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Research Article

A New Methodology of Solving Fuzzy Linear Equations Based on Simulation

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Abstract

We develop a method for solving an arbitrary general fuzzy linear system by using the weighted metric. By using the distance between two fuzzy numbers, the best and nearest approximation fuzzy symmetric vector can be obtained. Defuzzification can replace the $n \times n$ fuzzy linear system with two $n \times n$ crisp linear systems. Consequently, to obtain the best approximation of the solution for an $n \times n$ fuzzy linear system, one must solve two $n \times n$ crisp linear systems. The existence and uniqueness of the fuzzy solution to the $n \times n$ linear fuzzy system are considered. Numerical examples are presented to illustrate the proposed model.

Keywords : Fuzzy number; Defuzzification; Fuzzy linear system; Distance; Weighted metric.

1 Introduction

More and more, modeling techniques, control problems and operation research algorithms have been designed for fuzzy data since the concept of fuzzy number and arithmetic operations with these numbers was introduced and investigated first by Zadeh. Whereas systems of simultaneous linear equations play major roles in various areas, it is immensely important to establish mathematical models and numerical procedures for fuzzy linear systems and solve them. A general model for solving an arbitrary $n \times n$ fuzzy linear system whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy number vector was first proposed by Friedman et al. [5]. They also studied duality fuzzy linear systems in [6]. Moreover, they used the embedding method given in [7] and replaced the original fuzzy linear system with a $2n \times 2n$ crisp linear system. The numerical solution of fuzzy linear systems is studied in [1–3]. The solution of an $m \times n$ original fuzzy linear system by a $2m \times 2n$ crisp system is done in [4]. Many authors obtained the least square symmetric solution of $m \times n$ fuzzy general linear systems in which $m \neq n$.

In this study, the researcher gives a new method for solving an $n \times n$ fuzzy linear system, whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy number vector, by using the weighted metric. First of all, this study uses the concept of the symmetric triangular fuzzy number and introduces an approach to defuzzify a general linear system. Hence, defuzzification [8–10, 14] can make a map-

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ping from the set of all fuzzy numbers to the positive half-plane $\mathbb{R}^2$, and this enables us to replace an $n\times n$ fuzzy system with two $n\times n$ crisp linear systems and to solve an $n\times n$ fuzzy linear system in n-dimensional space. It is clear that solving an $n\times n$ linear system in large systems is better than solving a $2n\times 2n$ linear system.

Since perturbation analysis is very important in numerical methods, the authors in [11] have presented the perturbation analysis for a class of fuzzy linear systems which could be solved by an embedding method. Now, according to the presented method in this study, the researcher can investigate perturbation analysis in two $n\times n$ crisp linear systems instead of one $2n\times 2n$ linear system as the authors of [11] have done.

This paper is organized as follows. In Section 2, the researcher recalls some fundamental results on fuzzy numbers. In Section 3, this article obtains the nearest symmetric triangular fuzzy vector. In this section, some theorems and remarks are proposed and illustrated. It is explained with two examples in Section 4. The paper ends with conclusions in Section 5.

2 Preliminaries

The basic definition of a fuzzy number given in [12, 14, 15] is as follows:

Definition 2.1. *A fuzzy number is a mapping $u:\mathbb{R}\to[0,1]$ with the following properties:*

1. *u is an upper semi-continuous function on $\mathbb{R}$,*
2. *$u(x)=0$ outside of some interval $[a_1,b_2]\subset\mathbb{R}$,*
3. *There are real numbers a_2,b_1 such that $a_1\le a_2\le b_1\le b_2$ and*
   - *(a) $u(x)$ is a monotonic increasing function on $[a_1,a_2]$,*
   - *(b) $u(x)$ is a monotonic decreasing function on $[b_1,b_2]$,*
   - *(c) $u(x)=1$ for all x in $[a_2,b_1]$.*

Let $\mathbb{R}$ be the set of all real numbers. The researcher assumes a fuzzy number u that can be expressed for all $x\in\mathbb{R}$ in the form

$$u(x)=\begin{cases} g(x) & \text{when } x\in[a,b),\\ 1 & \text{when } x\in[b,c],\\ h(x) & \text{when } x\in(c,d],\\ 0 & \text{otherwise.}\end{cases} \tag{2.1}$$

where a,b,c,d are real numbers such that $a<b\le c<d$, and g is a real-valued function that is increasing and right continuous, and h is a real-valued function that is decreasing and left continuous.

Definition 2.2. *A fuzzy number u in parametric form is a pair $(\underline{u},\overline{u})$ of functions $\underline{u}(r)$ and $\overline{u}(r)$, $0\le r\le 1$, which satisfies the following requirements:*

1. *$\underline{u}(r)$ is a bounded monotonic increasing left continuous function,*
2. *$\overline{u}(r)$ is a bounded monotonic decreasing left continuous function,*
3. *$\underline{u}(r)\le\overline{u}(r),\ 0\le r\le 1$.*

Definition 2.3. *The symmetric triangular fuzzy number $u=(x_0,\sigma)$, with defuzzifier x_0 and fuzziness σ, is a fuzzy set where the membership function is*

$$u(x)=\begin{cases} \frac{1}{\sigma}(x-x_0+\sigma) & x_0-\sigma\le x\le x_0,\\ \frac{1}{\sigma}(x_0-x+\sigma) & x_0\le x\le x_0+\sigma,\\ 0 & \text{otherwise.}\end{cases} \tag{2.2}$$

The parametric form of the symmetric triangular fuzzy number is

$$\underline{u}(r)=x_0-\sigma+\sigma r,\quad \overline{u}(r)=x_0+\sigma-\sigma r. \tag{2.3}$$

Definition 2.4. *For a fuzzy set u, the support function is defined as follows:*

$$\operatorname{supp}(u)=\overline{\{x\mid u(x)>0\}},$$

where $\overline{\{x\mid u(x)>0\}}$ is the closure of the set $\{x\mid u(x)>0\}$.

The addition and scalar multiplication of fuzzy numbers are defined by the extension principle and can be equivalently represented as follows. For arbitrary fuzzy numbers $u = (\underline{u}, \overline{u})$ and $v = (\underline{v}, \overline{v})$, this article defines addition $(u + v)$ and multiplication by scalar $k > 0$ as

$$(\underline{u+v})(r) = \underline{u}(r)+\underline{v}(r), \quad (\overline{u+v})(r) = \overline{u}(r)+\overline{v}(r), \tag{2.4}$$

$$(\underline{ku})(r) = k\underline{u}(r), \quad (\overline{ku})(r) = k\overline{u}(r). \tag{2.5}$$

To emphasize, the collection of all fuzzy numbers with addition and multiplication as defined above is denoted by F, which is a convex cone.

Definition 2.5 (16). *For two arbitrary fuzzy numbers $u = (\underline{u}, \overline{u})$ and $v = (\underline{v}, \overline{v})$, this study calls*

$$d_w(u, v) = \left(\int_0^1 f(r)d^2(u, v)\, dr\right)^{\frac{1}{2}},$$

the weighted distance between fuzzy numbers u and v, where

$$d^2(u, v) = (\underline{u}(r) - \underline{v}(r))^2 + (\overline{u}(r) - \overline{v}(r))^2,$$

and the function $f(r)$ is nonnegative and increasing on $[0, 1]$ with $f(0) = 0$ and $\int_0^1 f(r)\, dr = \frac{1}{2}$.

The function $f(r)$ is also called a weighting function. Both conditions $f(0) = 0$ and $\int_0^1 f(r)dr = \frac{1}{2}$ ensure that the distance defined by Eq. (2.4) is the extension of the ordinary distance in $\mathbb{R}$.

Definition 2.6. *The $n\times n$ linear system of equations*

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = y_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = y_2, \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = y_n, \end{cases} \tag{2.6}$$

where the coefficient matrix $A = (a_{ij})$, $1 \le i, j \le n$, is a crisp $n \times n$ matrix and $y_i \in F$, $1 \le i \le n$, is called a fuzzy linear system (FLS).

Definition 2.7. *A fuzzy number vector $(x_1, x_2, \cdots, x_n)^t$ given by*

$$x_j = (\underline{x}_j(r), \overline{x}_j(r)), \quad 1 \le j \le n, \quad 0 \le r \le 1,$$

is called a solution of (2.6) if

$$\begin{cases} \sum_{j=1}^n a_{ij}\underline{x}_j = \underline{y}_i, & i = 1, 2, \cdots, n, \\ \sum_{j=1}^n a_{ij}\overline{x}_j = \overline{y}_i, & i = 1, 2, \cdots, n. \end{cases} \tag{2.7}$$

Finally, this section concludes with a review of the proposed method for solving fuzzy linear systems in [5]. The authors in [5] wrote the linear system of Eq. (2.6) as follows:

$$SX = Y, \tag{2.8}$$

where the elements of $S = (s_{ij})$, $1 \le i, j \le 2n$, were as follows:

$$\begin{cases} a_{ij} \ge 0 \implies s_{ij} = a_{ij}, \quad s_{i,j+n} = a_{ij}, \\ a_{ij} < 0 \implies s_{i,j+n} = -a_{ij}, \quad s_{i+n,j} = -a_{ij}, \end{cases} \tag{2.9}$$

and any s_{ij} not determined by Eq. (2.9) is zero. The unknowns and the right-hand side column were

$$X = (\underline{x}_1, \underline{x}_2, \cdots, \underline{x}_n, -\overline{x}_1, -\overline{x}_2, \cdots, -\overline{x}_n)^t, \tag{2.10}$$

$$Y = (\underline{y}_1, \underline{y}_2, \cdots, \underline{y}_n, -\overline{y}_1, -\overline{y}_2, \cdots, -\overline{y}_n)^t, \tag{2.11}$$

respectively.

The structure of S implies that $s_{ij} \ge 0$, $1 \le i, j \le 2n$, and that

$$S = \begin{pmatrix} B & C \\ C & B \end{pmatrix}, \tag{2.12}$$

where B contains the positive entries of A and C the absolute values of the negative entries of A, and $A = B - C$.

Theorem 2.1 (5). *The matrix S is nonsingular if and only if the matrices $A = B - C$ and $B + C$ are both nonsingular.*

Theorem 2.2 (5). *If S^{-1} exists, it must have the same structure as S, i.e.,*

$$S^{-1} = \begin{pmatrix} D & E \\ E & D \end{pmatrix},$$

where $D = \frac{1}{2}[(B + C)^{-1} + (B - C)^{-1}]$ and $E = \frac{1}{2}[(B + C)^{-1} - (B - C)^{-1}]$.

It is known that if S is nonsingular then

$$X = S^{-1}Y,$$

but this solution may still not be an appropriate fuzzy vector, as shown in the following example.

Example 2.1. *Consider the* 3×3 *fuzzy system*

$$\begin{cases} 4x_1 + 2x_2 - x_3 = (-27 + 7r, -7 - 13r), \\ 2x_1 + 7x_2 + 6x_3 = (1 + 15r, 40 - 24r), \\ -x_1 + 6x_2 + 10x_3 = (26 + 18r, 47 - 3r). \end{cases}$$

This fuzzy system will not yield a fuzzy solution if the method of [5] is used.

In the next section, the researcher will investigate the fuzzy linear system and define a solution fuzzy vector for the linear system.

3 The Proposed Model

In this section, the researcher wants to obtain the symmetric triangular fuzzy vector $x = (x_1, x_2, \cdots, x_n)^t$ which is the nearest to the solution of the fuzzy linear system $Ax = y$. To carry out this purpose, this study minimizes the following functions:

$$d_w\left(\sum_{j=1}^{n} a_{ij}x_j, y_i\right) = \left[\int_0^1 f(r)\left(\sum_{j=1}^{n} a_{ij}\underline{x}_j(r) - \underline{y}_i(r)\right)^2 dr + \int_0^1 f(r)\left(\sum_{j=1}^{n} a_{ij}\overline{x}_j(r) - \overline{y}_i(r)\right)^2 dr\right]^{\frac{1}{2}}, \tag{3.13}$$

with respect to x_{0j} and σ_j, since the parametric form of components x_j is as follows:

$$\underline{x}_j(r) = x_{0j} + \sigma_j(r-1), \quad \overline{x}_j(r) = x_{0j} + \sigma_j(1-r). \tag{3.14}$$

In order to minimize $d_w\left(\sum_{j=1}^{n} a_{ij}x_j, y_i\right)$, it suffices to minimize the function

$$d_i(x_{01}, \cdots, x_{0n}, \sigma_1, \cdots, \sigma_n) = d_w^2\left(\sum_{j=1}^{n} a_{ij}x_j, y_i\right), \quad \forall i = 1, 2, \cdots, n. \tag{3.15}$$

Therefore, we minimize the following functions:

$$d_i(x_{01}, \cdots, x_{0n}, \sigma_1, \cdots, \sigma_n) = \int_0^1 f(r)\left(\sum_{a_{ij}>0} a_{ij}\underline{x}_j(r) + \sum_{a_{ij}<0} a_{ij}\overline{x}_j(r) - \underline{y}_i(r)\right)^2 dr + \int_0^1 f(r)\left(\sum_{a_{ij}>0} a_{ij}\overline{x}_j(r) + \sum_{a_{ij}<0} a_{ij}\underline{x}_j(r) - \overline{y}_i(r)\right)^2 dr, \tag{3.16}$$

with respect to x_{0j} and σ_j for each $1 \le i, j \le n$.

In order to minimize $d_i(x_{01}, \cdots, x_{0n}, \sigma_1, \cdots, \sigma_n)$, the researcher considers

$$\frac{\partial d_i}{\partial x_{0j}} = 2a_{ij} \times \int_0^1 \left(2\sum_{j=1}^{n} a_{ij}x_{0j} - (\underline{y}_i(r) + \overline{y}_i(r))\right) f(r)\, dr,$$

$$\frac{\partial d_i}{\partial \sigma_j} = 2a_{ij} \times \int_0^1 \left(2\sum_{j=1}^{n} a_{ij}\sigma_j(1-r)^2 - (\overline{y}_i(r) - \underline{y}_i(r))(1-r)\right) f(r)\, dr,$$

for $1 \le j \le n$.

Therefore, a stationary point $(x_{01}, \cdots, x_{0n}, \sigma_1, \cdots, \sigma_n)$ ought to be found for which

$$\begin{cases} \frac{\partial d_i}{\partial x_{0j}} = 0, \\ \frac{\partial d_i}{\partial \sigma_j} = 0. \end{cases} \tag{3.17}$$

Thus, the researcher obtains two $n \times n$ normal equation systems as follows:

$$\int_0^1 \left(2\sum_{j=1}^{n} a_{ij}x_{0j} - (\underline{y}_i(r) + \overline{y}_i(r))\right) f(r)\, dr = 0, \tag{3.18}$$

$$\int_0^1 \left(2\sum_{j=1}^{n} |a_{ij}|\sigma_j(1-r)^2 - (\overline{y}_i(r) - \underline{y}_i(r))(1-r)\right) \times f(r)\, dr = 0, \tag{3.19}$$

which yield

$$\sum_{j=1}^{n} a_{ij}x_{0j} = \int_0^1 (\underline{y}_i(r) + \overline{y}_i(r))f(r)\,dr, \tag{3.20}$$

$$\sum_{j=1}^{n} |a_{ij}|\sigma_j = \frac{\int_0^1 (\overline{y}_i(r) - \underline{y}_i(r))(1-r)f(r)\,dr}{2\int_0^1 (1-r)^2 f(r)\,dr}, \tag{3.21}$$

for $i = 1, 2, \cdots, n$. The matrix form of these equations is

$$Ax_0 = p, \quad B\sigma = q, \tag{3.22}$$

where $B = (b_{ij})$ contains the absolute values of the elements of A, i.e.,

$$b_{ij} = |a_{ij}| \quad \text{for each } 1 \leq i, j \leq n, \tag{3.23}$$

and

$$p_i = \int_0^1 (\underline{y}_i(r) + \overline{y}_i(r))f(r)\,dr, \tag{3.24}$$

$$q_i = \frac{\int_0^1 (\overline{y}_i(r) - \underline{y}_i(r))(1-r)f(r)\,dr}{2\int_0^1 (1-r)^2 f(r)\,dr}. \tag{3.25}$$

If we assume $f(r) = r$, then

$$p_i = \int_0^1 (\underline{y}_i(r) + \overline{y}_i(r))r\,dr, \tag{3.26}$$

$$q_i = 6\int_0^1 (\overline{y}_i(r) - \underline{y}_i(r))(1-r)r\,dr. \tag{3.27}$$

Then, to obtain the nearest symmetric triangular fuzzy vector, two $n \times n$ crisp linear systems should be solved. As a consequence, for obtaining the nearest approximation symmetric triangular fuzzy vector of the solution of an $n \times n$ fuzzy linear system, one must solve linear systems in n-dimensional space, and this decreases the computation time.

Definition 3.1. *Let $x_0 = (x_{01}, x_{02}, \cdots, x_{0n})^t$ and $\sigma = (\sigma_1, \sigma_2, \cdots, \sigma_n)^t$ denote the unique solutions of the two equations in (3.22). The vector $x = (x_1, x_2, \cdots, x_n)^t$, whose parametric form of x_i is defined by*

$$\underline{x}_i(r) = x_{0i} + |\sigma_i|(r-1), \quad \overline{x}_i(r) = x_{0i} + |\sigma_i|(1-r), \tag{3.28}$$

is called the nearest fuzzy solution of the fuzzy linear system (2.6). If $\sigma_i \geq 0$ for $1 \leq i \leq n$, then

$$\underline{x}_i(r) = x_{0i} + \sigma_i(r-1), \quad \overline{x}_i(r) = x_{0i} + \sigma_i(1-r),$$

and x is called a strong nearest symmetric fuzzy solution. Otherwise, x is a weak nearest symmetric fuzzy solution.

Theorem 3.1. *The nearest symmetric fuzzy solution of the fuzzy system exists if and only if the solution of the crisp system $B\sigma = q$ is positive, i.e.,*

$$\sigma_i \geq 0, \quad 1 \leq i \leq n.$$

Proof. By noting the definition of a fuzzy number, the proof is evident. □

Theorem 3.2. *Let $D = (d_{ij})$ be the inverse of B. The nearest solution X of (2.6) is a fuzzy vector for each arbitrary Y if and only if D is nonnegative, i.e.,*

$$d_{ij} \geq 0, \quad 1 \leq i \leq n, \quad 1 \leq j \leq n.$$

Proof. By the definition of q_i in Eq. (3.25), the proof is evident. □

Theorem 3.3 (7)**.** *The inverse of a non-negative matrix A is non-negative if and only if A is a generalized permutation matrix.*

According to Theorem 3.3, since B is non-negative, then D is non-negative if and only if D is a generalized permutation matrix.

4 Examples

In this section, several types of fuzzy solutions are discussed by means of examples. Throughout this section, the researcher assumes that $f(r) = r$.

Example 4.1. *Consider the 2×2 fuzzy linear system:*

$$\begin{cases} x_1 - x_2 = (-7 + 2r, -3 - 2r), \\ x_1 + 3x_2 = (19 + 4r, 27 - 4r). \end{cases} \tag{4.29}$$

Two crisp linear systems should be solved as follows:

$$\begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_{01} \\ x_{02} \end{bmatrix} = \begin{bmatrix} -5 \\ 23 \end{bmatrix},$$

and

$$\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}.$$

The solutions of Eq. (3.22) *are*

$$x_{01} = 2, \quad x_{02} = 7, \quad \sigma_1 = 1, \quad \sigma_2 = 1.$$

Therefore, the fuzzy solution is

$$\underline{x}_1 = 2 + (r-1), \quad \overline{x}_1 = 2 + (1-r),$$
$$\underline{x}_2 = 7 + (r-1), \quad \overline{x}_2 = 7 + (1-r).$$

This is a strong nearest solution.

Example 4.2 (4)**.** *Consider the* 5×5 *fuzzy linear system:*

$$\begin{cases} 6x_1 + x_2 + 3x_3 - x_4 + 6x_5 = (1+r, 3-r), \\ 5x_1 + 9x_2 + x_3 + 2x_4 + 3x_5 = (6+r, 8-r), \\ 2x_1 + 3x_2 + 9x_3 + 2x_4 + 3x_5 = (5+r, 7-r), \\ -x_1 + x_2 + 3x_3 + 8x_4 + 3x_5 = (3+r, 5-r), \\ x_1 + 2x_2 + 2x_3 + x_4 + 9x_5 = (2+r, 4-r). \end{cases} \tag{4.30}$$

The exact solutions are

$$x_1 = (-0.040 + 0.046r,\ 0.051 - 0.046r),$$
$$x_2 = (0.613 + 0.038r,\ 0.690 - 0.038r),$$
$$x_3 = (0.319 + 0.046r,\ 0.412 - 0.046r),$$
$$x_4 = (0.185 + 0.067r,\ 0.319 - 0.067r),$$
$$x_5 = (-0.001 + 0.079r,\ 0.158 - 0.079r).$$

By using the proposed method, two crisp linear systems should be solved as follows:

$$\begin{bmatrix} 6 & 1 & 3 & -1 & 6 \\ 5 & 9 & 1 & 2 & 3 \\ 2 & 3 & 9 & 2 & 3 \\ -1 & 1 & 3 & 8 & 3 \\ 1 & 2 & 2 & 1 & 9 \end{bmatrix} \begin{bmatrix} x_{01} \\ x_{02} \\ x_{03} \\ x_{04} \\ x_{05} \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 6 \\ 4 \\ 3 \end{bmatrix},$$

and

$$\begin{bmatrix} 6 & 1 & 3 & 1 & 6 \\ 5 & 9 & 1 & 2 & 3 \\ 2 & 3 & 9 & 2 & 3 \\ 1 & 1 & 3 & 8 & 3 \\ 1 & 2 & 2 & 1 & 9 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

The solutions of Eq. (3.22) *are*

$$x_{01} = 0.0053, \quad x_{02} = 0.6519, \quad x_{03} = 0.3659,$$
$$x_{04} = 0.2525, \quad x_{05} = 0.0785,$$
$$\sigma_1 = 0.0462, \quad \sigma_2 = 0.0388, \quad \sigma_3 = 0.0465,$$
$$\sigma_4 = 0.0671, \quad \sigma_5 = 0.0796.$$

Therefore, the fuzzy solutions are

$$\underline{x}_1 = 0.005 + 0.046(r-1), \quad \overline{x}_1 = 0.005 + 0.046(1-r),$$
$$\underline{x}_2 = 0.651 + 0.038(r-1), \quad \overline{x}_2 = 0.651 + 0.038(1-r),$$
$$\underline{x}_3 = 0.365 + 0.046(r-1), \quad \overline{x}_3 = 0.365 + 0.046(1-r),$$
$$\underline{x}_4 = 0.252 + 0.067(r-1), \quad \overline{x}_4 = 0.252 + 0.067(1-r),$$
$$\underline{x}_5 = 0.078 + 0.079(r-1), \quad \overline{x}_5 = 0.078 + 0.079(1-r).$$

According to the fact that $\underline{x}_i \le \overline{x}_i$, $i = 1, 2, \cdots, 5$, *are monotonic functions, the fuzzy solutions* $x_i = (\underline{x}_i, \overline{x}_i)$, $i = 1, 2, \cdots, 5$, *constitute a strong fuzzy solution.*

Example 4.3 (5)**.** *Consider the* 3×3 *fuzzy linear system:*

$$\begin{cases} x_1 + x_2 - x_3 = (r, 2-r), \\ x_1 - 2x_2 + x_3 = (2+r, 3), \\ 2x_1 + x_2 + 3x_3 = (-2, -1-r). \end{cases} \tag{4.31}$$

The matrix forms of Eqs. (3.22) *are as follows:*

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_{01} \\ x_{02} \\ x_{03} \end{bmatrix} = \begin{bmatrix} 1 \\ 2.83 \\ -1.83 \end{bmatrix},$$

and

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix}.$$

The solutions of Eq. (3.22) *are*

$$x_{01} = 1.2685, \quad x_{02} = -1.2931, \quad x_{03} = -1.0246,$$
$$\sigma_1 = 3.5, \quad \sigma_2 = -0.5, \quad \sigma_3 = -2.$$

Therefore, the solution is

$$\underline{x}_1 = 1.268 + 3.5(r-1), \quad \overline{x}_1 = 1.268 + 3.5(1-r),$$
$$\underline{x}_2 = -1.293 - 0.5(r-1), \quad \overline{x}_2 = -1.293 - 0.5(1-r),$$
$$\underline{x}_3 = -1.024 - 2(r-1), \quad \overline{x}_3 = -1.024 - 2(1-r).$$

Since $\sigma_2, \sigma_3 < 0$, *then* x_2 *and* x_3 *are not fuzzy numbers. Therefore, the fuzzy solution in this case is a weak weighted solution as follows:*

$$x_1 = (1.2685 + 3.5(r-1),\ 1.2685 + 3.5(1-r)),$$
$$x_2 = (-1.2931 + 0.5(r-1),\ -1.2931 + 0.5(1-r)),$$
$$x_3 = (-1.0246 + 2(r-1),\ -1.0246 + 2(1-r)).$$

5 Conclusion

The researcher proposed a model for solving fuzzy linear systems, and the original system is replaced by two $n \times n$ crisp linear systems in this paper. Moreover, an interesting approach to symmetric triangular approximation of the solution of a fuzzy linear system is suggested. The proposed method leads to the triangular fuzzy number which is the best one with respect to a certain measure of weighted distance among fuzzy numbers, $d_w(u, v)$. This method is simple, natural, and applicable anywhere.

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