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Spectra of heavy quarkonia in a magnetized-hot medium in the framework of fractional non-relativistic quark model

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Abstract
In the fractional nonrelativistic potential model, the decomposition of heavy quarkonium in a hot magnetized medium is investigated. The analytical solution of the fractional radial Schrödinger equation for the hot-magnetized interaction potential is displayed by using the conformable fractional Nikiforov-Uvarov method. Analytical expressions for the energy eigenvalues and the radial wave function are obtained for arbitrary quantum numbers. Next, we study the charmonium and bottmonium binding energies for different values of the magnetic field in the thermal medium. The effect of the fractional parameter on the decomposition temperature is also analyzed for charmonium and bottomonium in the presence of hot magnetized media. We conclude that the dissociation of heavy quarkonium in the fractional nonrelativistic potential model is faster than the classical nonrelativistic potential model.

Keywords
Strong magnetic field, Heavy qurakonium, Fractional Schrödinger equation.

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1. Introduction

Quantum chromodynamics theory calculates that at sufficiently high temperatures and densities, the gluons and quarks confined inside the hadrons are freed into a medium of gluons and quarks. Recent works have focused on producing and identifying this new state of matter theoretically [1–7] and experimentally in ultra-relativistic heavy-ion collisions (URHIC) with the increasing center of mass energies in the BNL AGS, CERN SPS, BNL RHIC, and CERN LHC experiments. However, for the noncentral events in URHICs, the powerful magnetic field is generated at the collisions' initial stages due to very high relative velocities of the spectator quarks concerning the fireball [8, 9].
There are numerous research studies to investigate baryon properties to descript the ground states and excitation spectra in the non-relativistic models such as [10–14]. The research studies are extended to study the quarkonium in the magnetic field, in which the three-dimensional Schrödinger equation (SE) numerically solved with the Cornell and the QCD Coulomb potentials [15]. In Ref. [16], the properties of quarkonium states have been studied in the presence of strong magnetic field. Two methods were used to calculate the critical value of the magnetic field for both charmonium and bottomonium states. Bagchi et al. inferred that in the presence of a magnetic field, the bound states J/ψ and $\Upsilon(1S)$ become more firmly bound than in a pure thermal QGP owing to the alteration of the heavy quark potential [17]. In Ref. [18], the authors studied the effect of a strong external magnetic field on quarkonium states $c\bar{c}$ and $b\bar{b}$ in the framework of a non-relativistic quark model. Furthermore, the authors included in their calculations anisotropies through static quark-antiquark potential in agreement with recent lattice studies. In Ref. [19], the dissociation of heavy quarks in hot QCD plasma in the presence of a strong magnetic field is studied by using Nikiforov-Uvarov (NU) method.
Fractional calculus has drawn interest in a variety of physics fields [20–25]. The analytical-exact iteration method is extended to the conformable fractional form to obtain the analytical solutions of the N-dimensional radial SE [20] with its applications on heavy mesons. The generalized NU method is extended to the fractional domain of high-energy physics by using the radial SE [21]. The fractional concept of NU was used to solve fractional radial SE for different interaction potentials such as the oscillator potential, Woods-Saxon potential, and Hulthen potential [22]. Herrmann applied a derivative Caputo fractional Schrödinger wave equation using quantitative of the classical nonrelativistic Hamiltonian [23].
The conformable fractional form is extended to a finite temperature medium to study the binding energy and dissociation

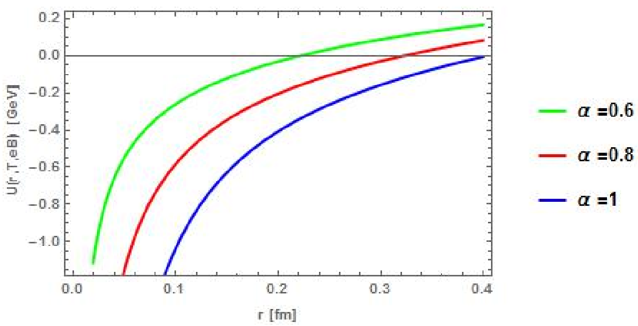

Figure 1. In left panel, the potential interaction is plotted as a function of (r) for the different parameters α.

of temperature [24].
The aim of the present work. It will show that the fractional model plays an essential role in studying the binding energy and the dissociation temperature of quarkonia in the hot-magnetized medium, which are not considered in the previous works.
This paper is organized as follows: In Sec. 2, we provide the theoretical method. In Sec.3, the method is given in detail to solve the N-dimensional SE. In Sec. 4, we discuss the obtained results. The conclusion is given in Sec. 5.

2. Theoretical model

Fractional derivative plays an important role in the applied science. Important mathematical tools for working with fractional models and solving fractional differential equations, such as a generalization of Stirling numbers in the framework of fractional calculus and a set of efficient numerical methods, and the analytic-exact methods that employed well known definitions such as Riemann-Liouville, Riesz and Caputo, and the comfortable fractional derivative, in which gave an elegant formula that allows applying boundary and initial conditions

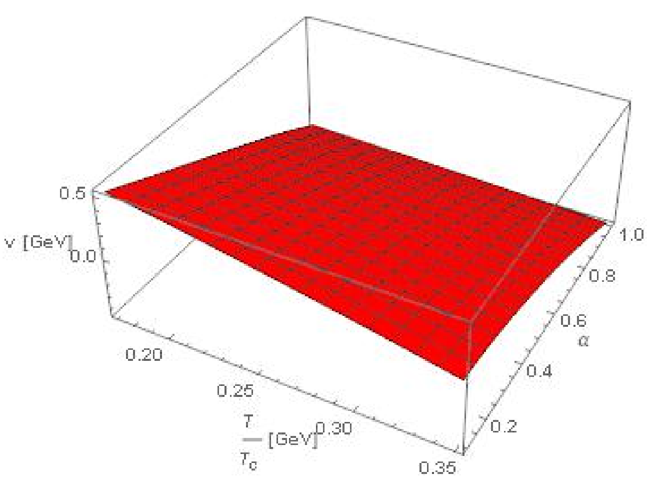

Figure 2. Interaction potential is plotted as a function of temperature ratio and fractional parameter.

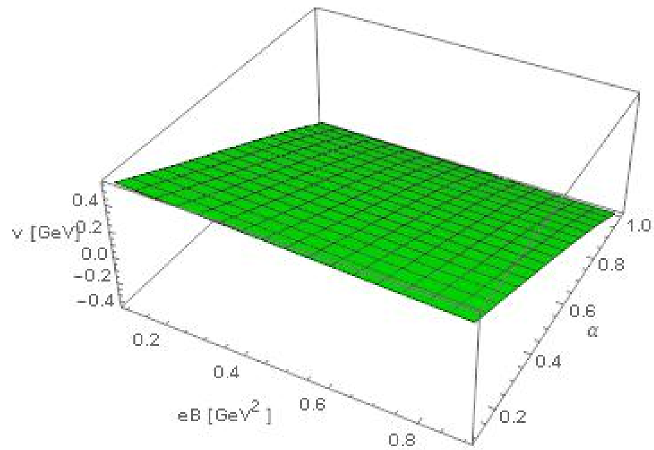

Figure 3. Interaction potential is plotted as a function of magnetic field and fractional parameter.

as in Ref. [25].

$$D_r^{\alpha}(r) = \int_{r_0}^{r} K_a(r-s) f^{(n)}(s) d(s), r > r_0 \tag{1}$$

with

$$K_a(r-s) = \frac{(r-s)^{n-\alpha-1}}{\Gamma(n-\alpha)} \tag{2}$$

where, $f^{(n)}$ is the n the derivative of the function $f(r)$, and $K_a(r-s)$ is the kernel, which is fixed for a given real number α. The kernel $K_a(r-s)$ has singularity at $r = s$. Caputo and Fabrizio [26] suggested a new formula of the fractional derivative with smooth exponential kernel of the form to avoid

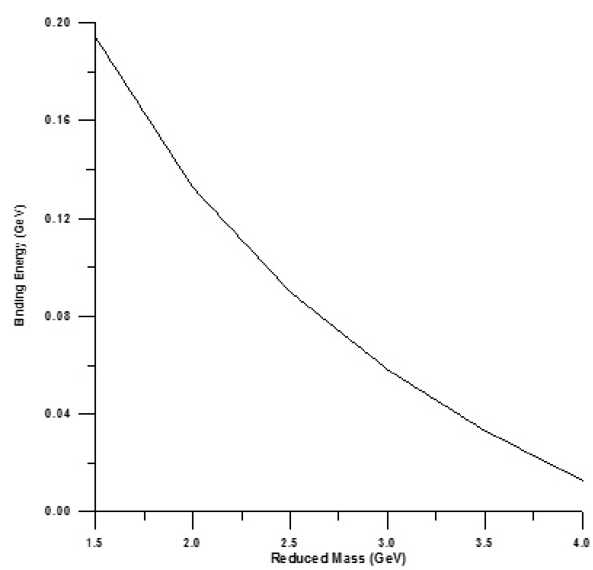

Figure 4. The binding energy is plotted as a function of reduced mass at T=0 and $\alpha = 1$.

the difficulties that found in Eq. (1)

$$D_r^{\alpha} = \frac{M(a)}{1-\alpha}\int_{r_0}^{r} e^{(\frac{\alpha(t-s)}{1-\alpha})}\dot{y}(s)d(s). \tag{3}$$

where M(a) is a normalization function with $M(0) = M(1) = 1$.

A new formula of fractional derivative called conformable fractional derivative (CFD) is proposed by Khalil et al. [27].

$$D_t^{\alpha} f(r)_{\varepsilon\to 0} \lim \frac{f(r-\varepsilon r^{1-\alpha}) - f(r)}{\varepsilon} \tag{4}$$

$$f(0)_{\varepsilon\to 0} = \lim f(r) \tag{5}$$

where,

$$D^{\alpha}[f_{nl}(r)] = r^{1-\alpha}\grave{f}_{nl}(r) \tag{6}$$

$$D^{\alpha}[D^{\alpha} f(r)] = (1-\alpha)r^{1-2\alpha}\grave{f}_{nl}(r) + r^{2-2\alpha}\acute{\grave{f}}_{nl}(r) \tag{7}$$

with $0 < \alpha \leq 1$. This a new definition is simple and provides a natural extension of differentiation with integer order $n \in z$ to fractional order $\alpha \in C$. Moreover, the CFD operator is linear and satisfies the interesting properties that traditional fractional derivatives do not, such as the formula of the derivative of the product or quotient of two functions and the chain rule [28].

3. Real part of the potential in a magnetic field

In Ref. [29], the medium-modification to the vacuum potential in the presence of magnetic field by correcting both its short and long-distance part with a dielectric function $\varepsilon(q)$ as

$$V(r,T,B) = \int \frac{d^3q}{(2\pi)^{\frac{3}{2}}}(e^{ik\cdot r}-1)\frac{V(q)}{\varepsilon(q)} \tag{8}$$

where the r-independent term has subtracted to renormalize the heavy quark free energy, which is the perturbation free energy of quarkonium at infinite separation. The Fourier transform, $V(q)$ of the perturbative part of the Cornell potential ($V(r) = -4\alpha_s/3r$) is given ($V(q) = -4/3(2/\pi)^{1/2}(\alpha_s/q^2)$) and the dielectric permittivity, $\varepsilon(q)$ embodies the effects of confined medium in the presence of magnetic field. $\varepsilon(q)$ is defined by the static limit of "00"-component of resummed gluon propagator from the linear response theory

$$\frac{1}{\varepsilon(q)} = \lim_{q_0\to 0} q^2 D^{00}(q_0,q) \tag{9}$$

The real parts of the nonperturbative (NP) term by using the dimension two gluon condensate are given as follows

$$ReD_{NP}^{00}(q_0=0,q) = -\frac{m_G^2}{(q^2+M_D^2)^2} \tag{10}$$

where m_G^2 is a dimensional constant, which can be related to the string tension through the relation $\sigma = \alpha m_G^2/2$. Thus, the real part of the "00"-component of the resummed gluon propagator that consists of both the HTL and the NP contributions can be written as follows

$$ReD_{NP}^{00}(q_0=0,q) = -\frac{1}{q^2M_D^2} - \frac{m_G^2}{(q^2+M_D^2)^2} \tag{11}$$

Now substituting Eq. (11) into Eq. (9) gives the real part of the dielectric permittivity, respectively

$$\frac{1}{Re\varepsilon(q)} = \frac{q^2}{q^2+M_D^2} + \frac{q^2m_G^2}{(q^2+M_D^2)^2} \tag{12}$$

The real-part of the dielectric permittivity in Eq. (12) is substituted into the definition of potential in Eq. (8) to obtain the real-part of $Q\bar{Q}$ potential in the presence of magnetic field. This potential depends on the radial distance. The effect of magnetic field will be appearing through the Debye mass. In addition, the anisotropy in the present potential with respect to the direction of magnetic field is not breaks the translational invariance of space, thus, we can write the potential interactions as follows (see Ref. [16], for detail).

$$V(r) = -\frac{4}{3}\alpha\left(\frac{e^{-m_D r}}{r} + m_D\right) + \frac{4}{3}\frac{\sigma}{m_D}(1-e^{-m_D r}) \tag{13}$$

where, the string tension σ= 0.18 GeV^2 and

$$\alpha = \frac{12\pi}{11N_c \ln\left(\frac{\mu_0^2+M_B^2}{\Lambda_V^2}\right)} \tag{14}$$

where, N_c is the number of colors, M_B($\sim$1 GeV) is an infrared mass which is interpreted as the ground state mass of the two gluons bound to by the basic string, μ_0 =1.1 (GeV), Λ_V =0.385 (GeV) as in Refs. [30–33] and the Debye mass [33] becomes as:

$$m_D^2 = \acute{g}^2T^2 + \frac{g^2}{4\pi^2T}\sum_f |q_f B| \int_0^{\infty} \frac{e^{\beta\sqrt{p_z^2+m_f^2}}}{(1+e^{\beta\sqrt{p_z^2+m_f^2}})^2} dp_z \tag{15}$$

where, the first term is the contribution from the gluon loops and dependent on temperature and the magnetic field doesn't affect it. The second term is this term strongly depends on the eB and is not much sensitive to the T of the medium. In the first term, where $\acute{g}$ is the running strong coupling and is given by

$$\acute{g} = 4\pi\acute{\alpha}_s(T) \tag{16}$$

where, $\acute{\alpha}_s(T)$ is the usual temperature-dependent running coupling. It is given by

$$\acute{\alpha}_s(T) = \frac{2\pi}{(11-\frac{2}{3}N_f)\ln\left(\frac{\Lambda}{\Lambda_{QCD}}\right)} \tag{17}$$

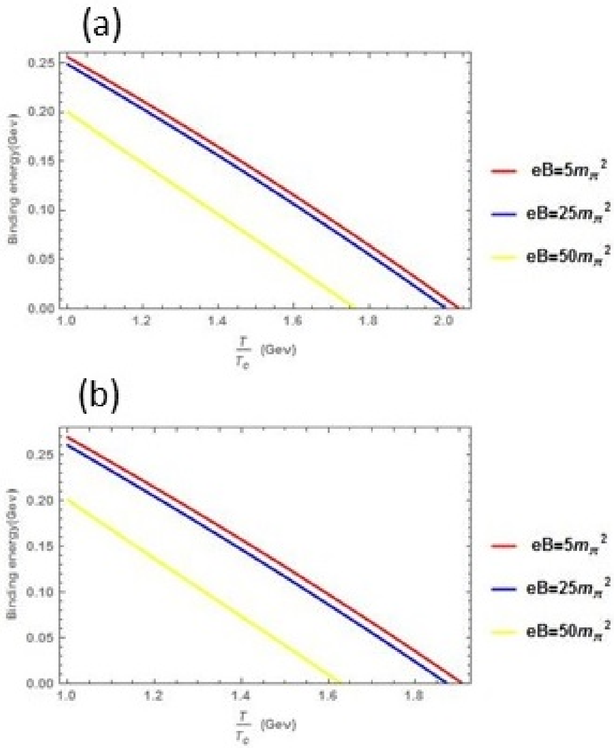

Figure 5. a) The Binding energy of charmonium as a function of the T in the thermal medium in the presence of the eB for the different magnetic field values at N_f=2 and α=1. b) The Binding energy of charmonium as a function of the T in the thermal medium in the presence of the eB for the different magnetic field values at N_f=2 and α=0.5.

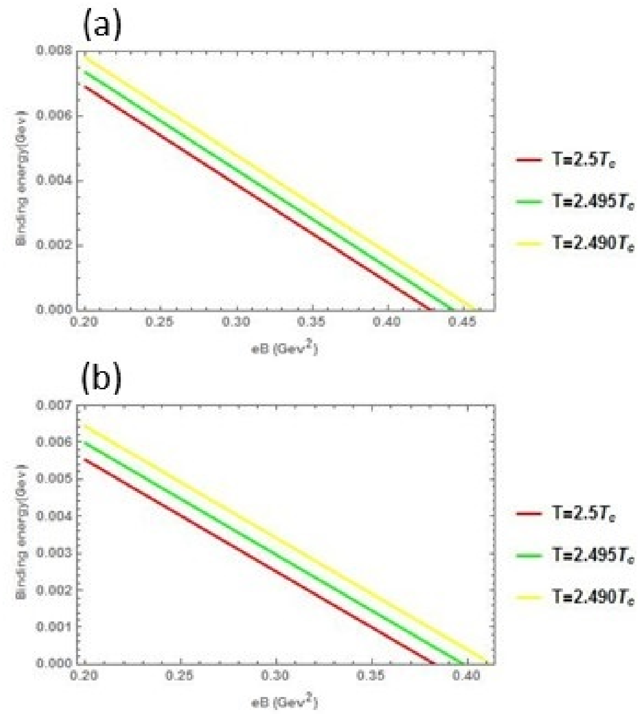

Figure 6. a) The binding energy of charmonium as a function of the magnetic field in the thermal medium for different values of the temperature at N_f=2 and α=1. b) The binding energy of charmonium as a function of the magnetic field in the thermal medium for different values of the temperature at N_f=2 and α=0.5.

where, N_f is the number of flavors, Λ is the renormalization scale is taken as $2\pi T$ and Λ_{QCD} ~0.2 (GeV) as in Ref. [34]. The second term is g=3.3, q_f is the quark charge flavor $f = u$ and d, B is the magnetic field, β is the inverse of temperature and quark mass massive m_f=0.307 (GeV) as in Ref. [30]. In Eq. (13), $e^{-m_D r}$ is extend if $m_D r \ll 1$ is considered in Ref. [35]. We rewrite Eq. (13) as follows

$$V(r) = a_1 r^2 + a_2 r + \frac{a_3}{r} \tag{18}$$

where,

$$a_1 = -\frac{2}{3}\sigma m_D \tag{19}$$

$$a_2 = -\frac{4}{3}\alpha m_D^2 + \frac{4}{3}\sigma \tag{20}$$

$$a_3 = -\frac{4}{3}\alpha \tag{21}$$

In Eq. (18), the first term is a harmonic potential that dominates the confinement force and the second term is a linear potential that also dominates the confinement force at long distances. A Coulombic potential that dominates the Coulombic force at small distances as in the third term. The second and third terms called Cornell potential [36,37]. To find energy eigenvalue and wave function, we used radial Schrödinger equation as in Refs. [16,29,30]. As in Ref. [37], in the N-dimensional space, the for two particles Schrödinger equation which interact with symmetrical potentials takes form

$$[\frac{d^2}{dr^2} + \frac{N-1}{r}\frac{d}{dr} - \frac{l(l+N-2)}{r^2} + 2\mu(E - V(r))]\Psi(r) = 0 \tag{22}$$

where l, N and μ are the angular momentum quantum number, the dimensional number, and the reduced mass of the system. The following radial SE is obtained by applying the wave function $\Psi(r) = r^{\frac{1-N}{2}} R(r)$

$$[\frac{d^2}{dr^2} - 2\mu(E - V(r) - \frac{(l+\frac{N-2}{2})^2 - \frac{1}{4}}{2\mu r^2})R(r)] = 0 \tag{23}$$

to put Eq. (23) in the fractional form. Firstly, we put it in the dimensionless form as follows

$$[\frac{d^2}{dz^2} - 2\mu^1(E^1 - V(z) - \frac{(l+\frac{N-2}{2})^2 - \frac{1}{4}}{2\mu z^2})R(z)] = 0 \tag{24}$$

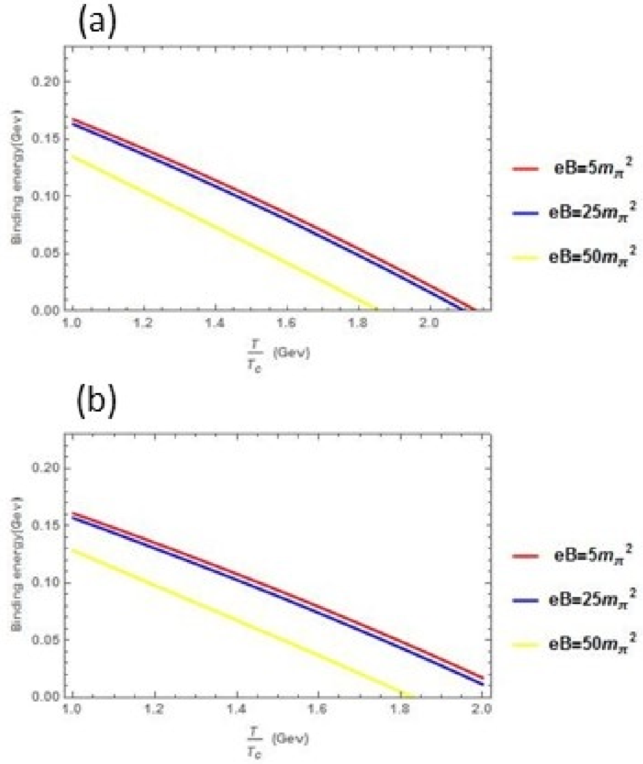

Figure 7. a) The Binding energy of bottomonium as a function of the temperature in the thermal medium in the presence of magnetic field for the different magnetic field values at N_f=2 and α=1. b) The binding energy of bottomonium (in GeV) as a function of the temperature in the thermal medium in the presence of magnetic field for the different magnetic field values at N_f=2 and α=0.5.

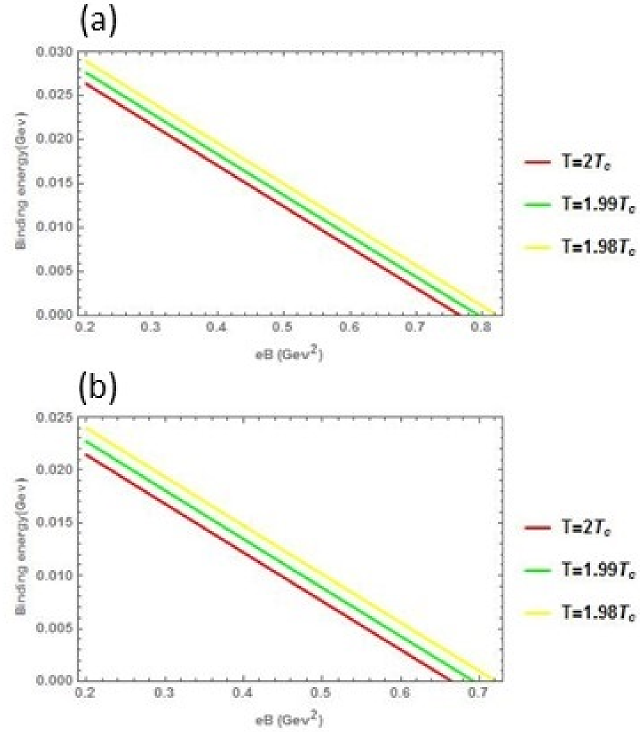

Figure 8. a) The binding energy of bottomonium (in GeV) as a function of the magnetic field in the thermal medium for different values of the temperature at N_f=2 and α=1. b) The binding energy of bottomonium (in GeV) as a function of the magnetic field in the thermal medium for different values of the temperature at N_f=2 and α=0.5.

where,

$$V(z) = a_1^1 z^2 + a_2^1 z + \frac{Aa_3}{z} \tag{25}$$

where, $a_1^1 = a_1/A^3$, $a_2^1 = a_2/A^2$, $\mu^1 = \mu/A$, $E^1 = E/A$, $z = rA$, since A is a dimensional unit equals 1 GeV. Therefore, we can write Eq.. (24) in the fractional form as follows;

$$D^\alpha[D^\alpha\Psi(z^\alpha)] + [2\mu^1(E^1 - V(z^\alpha)) - \frac{(l+\frac{N-2}{2})^2 - \frac{1}{4}}{2\mu^1 z^{2\alpha}}]\Psi(z^\alpha) = 0 \tag{26}$$

By applying NU method (For detail, see Refs. [21,24]). We obtain the spectrum of energy in dimensionless form

$$E^1 = \frac{6a_1^1}{\delta^{12}} + \frac{3a_2^1}{\delta^1} - \frac{2\mu^1(\frac{8a_1^1}{\delta^3} + \frac{3a_2^1}{\delta^2} - a_3)^2}{[(2n+1) \pm \sqrt{w + 8\mu^1(\frac{3a_1^1}{\delta^{14}} + \frac{a_2^1}{\delta^{13}} + \frac{(l+\frac{N-2}{2})^2 - \frac{1}{4}}{2\mu^1})}]^2} \tag{27}$$

then, we rewrite Eq. (27) in the dimensional form as follows

$$E_{n,l} = \frac{6a_1}{\delta^2} + \frac{3a_2}{\delta} - \frac{2\mu(\frac{8a_1}{\delta^3} + \frac{3a_2}{\delta^2} - a_3)^2}{[(2n+1) \pm \sqrt{w + 8\mu(\frac{3a_1}{\delta^4} + \frac{a_2}{\delta^3} + \frac{(l+\frac{N-2}{2})^2 - \frac{1}{4}}{2\mu})}]^2} \tag{28}$$

where,

$$w = (2n\alpha)^2 - 4(n(3\alpha - \alpha^2) + \frac{1}{2}n(n-1)\alpha(\alpha+1) + \alpha - 1) \tag{29}$$

The radial of wave function takes the following form:

$$R_{nl}(r^\alpha) = C_{nl} r^{-(\frac{B_1}{\sqrt{2A_1}} - 1)\alpha} e^{\sqrt{2A_1} r^\alpha} (-r^{2\alpha} D)^n r^{(-2n + \frac{B_1}{\sqrt{2A_1}})\alpha} e^{-2\sqrt{2A_1} r^{2\alpha}} \tag{30}$$

C_{nL} is the normalization constant, also,

$$A_1 = -\mu(E - \frac{6a_1}{\delta^2} - \frac{3a_2}{\delta}) \tag{31}$$

Table 1. Dissociation temperature (T_D) for charmonium.

Sate	eB=5m_π^2	eB=25m_π^2	eB=50m_π^2
$\alpha = 1$	1.575 T_c	1.52T_c	1.283T_c
$\alpha = 0.5$	1.118T_c	1.11T_c	1.014T_c

Table 3. Dissociation of charmonium in the magnetic field.

$c\bar{c}$	$T = 2.5T_c$	$T = 2.495T_c$	$T = 2.490T_c$
$\alpha = 1$	eB=22.5m_π^2	eB=23m_π^2	eB=24m_π^2
$\alpha = 0.5$	eB=20m_π^2	eB=21m_π^2	eB=22m_π^2

$$B_1 = \mu(\frac{8a_1}{\delta^3} + \frac{3a_2}{\delta^2} - a_3) \quad (32)$$

$$C_1 = \mu(\frac{3a_1}{\delta^4} + \frac{a_2}{\delta^3} + \frac{(l + \frac{N-2}{2})^2 - \frac{1}{4}}{2\mu}) \quad (33)$$

4. Results and discussion

In Fig. (1), in the left panel, we have plotted the real part of the potential as a function of r for different values of fractional parameter α=0.6, α=0.8, and α=1 at fixed eB=10m_π^2 and $T = T_c$. By increasing fractional parameter, we note the real-part is more screened. Beside, we note that the fractional parameter has an effect on the linear term of potential. In addition, a further increase temperature at $T = 2T_c$ that the potential becomes more attractive as in the right panel. As a result, the real part of the potential was found to be more screened by increasing value of both T and α.
In Fig. (2), we have plotted the real part of potential as a function of the temperature ratio and the fractional parameter for the fixed value of r=0.2 fm. By taking the temperature range T=0.17-0.3 GeV, we notice that potential is more attractive with increasing fractional parameter than temperature.
In Fig. (3), we see that the potential interaction is more screened by increasing magnetic field and fractional parameter. Thus, we deduce that the fractional parameter play a role in the hot medium at fixed magnetic field and the magnetized medium at fixed temperature.

4.1 Binding energy

By solving the radial Schrödinger equation, we obtain the energy eigen value $E_{n,l}$ of $c\bar{c}$ and $b\bar{b}$. Spectarl function method defines binding energy of quarkonium $E_{bin} = 2m_q + V(r \rightarrow \infty)$–$E_{n,l}$ with M being resonance mass. In our case, the energy eigenvalues are known as ionization potential or binding energy as in Refs. [38, 39]. In Fig. (4), we note that the binding energy decreases with increasing the reduced mass and this finding is agreement with Ref. [**?**].
We see the change of the binding energy under the effect of fractional parameter in the hot-magnetized medium. Charmonium binding energy is plotted as a function of T for three cases eB=5m_π^2, eB=25m_π^2 and eB=50m_π^2 in Fig. (5). In the left panel, we note that the binding energy of charmonium decreases with increasing temperature also the binding energy shifts to lower values by increasing magnetic field at fixed α=1. By decreasing fractional parameter at of α=0.5, we note that the binding energy faster than the dissociation at α=1. Therefore, the dissociation of temperature will be affected with considering fractional parameter.
Similarly, In Fig. (6), we have plotted the binding energy of charmonium at temperature T=2.5T_c, T=2.495T_c, and T=2.490T_c as a function of the magnetic field. We find that binding energy decreases when the magnetic field increases. Also, the binding energy shifts to slightly lower values by increasing temperature of medium. Thus, the binding temperature tends to zero when temperature increases. By decreasing fractional parameter from α=1 to α=0.5, we note the binding energy is faster to tends to zero.
In Fig. (7), Bottmonium binding energy is plotted as a function of T for three cases eB=5m_π^2, eB=25m_π^2 and eB=50m_π^2. By increasing temperature, we notice that the binding energy of 1S bottomonium decreases. In the left panel, we took N_f=2 and α=1. Besides, the binding energy decreases by increasing magnetic field. In right panel, the figure shows that the binding energy tends to zero faster the curves in the left panel when we took α=0.5 .
In Fig. (8), we have plotted the binding energy of bottomonium at temperature T=1.98T_c, T=1.99T_c, and T=2T_c as a function of the eB. Note that BE decreases with increasing temperature and magnetic field. In the right panel, the binding tends to zero faster the left panel.

Table 2. Dissociation temperature (T_D) for bottomonium.

Sate	eB=5m_π^2	eB=25m_π^2	eB=50m_π^2
$\alpha = 1$	1.94 T_c	1.9T_c	1.66T_c
$\alpha = 0.5$	1.92T_c	1.89T_c	1.65T_c

4.2 Dissociation temperature with fractional parameter

In the present work, we obtain the dissociation temperature at $E_b \simeq 0$, an approximation that provides good accuracy in calculating the dissociation temperature. In the current analysis, we also study influence of the fractional parameter on the dissociation temperature in the presence of hot-magnetized

medium for charmonium and bottomonium, using the calculated binding energies.
Table (1) shows the effect fractional parameter on the dissociation of temperature for different values of magnetic field. We note that the dissociation of temperature decreases by increasing magnetic field at $\alpha = 1$. By taking $\alpha = 0.5$, we note that the dissociation of temperature takes lower values than the values at $\alpha = 1$. Similarly, we note that dissociation of bottomonum decreases by decreasing fractional parameter. Besides, the dissociation of bottomonium is lower than the dissociation of charmonium as in Table (2).

4.3 Dissociation of heavy quarkonia in a magnetic field

We calculate the dissociation of charmonium and bottomonium at fixed temperature and fractional parameter when $E_b \simeq 0$.
In Table (3), By taking thermal medium at $T = 2.5T_c$, we note that the binding energy of charmonium dissociates as magnetic field increases $eB = 22.5m_\pi^2$. By decreasing the temperature of the medium up to $T = 2.49T_c$, we note that the binding energy dissociated at $eB = 24m_\pi^2$. By taking $\alpha = 0.5$, we note that the dissociation of temperature takes lower values than the values at $\alpha = 1$. A similar situation is noted for the dissociation of bottomonium however the dissociation of bottomonium is larger than charmonium as in Table (4). This conclusion is agreed with works such that [18, 19, 29, 30].

5. Conclusion

The SE is analytically solved by conformable fractional of the NU method, where the real fractional potential includes temperature T and eB. The eigenvalues of energy and corresponding wave functions are obtained, in which they depend on the fractional parameter $0 < \alpha \leq 1$. The study shows the effect of fractional parameter on the effective interaction potential, the binding energy, dissociation of qurakonium in which the interaction potential is screened by increasing the fractional parameter. The binding energy and the dissociation of temperature in the fractional quark model are lower than the classical quark model at α=1. We have also observed that the magnetic field is largely affected by large-distance interaction, as a result of which the real part of potential is more attractive. The sound representation of the fraction solution provides an efficient and elegant way to solve the specific problems on the physics of interest. Consequently, the studying of analytical solution of the modified fractional radial Schrödinger equation for the hot-magnetized interaction potential within the framework conformable fractional the Nikiforov-Uvarov method could provide valuable information on the quantum mechanical dynamics at nuclear, atomic and molecule physics and opens new window.

Table 4. Dissociation of bottomonium in the magnetic field.

$b\bar{b}$	$T = 2T_c$	$T = 1.99T_c$	$T = 1.98T_c$
$\alpha = 1$	eB=41m_π^2	eB=42m_π^2	eB=43m_π^2
$\alpha = 0.5$	eB=35m_π^2	eB=37m_π^2	eB=38m_π^2

Conflict of interest statement:
The authors declare that they have no conflict of interest.

References

[1] N. Salehi, H. Hassanabadi, and A.A. Rajabi. *Chinese Physics C*, **37**:113101, 2013.

[2] N. Salehi and A. A. Rajabi. *Modern Physics Letters A*, **62**:2875, 1991.

[3] S. Nasrin, A. Rajabi, and Z. Ghalenovi. *Acta Physica Polonica B*, **42**:6, 2011.

[4] N. Salehi, H. Hassanabadi, and A. Rajabi. *European Physical Journal Plus*, **128**:27, 2013.

[5] N. Salehi and N. Mohajery. *European Physical Journal Plus*, **133**:416, 2018.

[6] R. Alexander. *Physics Reports*, **858**:1, 2020.

[7] B. Jean-Paul and M. A. Escobedo. *Journal High Energy Physics*, **6**:1, 2018.

[8] M. Abu-Shady, H. M. Mansour, and A. I. Ahmadov. *Advances in High Energy Physics*, **2019**:ID 4785615, 2019.

[9] M. Abu-Shady. *International Journal of Theoretical Physics*, **49**:2425, 2010.

[10] M. Abu-Shady and M. Soleiman. *Physics of Particles and Nuclei Letters*, **10**:683, 2013.

[11] M. Abu-Shady. *Modern Physics Letter A*, **29**:1450176, 2014.

[12] M. Abu-Shady and H. M. Mansour. *Physics Review C*, **85**:055204, 2012.

[13] V. Skokov, A. Illarionov, and V. Toneev. *International Journal of Modern Physics A*, **24**:5925, 2009.

[14] V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya, V. P. Konchakovski, and S. A. Voloshin. *Physics Review C*, **83**:054911, 2011.

[15] C. S. Machado, F. S. Navarra, E. G. de Oliveria, and J. Noronha. *Physics Review D*, **88**:034009, 2013.

[16] M. Hasana, B. Chatterjee, and B. K. Patra. *European Physical Journal C*, **77**:767, 2017.

[17] B. Chatterjee S. P. Adhya P. Bagchi, N. Dutta. *Journal of Alloys and Compounds*, **690**:799, 2017.

[18] C. Bonati, M. D́'Elia, and A. Rucci. *Physics Review D*, **92**, 2015.

[19] M. Abu-shady and H. M. Fath-Allah. *Oriental Journal of Physical Sciences*, **6**, 2021.

[20] Abu-Shady and Sh Y. Ezz-Alarab. *Few-Body Systems*, **62.2**:1, 2021.

[21] Al-Jamel. *International Journal of Modern Physics A*, **34**:1950054, 2019.

[22] T. Abdeljawad. *Journal of Computational and Applied Mathematics*, **259**:57, 2015.

[23] R. Herrmann. *Biomass and Bioenergy*, **59**:380, 2006.

[24] M. Abu-Shady. *International Journal of Modern Physics A*, **34.31**:1950201, 2019.

[25] I. Podlubny. *Fractional Differential Equations*. Academic Press, 1th edition, 1999.

[26] M. Caputo and M. Fabrizio. *Progress in Fractional Differentiation Applications*, **1**:73, 2015.

[27] R. Khalil, M. A. Horani, A. Yousef, and M. Sababheh. *Journal of Computational and Applied Mathematics*, **264**:65, 2014.

[28] H. Karayer, D. Demirhan, and F. B¨üy¨ükkiliç. *Communications in Theoretical Physics*, **66**:12, 2016.

[29] M. Hasan and B. K. Patra. *Physics Review D*, **102**:036020, 2020.

[30] M. Hasan, B. K. Patra, and P. Bagchi. *Nuclear Physics A*, **955**:121688, 2020.

[31] E. J. Ferrer, V. de la Incera, and X. J. Wen. *Physics Review D*, **91**:054006, 2015.

[32] Yu. A. Simonov. *Physics of Atomic Nuclei*, **58**:107, 1995.

[33] M. A. Andreichikov, V. D. Orlovsky, and Yu. A. Simonov. *Physical Review Letters*, **110**:162002, 2013.

[34] B. Chatterjee M. Hasan, B. K. Patra and P. Bagchi. *Journal of Non-Crystalline Solids*, **433**:60, 2018.

[35] V. K. Agotiya, V. Chandra, M. Y. Jamal, and I. Nilima. *Physics Review D*, **94**:094006, 2016.

[36] M. Abu-Shady. *International Journal of Applied Mathematics and Theoretical Physics*, **2**:16, 2015.

[37] F. Karsch, M. T. Mehr, and H. Satz. *Z. Phys. C-particles and fields*, **37**:617, 1988.

[38] F. Brau and F. Buisseret. *Phys. Rev. C*, **76**:065212, 2007.

[39] V. Agotiya, V. Chandra, and B. K. Patra. *Phys. Rev. C*, **80**:025210, 2009.