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# Nonlinear responses of inhomogeneous collisional plasma to pondermotive effect in the laser-plasma interaction

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**Abstract**
The pondermotive force causes interesting nonlinear processes in the interaction of the intense laser beam with plasma. The pondermotive effect modifies the profile density, and in turn, affects the propagation of the laser beam through the plasma medium. Here, it is considered the inhomogeneous collisional plasma, and assumed that a high-intensity laser beam passes through the plasma. The nonlinear electric field solutions are obtained by applying numerical methods. Due to the nonlinear ponderomotive force, the changes in the physical characteristics of the electromagnetic wave are investigated. The wave profile of laser beam and the initial electron density have an important effect on the passing of the electromagnetic field through the plasma. Here, the Gaussian and the plane wave for the wave profile and the Gaussian and linear profiles for the initial electron density are considered. The obtained numerical results show that the profile of the incident wave and also the initial electron density have significant effects on the laser-plasma interaction. It is also demonstrated that the wave profile of laser beam incident to the inhomogeneous plasma plays an important role in its absorption by the plasma.

**Keywords**
Under-dense plasma, Over-dense plasma, Pondermotive force, Intense laser, Collisional plasma.

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## 1. Introduction

Transfer of a large number photons energy to matter in small volumes and in very short time scales leads to a strong nonlinear process in the intense laser-plasma interactions [1]. The considerable theoretical and experimental studies have been devoted in the last years to the fundamental aspect of the laser-plasma interaction in technology and experimental physics [2,3]. In particular, because of the unique and useful properties of the high-intensity lasers they can be used for instance in particle accelerators [4,5], x-ray generators [6], and inertial confinement fusion experiments [7].
When the amplitude of the propagating wave in plasma is increased beyond the linear regime, the plasma density profile is changed due to a nonlinear force called Ponderomotive force [8,9]. It has been revealed that the nonlinear response of the fluids is responsible for the appearance of the Ponderomotive force in the wave-plasma interaction [8]. The interaction of high-intensity laser beam with plasma leads to numerous nonlinear phenomena in the plasma. The modification induced to the electron density and the electric field is a kind of these nonlinearity phenomena [8,10]. In this process, the electrons are trapped by ponderomotive potential; consequently, the electron density becomes dependent on the electromagnetic field profile. As it has been shown in [10], charge separation in the plasma subjected to the laser beam is proportional to the gradient of the laser pulse intensity.
The ponderomotive force has significant impacts on the density and the propagation of the laser beam on the plasma. Therefore, the study of the nonlinear ponderomotive effect has attracted attention for its unique effects on the laser-plasma interaction which arises from the nonlinearity in the plasmas with different conditions. The effects of ponderomotive force have been investigated in the magnetized plasma in three and two dimensions [11]. It has been shown that in the interaction of high power laser and the inhomogeneous under-dense plasma, the relativistic ponderomotive force produces a self-defocusing effect on laser beams passing through this plasma [4]. Moreover, some useful theoretical proofs are suggested for the design of new accelerators using the high-energy electrons that can be obtained by the relativistic ponderomotive force. Also, it is shown that by considering the ohmic heating of plasma electrons and the ponderomotive force, modification of the electron density and absorption of the short laser pulse has occurred when an intense laser pulse propagates in the isothermal and non-isothermal under-dense collisional plasmas [8–12].
To investigate the laser-plasma interaction, the laser beam

should be considered as a Gaussian profile. But since in the linear approximation, the processes created in the laser-plasma interaction are usually very complicated, the laser wave is considered as a plane wave [12–15]. However, in some articles the system is perused as a Gaussian wave [16].

Several interesting phenomena have been observed when high intense electromagnetic wave propagates through the collisionless under-dense plasma in the presence of the external magnetic field and the ponderomotive force, with consideration of the beam wave as a plane wave [2, 3, 9, 13, 14]. It has been reported in the literature that for the interaction of high-intensity laser beam and unmagnetized inhomogeneous underdense plasma with a linear density profile, the profile of the electric and magnetic fields and the electron density can be changed strongly by considering the ponderomotive force [15]. The effect of collision and the dissipation factor in the nonlinear response of the different plasmas has not been largely investigated yet [12].

In the interaction of laser-plasma, the propagation of wave through the plasma depends on the density of plasma and the frequency of incident wave. If the plasma is under-dense the wave can pass through the plasma otherwise the wave cannot pass the plasma as usual. Because in the case of the intense laser-plasma, due to the nonlinear ponderomotive force, the laser beam and plasma density affect each other [17–19]. Among this interaction the plasma density will be changed as a result of the propagation of the wave through the plasma. The propagation of the wave through the overdense plasma slab takes place due to the excitation of the plasma surface waves [20–24].

This work devoted to study the behavior of the dissipative and magnetized inhomogeneous plasma subjected to the high-intensity laser pulses with both Gaussian beam profile and the plane wave profile. By considering the collision in the plasma, the linear momentum equation is derived theoretically. It is assumed that the characteristic length is greater than the Debye length and we obtain the electron density of the plasma based on a balance between the pressure gradient of the plasma and the pondermotive force. To achieve the electric field profile, we apply the nonlinear plasma permittivity to the Maxwell's equation. Here, the dispersion relation is so important in the propagation of the wave. By obtaining the wave number, we try to investigate the effect of the collision on the magnetic field and also the type of the laser wave profile on the wave number. Then, the effect of different parameters on the interaction of electron density with laser, such as the profile of initial electron density and the dissipative factors are studied [22, 23]. Since there is usually no sharp boundary between plasma and vacuum, different density profiles are considered. Due to the fact that in the physical system we have considered, the passage of the laser wave field due to the nonlinear force of the Pondermotive has a significant effect on the plasma density. Different density profiles can have an effect on the wave propagation and plasma density change. The initial electron density of the inhomogeneous plasma is

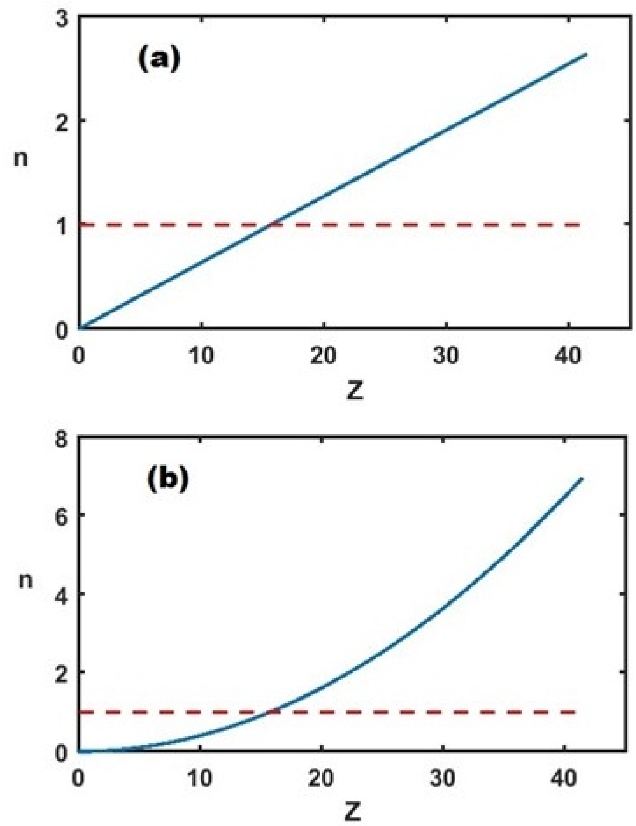


**Figure 1.** (a) The linear profile. (b) the Gaussian profile for the initial density of inhomogeneous plasma as a function of $z$.

considered with both Gaussian profile and the linear profile. Hence, the effect of the interaction of the laser beam with collisional inhomogeneous plasma on the local variation of the electron density can be studied and leads to interesting results. Also, the propagation of the electric field in the plasma is examined. As a significant feature of this study, since the laser beam is considered with both profiles of the Gaussian and the plane separately, the consequences of both approaches are achieved and a thoroughly comparison between the results is made and the differences are discussed.

## 2. The theoretical details and the model

We consider the intense laser beam propagation inside collisional inhomogeneous plasma. The response of this system is described by the momentum equation,

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\frac{e}{m}\mathbf{E} - \nu \mathbf{V} \tag{1}$$

where $\nu$, $\mathbf{E}$, $\mathbf{V}$, $e$, and $m$ are the collision frequency, the electric field, the electron velocity, the electron charge, and electron mass, respectively. It should be mentioned that the ions are considered immobile due to their high mass. Assuming linear polarization of $\mathbf{V}$, and neglecting the second term on the left-hand side of Eq.(1) to linearize the momentum equation, we can find

$$\mathbf{V} = \frac{e\mathbf{E}}{m(i\omega - \nu)} \tag{2}$$

Here $\omega$ is the laser beam frequency. The electric field pattern is driven by solving coupled Maxwell's equations for the linear polarization of the laser beam as

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{3}$$

$$\nabla \times \mathbf{B} = -\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi n_{e0}(z)e\mathbf{V}}{c} \tag{4}$$

Then, by using relations (2)-(4), the wave equation can be given

$$\frac{d^2E}{dz^2} + (\frac{\omega}{c})^2(1 - \frac{4\pi n_e(z)e^2}{m\omega(i\omega - \nu)})E = 0 \tag{5}$$

where $c$ and, $n_e$ refer to the light velocity in a vacuum and the equilibrium electron density, respectively. This equation describes the propagation of optical pulses through a dissipative plasma medium in which the third term on the left-hand side shows how $\nu$ affects the electric field. To analyze the behavior of the magnetic field, we also obtain the magnetic field $B$ from faraday's law as follows

$$\frac{dE}{dz} = \frac{i\omega}{c}B \tag{6}$$

We assume that the characteristic length is greater than the Debye length, then the following equation is derived based on a balance between the pressure gradient of the plasma and the pondermotive force [12]

$$\frac{1}{n_e}\nabla n_e = \frac{e^2}{mT_e\omega^2}\nabla^2 E \tag{7}$$

where $T_e$ refers to the electron temperature. The wave propagates on the $z$ direction therefore after integrating Eq.(7) in the inhomogeneous plasma medium, $n_e$ will be

$$n_e(z,E) = n_{e0}(z)e^{(\frac{e^2E^2}{2mT_e\omega^2})} \tag{8}$$

The dielectric permittivity of free electron contribution in a cold plasma is the following well known form

$$\varepsilon = 1 - (\frac{\omega_p}{\omega})^2 \tag{9}$$

where $\omega_p$ is the plasma frequency and is given by

$$\omega_p^2 = \frac{4\pi n_e e^2}{m} \tag{10}$$

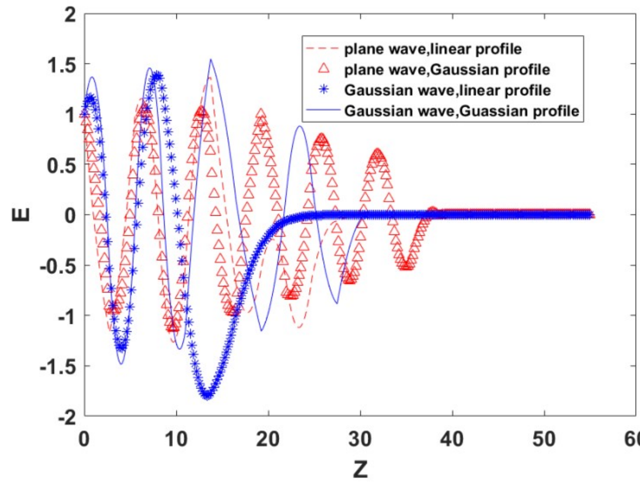


**Figure 2.** The electric field of the laser beam passing through the inhomogeneous plasma as a function of $z$.

According to Eq.(8), the permittivity of plasma is rewritten as

$$\varepsilon = 1 - (\frac{\omega_p}{\omega})^2 e^{(\frac{e^2E^2}{mT_e\omega^2})} \tag{11}$$

To study the nonlinear effect of the ponderomotive force on the laser- plasma interaction with a definite profile of the wave, it is required to solve Eq. (5). In what follows we present these solutions for two different wave profiles namely the plane and Gaussian wave.

a) **Plane wave**

Generally, the profile of the laser beam should be represented as a Gaussian wave but in the regions where the laser beam is quite collimated, the light can be considered as a plane wave. By using the relation $f(z) \sim A(z)\exp(-i\omega t)$ as a plane wave, with $A(z)$ being the wave amplitude, Eq. (5) can be rewritten as

$$\frac{d^2E}{dz^2} + (\frac{\omega}{c})^2(1 - (\frac{\omega_p}{\omega})^2 \frac{1}{1 + i\frac{\nu}{\omega}} e^{\frac{e^2E^2}{mT_e\omega^2}})E = 0 \tag{12}$$

where the complex factor $1/(1 + i\nu/\omega)$, is presented due to the electron collision during the interaction of an electromagnetic wave with the dissipative plasma.

b) **Gaussian wave**

The Gaussian profile of the laser beam is given by

$$f(r,z) \sim f(z)\frac{w_0}{w(z)} e^{-\frac{r^2}{w^2(z)}} e^{-i(\omega t + \frac{kr^2}{2R(z)} + \varphi(z))} \tag{13}$$

where, $\lambda$ is the laser beam wavelength in vacuum, $w_0 = w(z_0)$ is the waist radius, $r$ is the radial distance from the laser beam axis, $k = 2\pi/\lambda$ is the wave number, $z_R = \pi w_0^2 n/\lambda$ is the Raleigh length of the Gaussian beam, $n$ is the index of refraction of the medium in which the beam propagates. $w(z) = w_0\sqrt{1 - (z/z_R)^2}$ and $R(z) = z[1 + (z_0/z)^2]$ are the beam radius and the radius of curvature of the beam's wavefront, respectively. Finally, $\varphi(z)$ contains all the radially uniform phase variations and $f(z)$ is the electric field amplitude.

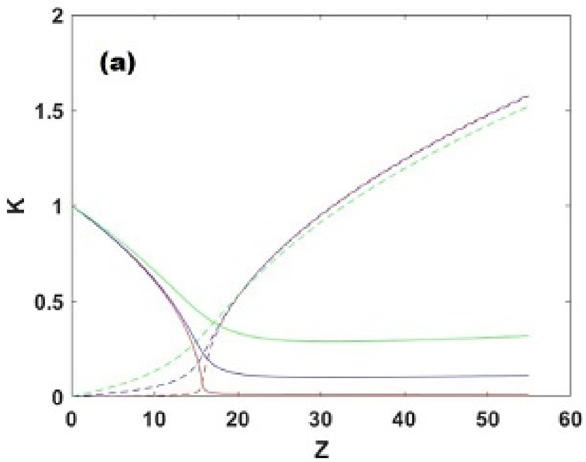


**Figure 3.** Wavenumber of the intense laser light waves passing through the inhomogeneous plasma (a) the plane wave and (b) the Gaussian wave as a function of $z$ at different collision frequencies: $\nu = 0.01$ (red), $\nu = 0.1$ (blue) and $\nu = 0.3$ (green). The dashed lines represent the real part of the wavenumber and the solid lines represent the imaginary part of the wavenumber.

Considering relation (13) in the Eq.(5), we get

$$\frac{d^2E}{dz^2}+\frac{1}{w(z)}(\frac{2}{w(z)}(\frac{dw(z)}{dz})^2-\frac{d^2w(z)}{dz^2})E$$

$$+(\frac{\omega}{c})^2(1-(\frac{\omega_p}{\omega})^2\frac{1}{1+i\frac{\nu}{\omega}}e^{(\frac{e^2E^2}{mT_e\omega^2})})E=0 \qquad (14)$$

In the next section, we present the numerical results of the electric field behavior of a laser beam propagating through the collisional and inhomogeneous plasma using Eq.(12) and Eq.(14). Since the plasma is inhomogeneous, we examine the linear profile for the initial density of plasma i.e., $n_{e0} = n_{cr}z/L$ and Gaussian profile i. e., $n_{e0} = n_{cr}z^2/L^2$ where $n_{cr}$ is the critical density and $L$ is the length of the underdense region.

## 3. Electric field of an intense laser propagating in plasma

The behavior of the electric field for the plane wave passing through the inhomogeneous and dissipative plasma medium are described by using Eq.(12) and Gaussian wave by using Eq.(14) for the linear (Fig.(1a)) and Gaussian density profile (Fig.(1b)). In these figures, the dash- red line shows the critical density point. Since these equations do not have an analytical solution, the PDE software is used to solve these equations. In all of the figures, the laser intensity and the length of inhomogeneity of plasma are $I = 5\times10^{17}$ w/cm$^2$ and $L$=20 $\mu$m, respectively.
Note that dielectric permittivity plays an important role in the propagation of the electromagnetic waves in the medium. As seen in the Eq.(11), the plasma permittivity depends on the density ($\omega_p^2 = 4\pi ne^2/m$), $\varepsilon = 1-\omega_p^2/\omega^2$. Therefore, when $\omega^2 > \omega_p^2$, $\varepsilon$ is greater than zero in which the plasma is called underdense and when $\omega^2 < \omega_p^2$, $\varepsilon$ is smaller than zero and the plasma is called overdense. Furthermore, for $\omega^2 = \omega_p^2$ in which the permittivity becomes zero, the density, $n_c$, is named the critical or cutoff density.
In order to study the effect of laser wave profile on the propagation of the laser beam through the dissipative and the inhomogeneous plasma, Eqs.(12) and (14) are solved for two different density profiles. As it shown in Fig.(2) the electric field of the laser beam oscillates along the $z$ axis in the underdense plasma before the cutoff layer. The results reveal when the laser beam is considered as the Gaussian wave and the density profile has a Gaussian form, the oscillations of the electric field are more damped after cut off layer in the over dense plasma region. Moreover, the computational results show that the skin depth for the plane wave increases compared to Gaussian wave. It means that the wave can penetrate into the opaque material when the electric field is the plane wave.

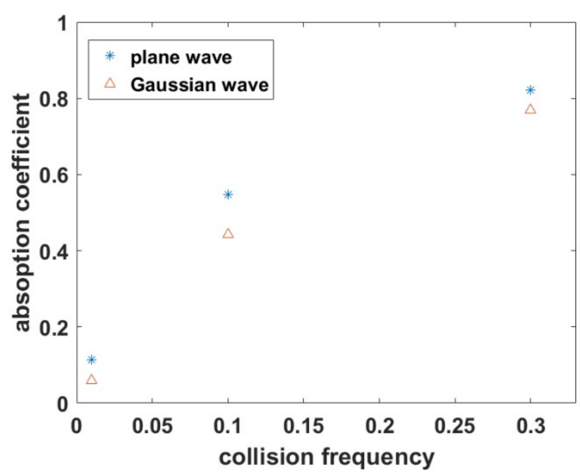


**Figure 4.** The absorption coefficient of incident laser light passing an inhomogeneous plasma for the Gaussian wave and the plane wave.

## 4. Wavenumber profile

In this section, we are interested to study the wave number of the electromagnetic filed propagation in the dissipative and the inhomogeneous plasma. The dispersion relation for this situation is obtained as:

$$K^2=(\frac{\omega}{c})^2(1-(\frac{\omega_p}{\omega})^2\frac{1}{s}e^{(\frac{e^2E^2}{mT_e\omega^2})}) \qquad (15)$$

$$s=1+i\frac{\nu}{\omega} \qquad (16)$$

Here $K = K_r + iK_i$ is the wave number of the propagated wave in the plasma, where $K_r$ is the real part and $K_i$ is the imaginary part of the wavenumber. The effect of the collisional frequencies on the wavenumber for the linear density profile and the Gaussian density profile of plasma by using the plane and Gaussian waves, are shown in Figs.(3a) and (3b). The computations were performed by setting the wavelength of laser $= 0.8 \times 10^{-4}$ (cm), power of laser $P = 5 \times 10^{24}$ (W), critical density of plasma $n_{cr} = 1.72 \times 10^{21}$ cm$^{-1}$, $w_0 = 10 \times 10^{-3}$ m and three collision frequencies: $\nu_1$ =0.01,$\nu_2$ =0.1,$\nu_3$ =0.3 in Eq.(15). In these figures, solid curves show the real part of the wavenumber, and dashed curves are the imaginary part of the wavenumber as a function of $z$. It can be seen that for both of the wave profiles when the collision frequency is approximately zero, the real part of the wavenumber decreases along the $z$-axis in the underdense region and tends to zero near the cutoff layer while the imaginary part of the wavenumber is zero and after the cutoff point increases.

The effect of collisional frequency on the wavenumber is also represented in Fig.(3a) and (3b) for the plane and the Gaussian waves, respectively. Fig.(3a) shows that for the plane wave, when the collisional frequency increases, the imaginary part of wavenumber in the underdense plasma region increases, and resulting in the absorption rate and damping are occurred in this region too. It means that by increasing the collision frequency, the place of cutoff layer is moved back up and the imaginary part of the wave number is also moved on this direction. On the other hand, the real part of wave number increases in the overdense plasma. Therefore, by taking into account the electron collisions in the plasma medium the electromagnetic waves in the overdense regions propagates. Comparing the plot of the wavenumber verses $z$ for the Gaussian and the plane waves reveals that the real part of wavenumber of the Gaussian wave exhibits a peak at the cut off layer and its imaginary part of wavenumber exhibits a valley at the cut off layer.

It should be noted that the plot of wavenumber versus position for the linear density profile and the Gaussian density profile are the same.

## 5. Absorption of the laser energy

As it is discussed in the previous section, deriving the imaginary part of the wavenumber, it is clear that the plasmas absorb energy of the wave in the underdense plasma region. When the laser propagates in the plasmas, it is assumed that the laser intensity is stationary, the change of density is very slow compared to the wavelength of the laser beam, and the dispersion relation is satisfied locally. The fraction of the absorption of the laser beam from the incidence to the plasma and back to the vacuum is derived by the following integration [23]:

$$\alpha_{abs} = 1 - e^{-2\int k_i dl} \tag{17}$$

where $k_i$ is the imaginary part of wavenumber and $dl$ is the element of distance. Fig.(4) shows the variation of the absorption coefficient of the incident light, passing an inhomogeneous plasma as a function of the collision frequency for the Gaussian and the plane waves. As it is depicted in this figure, the calculated absorption coefficient increases with the collision frequency for both mentioned waves. Comparing with the Gaussian wave, the laser beam with the plane wave profile has higher absorption coefficient when it passes through the inhomogeneous plasma.

## 6. Conclusions

In summary, the nonlinear behavior of dissipative and inhomogeneous plasma under high-intensity laser irradiation has been studied with two different wave profiles: a) The plane wave and b) The Gaussian wave. We have applied both the linear profile and the Gaussian profile to model the initial density of the plasma in the calculations. Our numerical results have showed that for both linear and Gaussian initial density of the plasma, the laser wave can penetrate more into the opaque material when it is a plane wave. Following the similar procedure, we have investigated the wavenumber of the propagated wave in the dissipative and inhomogeneous plasma. The results have showed that when the electron collisions are included, the electromagnetic waves can propagate in the overdense plasma. In addition, when the collisional frequency increases, the positive imaginary part of the wavenumber in the underdense plasma region resulting in the absorption of the laser beam and damping wave is seen. The calculated absorption coefficient of plasma demonstrated that the laser beam with the plane wave profile is more absorbed by the plasma in comparison to the laser beam with the Gaussian profile.

**Conflict of interest statement:**
The authors declare that they have no conflict of interest.

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