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DEA-Discriminant Analysis Using Goal Programming for More than Two Groups that Expand for Interval Data: Apply to Stock Companies

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ABSTRACT

One of the interesting subjects that engages the minds of researchers is hypothesizing the correct classification of a new sample by using available data. Data Envelopment Analysis and Discriminant Analysis can be used to classify data independently of each other. DEA classifies them as efficient and inefficient groups, and DA classifies the observations by their historical data. Merging these two methods is a powerful tool for classifying the data. Most of the methods represented are just useful for classifying observations into two groups. In this paper, we represent our new DEA-DA method by using Goal Programming to classify data into more than two groups. Since, in the real world, in many cases we do not have certain data, we present a method that can be used for certain and interval data. Then, we present an empirical example of our purpose method on the Iranian stock companies' data. We divided stock companies into four groups with certain and interval data. Since most of the classical DA models are used for two groups, the advantage of the proposed model is highlighted. The result shows that the model can predict and classify more than two groups with certain and interval data completely correctly.

1 Introduction

Discriminant Analysis (DA) is a classification method for predicting group membership of a new observation. In DA, at first, the observation data are classified into the specified group by some defined factors. By using the memberships of these observations, we can predict the membership of new

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observations. Data Envelopment Analysis (DEA) is a powerful tool in optimization, management, and decision science for classifying the units into two groups: efficient and inefficient. DEA has lots of applied models, such as BCC (Banker et al. [4]), CCR (Charnes et al. [5]), Additive, and so on. In (1999), the additive model of DEA was compared with the represented GP approach for DA. The comparison showed their likeness and discrepancy. Sueyoshi believed that combining DA and DEA in the framework of GP is so useful and helps us to specify the group membership of new observations more accurately. So, he represented his DEA-DA method by using GP in (1999), then he completed his model in (2001) (Sueyoshi [20,21]). In the real situation, sometimes we do not have access to certain data, so interval data or fuzzy data in these cases. In discriminant analysis, there are numerous papers in that the researcher used imprecise data. Some data should be expressed as an interval (Jahanshahloo et al. [11], Duarte Silva et al. [7-8], Angulo et al. [1]), and some data should be expressed as a fuzzy set because of their quiddity (Hosseinzadeh Lotfi et al. [9-10], Khalili-Damghani et al. [12], Omrani et al. [13]). DEA is one of the practical tools for portfolio selection and portfolio optimization. There are lots of researchers who want to find an easier and more accurate method in this field (Navidi et al. [14-18], and Banihashemi et al. [2-3]).

In portfolio cases, some data are imprecise (Peykani et al. [19], Tohidi et al. [22]). The DEA-DA method is used in lots of management cases. Most of the represented DEA-DA methods, by using GP, classify the observation into two groups. But, in the real world, we have more than two groups. For example, customer clubs usually separate their customers into the platinum card (loyal customers), gold card (good customers), silver card (average customers), and blue card (new customers). This classification helps the manager to present the best services appropriate to each customer. Another example, consider the capitalist who wants to establish a company.

The discriminant analysis helps him/her decide what is best. By using discriminant analysis, we can distribute available stock companies by their historical data into defined groups such as great, good, average, and weak stock companies. So, in this paper, we distribute available pharmaceutical stock companies by their historical data to defined groups (that are more than two groups) with certain and interval data; then, by using this information, we predict the group membership of new pharmaceutical stock companies. The remainder of this paper is organized as follows: In Section 2, we review some previous works in the DEA-DA method by using GP. Our proposed method is described in Section 3. The empirical example of our purpose method on the Iranian pharmaceutical stock companies is represented in Section 4. The conclusion is represented in section 5.

2 Background

In this section, we define previously represented models.

Sueyoshi [20] in 1999 represented his two-stage DEA-DA method for two groups. The first stage includes classification and overlaps identification. The second stage includes handling overlap.

Assume that there are n observations $j = (1, \dots, n)$ that belong to two groups (G_1 and G_2), Each observation is defined by k independent factors $i = (1, \dots, k)$ indicated by Z_{ij}.

The first stage, which involves classification and overlap identification, is formulated as follows:

$$
\begin{aligned}
& min \sum_{j\in G_1} S_{1j}^{+} + \sum_{j\in G_2} S_{2j}^{-} \\
& s.t. \sum_{i=1}^{k} \alpha_i Z_{ij} + S_{1j}^{+} - S_{1j}^{-} = d, \qquad\qquad j \in G_1 \\
& \sum_{i=1}^{k} \beta_i Z_{ij} + S_{2j}^{+} - S_{2j}^{-} = d - \eta, \qquad\qquad j \in G_2 \\
& \sum_{i=1}^{k} \alpha_i = 1 \\
& \sum_{i=1}^{k} \beta_i = 1 \\
& all\ slacks \geq 0\ , \quad \alpha_i \geq 0\ , \quad \beta_i \geq 0\ , \quad d{:}\ unrestricted
\end{aligned} \tag{1}
$$

Where S_{1j}^{+} ,$S_{1j}^{-}(j \in G_1)$ are the positive and negative aberrations of a piecewise linear discriminant function $\sum_{i=1}^{k} \alpha_i Z_{ij}$ from a discriminant score d of G_1, respectively. The positive aberration$(S_{1j}^{+} > 0\ ,\ j \in G_1)$, specifies an event of group misclassification on the jth observation in G_1, which comes in the objective of (1) to minimize its incorrect classification. Simultaneously, the negative aberration $(S_{1j}^{-} > 0\ ,\ j \in G_1)$, specifies an event of group correct classification, which comes within the constraints of (1) to avoid an infeasible solution. The ith weight of the discriminant function is α_i in (1).

The above explanation for G_1 expand to G_2. S_{2j}^{+} ,$S_{2j}^{-}(j \in G_2)$ are the positive and negative aberrations of a piecewise linear discriminant function $\sum_{i=1}^{k} \beta_i Z_{ij}$ from a discriminant score $d - \eta$ of G_2, respectively. The positive aberration $(S_{2j}^{+} > 0\ ,\ j \in G_2)$, specifies an event of group misclassification on the jth observation in G_2, it comes in the objective of (1) to minimize its incorrect classification. Simultaneously, the negative aberration $(S_{2j}^{-} > 0\ ,\ j \in G_2)$, specifies an event of group correct classification, which comes within the constraints of (1) to avoid an infeasible solution. The ith weight of the discriminant function is β_i in (1). η depends on the decision maker's intellectual (Sueyoshi [20] considered $\eta = 0.1$).
The new sample that is mth observation, whose value is defined by Z_{im}, can be classified by the following principle:

I. If $\sum_{i=1}^{k} \alpha_i^* Z_{im} > d^* \geq \sum_{i=1}^{k} \beta_i^* Z_{im}$ or $\sum_{i=1}^{k} \alpha_i^* Z_{im} \leq d^* < \sum_{i=1}^{k} \beta_i^* Z_{im}$ then $m \in G_1 \cap G_2$
II. If $\sum_{i=1}^{k} \alpha_i^* Z_{im} \geq d^*$ and $\sum_{i=1}^{k} \beta_i^* Z_{im} \geq d^*$ then $m \in G_1$
III. If $\sum_{i=1}^{k} \alpha_i^* Z_{im} < d^*$ and $\sum_{i=1}^{k} \beta_i^* Z_{im} < d^*$ then $m \in G_2$

(α_i^*, β_i^*, $d^*$are the optimal solutions of (1))

When the first situation happened ($m \in G_1 \cap G_2$) and an overlap is identified, we go to the second stage, where handling overlap is formulated as follows:

$$
\begin{aligned}
& \min \sum_{j\in G_1} S_{1j}^{+} + \sum_{j\in G_2} S_{2j}^{-} \\
& s.t. \sum_{i=1}^{k} \gamma_i Z_{ij} + S_{1j}^{+} - S_{1j}^{-} = d, \qquad j \in G_1 \\
& \sum_{i=1}^{k} \gamma_i Z_{ij} + S_{2j}^{+} - S_{2j}^{-} = d - \eta, \qquad j \in G_2 \\
& \sum_{i=1}^{k} \gamma_i = 1 \\
& all\ slacks \geq 0 \ , \quad \gamma_i \geq 0 \ , \quad d: unrestricted
\end{aligned} \tag{2}
$$

All observations that belonged to $G_1 \cap G_2$, can be classified to $G_1 or\ G_2$ by following two principles:

I. If $\sum_{i=1}^{k} \gamma_i^* Z_{ij} \geq d^*$ for $j \in G_1 \cap G_2$ then $j \in G_1$

II. If $\sum_{i=1}^{k} \gamma_i^* Z_{ij} < d^*$ for $j \in G_1 \cap G_2$ then $j \in G_2$

(γ_i^*, $d^*$are the optimal solutions of (2))

Sueyoshi [21] in 2001 represented his extended DEA-DA method for two groups.
The first stage, that is, classification and overlap identification, is formulated as follows:

$$
\begin{aligned}
& \min \sum_{j\in G_1} S_{1j}^{+} + \sum_{j\in G_2} S_{2j}^{-} \\
& s.t. \sum_{i=1}^{k} (\lambda_i^{+} - \lambda_i^{-}) Z_{ij} + S_{1j}^{+} - S_{1j}^{-} = d + 1, \qquad j \in G_1 \\
& \sum_{i=1}^{k} (\lambda_i^{+} - \lambda_i^{-}) Z_{ij} + S_{2j}^{+} - S_{2j}^{-} = d, \qquad j \in G_2 \\
& \sum_{i=1}^{k} (\lambda_i^{+} + \lambda_i^{-}) = 1 \\
& all\ slacks \geq 0 \ , \quad \lambda_i^{+} \geq 0 \ , \quad \lambda_i^{-} \geq 0 \ , \quad d: unrestricted
\end{aligned} \tag{3}
$$

All the observed factors Z_{ij} are connected by $\sum_{i=1}^{k} \lambda_i Z_{ij}$ where λ_i is a weight for the ith factor. These weights are limited in the way that the sum of the total values of $\lambda_i = (\lambda_i^{+} - \lambda_i^{-})$ for all $i = 1, \dots, k$ is unity. The new sample that is mth observation, whose value is defined by Z_{im}, can be classified by the following principle:

I. If $\sum_{i=1}^{k} \lambda_i^* Z_{im} \geq d^* + 1$ then $m \in G_1$

II. If $d^* + 1 > \sum_{i=1}^{k} \lambda_i^* Z_{im} > d^*$ then $m \in G_1 \cap G_2$

III. If $d^* \geq \sum_{i=1}^{k} \lambda_i^* Z_{im}$ then $m \in G_2$

($\lambda_i^* = ({\lambda_i^{+}}^{*} - {\lambda_i^{-}}^{*})$, d^* are the optimal solutions of (3))

For using these principles, the whole set G has been divided into the following subsets:

$$C_1 = \{j \in G_1 | \sum_{i=1}^{k} \lambda_i^* Z_{im} \geq d^* + 1\}$$

$$C_2 = \{j \in G_2 | \sum_{i=1}^{k} \lambda_i^* Z_{im} \leq d^*\}$$

$$D_1 = G_1 - C_1$$

$$D_2 = G_2 - C_2$$

When the overlap is identified, the second stage that handles overlap is used for two subgroups $(D_1 \cup D_2)$. The handling overlap is formulated as follows:

$$\begin{aligned}
&min \sum_{j \in D_1} S_{1j}^{+} + \sum_{j \in D_2} S_{2j}^{-} \\
&s.t. \sum_{i=1}^{k} (\lambda_i^{+} - \lambda_i^{-}) Z_{ij} \geq d + 1, && j \in C_1 \\
&\sum_{i=1}^{k} (\lambda_i^{+} - \lambda_i^{-}) Z_{ij} + S_{1j}^{+} - S_{1j}^{-} = c, && j \in D_1 \\
&\sum_{i=1}^{k} (\lambda_i^{+} - \lambda_i^{-}) Z_{ij} + S_{2j}^{+} - S_{2j}^{-} = c, && j \in D_2 \\
&\sum_{i=1}^{k} (\lambda_i^{+} - \lambda_i^{-}) Z_{ij} \leq d, && j \in C_2 \\
&\sum_{i=1}^{k} (\lambda_i^{+} + \lambda_i^{-}) = 1 \\
&d \leq c \leq d + 1 \\
&all\ slacks \geq 0\ , \quad \lambda_i^{+} \geq 0\ , \quad \lambda_i^{-} \geq 0\ , \quad d{:}\, unrestricted\ , \quad c{:}\, unrestricted
\end{aligned} \tag{4}$$

All correct classified observations are limited by constraints numbers 1 and 4 in the model (4).

$(\sum_{i=1}^{k} (\lambda_i^{+} - \lambda_i^{-}) Z_{ij} \geq d + 1,\ j \in C_1\ ;\ \sum_{i=1}^{k} (\lambda_i^{+} - \lambda_i^{-}) Z_{ij} \leq d,\ j \in C_2)$

The new discriminant score (c) is specified by minimizing the total aberration of observations in the overlap. The new sample that is rth observation which is identified as an overlap in the first stage can be classified by the following principle:

I. If $\sum_{i=1}^{k} \lambda_i^* Z_{ir} \geq c^*$ then $r \in G_1$

II. If $\sum_{i=1}^{k} \lambda_i^* Z_{ir} < c^*$ then $r \in G_2$

(For $\sum_{i=1}^{k} \lambda_i^* Z_{ir} = c^*$ model cannot determine accurately)

$(\lambda_i^* = (\lambda_i^{+^*} - \lambda_i^{-^*})$, $c^*$are the optimal solutions of (4))

3 Methodology

In this section, first we represent our model for certain data, then we expand it for interval data.

3.1 Proposed method

Based on the Sueyoshi [21] method, we propose our method that classifies observations into more than two groups, and then we expand it for interval data.

Assume that there are n observations $j=(1,\dots,n)$ that are belong to h groups $(g=1,\dots,h)$, Each observation is defined by k independent factors $i=(1,\dots,k)$ indicated by Z_{ij}.

The GP model of DEA-DA for more than two groups is formulated as follows:

$$
\begin{aligned}
& min \sum_{g=1}^{h} \sum_{s=1}^{2h-3} \sum_{j\in G_g} t_{sj} + \sum_{g=1}^{h} \sum_{s=1}^{2h-2} \sum_{j\in G_g} t_{(s+1)j} \\
& s.t. \sum_{i=1}^{k} \lambda_i Z_{ij} - c_g \geq -t_{sj}, \quad j\in G_g \ , \quad g=1,\dots,h-1 \ , \quad s=1,2,\dots,2h-3 \\
& \sum_{i=1}^{k} \lambda_i Z_{ij} - c_g \leq t_{(s+1)j}, \quad j\in G_{g+1} \ , \quad g=1,\dots,h-1 \ , \quad s=1,2,\dots,2h-2 \\
& \sum_{i=1}^{k} |\lambda_i| = 1 \\
& c_g(g=1,\dots,h-1): unrestricted \ , \quad \lambda_i: unrestricted \ , \quad t_{sj}\geq 0 \ , \quad t_{(s+1)j}\geq 0
\end{aligned}
\tag{5}
$$

All the observed factors Z_{ij} are connected by $\sum_{i=1}^{k}\lambda_i Z_{ij}$ where λ_i is a weight for the ith factor. These weights are limited in the way that the sum of total values of $|\lambda_i|$ for all $i=1,\dots,k$ is unity. The different h groups are separated with discriminant scores $c_g(g=1,\dots,h-1)$. The variables t_{sj} , $t_{(s+1)j}$ are the aberration of the discriminant function $\sum_{i=1}^{k}\lambda_i Z_{ij}$ from a discriminant score $c_g(g=1,\dots,h-1)$ to minimize the event of group misclassification (the s index is just used to make different, separate hyperplanes).

$(c_g^*(g=1,\dots,h-1)$ and $\lambda_i^*(i=1,\dots,k)$ are the optimal solutions of (5))

The new sample that is mth observation, whose value is defined by Z_{im}, can be classified by the following principle:

I. If $\sum_{i=1}^{k}\lambda_i^* Z_{im}\geq c_1^*$ then $m\in G_1$

II. If $c_{g-1}^*\geq \sum_{i=1}^{k}\lambda_i^* Z_{im}\geq c_g^*$ then $m\in G_g(g=2,\dots,h-1)$

III. If $\sum_{i=1}^{k}\lambda_i^* Z_{im}\leq c_{h-1}^*$ then $m\in G_h$

For a more comprehensive understanding, we bring Fig.1 and formulated model (5) widely for four groups as follows:

$$\min \sum_{j\in G_1} t_{1j} + \sum_{j\in G_2} t_{2j} + \sum_{j\in G_2} t_{3j} + \sum_{j\in G_3} t_{4j} + \sum_{j\in G_3} t_{5j} + \sum_{j\in G_4} t_{6j}$$

$$\begin{aligned}
s.t. \ &\sum_{i=1}^{k} \lambda_i Z_{ij} - c_1 \geq -t_{1j}, \qquad j \in G_1\\
&\sum_{i=1}^{k} \lambda_i Z_{ij} - c_1 \leq t_{2j}, \qquad j \in G_2\\
&\sum_{i=1}^{k} \lambda_i Z_{ij} - c_2 \geq -t_{3j}, \qquad j \in G_2\\
&\sum_{i=1}^{k} \lambda_i Z_{ij} - c_2 \leq t_{4j}, \qquad j \in G_3\\
&\sum_{i=1}^{k} \lambda_i Z_{ij} - c_3 \geq -t_{5j}, \qquad j \in G_3\\
&\sum_{i=1}^{k} \lambda_i Z_{ij} - c_3 \leq t_{6j}, \qquad j \in G_4\\
&\sum_{i=1}^{k} |\lambda_i| = 1\\
&c_1, c_2, c_3: unrestricted \ , \qquad \lambda_i: unrestricted \ ,\\
&t_{1j} \geq 0 \ , \quad t_{2j} \geq 0 \ , \quad t_{3j} \geq 0 \ , \quad t_{4j} \geq 0 \ , \quad t_{5j} \geq 0 \ , \quad t_{6j} \geq 0
\end{aligned} \tag{6}$$

The new sample that is mth observation, whose value is defined by Z_{im}, can be classified by the following principle:

I. If $\sum_{i=1}^{k} \lambda_i^* Z_{im} \geq c_1^*$ then $m \in G_1$
II. If $c_1^* \geq \sum_{i=1}^{k} \lambda_i^* Z_{im} \geq c_2^*$ then $m \in G_2$
III. If $c_2^* \geq \sum_{i=1}^{k} \lambda_i^* Z_{im} \geq c_3^*$ then $m \in G_3$
IV. If $\sum_{i=1}^{k} \lambda_i^* Z_{im} \leq c_3^*$ then $m \in G_4$

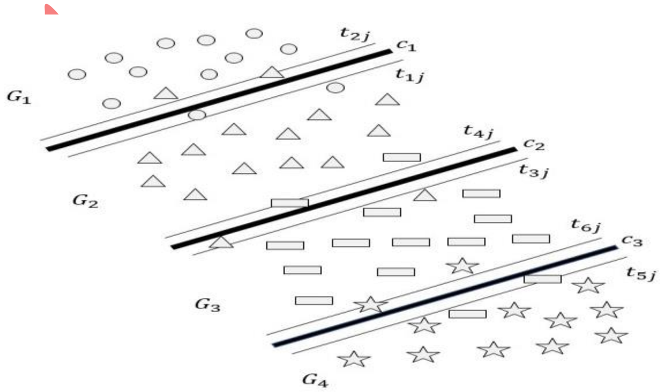

Fig. 1: Classification of four groups

As you see in Fig. 1, the observations of the first group (G_1) must be on the left side of its separator hyperplane (c_1) but we allow them to have a little tolerance (t_{1_j}). The observations of the second group (G_2) must be between its separator hyperplanes (c_1 and c_2) but we allow them to have a little tolerance (be between t_{2_j} and t_{3_j}). The observations of the third group (G_3) must be between its separator hyperplanes (c_2 and c_3) but we allow them to have a little tolerance (be between t_{4_j} and t_{5_j}). The observations of the fourth group (G_4) must be on the right side of its separator hyperplane (c_3) but we allow them to have a little tolerance (t_{6_j}).Consider that we can use model (5) for how many groups as we want ($g = 1, \dots, h$).

Theorem 1. The optimal value of the objective function of the model (5) is finite.

Proof. Let

$\lambda = e_1$

$c_g = 0 \;,\; (g = 1, \dots, h-1)$

$t_{sj} = -Z_{ij} \;,\; (j = 1, \dots, n) \;,\; (i = 1, \dots, k) \;,\; s = 1,2, \dots, 2h-3$

$t_{(s+1)j} = Z_{ij} \;,\; (j = 1, \dots, n) \;,\; (i = 1, \dots, k) \;,\; s = 1,2, \dots, 2h-2$

Then, with this select model (5), there is a feasible solution. On the other hand, we always have:

$$0 \le \theta = min \sum_{g=1}^{h} \sum_{s=1}^{2h-3} \sum_{j\in G_g} t_{sj} + \sum_{g=1}^{h} \sum_{s=1}^{2h-2} \sum_{j\in G_g} t_{(s+1)j}$$

Therefore, model (5) has a bounded optimal solution, and the proof is completed.

3.2 Proposed method for interval data

Since in the real world, we do not access certain data, the data might be an interval. Also, some data is expressed as an interval because of its properties. We can expand our model (5) for interval data. Assume that there are n observations $j = (1, \dots, n)$ that belong to h groups $(g = 1, \dots, h)$, Each observation is defined by k independent factors $i = (1, \dots, k)$ indicated by $Z_{ij} \epsilon \left[Z_{ij}^{L}, Z_{ij}^{U}\right]$ with permanent lower and upper bounds of the interval (for more explanation of imprecise data in DEA, see Cooper et al. [6]). Then we will have:

$$\theta = min \sum_{g=1}^{h} \sum_{s=1}^{2h-3} \sum_{j\in G_g} t_{sj} + \sum_{g=1}^{h} \sum_{s=1}^{2h-2} \sum_{j\in G_g} t_{(s+1)j} \tag{7}$$

$$s.t. \sum_{i=1}^{k} \lambda_i Z_{ij} - c_g \ge -t_{sj}, \quad j \in G_g \;, \quad g = 1, \dots, h-1 \;, \quad s = 1,2, \dots, 2h-3$$

$$\sum_{i=1}^{k} \lambda_i Z_{ij} - c_g \le t_{(s+1)j}, \quad j \in G_{g+1} \;, \quad g = 1, \dots, h-1 \;, \quad s = 1,2, \dots, 2h-2$$

$$\sum_{i=1}^{k} |\lambda_i| = 1$$

$$c_g (g = 1, \dots, h-1): unrestricted \;, \quad \lambda_i : unrestricted \;, \quad t_{sj} \ge 0 \;, \quad t_{(s+1)j} \ge 0$$

Where $Z_{ij} \epsilon [Z_{ij}^L, Z_{ij}^U]$.

All the observed factors $Z_{ij} \epsilon [Z_{ij}^L, Z_{ij}^U]$ are connected by $\sum_{i=1}^{k} \lambda_i [Z_{ij}^L, Z_{ij}^U]$ where λ_i is a weight for the ith factor. These weights are limited in the way that the sum of the total values of $|\lambda_i|$ for all $i = 1, \dots, k$ is unity. The different h groups are separated with discriminant scores $c_g(g = 1, \dots, h-1)$. The variables t_{sj}, $t_{(s+1)j}$ are the aberration of the discriminant function $\sum_{i=1}^{k} \lambda_i [Z_{ij}^L, Z_{ij}^U]$ from a discriminant score $c_g(g = 1, \dots, h-1)$ to minimize the event of group misclassification (The s index ($s = 1,2, \dots$) is just used to make different separate hyperplanes).

For solving model (7), we can solve models (8) and (9) that are its upper and lower bounds:

$$\begin{aligned}
&\theta^L = min \sum_{g=1}^{h} \sum_{s=1}^{2h-3} \sum_{j \in G_g} t_{sj} + \sum_{g=1}^{h} \sum_{s=1}^{2h-2} \sum_{j \in G_g} t_{(s+1)j} \\
&s.t. \sum_{i=1}^{k} \lambda_i Z_{ij}^L - c_g \geq -t_{sj}, \qquad j \in G_g, \quad g = 1, \dots, h-1, \quad s = 1,2, \dots, 2h-3 \\
&\sum_{i=1}^{k} \lambda_i Z_{ij}^L - c_g \leq t_{(s+1)j}, \qquad j \in G_{g+1}, \quad g = 1, \dots, h-1, \quad s = 1,2, \dots, 2h-2 \\
&\sum_{i=1}^{k} |\lambda_i| = 1 \\
&c_g(g = 1, \dots, h-1): unrestricted, \quad \lambda_i: unrestricted, \quad t_{sj} \geq 0, \quad t_{(s+1)j} \geq 0
\end{aligned} \tag{8}$$

$$\begin{aligned}
&\theta^U = min \sum_{g=1}^{h} \sum_{s=1}^{2h-3} \sum_{j \in G_g} t_{sj} + \sum_{g=1}^{h} \sum_{s=1}^{2h-2} \sum_{j \in G_g} t_{(s+1)j} \\
&s.t. \sum_{i=1}^{k} \lambda_i Z_{ij}^U - c_g \geq -t_{sj}, \qquad j \in G_g, \quad g = 1, \dots, h-1, \quad s = 1,2, \dots, 2h-3 \\
&\sum_{i=1}^{k} \lambda_i Z_{ij}^U - c_g \leq t_{(s+1)j}, \qquad j \in G_{g+1}, \quad g = 1, \dots, h-1, \quad s = 1,2, \dots, 2h-2 \\
&\sum_{i=1}^{k} |\lambda_i| = 1 \\
&c_g(g = 1, \dots, h-1): unrestricted, \quad \lambda_i: unrestricted, \quad t_{sj} \geq 0, \quad t_{(s+1)j} \geq 0
\end{aligned} \tag{9}$$

The objective function $\theta \in [\theta^L, \theta^U]$ is to minimize group misclassification.

The new sample that is mth observation, whose value is defined by $Z_{im} \epsilon [Z_{im}^L, Z_{im}^U]$, can be classified by the following principle:

I. If $\sum_{i=1}^{k} \lambda_i^{L^*} Z_{im}^L \geq c_1^{L^*}$ then $m \in G_1$

II. If $c_{g-1}^{L^*} \geq \sum_{i=1}^{k} \lambda_i^{L^*} Z_{im}^L \geq c_g^{L^*}$ & $c_{g-1}^{U^*} \geq \sum_{i=1}^{k} \lambda_i^{U^*} Z_{im}^U \geq c_g^{U^*}$ then $m \in G_g(g = 2, \dots, h-1)$

III. If $c_{g-1}^{L^*} \geq \sum_{i=1}^{k} \lambda_i^{L^*} Z_{im}^{L} \geq c_g^{L^*}$ & $c_g^{U^*} \geq \sum_{i=1}^{k} \lambda_i^{U^*} Z_{im}^{U} \geq c_{g+1}^{U^*}$ then $m \in G_g (g = 2, \dots, h-1)$

IV. If $\sum_{i=1}^{k} \lambda_i^{U^*} Z_{im}^{U} \leq c_{h-1}^{U^*}$ then $m \in G_h$

($\lambda_i^{L^*}$ and $c_g^{L^*}$ are the optimal solutions of model (8), $\lambda_i^{U^*}$ and $c_g^{U^*}$ are the optimal solutions of model (9))

Theorem 2. Let θ^*, θ^{L^*}, θ^{U^*} be the optimal solution for model (7), (8), (9) respectively. Then $\theta^{L^*} \leq \theta^* \leq \theta^{U^*}$.

Proof. Let θ^*, c_g^*, t_{sj}^*, $t_{(s+1)j}^*$ be the optimal solution of model (7). Because of $Z_{ij} \leq Z_{ij}^U$ and $\sum_{i=1}^{k} \lambda_i Z_{ij} \leq c_g + t_{(s+1)j}$

We have

$$\sum_{i=1}^{k} \lambda_i Z_{ij} \leq c_g^* + t_{(s+1)j}^*$$

$$\sum_{i=1}^{k} \lambda_i Z_{ij}^U \leq c_g^U + t_{(s+1)j}^U$$

$$c_g^* + t_{(s+1)j}^* \leq c_g + t_{(s+1)j} \leq c_g^U + t_{(s+1)j}^U$$

$$\sum_{i=1}^{k} \lambda_i Z_{ij} \leq \sum_{i=1}^{k} \lambda_i Z_{ij}^U$$

So

$$\sum_{i=1}^{k} \lambda_i Z_{ij} \leq c_g^U + t_{(s+1)j}^U$$

Therefore $\theta^* \leq \theta^U$, we can prove $\theta^L \leq \theta^*$ likewise.

4 Empirical examples

In this section, we apply our model to certain data and interval data separately.

4.1 Data collection for certain data

The dataset was collected from the 24 Iranian pharmaceutical stock companies from 2023 to 2024. The dataset was obtained from http://www.fipiran.com. All the stock companies are shown by their company's symbol. Table 2 represents Z_{ij} from Iranian pharmaceutical stock companies. In this research, we used 10 financial indices ($i1, \dots, i10$) that are expressed in Table 1, to distribute the 24 Iranian pharmaceutical stock companies to 4 groups. More details are in section 4.2.

Table 1: The Financial Indices

$i1$	Total Current Assets	$i6$	Total Assets
$i2$	Total Current Liabilities	$i7$	Total Liabilities
$i3$	Total Stockholder Equity	$i8$	Capital
$i4$	Profit Margin	$i9$	Retained Earnings
$i5$	Gross Profit Ratio	$i10$	Cash

Table 2: Financial Data of Iranian Pharmaceutical Stock Companies

Obs.	$i1$	$i2$	$i3$	$i4$	$i5$	$i6$	$i7$	$i8$	$i9$	$i10$
	Z_{1j}	Z_{2j}	Z_{3j}	Z_{4j}	Z_{5j}	Z_{6j}	Z_{7j}	Z_{8j}	Z_{9j}	Z_{10j}
DSOB1	0.102	0.016	0.477	0.376	0.116	0.216	0.015	0.293	0.427	0.047
FTIR1	0.058	0.029	0.126	0.125	0.114	0.068	0.029	0.07	0.117	0.095
PDRO1	0.178	0.165	0.158	0.435	0.589	0.144	0.149	0.037	0.22	0.306
JAMD1	0.018	0.008	0.022	0.019	0.015	0.014	0.007	0.021	0.022	0.072
DRZK1	0.281	0.244	0.164	0.237	0.217	0.21	0.221	0.104	0.212	0.505
IRDR1	0.083	0.067	0.059	0.03	0.049	0.064	0.064	0.093	0.025	0.118
AMIN1	0.15	0.112	0.162	0.129	0.121	0.144	0.11	0.146	0.157	0.171
ROZD1	0.072	0.066	0.068	0.018	0.027	0.082	0.078	0.123	0.039	0.129
DSIN1	0.143	0.177	0.149	0.192	0.184	0.173	0.163	0.07	0.223	0.34
ABDI1	0.355	0.281	0.297	0.278	0.45	0.306	0.265	0.295	0.275	0.282
DSNZ1	0.137	0.139	0.095	0.026	0.074	0.129	0.132	0.151	0.029	0.236
DALZ1	0.415	0.319	0.369	0.344	0.271	0.351	0.285	0.326	0.384	0.292
DJBR1	0.287	0.192	0.274	0.217	0.157	0.237	0.175	0.133	0.345	0.063
KIMI1	0.134	0.108	0.14	0.146	0.077	0.109	0.102	0.042	0.173	0.015
KSPZ1	0.157	0.16	0.085	0.139	0.126	0.122	0.144	0.026	0.144	0.155
DTDZ1	0.098	0.086	0.055	0.061	0.065	0.091	0.107	0.065	0.051	0.027
DDPK1	0.094	0.087	0.044	0.037	0.044	0.072	0.08	0.045	0.037	0.024
DLGM1	0.077	0.079	0.094	0.029	0.041	0.087	0.093	0.079	0.056	0.053
EXIR1	0.259	0.254	0.146	0.238	0.234	0.19	0.238	0.079	0.206	0.114
BRKT1	0.096	0.236	0.456	0.238	0	0.489	0.443	0.597	0.244	0.123
DPAK1	0.332	0.396	0.18	0.261	0.274	0.293	0.372	0.11	0.238	0.266
DFRB1	0.245	0.266	0.153	0.233	0.185	0.21	0.242	0.07	0.233	0.316
DZAH1	0.317	0.452	0.098	0.124	0.159	0.272	0.403	0.439	0.139	0.082
DAML1	0.082	0.107	0.059	0.031	0.045	0.076	0.097	0.161	0.025	0.022

4.2 Apply the proposed method on certain data

In this section, we apply our represented model (5) to our data. Table 3 presents the $\lambda_i^*(i = 1, \ldots, 10)$ and $c_g^*(g = 1,2,3)$.

Table 3: Weight Estimates and Discriminant Scores

c_1^*	0.01
c_2^*	0
c_3^*	-0.005
λ_1^*	0.12
λ_2^*	-0.32
λ_3^*	0.05
λ_4^*	0.02
λ_5^*	0.03
λ_6^*	-0.17
λ_7^*	0.21
λ_8^*	0.03
λ_9^*	0
λ_{10}^*	0.05

In this research, we used 10 financial indices to distribute the 24 Iranian pharmaceutical stock companies into 4 groups:
G_1= {Great pharmaceutical stock companies}
G_2= {Good pharmaceutical stock companies}
G_3= {Average pharmaceutical stock companies}
G_4= {Weak pharmaceutical stock companies}
Table 4 presents the group membership and prediction of the group membership of Iranian pharmaceutical stock companies that were achieved from using model (5).

Table 4: Classification

Obs	Group	Prediction
DSOB1	G_1	G_1
FTIR1	G_1	G_1
PDRO1	G_1	G_1
JAMD1	G_2	G_2
DRZK1	G_2	G_2
IRDR1	G_2	G_2
AMIN1	G_2	G_2
ROZD1	G_2	G_2
DSIN1	G_2	G_2
ABDI1	G_2	G_2
DSNZ1	G_2	G_2
DALZ1	G_2	G_2
DJBR1	G_2	G_2
KIMI1	G_3	G_3
KSPZ1	G_3	G_3
DTDZ1	G_3	G_3
DDPK1	G_3	G_3
DLGM1	G_3	G_3
EXIR1	G_3	G_3
BRKT1	G_3	G_3
DPAK1	G_4	G_4
DFRB1	G_4	G_4
DZAH1	G_4	G_4
DAML1	G_4	G_4

As you see in Table 4, all 24 pharmaceutical stock companies are classified as 100% correct. Model (5) is a simple and convenient model that can correctly predict group membership. By using model (5), we can predict the group membership of new pharmaceutical stock companies easily.

4.3 Data collection for interval data

The dataset was collected from the 24 Iranian pharmaceutical stock companies from 2020 to 2024. The dataset was obtained from http://www.fipiran.com. All the stock companies are shown by their company symbols. Table 5 presents $Z_{im} \epsilon [Z_{im}^L, Z_{im}^U]$ from Iranian pharmaceutical stock companies. In this research, we used 10 financial indices ($i1, \dots, i10$) to distribute the 24 Iranian pharmaceutical stock companies to 4 groups described in section 4.2.

Table 5: Financial Data of Iranian Pharmaceutical Stock Companies by Interval Data

Obs.	$i1$		$i2$		$i3$		$i4$		$i5$	
	Z_{1j}^L	Z_{1j}^U	Z_{2j}^L	Z_{2j}^U	Z_{3j}^L	Z_{3j}^U	Z_{4j}^L	Z_{4j}^U	Z_{5j}^L	Z_{5j}^U
DSOB1	0.111	0.112	0.006	0.006	0.382	0.436	0.47	0.492	0	0
FTIR1	0.062	0.067	0.02	0.024	0.089	0.105	0.115	0.124	0.106	0.157
PDRO1	0.183	0.222	0.152	0.163	0.115	0.139	0.215	0.365	0.555	0.58
JAMD1	0.015	0.017	0.007	0.008	0.02	0.023	0.024	0.029	0.017	0.018
DRZK1	0.281	0.306	0.244	0.274	0.164	0.166	0.237	0.267	0.217	0.232
IRDR1	0.07	0.083	0.057	0.065	0.045	0.046	0.029	0.038	0.051	0.054
AMIN1	0.116	0.125	0.09	0.114	0.145	0.153	0.084	0.098	0.099	0.1
ROZD1	0.045	0.06	0.065	0.072	0.076	0.087	0.004	0.016	0.028	0.031
DSIN1	0.139	0.163	0.078	0.158	0.154	0.176	0.25	0.259	0.206	0.21
ABDI1	0.246	0.265	0.207	0.215	0.182	0.242	0.181	0.233	0.365	0.432
DSNZ1	0.135	0.141	0.158	0.161	0.074	0.084	0.024	0.025	0.081	0.089
DALZ1	0.241	0.337	0.128	0.222	0.256	0.329	0.397	0.407	0.317	0.351
DJBR1	0.302	0.323	0.222	0.234	0.264	0.309	0.277	0.372	0.198	0.231
KIMI1	0.138	0.14	0.117	0.123	0.094	0.105	0.123	0.155	0.081	0.095
KSPZ1	0.15	0.17	0.142	0.15	0.079	0.092	0.119	0.139	0.153	0.154
DTDZ1	0.105	0.107	0.089	0.095	0.062	0.073	0.049	0.076	0.081	0.088
DDPK1	0.082	0.086	0.078	0.079	0.034	0.051	0.032	0.037	0.046	0.052
DLGM1	0.088	0.097	0.09	0.104	0.063	0.089	0.025	0.035	0.032	0.05
EXIR1	0.293	0.304	0.307	0.32	0.078	0.085	0.089	0.09	0.211	0.225
BRKT1	0.11	0.131	0.232	0.285	0.572	0.705	0.264	0.284	0	0
DPAK1	0.376	0.392	0.399	0.474	0.144	0.146	0.161	0.17	0.287	0.333
DFRB1	0.265	0.322	0.311	0.313	0.172	0.199	0.185	0.222	0.19	0.198
DZAH1	0.361	0.363	0.38	0.455	0.061	0.102	-0.189	0.169	0.17	0.204
DAML1	0.083	0.088	0.116	0.12	-0.039	0.008	-0.036	-0.017	0.047	0.052

Table 5: Continued

Obs.	$i6$		$i7$		$i8$		$i9$		$i10$	
	Z^L_{6j}	Z^U_{6j}	Z^L_{7j}	Z^U_{7j}	Z^L_{8j}	Z^U_{8j}	Z^L_{9j}	Z^U_{9j}	Z^L_{10j}	Z^U_{10j}
DSOB1	0.159	0.188	0.005	0.006	0.264	0.371	0.429	0.443	0.031	0.036
FTIR1	0.057	0.064	0.021	0.023	0.055	0.089	0.102	0.106	0.066	0.112
PDRO1	0.149	0.17	0.132	0.166	0.047	0.055	0.209	0.218	0.108	0.143
JAMD1	0.012	0.013	0.006	0.007	0.011	0.018	0.024	0.025	0.083	0.098
DRZK1	0.21	0.221	0.221	0.239	0.104	0.108	0.212	0.239	0.442	0.505
IRDR1	0.051	0.066	0.051	0.068	0.042	0.067	0.021	0.029	0.03	0.08
AMIN1	0.112	0.127	0.082	0.1	0.153	0.184	0.074	0.091	0.202	0.308
ROZD1	0.076	0.078	0.068	0.074	0.156	0.168	0.001	0.037	0.046	0.062
DSIN1	0.12	0.162	0.09	0.14	0.053	0.089	0.241	0.245	0.421	0.535
ABDI1	0.201	0.219	0.186	0.195	0.088	0.149	0.181	0.299	0.141	0.345
DSNZ1	0.121	0.125	0.136	0.146	0.1	0.1	0.027	0.033	0.075	0.12
DALZ1	0.175	0.257	0.111	0.19	0.119	0.207	0.367	0.403	0.101	0.164
DJBR1	0.238	0.251	0.193	0.204	0.1	0.168	0.406	0.414	0.064	0.08
KIMI1	0.109	0.114	0.103	0.109	0.053	0.063	0.123	0.132	0.019	0.03
KSPZ1	0.12	0.122	0.123	0.13	0.033	0.04	0.114	0.133	0.214	0.367
DTDZ1	0.084	0.099	0.082	0.116	0.061	0.082	0.031	0.061	0.023	0.033
DDPK1	0.06	0.065	0.069	0.07	0.022	0.057	0.036	0.037	0.026	0.046
DLGM1	0.1	0.111	0.104	0.115	0.059	0.1	0.049	0.058	0.035	0.073
EXIR1	0.208	0.215	0.263	0.275	0.04	0.1	0.068	0.082	0.106	0.148
BRKT1	0.553	0.626	0.494	0.529	0.756	0.895	0.231	0.261	0.029	0.059
DPAK1	0.327	0.33	0.363	0.417	0.111	0.132	0.128	0.152	0.138	0.373
DFRB1	0.218	0.254	0.273	0.275	0.079	0.089	0.259	0.268	0.331	0.38
DZAH1	0.262	0.274	0.327	0.405	0.053	0.063	-0.32	0.138	0.066	0.076
DAML1	0.072	0.074	0.113	0.116	0.025	0.028	-0.114	-0.081	0.009	0.017

4.4 Apply the proposed method on interval data

In this section, we apply our represented model (8), (9) to our interval data. Table 6 presents the $\lambda_i^*(i = 1,\dots,10)$ and $c_g^*(g = 1,2,3)$.

In this research, we used 10 financial indices to distribute the 24 Iranian pharmaceutical stock companies into 4 described groups.

Table 6: Weight Estimates and Discriminant Scores for Interval Data

c_1^*	0.01
c_2^*	0
c_3^*	-0.01
λ_1^*	0.1
λ_2^*	-0.11
λ_3^*	-0.08
λ_4^*	0.01
λ_5^*	0.04
λ_6^*	-0.29
λ_7^*	0.13
λ_8^*	0.19
λ_9^*	0.05
λ_{10}^*	0

Table 7 presents the group membership and prediction of the group membership of Iranian pharmaceutical stock companies.

Table 7: Classification of Interval Data

Obs.	Group	Prediction
DSOB1	G_1	G_1
FTIR1	G_1	G_1
PDRO1	G_1	G_1
JAMD1	G_2	G_2
DRZK1	G_2	G_2
IRDR1	G_2	G_2
AMIN1	G_2	G_2
ROZD1	G_2	G_2
DSIN1	G_2	G_2
ABDI1	G_2	G_2
DSNZ1	G_2	G_2
DALZ1	G_2	G_2
DJBR1	G_2	G_2
KIMI1	G_3	G_3
KSPZ1	G_3	G_3
DTDZ1	G_3	G_3
DDPK1	G_3	G_3
DLGM1	G_3	G_3
EXIR1	G_3	G_3
BRKT1	G_3	G_3
DPAK1	G_4	G_4
DFRB1	G_4	G_4
DZAH1	G_4	G_4
DAML1	G_4	G_4

As you see in Table 7, all 24 pharmaceutical stock companies are classified as 100% correct. Models (8) (9) are simple and convenient models that are used instead of solving model (7), which can correctly predict group membership. By using these models, we can predict the group membership of new pharmaceutical stock companies easily.

5 Conclusions

As we know, one of the important and useful subjects is hypothesizing the correct classification of a new sample by using available data. There are lots of models and methods represented in this field. But most of them are just useful for classifying observations into two groups. In this paper, we presented a simple and convenient model by using the DEA-DA method with GP that can classify observations into more than two groups, as many groups as we want. Also, it can be used for certain and interval data. We applied our purpose model on the Iranian pharmaceutical stock companies with certain and interval data. As shown in Table 4, our represented method predicted all the pharmaceutical stock companies' group membership 100% correctly for certain data. Also, our represented method predicted all of the pharmaceutical stock companies' group membership 100% correctly with interval data (Table 7).Future work can expand our framework to other alterations of the DEA methods. Besides, it can be expanded into an integrated numerical optimization using our framework.

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