\ &+ \left(\frac{Q_o^2}{16\pi\epsilon_o d^3}e^{-\kappa}\left(-1 - \kappa + \kappa^2\right) - \frac{9\mu_o m_o^2}{8\pi d^5}\right) \\ &\left[u_{m+1/2,n+\sqrt{3}/2} + u_{m-1/2,n+\sqrt{3}/2} + u_{m-1/2,n-\sqrt{3}/2} + u_{m+1/2,n-\sqrt{3}/2} - 4u_{m,n}\right] \\ &+ \left(\frac{\sqrt{3}}{4}\frac{Q_o^2}{4\pi\epsilon_o d^3}e^{-\kappa}\left(1 + \kappa + \kappa^2\right) - \frac{15\sqrt{3}\mu_o m_o^2}{8\pi d^5}\right) \\ &\left[v_{m+1/2,n+\sqrt{3}/2} - v_{m-1/2,n+\sqrt{3}/2} + v_{m-1/2,n-\sqrt{3}/2} - v_{m+1/2,n-\sqrt{3}/2}\right] + \left(\frac{3\mu_o m_o m_o^{'}}{4\pi d^4}\right)\left[z_{m+1,n} - z_{m-1,n}\right] \\ &+ \left(\frac{3\mu_o m_o m_o^{'}}{8\pi d^4}\right)\left[z_{m+1/2,n+\sqrt{3}/2} - z_{m-1/2,n+\sqrt{3}/2} - z_{m-1/2,n-\sqrt{3}/2} + z_{m+1/2,n-\sqrt{3}/2}\right] \end{aligned} \tag{7}$$

$$\begin{aligned} M\ddot{v}_{m,n} + 2\nu M\dot{v}_{m,n} &= \left(-\frac{Q_o^2}{4\pi\epsilon_o d^3}e^{-\kappa}(1 + \kappa) - \frac{3\mu_o m_o^2}{4\pi d^5}\right)\left(v_{m+1,n} + v_{m-1,n} - 2v_{m,n}\right) \\ &+ \left(\frac{Q_o^2}{16\pi\epsilon_o d^3}e^{-\kappa}\left(5 + 5\kappa + 3\kappa^2\right) - \frac{39\mu_o m_o^2}{8\pi d^5}\right)\left[v_{m+1/2,n+\sqrt{3}/2} + v_{m-1/2,n+\sqrt{3}/2} + v_{m-1/2,n-\sqrt{3}/2} + v_{m+1/2,n-\sqrt{3}/2} - 4v_{m,n}\right] \\ &+ \left(\frac{\sqrt{3}}{4}\frac{Q_o^2}{4\pi\epsilon_o d^3}e^{-\kappa}\left(2 + 2\kappa + \kappa^2\right) - \frac{15\sqrt{3}\mu_o m_o^2}{8\pi d^5}\right)\left[u_{m+1/2,n+\sqrt{3}/2} - u_{m-1/2,n+\sqrt{3}/2} + u_{m-1/2,n-\sqrt{3}/2} - u_{m+1/2,n-\sqrt{3}/2}\right] \\ &+ \left(\frac{3\sqrt{3}\mu_o m_o m'_o}{8\pi d^4}\right)\left[z_{m+1/2,n+\sqrt{3}/2} + z_{m-1/2,n+\sqrt{3}/2} - z_{m-1/2,n-\sqrt{3}/2} - z_{m+1/2,n-\sqrt{3}/2}\right] \end{aligned} \tag{8}$$

$$
\begin{aligned}
M\ddot{z}_{m,n} + 2\nu M\dot{z}_{m,n} = & \left(-\frac{Q_o^2 e^{-\kappa}(1+\kappa)}{4\pi\epsilon_o d^3} + \frac{9\mu_o m_o^2}{4\pi d^5}\right)(z_{m+1,n} + z_{m-1,n} + z_{m+1/2,n+\sqrt{3}/2} + z_{m-1/2,n+\sqrt{3}/2} \\
& + z_{m-1/2,n-\sqrt{3}/2} + z_{m+1/2,n-\sqrt{3}/2} - 6z_{m,n}) + \left(\frac{Q_o Q'_o e^{-\kappa}(1+\kappa)}{4\pi\epsilon_o d^2} - \frac{3\mu_o m_o m'_o}{4\pi d^4}\right. \\
& \left. + \frac{\mu_o m'^2_o}{4\pi d^3}\right)\left[u_{m+1,n} - u_{m-1,n}\right] + \left(-\frac{Q'^2_o e^{-\kappa}}{4\pi\epsilon_o d} + \frac{\mu_o m'^2_o}{4\pi d^3}\right)(z_{m+1,n} + z_{m-1,n} \\
& + z_{m+1/2,n+\sqrt{3}/2} + z_{m-1/2,n+\sqrt{3}/2} + z_{m-1/2,n-\sqrt{3}/2} + z_{m+1/2,n-\sqrt{3}/2}) \\
& + 6\left(-\frac{Q_o Q''_o e^{-\kappa}(1+\kappa)}{4\pi\epsilon_o d} + \frac{\mu_o m_o m''_o}{4\pi d^3}\right) z_{m,n} \\
& + \left(\frac{Q_o Q'_o e^{-\kappa}(1+\kappa)}{8\pi\epsilon_o d^2} - \frac{3\mu_o m_o m'_o}{8\pi d^4}\right)[u_{m+1/2,n+\sqrt{3}/2} - u_{m-1/2,n+\sqrt{3}/2} - u_{m-1/2,n-\sqrt{3}/2} \\
& + u_{m+1/2,n-\sqrt{3}/2}] + \left(\frac{\sqrt{3} Q_o Q'_o e^{-\kappa}(1+\kappa)}{8\pi\epsilon_o d^2} - \frac{3\sqrt{3}\mu_o m_o m'_o}{8\pi d^4}\right)[\nu_{m+1/2,n+\sqrt{3}/2} \\
& + \nu_{m-1/2,n+\sqrt{3}/2} - \nu_{m-1/2,n-\sqrt{3}/2} - \nu_{m+1/2,n-\sqrt{3}/2}] + \left(QE\right)'_o z_{m,n} + 2\alpha\left(BB'\right)'_o z_{m,n}
\end{aligned}
\tag{9}
$$

where μ, ν and z, are displacement components of the origin particle; m_o stands for the equilibrium magnetic moment of the grains. Subscript "0" denotes the equilibrium position z = 0. There is always a position where gravitation can be compensated by the electric and magnetic fields:

$$
Mg = Q_o E_o + 2\alpha B_o B'_o - 6\frac{Q_o Q'_o}{4\pi\epsilon_o d} + 6\frac{\mu_o m_o m'_o}{4\pi d^3} \tag{10}
$$

Assuming now that $u_{m,n}, \nu_{m,n}, z_{m,n}$ vary as $\propto$ exp $[i(kmd + knd + kld - \omega t)]$, then it yields from the equation of motion:

$$
\left[\omega^2 + 2i\nu\omega - D_{11}\right]u - D_{12}\nu - iD_{13}z = 0 \tag{11}
$$

$$
-D_{21}u + \left[\omega^2 + 2i\nu\omega - D_{22}\right]\nu - iD_{23}z = 0 \tag{12}
$$

$$
-iD_{31}u - iD_{32}\nu + \left[\omega^2 + 2i\nu\omega - D_{33} - \Omega^2_{conf}\right]z = 0 \tag{13}
$$

where D_{ij} are defined in Appendix.

If we set $Q' = 0$ and $B' = 0$ in Equations (11), (12), (13), the vertical oscillatory mode will be an independent mode, while two other modes are coupled yet. In this case the vertical component of equation of motion is

$$
\begin{aligned}
M\ddot{z} + 2\nu M\dot{z} = & -\Omega^2_{conf} z_{m,n} + \left(-\frac{Q_o^2}{4\pi\epsilon_o d^3} e^{-\kappa}(1+\kappa) + \frac{9\mu_o m_o^2}{4\pi d^5}\right) \\
& (z_{m+1,n} + z_{m-1,n} + z_{m+1/2,n+\sqrt{3}/2} + z_{m-1/2,n+\sqrt{3}/2} + z_{m-1/2,n} \\
& {}_{-\sqrt{3}/2} + z_{m+1/2,n-\sqrt{3}/2} - 6z_{m,n})
\end{aligned}
\tag{14}
$$

which is in accordance with Equation (1) of Vladimirov et al. [15], else second term in vertical frequency is due to dipole interactions.

Two another components of the equation of motion is same as Equations (14) and (15) of Farokhi et al. [14], approximately. These equations are include the effect of dipole-dipole interactions, and it leads to modified coefficients:

$$
\begin{aligned}
M\ddot{u} + 2\nu M\dot{u} = & \left(\frac{Q_o^2}{4\pi\epsilon_o d^3} e^{-\kappa}\left(2 + 2\kappa + \kappa^2\right) - \frac{3\mu_o m_o^2}{\pi d^5}\right)(u_{m+1,n} + u_{m-1,n} - u_{m,n}) \\
& + \left(\frac{Q_o^2}{16\pi\epsilon_o d^3} e^{-\kappa}\left(-1 - \kappa + \kappa^2\right) - \frac{9\mu_o m_o^2}{8\pi d^5}\right)\left[u_{m+1/2,n+\sqrt{3}/2} + u_{m-1/2,n+\sqrt{3}/2} + u_{m-1/2,n-\sqrt{3}/2} + u_{m+1/2,n-\sqrt{3}/2} - 4u_{m,n}\right] \\
& + \left(\frac{\sqrt{3}}{4}\frac{Q_o^2}{4\pi\epsilon_o d^3} e^{-\kappa}\left(1 + \kappa + \kappa^2\right) - \frac{15\sqrt{3}\mu_o m_o^2}{8\pi d^5}\right)\left[\nu_{m+1/2,n+\sqrt{3}/2} - \nu_{m-1/2,n+\sqrt{3}/2} + \nu_{m-1/2,n-\sqrt{3}/2} - \nu_{m+1/2,n-\sqrt{3}/2}\right]
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
M\ddot{v}+2\nu M\dot{v}=&\left(-\frac{Q_o^2}{4\pi\epsilon_o d^3}e^{-\kappa}(1+\kappa)-\frac{3\mu_o M_o^2}{4\pi d^5}\right)\left(v_{M+1,n}+v_{M-1,n}-v_{M,n}\right)\\
&+\left(-\frac{Q_o^2}{4\pi\epsilon_o d3}e^{-\kappa}(1+\kappa)+\frac{3}{4}\frac{Q_o^2}{4\pi\epsilon_o d^3}e^{-\kappa}\left(2+2\kappa+\kappa^2\right)-\frac{39\mu_o M_o^2}{8\pi d^5}\right)[v_{M+1/2,n+\sqrt{3}/2}\\
&+v_{M-1/2,n+\sqrt{3}/2}+v_{M-1/2,n-\sqrt{3}/2}+v_{M+1/2,n-\sqrt{3}/2}-4v_{M,n}]\\
&+\left(\frac{\sqrt{3}}{4}\frac{Q_o^2}{4\pi\epsilon_o d^3}e^{-\kappa}\left(2+2\kappa+\kappa^2\right)-\frac{15\sqrt{3}\mu_o M_o^2}{8\pi d^5}\right)[u_{M+1/2,n+\sqrt{3}/2}-u_{M-1/2,n+\sqrt{3}/2}\\
&+u_{M-1/2,n-\sqrt{3}/2}-u_{M+1/2,n-\sqrt{3}/2}]
\end{aligned}
\tag{16}
$$

However, if gradients of fields and charge to be account, Equations (11), (12), and (13) shows a coupling between three modes. Dispersion relation can obtain from simultaneous solution of these equations, so one can obtain the dispersion relation

$$
\det\begin{pmatrix}\omega^2+2i\nu\omega-D_{11} & -D_{12} & -iD_{13}\\ -D_{21} & \omega^2+2i\nu\omega-D_{22} & -iD_{23}\\ -iD_{31} & -iD_{32} & \omega^2+2i\nu\omega-D_{33}-\Omega_{conf}^2\end{pmatrix}=0 \tag{17}
$$

Three dust lattice modes are mixed, via Equation (17), which for study of coupling of modes it should be plotted and be compared with modes in absent of coupling. By using the new notations for characteristics frequencies, we write the dispersion relation of the DL modes in the form:

$$
\left(\omega^2+2i\nu\omega-D_+\right)\left(\omega^2+2i\nu\omega-D_-\right)\left(\omega^2+2i\nu\omega-D_z\right)+\omega_{coup}^4\left(\omega^2+2i\nu\omega-D_o\right)=0 \tag{18}
$$

where

$$
D_{+,-}=\frac{(D_{11}+D_{22})\pm\sqrt{(D_{11}-D_{22})^2+4D_{12}D_{21}}}{2},\quad D_z=\Omega_{conf}^2+D_{33}, \tag{19}
$$

The coupling frequency, $\omega_{coup}^4=D_{23}D_{32}+D_{13}D_{31}$, characterized magnitude of coupling between the modes. Also the mixing frequency, D_o, indicates on coupling between in-plane-modes, where $D_o=(D_{11}D_{23}D_{32}-D_{12}D_{23}D_{31}-D_{31}D_{21}D_{32}+D_{13}D_{22}D_{31})/(D_{23}D_{32}+D_{13}D_{31})$. Also $D_{+,-}$ indicates on acoustic-type modes and D_z is the optical-type mode. At larger wave numbers, the optical mode may cross with the acoustic modes. In this case, the magnitudes of the in-plane modes and of the mixing frequency are similar for any wave vector. Far from the cross point the coupling frequency is very small in comparison with dust lattice modes, so the all modes in accordance with Figure 2, are well separated. In vicinity of cross point, coupling of the in-plane and out-of-plane modes in a narrow proximity of the intersection line(s) ω_0 (k) becomes crucial. In this region the coupling gives rise to the *hybrid* mode with $\mathrm{Re}(\omega_{hyb})=\sqrt{(D_z+D_+)/2}$. The analysis of Equation (18) shows that the hybrid mode is always unstable. One can show that the coupling coefficient is positive only for the intersection of the modes D_z and D_+, which causes the hybridization and triggers the instability. Figure 3 indicates on this mode. The intersection of the out-of-plane mode with the in-plane mode D_- (as well as the crossing of the in-plane modes) does not result in the mode-coupling

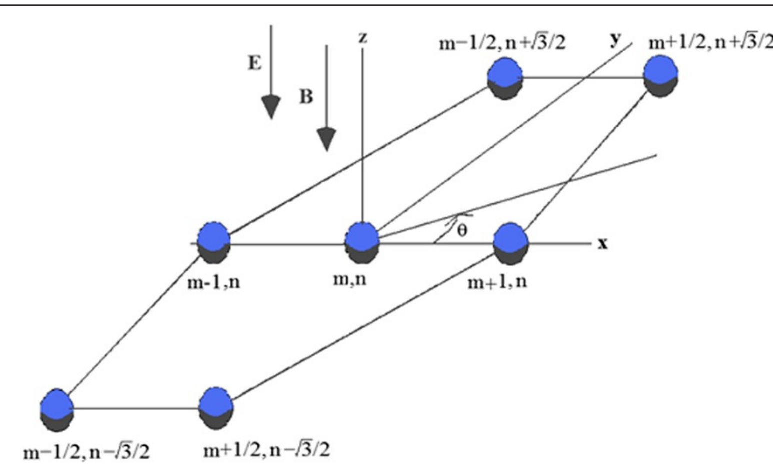

Figure 1 Hexagonal structure of dusty plasma crystal.

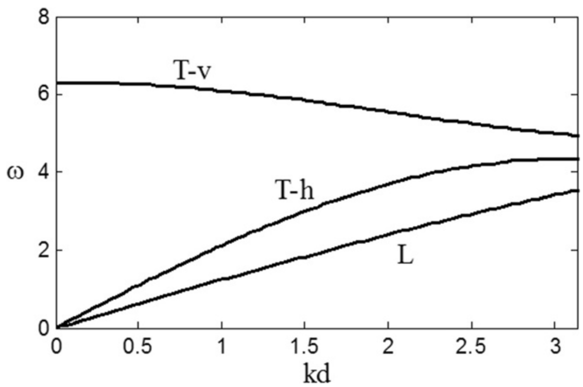

Figure 2 The dispersion relation of dust lattice modes for propagation along x-axis and $\Omega_{conf} = \sqrt{40}$.

instability. Mathematically, this is because the behavior of the modes in the vicinity of the intersection line is determined by the sign of the corresponding coupling term (last term in Equation (18)), (which for that numerical value of various parameters is in accordance with Ref [11]). The real part of frequency normalized to $\omega_o = Q^2\exp(-\kappa)/4\pi\epsilon_\circ Md^3$. The imaginary part (dashed line) has magnified with a 10 factor. Symbols L, T-h and T-v are used for the longitudinal mode, in-plane (horizontal) transverse mode and for the out-of-plane (vertical) transverse mode, respectively. Also "hyb" indicates the hybrid mode.

Conclusion

In summary the propagation of dust lattice modes in a hexagonal paramagnetic dust crystal has studied, including gradients of magnetic and electric fields and dust charge. Paramagnetic property of dusts, leads to

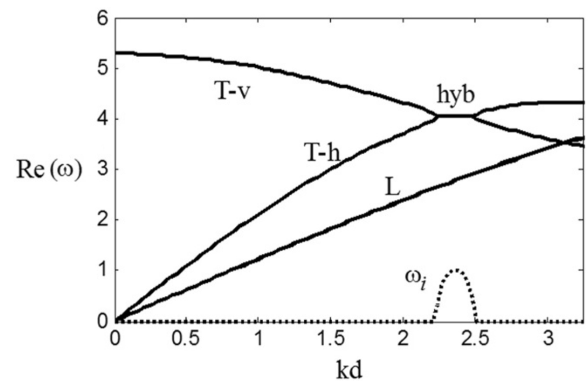

Figure 3 The Dispersion relations of the dust lattice modes for propagation along x-axis, $\Omega_{conf} = \sqrt{28}$ and *B* = *0.2 T*. The real part of frequency normalized to $\omega_o = Q^2\exp(-\kappa)/4\pi\epsilon_\circ Md^3$. The imaginary part (dashed line) has magnified with a 10 factor. Symbols L, T-h and T-v are used for the longitudinal mode, in-plane (horizontal) transverse mode and for the out-of-plane (vertical) transverse mode, respectively. Also "hyb" indicates the hybrid mode.

modification of frequencies of dust lattice waves. When these gradient taken into account, the main conclusion is coupling of three modes. Also these gradients modify the levitation condition and affect the frequencies of dust lattice waves. This implies that the characteristics of dust lattice modes coupling can be effectively controlled externally, due to gradients by experimental conditions.

The coupling between in-plane, D_+, and out-of-plane, D_z, modes gives rise to the *hybrid* mode, which the analysis of Equation (18) shows that this hybrid mode is always unstable. But the intersection of the in-plane mode D_- with other modes does not result in the mode-coupling instability. Also we calculated the critical

Appendix

$$D_{11} = 4\left(\frac{Q_o^2}{4\pi\epsilon_o Md^3}e^{-\kappa}\left(2+2\kappa+\kappa^2\right) - \frac{3\mu_o m_o^2}{4\pi Md^5}\right)\sin^2\left(\frac{kd\cos\theta}{2}\right)$$
$$+4\left(\frac{Q_o^2}{16\pi\epsilon_o Md^3}e^{-\kappa}\left(-1-\kappa+\kappa^2\right) - \frac{9\mu_o m_o^2}{8\pi Md^5}\right)\left[\sin^2\left(\frac{kd\cos(\theta+\pi/3)}{2}\right) + \sin^2\left(\frac{kd\cos(\theta-\pi/3)}{2}\right)\right]$$

$$D_{12} = \left(-\frac{\sqrt{3}}{2}\frac{Q_o^2}{4\pi\epsilon_o Md^3}e^{-\kappa}\left(1+\kappa+\kappa^2\right) + \frac{15\sqrt{3}\mu_o m_o^2}{4\pi Md^5}\right)\left[\cos(kd\cos(\theta+\pi/3)) - \cos(kd\cos(\theta-\pi/3))\right]$$

$$D_{13} = -\left(\frac{3\mu_o m_o m'_o}{2\pi Md^4}\right)\left[\sin(kd\cos(\theta)) + \frac{1}{2}\left(\sin(kd\cos(\theta+\pi/3)) - \sin(kd\cos(\theta-\pi/3))\right)\right]$$

$$D_{21} = \left(-\frac{\sqrt{3}}{2}\frac{Q_o^2}{4\pi\epsilon_o Md^3}e^{-\kappa}\left(2+2\kappa+\kappa^2\right) + \frac{15\sqrt{3}\mu_o m_o^2}{4\pi Md^5}\right)\left[\cos(kd\cos(\theta+\pi/3)) - \cos(kd\cos(\theta-\pi/3))\right]$$

$$D_{22} = -\left(\frac{Q_o^2}{\pi\epsilon_o Md^3}e^{-\kappa}(1+\kappa) + \frac{3\mu_o m_o^2}{\pi Md^5}\right)\sin^2\left(\frac{kd\cos\theta}{2}\right) + 4\left(\frac{Q_o^2}{16\pi\epsilon_o Md^3}e^{-\kappa}\left(5+5\kappa+3\kappa^2\right) - \frac{39\mu_o m_o^2}{8\pi Md^5}\right)$$
$$\left[\sin^2\left(\frac{kd\cos(\theta+\pi/3)}{2}\right) + \sin^2\left(\frac{kd\cos(\theta-\pi/3)}{2}\right)\right]$$

$$D_{23} = -\sqrt{3}\left(\frac{3\mu_o m_o m'_o}{4\pi M d^4}\right)[\sin(kd\cos(\theta+\pi/3)) + \sin(kd\cos(\theta-\pi/3))]$$

$$D_{31} = \left(\frac{3\mu_o m_o m'_o}{2\pi M d^4} - \frac{Q_o Q'_o e^{-\kappa}}{2\pi\epsilon_0 M d^2}(1+\kappa)\right)\left[\sin(kd\cos(\theta)) + \frac{1}{2}(\sin(kd\cos(\theta+\pi/3)) - \sin(kd\cos(\theta-\pi/3)))\right]$$

$$D_{32} = \sqrt{3}\left(\frac{3\mu_o m_o m'_o}{4\pi M d^4} - \frac{Q_o Q'_o e^{-\kappa}}{4\pi\epsilon_0 M d^2}(1+\kappa)\right)[\sin(kd\cos(\theta+\pi/3)) - \sin(kd\cos(\theta-\pi/3))]$$

$$D_{33} = 4\left(-\frac{Q_o^2 e^{-\kappa}}{4\pi\epsilon_0 M d^3}(1+\kappa) + \frac{9\mu_o m_o^2}{4\pi M d^5}\right)\left[\sin^2\left(\frac{kd\cos(\theta)}{2}\right) + \sin^2\left(\frac{kd\cos(\theta+\pi/3)}{2}\right) + \sin^2\left(\frac{kd\cos(\theta-\pi/3)}{2}\right)\right]$$
$$-\frac{Q'^2_o e^{-\kappa}}{2\pi\epsilon_0 M d}\cos(kd\cos(\theta)) + \frac{3Q_o Q''_o e^{-\kappa}}{2\pi\epsilon_0 M d} + \frac{Q'^2_o e^{-\kappa}}{2\pi\epsilon_0 M d}[\cos(kd\cos(\theta+\pi/3)) + \cos(kd\cos(\theta-\pi/3))]$$

$$\Omega^2_{conf} = 6\left(\frac{Q_o Q''_o e^{-\kappa}(1+\kappa)}{4\pi\epsilon_o M d} - \frac{\mu_o m_o m''_o}{4\pi M d^3}\right) - \frac{(QE)'_o}{M} - \frac{2\alpha(BB')'_o}{M}$$

frequency of the vertical confinement corresponding to the instability onset and determined its universal dependence on plasma parameters.

Competing interests
The author declares that he has no competing interests.

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