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The quantum fluctuations of charge and current in a driven nonlinear LC-circuit with a linear capacitor and a nonlinear inductor

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The quantum fluctuations of charge and current in a driven nonlinear LC-circuit with a linear capacitor and a nonlinear inductor

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Abstract

A nonlinear soft-core ferrite (ferromagnetic material) inductor that obeys of a polynomial current-magnetic flux relationship (typically a power series in the magnetic flux) is introduced. The quantum Hamiltonian of a nonlinear LC-circuit consisting of a linear capacitor and a nonlinear inductor under the influence of an external field is found. The energy spectrum is obtained and the quantum behavior of the nonlinear coefficients is studied numerically. The quantum fluctuations of electric charge and current are obtained as a function of the characteristic parameters then the time-dependent of the characteristic parameters and the digger squeezing is analyzed by numerical approach.

Keyword: nonlinear inductor, squeezing effect, quantum LC-circuit, quantum fluctuation, quantization, numerical solution, Duffing's electrical oscillator

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1.Introduction

Nonlinear inductors are described in terms of current through inductance and can be made with various core shaps. The core material of the inductors plays an important role and is usually made with ferromagnetic material which creates magnetic flux through the inductance coil. Ferromagnetic materials are characterized by the magnetic hysteresis. Magnetic hysteresis is the gap between the change in magnetic induction B and the change in magnetic field strength H.The nonlinear

behavior of the inductor using ferrite nuclei (ferromagnetic material) show in [1] and [2] experimentally. Non-linear inductors are one of the most basic devices of nonlinear electrical LC-circuits (inductance L and capacitance C). Such nonlinear electrical circuits were classically studied many years ago before by [3-6]. When this nonlinear electric circuit is driven by a sinusoidal voltage source, it is known as a Duffing electric oscillator [4-9]. As progress in experimental control at the nanometer scale in nanotechnology, mesoscopic physics and nanoelectronics is under going a dramatic development [10]. People are intrested in miniaturizing integrated circuit and components to atomic-scale dimensions [11]. When the transport dimension of electric devices in the electric circuits reaches to coherence length, the quantum effects must be taken in to account. Therfore, a suitable quantum theory for electronic devices in nanoelectronic integrated circuits is necessary. For this propose, by comparing the electron motion equation with a harmonic oscillator, the LC- design circuit was quantized by Louisell [12] and its use expanded in [13-15]. Also, with much progress in many characters, such as quantum fluctuation, squeezing effect etc, studies have been done for various mesoscopic RLC circuits in [16-19].

To make the discussion more concrete, let us imagine a nonlinear electrical LC-circuit with a nonlinear inductor fabricated with the technology of microelectronic chips. Typical values that can be easily considered for the inductance and capacitance are in the nanohanry and picofarad range, it leads to a resonant frequency in the microwave range. Recently, it has been obtained by miniaturization the dimensions of the inductor to self-induction coefficient of the nanohenry order in [20-21]. When the overall dimensions of the circuit do not exceed a few hundred micrometers, it is much smaller than the corresponding wavelength, the electrical circuit is well within the bulk element limit. This is a very good approximation in the 'bulk element' limit where the physical size of the electronical devices in the nonlinear LC - oscillator is much smaller than the wavelength of electromagnetic waves at the frequency of the oscillator i.e., the dimensions of the devices are about the length of the carrier coherence. In this range, the electron behavior is wave-like, therefore by importance of the inductor in the electrical circuit, it seems necessary study the quantum nonlinear inductor. According to the views mentioned, in this paper, we consider an electrical circuit consisting of a linear capacitor and nonlinear inductor under the influence of a sinusoidal external source. we consider current-dependence of the inductance up to the order fifth magnetic flux ϕ and find the quantum Hamiltonian of such nonlinear circuit. The quantum extended Duffing

equation is found by the Heisenberg motion equation. We find the energy spectrum and the quantum fluctuations of electric charge and current flow of this circuit by analytical and numerical methods and study squeezing effect in this quantum electrical circuit

2. Quantization of an electrical LC-Circuit with a nonlinear inductor

A nonlinear electric circuit including a linear capacitor and a nonlinear inductor under the influence of an external field (a sinusoidal voltage source) is shown in Figure 1. The nonlinear inductor nature in the such nonlinear electric circuit exhibits the Duffing's electrical oscillator that was classically studied by years ago in [5, 7-8].
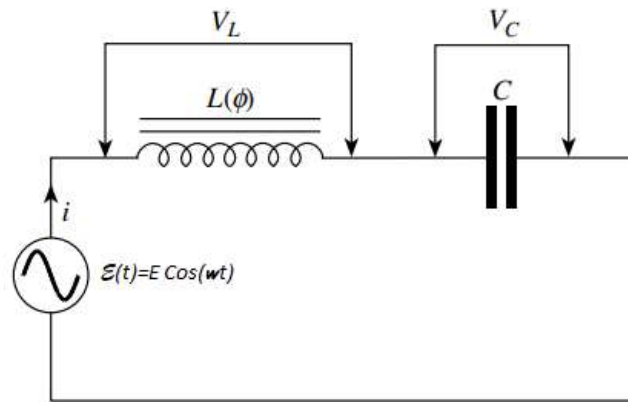

Figure 1. A nonlinear electric circuit including a linear capacitor and a nonlinear inductor under the influence of an external field.

When the inductor is assumed to be linear, the relationship between current I and magnetic flux ϕ in the inductor core is given as follows

$$I = \frac{\phi}{L} \tag{1}$$

where L is the self-induction coefficient of inductor or inductance. The voltage difference V_L across the inductor is equivalent to

$$V_L = N \frac{d\phi}{dt} \tag{2}$$

where N is the number of turns of the coil (inductor). Using the electrical laws, the differential equation of the electrical circuit under the action of the external time-dependent field (the potential difference across the capacitance C) $\varepsilon(t)$ is given by

$$N\frac{d\phi}{dt} + \frac{q}{C} = \varepsilon(t) \tag{3}$$

When the inductor acts as a nonlinear electric devices in the nonlinear circuit Figure 1, it has a ferromagnetic core that can be modeled using nonlinear characteristic if the hysteresis phenomenon is abstracted. In this paper, we consider the nonlinear relationship between the current and magnetic flux as a function of flux individual powers series based on representation Biermanns [4] and Hayashi [5]. Here, we expand this function up to the order fifth i.e.

$$I = \sum_{k=1}^{\infty} a_k \phi^k$$
$$I = a_1\phi + a_3\phi^3 + a_5\phi^5 \tag{4}$$

where a_1, a_3, a_5, ... are constants characterizing the core i.e., they depend on the type of the inductor [5, 22-23] and thier dimensional are $[a_1] = \dfrac{A^2}{J}, [a_3] = \dfrac{A^4}{J^3}, [a_5] = \dfrac{A^6}{J^5}$ respectively. Although theoretical inductors do not have any of these real-world nonlinearities, engineers dealing with realistic inductors usually need to consider these nonlinear effects to ensure that their design works properly in the expected conditions. In the differentiating equation (3), we consider $\varepsilon(t) = E\cos(\omega t)$ then

$$N\frac{d^2\phi}{dt^2} + \frac{I}{c} = \omega E\sin(\omega t) \tag{5}$$

where, the current I in the nonlinear circuit generates a magnetic field, which induces an electric current in the coil. Substituting Eq. (4) into Eq. (5), we find

$$\frac{d^2\phi}{dt^2} + \frac{1}{NC}(a_1\phi + a_3\phi^3 + a_5\phi^5) = \frac{E\omega}{N}\sin(\omega t) \tag{6}$$

Where $\dfrac{1}{L_n} = \dfrac{a_n}{N}$ and $n = 1, 3, 5$

When $n = 1$ the linear natural frequency of this circuit is given by $\omega = \dfrac{1}{\sqrt{L_1 C}}$. The Eq. (6) can be written as

$$\frac{d^2\phi}{dt^2} + \frac{1}{L_1 C}\phi + \frac{1}{L_3 C}\phi^3 + \frac{1}{L_5 C}\phi^5 = \frac{E\omega}{N}\sin(\omega t) \qquad (7)$$

Equation (7) occurs in several studies [24-27], Based on the classical equation of motion (7), we can be formulated the classical Hamiltonian of nonlinear electric circuit (nonlinear inductor) as follows

$$H = u_{nonlinear\ inductor} + u_{capacitor}$$
$$= \frac{p^2}{2L_1} + \frac{p^4}{4L_3} + \frac{p^6}{6L_5} + \frac{q^2}{2C} \qquad (8)$$

For n=1, parameters are $a_1 = \frac{N}{L_1}$, $a_3 = a_5 = 0$, then the Hamiltonian (8) becomes

$$H = \frac{p^2}{2L_1} + \frac{q^2}{2C} \qquad (9)$$

The classical equation of motion for an electric circuit of LC design is the same as that for a harmonic oscillator, Since the mesoscopic dissipation less LC circuit has been quantized by Louisell [12]. According to the standard canonical quantization principle the canonical conjugate quantities q and ϕ are replacing by the corresponding operators $\hat{q}$ and $\hat{\phi}$ as we know $p \equiv \phi$ then the quantum Hamiltonian operator (9) can be expressed as

$$\hat{H} = \alpha\hat{p}^2 + \lambda\hat{p}^4 + \gamma\hat{p}^6 + \frac{\hat{q}^2}{2C} \qquad (10)$$

where

$$\alpha = \frac{La_1^2}{2} = \frac{1}{2L}, \lambda = \frac{1}{4L_3}, \gamma = \frac{1}{6L_5} \qquad (11)$$

and $[\hat{q},\hat{p}] = i\hbar$.

We assume that thermal fluctuations are smaller than quantum fluctuations. As we know, higher order momentums are caused by the presence of nonlinear elements in the quantum electrical circuit, which are included in the Hamiltonian interaction term and can be the contribution of quantum noises.

It also promises the existence of the squeeze effect in quantum nonlinear circuits, which is only a quantum phenomenon.That is, this nonlinear crystal-like circuit can squzzing the light in one direction, but the uncertainty principle will not change. Higher order quantum theory is an extension of the quantum theory that is discussed in this work. In this way, the discussed higher order quantum theory has a fundamental value, which is a new perspective to analysis the properties of quantum theory. In a new method to study electrical circuits in the nano quantum scale by introducing a thermal momentum operator that is similar to the momentum operator that only introduces a nonlinear term [28-30].

We assume $a_1 = \dfrac{1}{L}$, as a result $\alpha = \dfrac{1}{2L}$, therefore, the quantum Hamiltonian (10) under the influence of an external field $\varepsilon(t)$ becomes

$$\hat{H} = \frac{1}{2L}\hat{p}^2 + \lambda\hat{p}^4 + \gamma\hat{p}^6 + \frac{\hat{q}^2}{2C} + \varepsilon(t)\hat{q} \qquad (12)$$

The current-dependence of the inductance in the Figure 1 makes the a Duffing electrical LC-oscillator (nonlinear LC- circuit). We can understand that the nonlinear LC- oscillator has a very good approximation only a single low energy degree of freedom, namely uniform divergence less current flow in the wire of the inductor which does not build up charge anywhere except on the plates of the capacitor. This is a very good approximation in the 'bulk element' limit where the physical size of the nonlinear LC-oscillator is much smaller than the wavelength of electromagnetic waves at the frequency of the oscillator. Therefore, the Duffing electrical LC-oscillator (nonlinear LC- circuit) of Figure 1 can now be treated quantum mechanically. It is known that the rigorous study and design optimization of such electronical devices with ferromagnetic cores in the quantum mechanic is difficult because of nonlinearity. Therfore, we introducing the usual annihilation and creation operators such that

$$\hat{a} = \frac{1}{\sqrt{2\hbar\omega L}}(\omega L\hat{q} + i\hat{p})$$

$$\hat{a}^\dagger = \frac{1}{\sqrt{2\hbar\omega L}}(\omega L\hat{q} - i\hat{p}) \qquad (13)$$

$$\left[\hat{a}, \hat{a}^\dagger\right] = 1 \qquad (14)$$

where $\omega = \dfrac{1}{\sqrt{LC}}$. The quantum Hamiltonian (12) in the terms of creation and annihilation operators is given by

$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}) + \varepsilon(t)\sqrt{\frac{\hbar}{2\omega L}}(\hat{a} + \hat{a}^\dagger) + \lambda\frac{(\hbar\omega L)^2}{4}(\hat{a} - \hat{a}^\dagger)^4 + \gamma\frac{(\hbar\omega L)^3}{8}(\hat{a} - \hat{a}^\dagger)^6 \qquad (15)$$

We can rewrite Hamiltonian (15) in the rotating-wave approximation [31-33] as

$$\hat{H} \approx \hbar[(\omega + \omega_1 - \omega_2)(\hat{a}^\dagger\hat{a})^1 + (\omega_1 - \omega_2)(\hat{a}^\dagger\hat{a})^2 - \omega_2(\hat{a}^\dagger\hat{a})^3] \qquad (16)$$

where

$$\omega_1 = \frac{3}{2}\lambda\hbar(\omega L)^2, \omega_2 = \frac{5}{2}\gamma\hbar^2(\omega L)^3 \qquad (17)$$

The Hamiltonian (16) is probably for study quantum nonlinear dynamics in such electrical nonlinear circuits. In the Hamiltonian (16) when $\omega_2 = 0$, it is well described nonlinear Kerr medium and show the nonclassical effects in the context of quantum optics [34]. A simple way to achieve a superposition of different number states is to displace the quantum oscillator with a classical external driving force so that the ground state is mapped to a so-called "coherent state". For this propose, we suppose that the initial state of the nonlinear circuit (16) is coherent state i.e.

$$|\alpha\rangle = exp(-\frac{1}{2}|\alpha|^2)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle \qquad (18)$$

With this choice, we can easily find the eigenvalues of energy as

$$E_n = \hbar(\omega + \omega_1 - \omega_2)n + \hbar(\omega_1 + \omega_2)n^2 - \hbar\omega_2 n^3 \qquad (19)$$

To determine the behavior of λ and γ nonlinear coefficients in the Eq. (19), we plotted the changes energy diagram for different levels of n by numerical approach. In Figure 2, we can appreciate, when the value λ increases, the amount of energy in the levels increases (Figure 2(a)) and by increasing γ energy decreasing (Figure 2(b)).
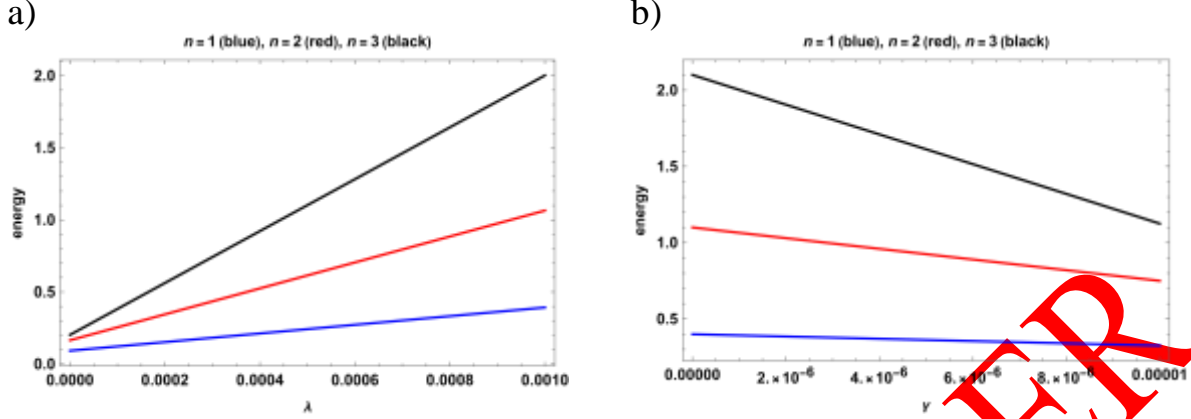
a)

$n=1$ (blue), $n=2$ (red), $n=3$ (black)

b)

$n=1$ (blue), $n=2$ (red), $n=3$ (black)

Figure 2. (a) Energy vs λ ($\hbar=1, L=100, C=1, \gamma=10^{-6}, n=1,2,3$) and (b) energy vs γ (
$\hbar=1, L=100, C=1, \lambda=10^{-3}, n=1,2,3$)

As we see in Figure 2 the nonlinear inductor, as any resonator in general, possesses a discrete energy spectrum. In this nonlinear electrical circuit, the energy difference between the ground and the first excited state is significantly different. (i.e. larger than the width of the levels than the difference between the first and the second excited states). Such LC-circuit resonator can be used as a qubit that is studied in [35].

3. The quantum dynamic of a LC-circuit with a nonlinear inductor

In order to study the quantum dynamical the electrical circuit, we employ the interaction picture and consider $\hbar((\omega_1-\omega_2)(\hat{a}^\dagger\hat{a})^2 - \omega_2(\hat{a}^\dagger\hat{a})^3)$ as the interaction part. Then the time-depending eigenvectors of Hamiltonian (16) are given by

$$|\psi(t)\rangle_I = exp(-\frac{1}{2}|\alpha|^2)\sum_{n=0}^{\infty}\exp(in^2(-\omega_1+\omega_2+n\omega_2)t)\frac{\alpha^n}{\sqrt{n!}}|n\rangle \qquad (20)$$

where $\hbar=1$. The Heisenberg equation of motion for the annihilation operator a is

$$\frac{d\hat{a}}{dt} = i\left[\hat{H},\hat{a}\right] = -i\{(\omega+\omega_1-\omega_2)+2(\omega_1-\omega_2)\hat{a}^\dagger\hat{a}-3(\hat{a}^\dagger\hat{a})^2\}\hat{a} \qquad (21)$$

Since $\hat{a}^\dagger\hat{a}$ is a constant of motion with the exact solution (21) we have

$$\hat{a}(t) = \exp(-it((\omega+\omega_1-\omega_2)+2(\omega_1-\omega_2)\hat{a}^\dagger\hat{a}-3(\hat{a}^\dagger\hat{a})^2))\hat{a} \qquad (22)$$

Also, the evolution equations for the operators $\hat{q}$ and $\hat{p}$ in the Heisenberg representation using the quantum Hamiltonian (12), can be computed explicitly as

$$\hat{I} = \dot{\hat{q}} = \frac{1}{i\hbar}\left[\hat{q}, \hat{H}\right]$$
$$= \frac{\hat{p}}{L} + 4\lambda \hat{p}^3 + 6\gamma \hat{p}^5 \tag{23}$$

and

$$\dot{\hat{p}} = \frac{1}{i\hbar}\left[\hat{p}, \hat{H}\right]$$
$$= -\frac{\hat{q}}{C} \tag{24}$$

By differentiating from relation (24) and substituting Eq. (23) into it, we have

$$\ddot{\hat{p}} + \frac{1}{LC}\hat{p} + \frac{4\lambda}{C}\hat{p}^3 + \frac{6\gamma}{C}\hat{p}^5 = 0 \tag{25}$$

Equation (25) is the fifth order quantum Duffing equation or quantum extended Duffing equation[36-38] .

To study the time behavior the quantum differential Eq (25) one needs to calculate expected value the operators in the Eq (25). For this Purpose, quadrature operators are introduced as follows [31-33]

$$X_j = \frac{1}{2}(\hat{a}e^{i\theta_j} + \hat{a}^\dagger e^{-i\theta_j}), \ (j = 1, 2) \tag{26}$$

where $\theta_1 = 0, \theta_2 = -\pi/2$, then $X_j \approx \hat{q}, \hat{p}$

Now using the Eqs (20) and (22) we can obtain

$$\langle X_j \rangle = \text{Re}\{|\alpha| e^{-it(\omega+\omega_1-\omega_2)-i\pi/2+\theta_j} \exp(|\alpha|^2 (e^{it(2(\omega_1-\omega_2)-3\omega_2)} - 1))\} \tag{27}$$

and

$$\langle X_j^2 \rangle = \frac{1}{4}(1 + 2|\alpha|^2 + 2\text{Re}\{|\alpha|^2 e^{-2it(\omega+3\omega_1-6\omega_2)-i\pi+2i\theta_j} \exp(|\alpha|^2 (e^{it(4(\omega_2-\omega_1)-6\omega_2)} - 1))\}) \tag{28}$$

3-1. The degree of squeezing

Squeezing states have no classical counterpart and states are non-classical. Squeezed light is produced by various nonlinear processes, and as a means of reducing noise in optical interference, it has attracted much attention in the creation of optical communication networks. Because even in an ideal laser that works in

coherence state, it still has quantum noise due to zero-point fluctuations [39-42]. So, prliminary experimental research is reported in references [43-44]. Therefore, the squeezed states are important because they have less quantum noise than coherent states. The nonlinear LC-circuit with L-nonlinear is suitable to study Squeezing states. To study the squeezing effect, we need to analyze the quantum fluctuations of the quadrature operators. Because the quantum fluctuations in a coherent state are equal to the zero-point fluctuations. We can find the time varying behavior of the quadrature variance using Eqs. (27) and (20) as

$$
<(\Delta\hat{q})^2> = 2\omega\mu\{(\frac{1}{4}+\frac{|\alpha|^2}{2}-\frac{|\alpha|^2}{2}\exp(|\alpha|^2(-1+\cos t(4\omega_1+2\omega_2)))
$$
$$
\cos(-2t(\omega+2\omega_1-5\omega_2)-|\alpha|^2\sin t(4\omega_1+2\omega_2)))-
$$
$$
(-|\alpha|\exp(|\alpha|^2(-1+\cos t(2\omega_1-5\omega_2)))\sin(t(\omega+\omega_1-2\omega_2)-|\alpha|^2\sin t(2\omega_1-5\omega_2)))^2\}
$$

(29)

$$
<(\Delta\hat{p})^2> = \frac{1}{2\omega\mu}\{(\frac{1}{4}+\frac{|\alpha|^2}{2}-\frac{|\alpha|^2}{2}\exp(|\alpha|^2(-1+\cos t(4\omega_1+2\omega_2)))
$$
$$
\cos(-\pi-2t(\omega+2\omega_1-5\omega_2)-|\alpha|^2\sin t(4\omega_1+2\omega_2)))-
$$
$$
(-|\alpha|\exp(-\frac{\pi}{2}+|\alpha|^2(-1+\cos t(2\omega_1-5\omega_2)))\sin(t(\omega+\omega_1-2\omega_2)-|\alpha|^2\sin t(2\omega_1-5\omega_2)))^2\}
$$

(30)

The quadrature variances (29) and (30) satisfy the following Heisenberg's uncertainty principle relationship

$$
<(\Delta\hat{q})^2><(\Delta\hat{p})^2> \geq \frac{1}{16}
$$

(31)

The Heisenberg's uncertainty principle in the measured values of these quantities play a central in understanding quantum noise [45]. For coherent states obey (or for the special case of the vacuum state),

$$
<(\Delta\hat{q})^2> = <(\Delta\hat{p})^2> = \frac{1}{4}
$$

(32)

We illustrate the time- evolutions of quantum fluctuation $<(\Delta\hat{q})^2>$ and $<(\Delta\hat{p})^2>$ by numerical approach in Figures 3(a), 3(b).

a)

b)
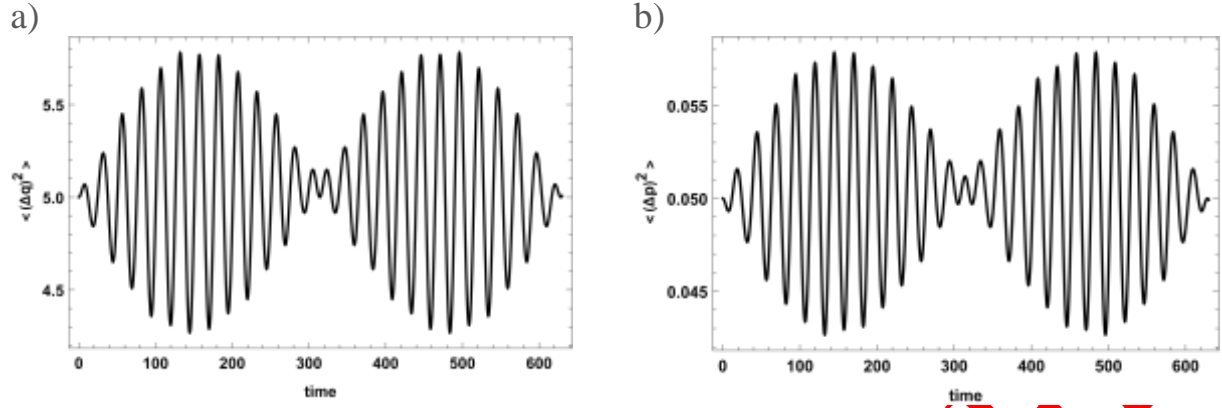
Figure 3. (a) $<(\Delta\hat{q})^2>$ vs time and (b) $<(\Delta\hat{p})^2>$ vs time

$$\omega_1/\omega = 0.3, \omega_2/\omega = 0.1, |\alpha| = 0.2$$

As shown in Figure 3, both variances show oscillatory behavior with a minimum indicating a significant amount of squeezing.

Heisenberg's uncertainty principle is one of the basic concepts in quantum systems and its investigation is also important in quantum electrical circuits. As we know, there is a limit to their simultaneous measuring, which is governed by Heisenberg's uncertainty principle. However, the uncertainty values of each of these observables can be arbitrarily small, but there is a limit in their product [46-49]. According to equations (29) and (30), the uncertainty principle can be concluded and we are expecting a generalization. Figure 4, the graph shows the uncertainty principle.
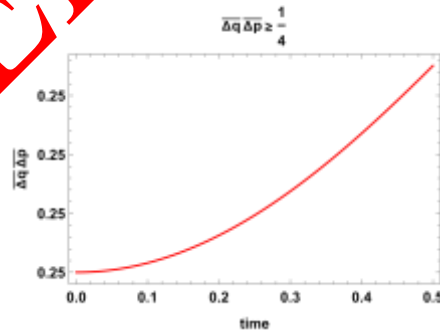

Figure 4. Uncertainty principle in this system ($\omega_1/\omega = 0.3, \omega_2/\omega = 0.1, |\alpha| = 0.2$)

 It is can also be seen from the figures that the fluctuations are exact periodic

functions, when $\dfrac{\omega}{\omega_1 - \omega_2}$ and $\dfrac{\omega + \omega_1}{\omega_2}$ are an integer. The squeezing effect occurs

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with the evolution of time, this behavior is compatible with the results found in [41,50].

With the numerical conditions that we have defined in Figure 3, the amounts of $<(\Delta\hat{p})^2>$ is less than 0.25 and $<(\Delta\hat{q})^2>$ is larger than 0.25, therefore, the squeezing is happing in the direction of p component (The squeezing condition for the p quadrature is that its variance should be less than the value of vacuum). As a result, if the quantum fluctuation in a component decrease, in the conjugate component increases correspondingly. For more attention, let us to check the degree of squeezing, for the p component , which is defined as follows [31]

$$S_j^{(2N)} = \frac{<(\Delta X_j)^{2N}> - (2N)!/(N!2^{3N})}{(2N)!/(N!2^{3N})} \tag{33}$$

In this equation, when $S_j^{(2N)} < 0$ the state is squeezed to the 2Nth order. By plotting S_q^2 and S_p^2 on a long-time scale p-component is squeezed, the quantum fluctuation in the p-component can be smaller of than zero-point fluctuation (Figure 5(a)).

Figure 5(b) shows the S_p^2 for different values of the squeezing constant. As we can appreciate, by increasing the amount of α the digger squeezing in the direction of p component disappears.
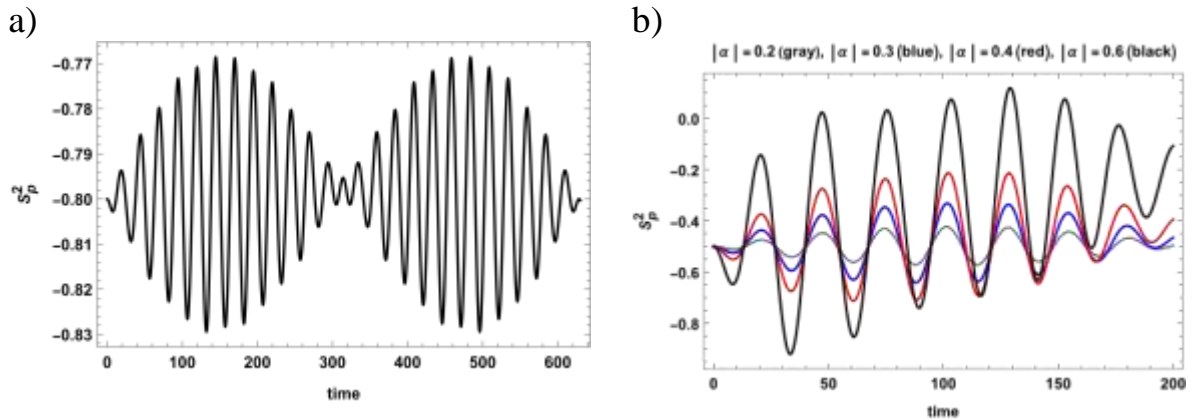

Figure 5. Squeezing p vs time, (a) $\omega_1/\omega = 0.3, \omega_2/\omega = 0.1, |\alpha| = 0.2$ and (b) by changing $|\alpha|$ ($\omega_1/\omega = 0.1, \omega_2/\omega = 0.01$)

Therefore, it can be concluded that the quantum electrical circuit with a nonlinear inductor and a linear capacitor can be considered similar to the crystal with nonlinear structure, which has removed the uncertainty from the symmetric state and has led to the creation of squeezed states.

3-2. Time-dependent the persistent current

In the following, in order to study of the behavior persistent current as function of the nonlinear coefficients γ and λ in the Eq. (23). We have plotted the diagram of the time-dependent current in the Figure 6, by numerical approach. According to Figures 6(a) and 6(b), it can be result that with increasing λ and γ the current amplitude increases. By comparing the two graphs, it can be seen that the rate of change of λ is greater than γ.
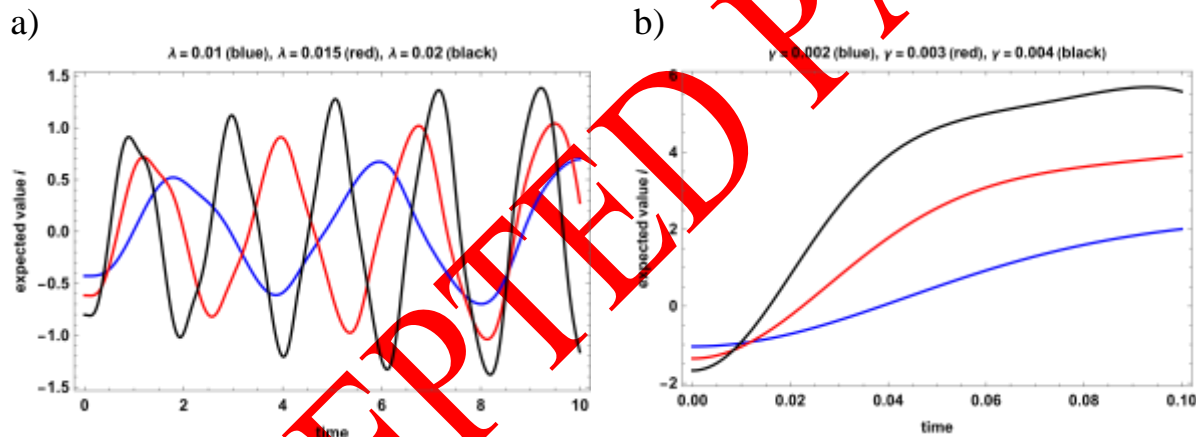
a)

b)

Figure 6. (a) Expected values of electric current vs time by changing λ ($\omega = 0.1, |\alpha| = 0.5, \gamma = 0.00001$) and (b) Expected values of electric current vs time by changing γ ($\omega = 0.1, |\alpha| = 0.5, \lambda = 0.01$)

4. Conclusion

The progress in experimental control on the nanometer scale in nanoelectronics and nanotechnology have enabled the study of electronic devices in the nano-scale. One of these devices are nonlinear inductors that with the presence of a ferromagnetic core as the main component of many electronic devices. Soft ferrites demonstrate properties very useful and appropriate for use in electronic devices applications, therefore, there is need to a quantum model that can accurately describe the operation of nonlinear inductors. Accordingly, we quantized a nonlinear LC-circuit consisting of a linear capacitor and a nonlinear inductor under the influence of an external field

and found the quantum Hamiltonian of this nonlinear LC-circuit. Unfortunately, the nature nonlinear inductors and the wide variety of core topologies make modeling difficult to study the quantum behavior the characteristic parameters. So, we studied the quantum dynamical effect of the nonlinear parameters in the rotating-wave approximation. The energy spectrum equation as a function of the linear and nonlinear coefficients for such electrical circuit is found and by numerical approach, it was shown that such a nonlinear electrical LC-circuit can to behave like qubits. By quantum dynamical approach, the quantum fluctuations of the quadrature operators current and charge in a coherent state is studied then the time-behavior characteristic parameters λ and γ and the digger squeezing are discussed numerically. Our formulation presented a method from a new point of view to the analysis of quantum effects in the quantum nonlinear electrical circuits with a nonlinear inductor device. We are well aware that, the theoretical study shows the numerical strength and computational efficiency.

Conflict of Interest The authors have reported that there is no conflict of interest.

Data Availability Data sharing is not appropriate for this research because no datasets were created or analyzed.

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