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Effect of general rotation on Rayleigh–Taylor instability of two superposed fluids with suspended particles

G. A. Hoshoudy · Pardeep Kumar

Abstract The Rayleigh–Taylor instability of a heavy fluid supported by a lighter one is investigated, with the suspended dust particles and small uniform general rotation. The fluids are assumed to be incompressible. The solutions of the linearized equations of motion using the boundary conditions lead to deriving the dispersion equation in complex formula. The real formula of dispersion relation has been analyzed and the behavior of growth rate with respect to the suspended dust particles and components of rotation have been examined. The results show that relaxation frequency of the suspended particles beside the general rotation will bring about more stability on the growth rate of unstable configuration.

Keywords Rayleigh–Taylor instability · General rotation · Suspended particles

Introduction

The Rayleigh–Taylor instability (RTI) [1, 2] occurs when heavy fluid lies over a lighter one, in the presence of a gravitational field. The RTI occurs in some natural phenomenon in a variety of astrophysical contexts, including supernova explosions [3], the interaction of shock waves with dense clouds present in the interstellar medium [4] and in the strong shocks in young supernova remnants [5, 6]. Also RTI takes place in inertial confinement fusion (ICF) [7–9]. For example, in ICF a directed high energy density provided by a set of laser beams is used to strongly compress a small pellet filled with deuterium–tritium in order to initiate nuclear burn. The perturbations which are generated in various locations in the pellet may grow with time through RT-type instabilities.

Several attempts to determine the effect of rotation on stability of one, two or three fluids have been studied alone or in the presence of other different factors. The effect of rotation (about the z-axis) on the interface between two superposed incompressible, inviscid fluids has been studied by Bjerknes et al. [10], Chandrasekhar [11] and then by Chakaraborty and Chandra [12]. The effect of vertical rotation and horizontal magnetic field on the RTI problem has been considered by Talwar [13] and Chakraborty [14]. The effect of rotation making an angle (the angular velocity about x-axis and z-axis) on the RTI has been considered by Hide [15, 16]. The RTI of rotating compressible inviscid fluids has been studied by Verma and Pratibha [17]. The RTI of two superposed non-viscous fluids under imposed horizontal and parallel rotation and horizontal magnetic fields has been considered by Davalos-Orozco [18]. The RTI of continuously stratified fluid under a general rotation has been studied by Davalos-Orozco and Aguilar-Rosas [19], while the effect of general rotation on RTI of two fluids has been studied by Davalos-Orozco [20]. The RTI of two-fluid layer system under the effect of a general rotation field and horizontal magnetic field has been investigated by Davalos-Orozco [21]. The effect of rotation on RTI for three layers has been considered by

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Chakaraborty and Chandra [22], Khater and Obied Allaah [23] and Obied Allah [24]. El-Ansary et al. [25] studied the influence of uniform vertical rotation with the surface tension on RTI of three fluids.

In the visible Universe situations, the fluids are often not pure but contain suspended particles. The suspended particles may play an important role in the fluids' stability problems, like, RTI, Kelvin–Helmholtz instability. Also, several attempts to determine the effect of suspended particles on this stability have been considered with various factors. Scanlon and Segel [26] considered the effect of suspended particles on the onset of Bénard convection. They found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. Sharma et al. [27] studied the effect of suspended particles on the onset of Bénard convection in hydromagnetics. Also, they found that the suspended particles have a destabilizing role on their selected problem. The RTI of two superposed conducting fluids in the presence of suspended particles and horizontal magnetic field is studied by Sharma et al. [28, 29]. The RTI of the plane interface separating the two partially-ionized superposed fluids through porous medium in the presence of a variable horizontal magnetic field is studied by Vaghela and Chhajlani [30]. The RTI of a Newtonian viscous fluid overlying a Rivlin–Ericksen viscoelastic fluid containing suspended particles in a porous medium is considered by Kumar [31]. The RTI of a Newtonian viscous fluid overlying Walters B' viscoelastic fluid containing suspended particles in porous medium has been studied by Kumar and Sharma [32]. Electro-hydrodynamic Kelvin–Helmholtz instability of two superposed Rivlin–Ericksen viscoelastic dielectric fluids containing suspended particles in a porous medium is considered by El-Sayed [33]. The effect of viscosity, finite ion Larmor radius and suspended particles on Kelvin–Helmholtz instability of two superposed incompressible fluids in the presence of a uniform magnetic field is considered by El-Sayed [34]. The stability of the plane interface separating two viscoelastic (Rivlin–Ericksen) superposed fluids in the presence of suspended particles are studied by Kumar and Singh [35]. The instability of two rotating viscoelastic (Walters B') superposed fluids permeated with suspended particles in porous medium is considered by Kumar and Singh [36]. The RTI of a Newtonian viscous fluid overlying an Oldroydian viscoelastic fluid containing suspended particles is considered by Kumar and Singh [37]. The stability of the plane interface separating two Rivlin–Ericksen viscoelastic superposed fluids permeated with suspended particles and uniform horizontal magnetic field is considered by Kumar and Abhilasha [38]. The stability of stratified Oldroyd viscoelastic fluid of the depth (d) in the presence of suspended particles and variable magnetic field

in porous medium has been studied by Singh and Dixit [39]. The RTI of two superposed incompressible fluids of different densities in the presence of small rotation, surface tension and suspended dust particles is investigated by Sharma et al. [40].

In all the papers mentioned above, whereas the rotation with the suspended particles is considered, it was in the horizontal or vertical direction only. In this paper, the RTI problem for two incompressible fluids in the presence of general rotation is considered and the system consists of suspended (or dust) particles. The goal is to obtain the dispersion relation that determines the growth rate as a function of the physical parameters of the system considered and the role of these parameters are determined, whereas, the RTI model may be important in the determination of the instability of planetary interiors, in particular, for the external core of Uranus that may be containing suspended particles [41, 42]. The appearance of such instabilities in previous topics has inspired us to study it and this is the main motivation of this work.

Formulation of the problem and perturbations equations

Consider a fluid of density ρ, stratified in the vertical z-direction. It is assumed that the fluid is permeated with suspended dust particles of uniform shape and size. The density of fluid is greater than the density of dust particles. The fluid is assumed to be infinitely extending having the free horizontal surface in the x–y plane. Also, the fluid is acted upon by a general rotation $\vec{\Omega} = (\Omega_x, \Omega_y, \Omega_z)$. Under the foregoing assumptions, equations of momentum and continuity can be written as (see Refs. [18–21, 40]):

$$\rho\left(\frac{\partial}{\partial t} + \vec{U} \cdot \vec{\nabla}\right)\vec{U} = -\vec{\nabla}P + \rho\,\vec{g} + 2\rho\left(\vec{U} \times \vec{\Omega}\right) + KN\left(\vec{V} - \vec{U}\right), \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho\,\vec{U}) = 0. \tag{2}$$

Here $\vec{U}(x, y, z, t)$ is the velocity of the fluid, ρ is the density, p the fluid pressure, $\vec{g} = (0, 0, -g)$ is the gravitational acceleration. $\vec{V}(x, y, z, t)$ and $N(x, y, z, t)$ denote the velocity and number density of the suspended particles, where $K = 6\pi\rho va$, v denotes the kinematic viscosity of the clean fluid, a the particle radius, is the Stokes' drag coefficient.

In the equations of motion (1), by assuming a uniform spherical particle shape and small relative velocities between the fluid and the particles, the presence of particles adds an extra force term proportional to the velocity

difference between the particles and the fluid. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion of the particles. The distances between particles are assumed quite large compared with their diameter, so that inter-particle reactions are ignored. The effects of pressure and gravity force on the suspended particles are negligibly small and therefore ignored. If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions are

$$mN\left[\frac{\partial}{\partial t} + \left(\vec{V} \cdot \vec{\nabla}\right)\vec{V}\right] = KN\left(\vec{U} - \vec{V}\right) + F_{\text{Cor}} + F_{\text{trans}} + F_{\text{Cent}},$$

(3)

$$\frac{\partial N}{\partial t} + \vec{\nabla} \cdot \left(N\vec{V}\right) = 0.$$

(4)

Where $F_{\text{Cor}} = -2\,m\Omega \times r^{\bullet}$, $F_{\text{trans}} = -m\Omega \times r$, $F_{\text{cent}} = -m\Omega \times (\Omega \times r)$ are the Coriolis force, transverse force and centrifugal force, respectively. r is position vector of the particle. They are present only if a particle is moving in a rotating coordinate system. The Coriolis force direction is always perpendicular to the velocity vector of the particle in the moving system. The Coriolis force thus seems to deflect a moving particle at right angles to its direction of motion. The transverse force is present only if there is an angular acceleration of the rotating coordinate system. This force is always perpendicular to the radius vector r, hence the name transverse. Finally, the centrifugal force is the familiar force arising from rotation about an axis. This force is always directed outward away from the axis of rotation and is perpendicular to that axis. The solution of our problem exploits the fact that the system is in the initial stage at rest (static case), then the terms F_{Cor}, F_{trans} and F_{cent} in the Eq. (3) will vanish.

Now, to investigate the stability of hydrodynamic motion, we ask how the motion responds to a small fluctuation in the value of any of the flow variables appearing in the Euler equations. If the fluctuation grows in amplitude so that the flow never returns to its initial state, we say that the flow is unstable with respect to fluctuations of that type. Accordingly, we replace the variables in Eqs. (1, 2, 3, 4) as follows: $\vec{U} = \vec{U}_0 + \vec{U}_1, \vec{V} = \vec{V}_1 + \vec{V}_0, \rho = \rho_0 + \rho_1$, $N = N_1 + N_0$, and $p = p_0 + p_1$. The quantities with subscripts "0" represent the unperturbed or "zeroth-order" motion of the fluid, while the quantities with subscripts "1" represent a small perturbation about the zeroth-order quantities (first-order or linearized quantities). Substituting these expressions into Eqs. (1, 2, 3, 4) and in particular example of RTI we consider the fluid is initially at rest (this

means that $\vec{U}_0 = 0$ and $\vec{V}_0 = 0$). Then the relevant linearization perturbation equations may be written from Eqs. (1, 2, 3, 4) as

$$\rho_0 \frac{\partial \vec{U}_1}{\partial t} = -\vec{\nabla}p_1 + \rho_1\vec{g} + 2\rho_0\left(\vec{U}_1\,x\vec{\Omega}\right) + KN_0\left(\vec{V}_1 - \vec{U}_1\right),$$

(5)

$$\frac{\partial \rho_1}{\partial t} + \vec{\nabla} \cdot \left(\rho_0\vec{U}_1\right) = 0,$$

(6)

$$mN_0\frac{\partial \vec{V}_1}{\partial t} = KN_0\left(\vec{U}_1 - \vec{V}_1\right), or \left(\tau\frac{\partial}{\partial t} + 1\right)\vec{V}_1 = \vec{U}_1,$$

(7)

$$\frac{\partial N_1}{\partial t} + N_0\vec{\nabla} \cdot \vec{V}_1 = 0, or \frac{\partial M_1}{\partial t} + \vec{\nabla} \cdot \vec{V}_1 = 0,$$

(8)

Here $\tau = \frac{m}{K}$ (relaxation time for the suspended dust particles) and $M_1 = \frac{N_1}{N_0}$. We now appeal to the fact that for many situations of interest in ICF, unstable flow occurs at velocities much smaller than the local sound speed. This has the effect that accelerations in the flow are not strong enough to change the density of a fluid element significantly, so the fluid moves without compressing or expanding. In such a situation we call the flow incompressible. Provided that we are well away from shock waves or centers of convergence, the assumption of incompressible flow is often valid. To say that fluid elements move without changing density is to say that the Lagrangian total derivative of density is zero, that is

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \left(\frac{\partial}{\partial t} + \vec{U} \cdot \vec{\nabla}\right)\rho = 0$$

(9)

We also linearize this equation, where the first-order quantities become

$$\frac{\partial \rho_1}{\partial t} + \left(\vec{U}_1 \cdot \vec{\nabla}\right)\rho_0 = 0.$$

(10)

Comparing this equation to Eq. (6), we can rewrite in expanded form as

$$\frac{\partial \rho_1}{\partial t} + \left(\vec{U}_1 \cdot \vec{\nabla}\right)\rho_0 + \rho_0\vec{\nabla} \cdot \vec{U}_1 = 0,$$

(11)

we see that subtracting Eq. (10) from Eq. (11) yields

$$\vec{\nabla} \cdot \vec{U}_1 = 0.$$

(12)

This is a consequence of the assumption of incompressible flow. We can use either Eq. (10) or Eq. (12) to replace the linearized continuity equation Eq. (6) under this assumption. One can see that the set of Eqs. (5, 7, 8, 10) and (12) is complete for describing the suspended dust particles effects on the RTI of incompressible rotating fluid.

Now, Let $\vec{U}_1 = (u_{x1}, u_{y1}, u_{z1})$, and $\vec{V}_1 = (v_{x1} + v_{y1}\ v_{z1})$. The fluid is arranged in horizontal strata, then ρ_0 is

a function of the vertical coordinate only [$\rho_0 = \rho_0(z)$], and $p_0 = p_0(z)$. So, the system of Eqs. (5, 7, 10 and 12, we take curl of Eq. 5) become

$$\left\{\frac{\partial}{\partial t}\left[\rho_0 \frac{\partial u_{z1}}{\partial y} - \frac{\partial}{\partial z}(\rho_0 u_{y1})\right]\right\} = -g\left\{\frac{\partial \rho_1}{\partial y}\right\}$$
$$+ 2\left\{\rho_0 \frac{\partial}{\partial y}[\Omega_y u_{x1} - \Omega_x u_{y1}] - \frac{\partial}{\partial z}[\rho_0(\Omega_x u_{z1} - \Omega_z u_{x1})]\right\}$$
$$+ \left\{\frac{\partial}{\partial y}[KN_0(v_{z1} - u_{z1})] - \frac{\partial}{\partial z}[KN_0(v_{y1} - u_{y1})]\right\} \quad (13)$$

$$\left\{\frac{\partial}{\partial t}\left[\frac{\partial}{\partial z}(\rho_0 u_{x1}) - \rho_0 \frac{\partial u_{z1}}{\partial x}\right]\right\} = g\left\{\frac{\partial \rho_1}{\partial x}\right\}$$
$$- 2\left\{\rho_0 \frac{\partial}{\partial x}[\Omega_y u_{x1} - \Omega_x u_{y1}] - \frac{\partial}{\partial z}[\rho_0(\Omega_y u_{z1} - \Omega_z u_{y1})]\right\}$$
$$- \left\{\frac{\partial}{\partial x}[KN_0(v_{z1} - u_{z1})] - \frac{\partial}{\partial z}[KN_0(v_{x1} - u_{x1})]\right\} \quad (14)$$

$$\rho_0 \frac{\partial}{\partial t}\left\{\frac{\partial u_{y1}}{\partial x} - \frac{\partial u_{x1}}{\partial y}\right\}$$
$$= 2\rho_0\left\{(\Omega_x \frac{\partial}{\partial x} + \Omega_y \frac{\partial}{\partial y})u_{z1} - \Omega_z(\frac{\partial u_{x1}}{\partial x} + \frac{\partial u_{y1}}{\partial y})\right\}$$
$$+ \left\{\frac{\partial}{\partial x}[KN_0(v_{y1} - u_{y1})] - \frac{\partial}{\partial y}[KN_0(v_{x1} - u_{x1})]\right\} \quad (15)$$

$$\frac{\partial u_{x1}}{\partial x} + \frac{\partial u_{y1}}{\partial y} + \frac{\partial u_{z1}}{\partial z} = 0, \quad (16)$$

$$\frac{\partial \rho_1}{\partial t} + u_{1z}\frac{d\rho_0}{dz} = 0 \quad (17)$$

$$\left\{\tau\frac{\partial}{\partial t} + 1\right\}v_{x1} = u_{x1}, \quad \left\{\tau\frac{\partial}{\partial t} + 1\right\}v_{y1}$$
$$= u_{y1}, \quad \left\{\tau\frac{\partial}{\partial t} + 1\right\}v_{z1} = u_{z1}, \quad (18)$$

Now, we assume that the perturbation in any physical quantity is dependent on space coordinate (x, y, z) and time t in the form

$$\psi(x, y, z, t) = \psi(z)\exp(ik_x x + ik_y y + nt) \quad (19)$$

where k_x, k_y ($k^2 = k_x^2 + k_y^2$) are horizontal wave numbers and n denotes the rate at which the system departs from the equilibrium. Using expression (19) in the system of Eqs. (13, 14, 15, 16, 17, 18), we have

$$n\{i\rho_0 k_y u_{z1} - D(\rho_0 u_{y1})\} = -i\rho_1 g k_y$$
$$+ 2\{i\rho_0 k_y[\Omega_y u_{x1} - \Omega_x u_{y1}] - D[\rho_0\Omega_x u_{z1} - \Omega_z u_{x1}]\}$$
$$+ \{ik_y[KN_0(v_{z1} - u_{z1})] - D[KN_0(v_{y1} - u_{y1})]\} \quad (20)$$

$$n\{D(\rho_0 u_{x1}) - i\rho_0 k_x u_{z1}\} = i\rho_1 g k_x$$
$$+ 2\{D[\rho_0(\Omega_z u_{y1} - \Omega_y u_{z1})] - i\rho_0 k_x[\Omega_y u_{x1} - \Omega_x u_{y1}]\}$$
$$+ \{D[KN_0(v_{x1} - u_{x1})] - ik_x[KN_0(v_{z1} - u_{z1})]\} \quad (21)$$

$$\rho_0 n\{ik_x u_{y1} - ik_y u_{x1}\}$$
$$= 2\rho_0\{(ik_x\Omega_x + ik_x\Omega_y)u_{z1} - ik_y(\Omega_z u_{y1} - \Omega_y u_{z1}\}$$
$$+ \{ik_x[KN_0(v_{y1} - u_{y1})] - ik_y[KN_0(v_{x1} - u_{x1})]\} \quad (22)$$

$$ik_x u_{x1} + ik_y u_{y1} + D u_{z1} = 0, \quad (23)$$

$$n\rho_1 + u_{z1}\frac{d\rho_0}{dz} = 0, \quad (24)$$

$$\{\tau n + 1\}v_{x1} = u_{x1}, \quad \{\tau n + 1\}v_{y1}$$
$$= u_{y1}, \quad \{\tau n + 1\}v_{z1} = u_{z1}. \quad (25)$$

Now, we eliminate some variables from the system of Eqs. (20, 21, 22, 23, 24, 25) and get the following differential equation in u_{z1}

$$n'\{D(\rho_0 D u_{z1}) - k^2\rho_0 u_{z1}\} + \left\{\frac{gk^2}{n} + 2i\Omega^-\right\}(D\rho_0)u_{z1}$$
$$+ \frac{4i\rho\,\Omega^+}{n'}\{i\Omega^+ u_{z1} + \Omega_z D u_{z1}\}$$
$$+ \frac{4\Omega_z}{n'}D\{\rho_0(i\Omega^+ u_{z1} + \Omega_z D u_{z1})\} = 0, \quad (26)$$

$$\Omega^+ = k_x\Omega_x + k_y\Omega_y, \quad \Omega^- = k_y\Omega_x - k_x\Omega_y,$$

$$n' = n + \frac{mN_0 n}{\rho_0(1 + n\tau)}, \quad D = \frac{d}{dz}.$$

The instability for two layers

In this section we consider two incompressible fluids of densities ρ_1, ρ_2. The fluids are acted upon by a general rotation field. Moreover, the density is constant in each region, i.e., we specialize to the case of constant densities, which are defined as:

$$\rho = \rho_1, \quad z < 0,$$
$$\rho = \rho_2, \quad z > 0. \quad (27)$$

For the case of constant density, Eq. (26) becomes

$$\left\{n'^2 + 4\Omega_z^2\right\}D^2 u_{z1} + 8i\Omega^+\Omega_z D u_{z1}$$
$$- \left\{k^2 n'^2 + 4(\Omega^+)^2\right\}u_{z1} = 0, \quad (28)$$

The general solution of Eq. (28) in each region can easily be found as

$$u_{z1} = C\exp(q_1 z), \quad z < 0,$$
$$u_{z1} = \bar{C}\exp(q_2 z), \quad z > 0, \quad (29)$$

where C and $\bar{C}$ are constants,

$$q_1 = \frac{1}{n'^2 + 4\Omega_z^2}\left\{-4i\Omega^+\Omega_z + \left[n'^2 k^2(n'^2 + 4\Omega_z^2) + 4n'^2(\Omega^+)^2\right]^{\frac{1}{2}}\right\}$$

$$q_2 = \frac{-1}{n'^2 + 4\Omega_z^2}\left\{4i\Omega^+\Omega_z + \left[n'^2 k^2(n'^2 + 4\Omega_z^2) + 4n'^2(\Omega^+)^2\right]^{\frac{1}{2}}\right\}. \quad (30)$$

The boundary conditions which are to be satisfied at the interface between the two fluids are

1. At the interface between the two fluids u_{z1} is continuous at $z = 0$.
2. From Eq. (26), the jump condition at the interface $z = 0$ is given as

$$
n' \left\{ (\rho_0 D u_{z1})_{z>0} - (\rho_0 D u_{z1})_{z<0} \right\}
$$
$$
+ \left\{ \frac{gk^2}{n} + 2i\,\Omega^- \right\} \left\{ (\rho_0)_{z>0} - (\rho_0)_{z<0} \right\} (u_{z1})_s
$$
$$
+ \frac{4\Omega_z}{n'} \left\{ \left[\rho_0 \left(i\,\Omega^+ u_{z1} + \Omega_z D u_{z1} \right) \right]_{z>0} \right.
$$
$$
\left. - \left[\rho_0 \left(i\,\Omega^+ u_{z1} + \Omega_z D u_{z1} \right) \right]_{z<0} \right\} = 0, \tag{31}
$$

Using the above boundary conditions and eliminating the constants C and $\bar{C}$, the dispersion relation is given by equation

$$
n' k \left[1 + 4 \left(\frac{\Omega_z^2}{n'^2} + \frac{\Omega^{+2}}{n'^2 k^2} \right) \right]^{\frac{1}{2}} = \left[\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \right] \left[\frac{gk^2}{n} + 2i\,\Omega^- \right], \tag{32}
$$

in the absence of horizontal component of the rotation $(\Omega_x = \Omega_y = 0 \rightarrow \Omega^+ = \Omega^- = 0)$, and we put $\Omega_z = \Omega$, the dispersion relation (32) becomes

$$
n' n \left[1 + \frac{4\Omega^2}{n'^2} \right]^{\frac{1}{2}} = gk \left[\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \right] \tag{33}
$$

This equation is similar to Eq. (21), in the absence of surface tension that has been derived by Sharma et al. [40].

In the case of slow rotating fluids to discuss the RTI and stability we assume parametric limit $4 \left(\frac{\Omega_z^2}{n'^2} + \frac{\Omega^{+2}}{n'^2 k^2} \right) \ll 1$. Then the dispersion relation (32) takes the form

$$
n' k \left[1 + 2 \left(\frac{\Omega_z^2}{n'^2} + \frac{\Omega^{+2}}{n'^2 k^2} \right) \right] = \left[\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \right] \left[\frac{gk^2}{n} + 2i\,\Omega^- \right], \tag{34}
$$

Substituting the expression of n' into Eq. (34) and by rearranging the new equation, the dispersion relation is given by

$$
D(n,k) = \tau^2 n^4 + \left\{ 2\tau(1+\alpha) - \frac{\tau^2 A (2i\Omega^-)}{k} \right\} n^3
$$
$$
+ \left\{ (1+\alpha)^2 + \frac{2\tau^2}{k^2} (\Omega^{+2} + k^2 \Omega_z^2) \right.
$$
$$
\left. - \frac{\tau A (2i\Omega^-)(2+\alpha)}{k} - \tau^2 A gk \right\} n^2
$$
$$
+ \left\{ \frac{4\tau}{k^2} (\Omega^{+2} + k^2 \Omega_z^2) - \tau gkA(2+\alpha) - \frac{A(2i\Omega^-)(1+\alpha)}{k} \right\} n
$$
$$
+ \frac{2}{k^2} (\Omega^{+2} + k^2 \Omega_z^2) - gkA(1+\alpha) = 0, \tag{35}
$$

where $A = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$ is the Atwood number and $\alpha = \frac{mN_0}{\rho_2 + \rho_1}$ is the mass concentration of suspended dust particles.

Again, in the absence of horizontal component of the rotation $\Omega_x = \Omega_y = 0 \rightarrow \Omega^+ = \Omega^- = 0$, and we put $\Omega_z = \Omega$, the dispersion relation (35) becomes

$$
D(n,k) = \tau^2 n^4 + 2\tau(1+\alpha)n^3
$$
$$
+ \left\{ (1+\alpha)^2 + 2\tau^2 \Omega^2 - Agk \right\} n^2
$$
$$
+ \left\{ 4\tau \Omega^2 - \tau gkA(2+\alpha) \right\} n
$$
$$
+ 2\Omega^2 - gkA(1+\alpha) = 0, \tag{36}
$$

This equation is similar to Eq. (23), in the absence of surface tension that has been derived by Sharma et al. [40]

Now, one can see that the roots of Eq. (35) are complex, which means that physically the perturbation will have an oscillating behavior with respect to time. So, to discuss the role of our selected parameters we suppose that the rate between the horizontal components of rotation (Ω_x, Ω_y) and horizontal wave numbers components (k_x, k_y) is equivalent (i.e., $\frac{\Omega_x}{\Omega_y} = \frac{k_x}{k_y}$) and if we consider the wave number of perturbation makes an angle θ with respect to the x-axis and the horizontal component of the angular velocity makes an angle Φ with respect to the x-axis. Then (k_x, k_y) = ($k\sin\theta$, $k\cos\theta$) (Ω_x, Ω_y) = ($\Omega\sin\Phi$, $\Omega\cos\Phi$) $\Rightarrow$ $\Omega^+ = k_x\Omega_x + k_y\Omega_y = k\Omega\cos(\theta-\Phi)$ and $\Omega^- = k_y\Omega_x - k_x\Omega_y = k\Omega\sin(\theta-\Phi)$. This leads to $k_y\Omega_x - k_x\Omega_y = \Omega^- = 0$ (this means that the perturbation wave vector is parallel to the horizontal component of rotation) $\Rightarrow \theta = \Phi \Rightarrow^+ = k\Omega$. Then the dispersion relation (35) becomes in the formula

$$
D(n,k) = \tau^2 n^4 + 2\tau(1+\alpha)n^3
$$
$$
+ \left\{ (1+\alpha)^2 + \frac{2\tau^2}{k^2}(\Omega^{+2} + k^2\Omega_z^2) - \tau^2 A gk \right\} n^2
$$
$$
+ \left\{ \frac{4\tau}{k^2}(\Omega^{+2} + k^2\Omega_z^2) - \tau gkA(2+\alpha) \right\} n
$$
$$
+ \frac{2}{k^2}(\Omega^{+2} + k^2\Omega_z^2) - gkA(1+\alpha) = 0, \tag{37}
$$

Introducing the relaxation frequency parameter $f_s^* = \frac{1}{\tau}$ of the suspended particles, the above equation becomes

$$
D(n,k) = n^4 + 2f_s^*(1+\alpha)n^3
$$
$$
+ \left\{ f_s^{*2}(1+\alpha)^2 + \frac{2}{k^2}(\Omega^{+2} + k^2\Omega_z^2) - Agk \right\} n^2
$$
$$
+ f_s^* \left\{ \frac{4}{k^2}(\Omega^{+2} + k^2\Omega_z^2) - gkA(2+\alpha) \right\} n
$$
$$
+ f_s^{*2} \left\{ \frac{2}{k^2}(\Omega^{+2} + k^2\Omega_z^2) - gkA(1+\alpha) \right\} = 0, \tag{38}
$$

This equation may have real positive roots, whereas the expression $\Omega^+ = k_x\Omega_x + k_y\Omega_y$ represents a measure of the horizontal components of rotation effects on the rate of

growth. Also, if the parameters' problem (general rotation and relaxation frequency of suspended dust particles) are neglected, then we return to the classical case, where the growth rate is given by $n = \sqrt{Agk}$.

Stability discussion

To check up the behavior of the growth rate in (38) ascribable to the relaxation frequency of suspended dust particles and the general rotation we need to calculate the derivative of the growth rate of unstable R–T mode n_0 with relaxation frequency of the suspended dust particles (f_s^*) and components of the rotation (Ω_z, Ω^+) (i.e. $\frac{\partial n_0}{\partial f_s^*}, \frac{\partial n_0}{\partial \Omega_z}, \frac{\partial n_0}{\partial \Omega^+}$) that will be given in the subsections ("At $\Omega^+ = \Omega_z = 0$, $f_s^* \neq 0$", "At $\Omega_z \neq 0$, $\Omega^+ = 0$, $f_s^* \neq 0$", "$\Omega^+ \neq 0$, $\Omega_z = 0$, $f_s^* \neq 0$", "At $\Omega^+ \neq 0$, $\Omega_z \neq 0$, $f_s^* \neq 0$"). Through this subsections some numerical calculations are presented in Figs. 1, 2, 3, 4, 5, 6, 7 and 8. The parameters in Eq. (38) take the next units $g = 9.8$ km/s^2 or $g = 0.98 \cong 1$ km/s^2, $1 \leq k \leq 10$ km^{-1}, growth rate $0 \leq n \leq 2.5$ s^{-1}, the horizontal and vertical components of rotation are $0 \leq \Omega_z \leq 2$ s^{-1}, $0 \leq \Omega_x \leq 20$ s^{-1}, relaxation frequency of suspended dust particles $0 \leq f_s^* = \frac{1}{\tau} \leq 20 km^{-1} s^{-1}$, $A = 0.5$ kg^{-1} km^{-3} and $\alpha = 0.5$ km^{-3}.

At $\Omega^+ = \Omega_z = 0, f_s^* \neq 0$

This means that we discuss the role of relaxation frequency of suspended dust particles in the absence of both vertical and horizontal rotation components, where from Eq. (38) we find

$$\frac{\partial n_0}{\partial f_s^*} = -\frac{2(1+\alpha)n_0^3 + 2f_s^*(1+\alpha)^2 n_0^2 - gkA\left[(2+\alpha)n_0 + 2f_s^*(1+\alpha)\right]}{\left\{4n_0^3 + 6f_s^*(1+\alpha)n_0^2 + 2f_s^{*2}(1+\alpha)^2 n_0 - Agk[2n_0 + f_s^*(2+\alpha)]\right\}}.$$
(39)

From Eq. (39) $\frac{\partial n_0}{\partial f_s^*}$ will be negative under the two conditions

$$\left\{2(1+\alpha)n_0^3 + 2f_s^*(1+\alpha)^2 n_0^2\right\}$$
$$- gkA\left\{(2+\alpha)n_0 + 2f_s^*(1+\alpha)\right\} > 0,$$
(40)

$$\left\{4n_0^3 + 6f_s^*(1+\alpha)n_0^2 + 2f_s^{*2}(1+\alpha)^2 n_0\right\}$$
$$- Agk\{2n_0 + f_s^*(2+\alpha)\} > 0,$$
(41)

or under the two conditions

$$\left\{2(1+\alpha)n_0^3 + 2f_s^*(1+\alpha)^2 n_0^2\right\}$$
$$- gkA\left\{(2+\alpha)n_0 + 2f_s^*(1+\alpha)\right\} < 0,$$
(42)

$$\left\{4n_0^3 + 6f_s^*(1+\alpha)n_0^2 + 2f_s^{*2}(1+\alpha)^2 n_0\right\}$$
$$- Agk\{2n_0 + f_s^*(2+\alpha)\} < 0,$$
(43)

In this case the relaxation frequency of suspended dust particles has a stabilizing influence on the growth rate of unstable configuration (see refs. [34, 40]), which (the stabilizing influence) increases with the relaxation frequency of suspended dust particles increasing. This case has been plotted in Fig. 1, where the magnitude of the growth rate decreases with relaxation frequency of suspended dust particles increasing and these values decreases with wave number decreasing. Also it observes that the magnitudes of growth rate in the presence of relaxation frequency of suspended dust particles are less than their counterpart in the classical case ($\Omega^+ = \Omega_z = 0, f_s^* = 0$).

For our suggested values of Fig. 1 (relaxation frequency of suspended dust particles f_s^*, $\Omega^+ = \Omega_z = 0$ and the values of growth rate generated, that has been generated) both conditions (40) and (41) have been satisfied as in Fig. 2a and b, where the values of these conditions are positive and these values increase with wave number increasing.

At $\Omega_z \neq 0, \Omega^+ = 0, f_s^* \neq 0$

This means that we discuss the role of vertical rotation component in the presence of relaxation frequency of suspended dust particles, where from Eq. (38) we see that
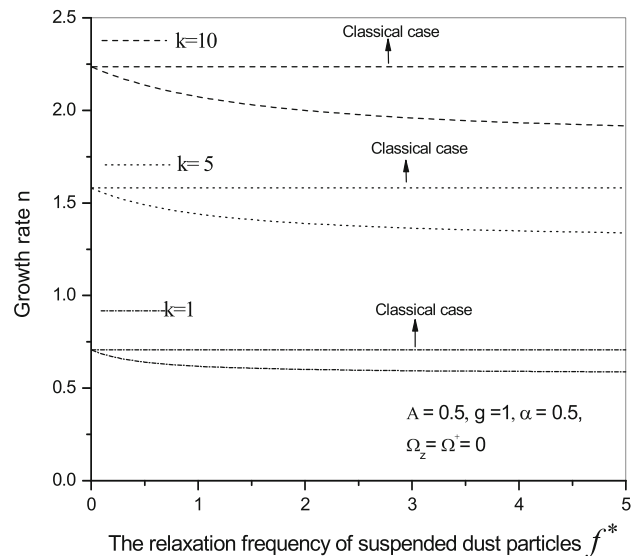

Fig. 1 The role of relaxation frequency of suspended dust particles on the growth rate in the absence of both vertical and horizontal rotation components ($\Omega^+ = \Omega_z = 0$), where the other parameters are $A = 0.5$, $\alpha = 0.5$
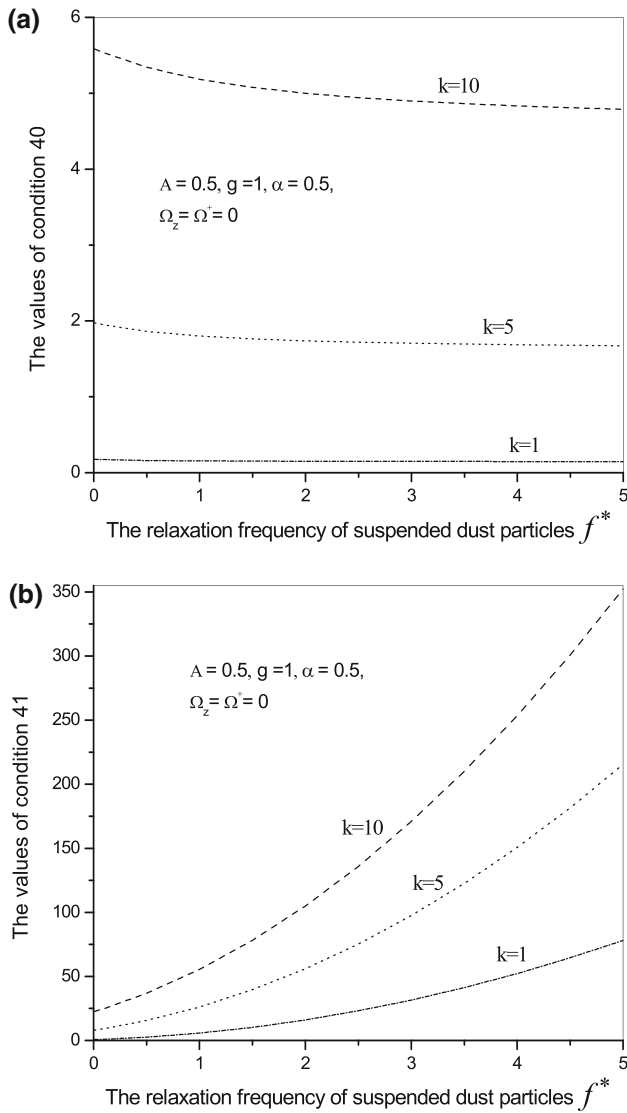
(a)

(b)

Fig. 2 The stability condition of **a** Eq. (40) and **b** Eq. (41)

$$\frac{\partial n_0}{\partial \Omega_z} = \frac{-4\Omega_z \left[n_0^2 + 2n_0 f_s^* + f_s^{*2} \right]}{\left[\begin{array}{c} 4n_0^3 + 6f_s^*(1+\alpha)\,n_0^2 + \left\{ f_s^{*2}(1+\alpha)^2 + 2\Omega_z^2 \right\} 2n_0 + \\ 4\Omega_z^2 f_s^* - gkA\left\{ 2n_0 + f_s^*(2+\alpha) \right\} \end{array} \right]}$$

(44)

From Eq. (44), $\frac{\partial n_0}{\partial \Omega_z}$ will be negative under the condition

$$\left[\begin{array}{c} 4n_0^3 + 6f_s^*(1+\alpha)\,n_0^2 + \left\{ f_s^{*2}(1+\alpha)^2 + 2\Omega_z^2 \right\} 2n_0 \\ + 4f_s^*\Omega_z^2 \\ - gkA\left\{ 2n_0 + f_s^*(2+\alpha) \right\} > 0, \end{array} \right]$$

(45)

This indicates that the growth rate of unstable R–T mode is decreased with increase in the vertical component of rotation. Then under the restriction (45), the vertical

component of rotation has a stabilizing influence with relaxation frequency of suspended dust particles kept constant. For the second time, this case is plotted in Fig 3, where the growth rate is plotted against the vertical components of rotation. It can be seen that the values of growth rate decreases with increasing Ω_z, that is less than their counterpart in the absence of Ω_z (classical case) and that decreases with increasing k. Numerical, the condition (45) clears in Fig. 4, where the counterpart values of Fig. 3 are greater than zero and that decreases as k decreases
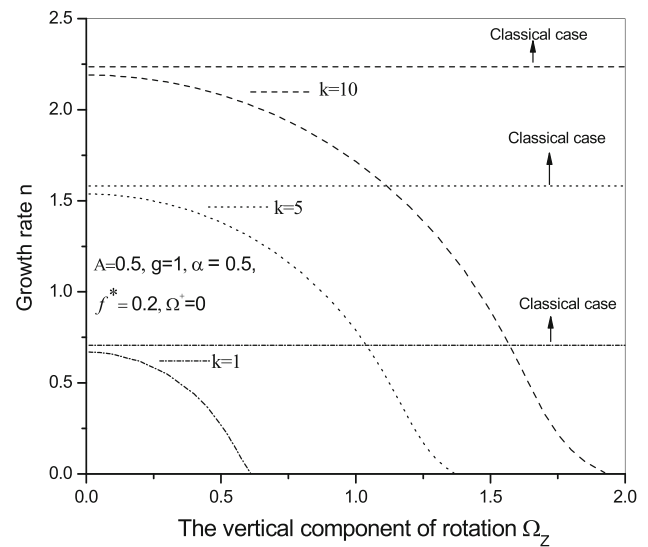

Fig. 3 The role of vertical rotation components on the growth rate, where the other parameters are $f_s^* = 0$, $\Omega^+ = 0$, $A = 0.5$, $\alpha = 0.5$
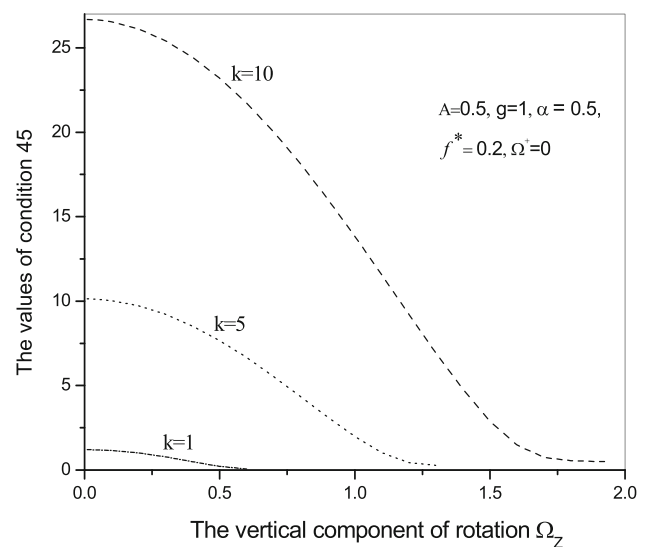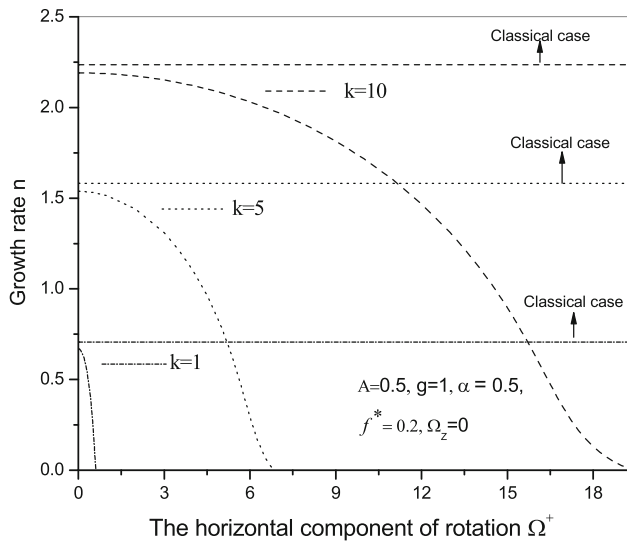

Fig. 4 The stability condition of Eq. (45)

Fig. 5 The role of horizontal rotation components on the growth rate, where the other parameters are $f_s^* = 0$, $\Omega_z = 0$, $A = 0.5$, $\alpha = 0.5$

Fig. 6 The stability condition of Eq. (47)

$\Omega^+ \neq 0$, $\Omega_z = 0$, $f_s^* \neq 0$

This means that we discuss the role of horizontal rotation components in the presence of relaxation frequency of suspended dust particles, where from Eq. (38) we have

$$\frac{\partial n_0}{\partial \Omega^+} = \frac{-4\Omega^+\left[n_0^2 + 2n_0 f_s^* + f_s^{*2}\right]}{k^2\left[\begin{array}{c} 4n_0^3 + 6f_s^*(1+\alpha)n_0^2 + \left\{f_s^{*2}(1+\alpha)^2 + \dfrac{2}{k^2}\Omega^{+2}\right\}2n_0 + \\[2mm] \dfrac{4}{k^2}\Omega^{+2}f_s^* - gkA\left[2n_0 + f_s^*(2+\alpha)\right] \end{array}\right]}.$$

(46)

From Eq. (46) $\frac{\partial n_0}{\partial \Omega^+}$ will be negative under the condition

$$\left\{4n_0^3 + 6f_s^*(1+\alpha)n_0^2 + \left\{f_s^{*2}(1+\alpha)^2 + \frac{2}{k^2}\Omega^{+2}\right\}2n_0 \right.$$
$$\left. + \frac{4}{k^2}\Omega^{+2}f_s^*\right\} - gkA\left\{2n_0 + f_s^*(2+\alpha)\right\} > 0,$$

(47)

Again, this indicates that the growth rate of unstable R–T mode is decreased with increase in the horizontal component of rotation. This means that under the restriction (47),
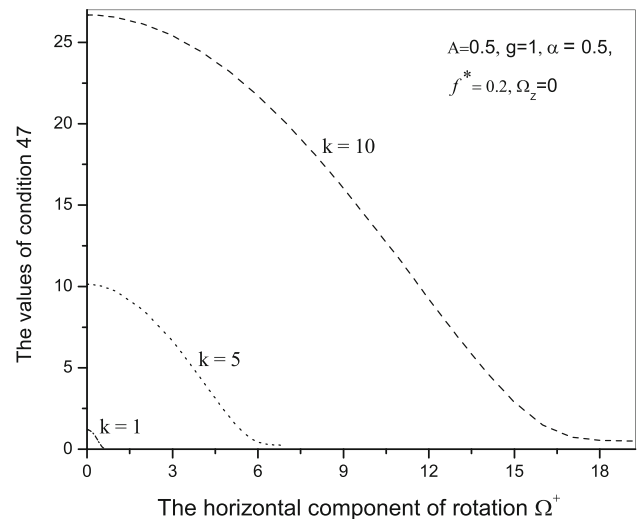
the horizontal component of rotation have a stabilizing with relaxation frequency of suspended dust particles kept constant. This role holds in Fig. 5, where the values of growth rate decreases with increasing of Ω^+ and the stability condition (Eq. 47) clears in Fig. 6, where the values of condition (47) and that is counterpart of Fig. 5 are positive.

From Figs. (3) and (5) and if we compare between the above results in "At $\Omega_z \neq 0$, $\Omega^+ = 0$, $f_s^* \neq 0$" and "$\Omega^+ \neq 0$, $\Omega_z = 0$, $f_s^* \neq 0$", it notices that the influence of vertical component of rotation is felt more than horizontal component, where at $k = 10$, the vertical component of rotation suppresses the instability completely at $\Omega_z \approx 2$, while for Ω^+ the complete stability happens at $\Omega^+ \approx 19$. The same behavior holds for both vertical and horizontal components of rotation at $k = 5, 1$.

At $\Omega^+ \neq 0$, $\Omega_z \neq 0$, $f_s^* \neq 0$

This means that we discuss the role of relaxation frequency of suspended dust particles in the presence of both vertical and horizontal rotation components, where from Eq. (38), we find that

$$\frac{\partial n_0}{\partial f_s^*} = \frac{-\left\{\begin{array}{c} 2(1+\alpha)n_0^3 + 2f_s^*(1+\alpha)^2 n_0^2 + \left\{\dfrac{4}{k^2}\left(\Omega^{+2} + k^2\Omega_z^2\right)\right\}n_0 + \\[2mm] \left[\dfrac{4f_s^*}{k^2}\left(\Omega^{+2} + k^2\Omega_z^2\right)\right] - gkA\left[(2+\alpha)n_0 + 2(1+\alpha)f_s^*\right] \end{array}\right\}}{\left\{\begin{array}{c} 4n_0^3 + 6f_s^*(1+\alpha)n_0^2 + 2\left[f_s^{*2}(1+\alpha)^2 + \dfrac{2}{k^2}\left(\Omega^{+2} + k^2\Omega_z^2\right)\right]n_0 + \\[2mm] f_s^*\left[\dfrac{4}{k^2}\left(\Omega^{+2} + k^2\Omega_z^2\right)\right] - Agk\left[2n_0 + f_s^*(2+\alpha)\right] \end{array}\right\}}.$$

(48)

Then $\frac{\partial n_0}{\partial f_s^*}$ will be negative under the two conditions

$$\left\{\begin{array}{l} 2(1+\alpha)\,n_0^3 + 2f_s^*(1+\alpha)^2 n_0^2 + \left\{\dfrac{4}{k^2}\left(\Omega^{+2}+k^2\Omega_z^2\right)\right\}n_0 \\[2mm] + \left[\dfrac{4f_s^*}{k^2}\left(\Omega^{+2}+k^2\Omega_z^2\right)\right] \\[3mm] -\, gkA\left\{\begin{array}{l}(2+\alpha)n_0 \\ +\,2\,(1+\alpha)f_s^*\end{array}\right\} > 0, \end{array}\right. \tag{49}$$

$$\left\{\begin{array}{l} 4n_0^3 + 6f_s^*(1+\alpha)\,n_0^2 + 2\left[f_s^{*2}(1+\alpha)^2 + \dfrac{2}{k^2}\left(\Omega^{+2}+k^2\Omega_z^2\right)\right]n_0 \\[2mm] + \dfrac{4f_s^*}{k^2}\left(\Omega^{+2}+k^2\Omega_z^2\right) \\[2mm] -\,A\,gk\{2n_0 + f_s^*(2+\alpha)\} > 0. \end{array}\right. \tag{50}$$

In this case the growth rate of unstable R–T mode is decreased with increase in relaxation frequency of suspended dust particles in the presence of general rotation. This means that under the conditions (49) and (50) the relaxation frequency of suspended dust particles has a stabilizing influence. Also, in the presence of general rotation, the relaxation frequency of suspended dust particles has the same stabilizing role under the two conditions:

$$\left\{\begin{array}{l} 2(1+\alpha)\,n_0^3 + 2f_s^*(1+\alpha)^2 n_0^2 + \left\{\begin{array}{l}\dfrac{4}{k^2}\left(\Omega^{+2}+\right. \\ \left. k^2\Omega_z^2\right)\end{array}\right\}n_0 \\[4mm] + \left[\dfrac{4f_s^*}{k^2}\left(\Omega^{+2}+k^2\Omega_z^2\right)\right] \\[3mm] -\, gkA\left\{\begin{array}{l}(2+\alpha)n_0 \\ +\,2\,(1+\alpha)f_s^*\end{array}\right\} < 0, \end{array}\right. \tag{51}$$

and

$$\left\{\begin{array}{l} 4n_0^3 + 6f_s^*(1+\alpha)\,n_0^2 + 2\left[f_s^{*2}(1+\alpha)^2 + \dfrac{2}{k^2}\left(\Omega^{+2}+k^2\Omega_z^2\right)\right]n_0 \\[2mm] + \dfrac{4f_s^*}{k^2}\left(\Omega^{+2}+k^2\Omega_z^2\right)\end{array}\right\} - A\,gk\{2n_0 + f_s^*(2+\alpha)\} < 0. \tag{52}$$

In the general case, the role of relaxation frequency of suspended dust particles in the presence of general rotation is plotted in Fig. 7, where if we compare between the values of growth rate in Figs. 1 and 7. It can be seen that the values of growth rate in the presence of general rotation (Fig. 7) are less than their counterpart in the absence of general rotation (Fig. 1). The stability role in the general case stratifies under the two conditions (49) and (50). They are shown in Fig. 8a and b, where the values of Eqs. (49) and (50) and that counterpart of values of growth rate of Fig. 7 are positive.

(a)

(b)

Fig. 8 The stability condition of **a** Eq. (49) and **b** Eq. (50)
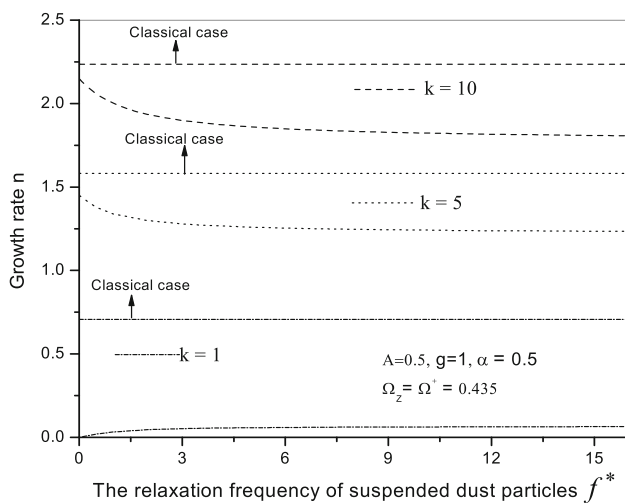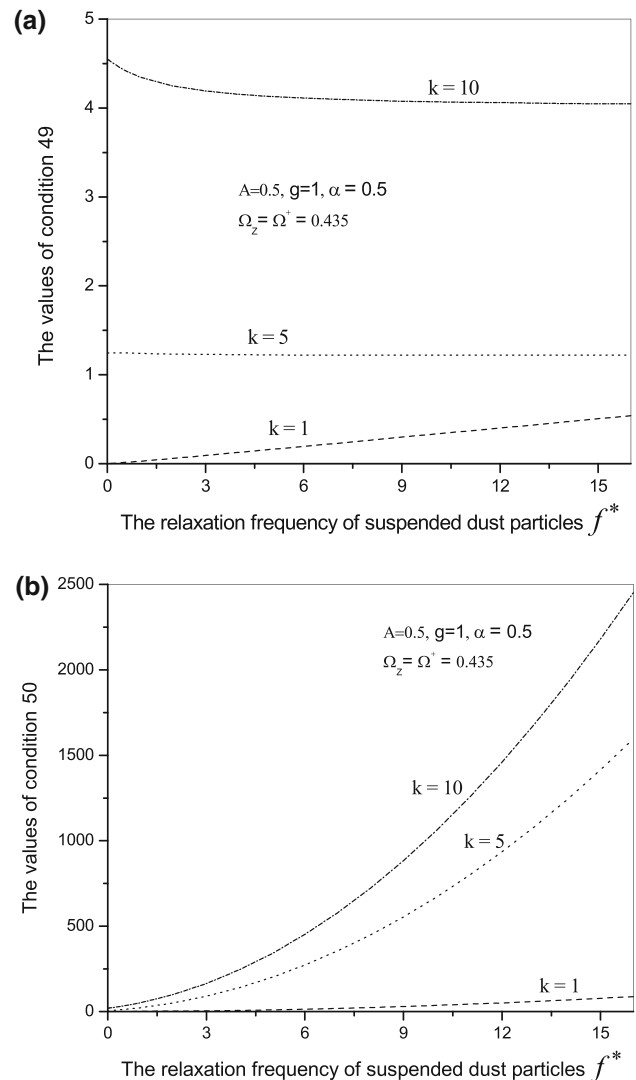

Fig. 7 The role of relaxation frequency of suspended dust particles on the growth rate in the presence of both vertical and horizontal rotation components ($\Omega^+ = \Omega_z = 0.435$), where the other parameters are $A = 0.5$, $\alpha = 0.5$

Conclusion

Finally, we have presented the analytical results of the RTI in the presence of small rotation (vertical and horizontal rotation components) and suspended dust particles of two incompressible fluids. The dispersion relation is derived as a function of the physical parameters of the system considered in Eq. (35) (complex formula). A real formula that happens at the rate between horizontal components of rotation (Ω_x, Ω_y) and horizontal wave number components (k_x, k_y) given in the form ($\frac{\Omega_x}{\Omega_y} = \frac{k_x}{k_y}$), is considered in Eq. (38). According to Eq. (38) and at $\rho_2 > \rho_1$ the condition of RTI can be obtained easily from the constant term and it is given as $\left(\Omega^{+^2} + k^2\Omega_z^2\right) < \frac{1}{2}gk^3A(1+\alpha)$, where the system remains unstable for all the values of rotation smaller than this value. Some special cases from Eq. (38) that isolate the effect of various parameters on the growth rate of the RTI are discussed in ("At $\Omega^+ = \Omega_z = 0$, $f_s^* \neq 0$", "At $\Omega_z \neq 0$, $\Omega^+ = 0$, $f_s^* \neq 0$", "$\Omega^+ \neq 0$, $\Omega_z = 0$, $f_s^* \neq 0$", "At $\Omega^+ \neq 0$, $\Omega_z \neq 0$, $f_s^* \neq 0$"). The numerical calculations have shown that both Ω_z and Ω^+ have a critical strength to suppress the instability completely. The stabilizing role of vertical component of rotation is greater than the stabilizing role of horizontal component. The system was more stable in the presence of both suspended dust particles, vertical and horizontal rotation components. There are two stability conditions in the presence of suspended dust particles, while in the presence of rotation only (vertical or horizontal component) there is one stability condition.

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