Journal of Mechanical Research and Application (JMRA), Vol. 15 No.1, 1404 (2025),1-26

JMRA

Journal of Mechanical Research and Application

ISSN: 2251-7383, eISSN: 2251-7391

Multi-Objective Optimization of Strength and Thickness for Delaminated Composite Shells Undergoing Large-Amplitude Oscillations Using NSGA-II

Abbas Kamaloo*, Mohammad Abbaszadeh

3 km, Daneshgah Boulevard, Minab, Hormozgan, Iran, Minab National Skilling University

Abstract

This study presents the multi-objective optimization of delaminated multi-ply cylindrical and conical shells to simultaneously maximize strength and minimize thickness using the Non-dominated Sorting Genetic Algorithm II (NSGA-II). The shells, featuring a through-the-circumference delamination, are subjected to large-amplitude oscillations. The optimization aims to identify optimal strength-to-thickness ratios by treating fiber orientation angles, individual layer thicknesses, and layup configurations (cross-ply, angle-ply, and off-axis) as design variables. Results indicate that cross-ply layups achieve the highest performance in terms of strength-to-thickness ratio under nonlinear vibrations, followed by angle-ply and off-axis configurations, respectively. Furthermore, increasing the initial population size improves the optimization results for both shell types. However, while a higher number of iterations enhances outcomes for cylindrical shells, it adversely affects the optimization performance for conical shells.

Keywords: NSGA-II; throughout-circumference delamination; delaminated conical composite shells; delaminated cylindrical composite shells; nonlinear vibration; stacking sequence optimization

---

1. Introduction

Laminated composite materials are widely used in industries such as aerospace, marine, automotive, and civil engineering due to their excellent mechanical performance, including high stiffness-to-weight and strength-to-weight ratios, superior corrosion resistance, and design flexibility. Among these applications, cylindrical and conical composite shells serve as fundamental elements in thin-walled structures such as fuselage sections, rocket motor casings, submarine hulls, and pressure vessels. Their vibration characteristics, therefore, have a direct impact on structural integrity, service life, and safety.

Despite extensive research, predicting the nonlinear vibrational response of laminated shells remains a challenge. Discrepancies persist between theoretical formulations and experimental

* Corresponding Author. Tel: +989123491829

Email Address: st_a_kamaloo @azad.ac.ir

outcomes, partly due to complexities such as geometric nonlinearity, material anisotropy, and manufacturing defects. Nonlinear behavior may manifest as *hardening springs*, where the natural frequency increases with oscillation amplitude, or as *softening springs*, where it decreases. Correctly capturing these effects is essential for accurate design and failure prevention.

Delamination represents one of the most critical damage mechanisms in laminated composites. It can originate during manufacturing due to residual stresses, voids, or improper curing, and during service due to impact loads, fatigue, or environmental degradation. Because delamination disrupts load transfer between plies and reduces both stiffness and strength, failing to incorporate it into analytical and numerical models can lead to unsafe overestimations of performance. This issue is particularly important in thin-walled multi-ply shells, where even minor delamination can significantly alter dynamic behavior.

Optimization in engineering design aims to identify the most favorable configuration of parameters subject to performance, manufacturing, and operational constraints. It reflects the universal principle of attaining maximum efficiency with minimum resource expenditure. In the context of composite shells, optimization must balance factors such as stiffness, strength, weight, cost, and damage tolerance. Given the large number of design variables—fiber orientations, layer thicknesses, material selection, and stacking sequences—traditional trial-and-error approaches are inefficient and prone to suboptimal results.

Over time, a wide range of computational optimization techniques has been employed, from gradient-based methods suitable for smooth search spaces to metaheuristic algorithms such as genetic algorithms, particle swarm optimization, and simulated annealing, which are effective for nonlinear, multi-objective, and highly constrained problems. In particular, evolutionary multi-objective algorithms, such as Deb's Non-Dominated Sorting Genetic Algorithm II (NSGA-II), have gained prominence for their ability to identify a diverse set of Pareto-optimal solutions in a single run.

Research into the nonlinear vibrations of cylindrical and conical composite shells dates back several decades. Landmark contributions include Chu (1961) on self-excited vibrations, Chen and Babcock (1975) on high-amplitude oscillations, and Hirano (1989) on amplitude–frequency relationships in cylindrical and conical shells. Subsequent works employed classical shell theories—Donnell's, Flügge's, and higher-order shear deformation formulations—combined with numerical approaches like Galerkin's method and Incremental Harmonic Balance to model both free and forced nonlinear vibrations. More recent studies have addressed delamination propagation, the effects of ply layup, and dynamic instability under various loading conditions.

In parallel, optimization approaches have been applied to composite design problems. Ohta (2010) examined stacking sequence optimization, Sasidhar et al. (2013) targeted weight and deflection reduction, and Akbulut and Sonmez (2008) pursued thickness minimization. However, the combination of delamination effects, large-amplitude vibration modeling, and multi-objective optimization using NSGA-II—particularly for both cylindrical and conical shells—remains comparatively underexplored.

The present work addresses this gap. Using the NSGA-II algorithm, delaminated cylindrical and conical graphite/epoxy shells are optimized for two conflicting objectives: maximizing the nonlinear free vibration frequency (as a strength-related performance measure that accounts for delamination) and minimizing thickness (for material and weight efficiency). The study investigates three layup configurations—cross-ply, angle-ply, and off-axis—while varying fiber orientation angles and individual ply thicknesses as primary design variables. The findings not only advance understanding of how delamination alters vibration characteristics but also provide practical guidelines for designing lightweight, high-performance composite shells under severe dynamic conditions.

1.1 Literature Review

Over the past several decades, researchers have investigated the nonlinear vibration behavior of laminated composite shells from different perspectives, including analytical formulations, numerical methods, experimental validation, and optimization. Table 1 summarizes representative works, emphasizing shell type, vibration regime (linear/nonlinear), the presence of delamination, analytical or numerical approaches, and whether optimization techniques were employed.

This review highlights that while nonlinear vibrations of laminated shells have been addressed in the past, most studies have either neglected delamination or addressed it without multi objective optimization. Moreover, investigations that simultaneously consider large amplitude nonlinear effects, delamination throughout the circumference, and both cylindrical and conical shells—optimized via an evolutionary multi objective approach such as NSGA II—are noticeably scarce. This gap forms the motivation and novelty of the current research.

Author(s) & Year	Shell Type(s) Studied	Vibration Regime	Delamination Considered	Methodology / Theory Used	Optimization Applied	Key Findings / Limitations
Chu (1961)	Cylindrical	Nonlinear	No	Early analytical models of self-excited vibrations	No	Established foundational nonlinear vibration theory for thin shells; no composite focus
Chen & Babcock (1975)	Cylindrical, isotropic	Nonlinear	No	Experimental + theoretical high-amplitude oscillations	No	Showed amplitude–frequency and damping effects; did not address composites
Hirano (1989)	Cylindrical & conical, laminated	Nonlinear	No	Donnell shell theory, analytical solutions	No	Provided amplitude–frequency relationships; ignored damage effects
Abrate (1998)	Laminated plates & shells	Nonlinear	Yes	Analytical damage mechanics	No	Investigated delamination influence; limited to small-amplitude cases
Wang & Chia (2000)	Conical, laminated	Nonlinear	No	Higher-order shear deformation theory	No	Improved accuracy for thick shells; no damage consideration
Akbulut & Sonmez (2008)	Laminated cylindrical	Linear	No	FEM + Genetic Algorithm	Yes	Optimized stacking for thickness reduction; ignored vibration & delamination
Ohta (2010)	Cylindrical, laminated	Linear	No	FEM + NSGA-II	Yes	Multi-objective layup optimization; no damage or nonlinear effect
Sasidhar et al. (2013)	Laminated plates	Linear	No	FEM + GA	Yes	Multi-objective weight & deflection optimization; no shell or delamination case
Ahmed et al. (2016)	Cylindrical laminated shells	Nonlinear	Yes	FEM, delamination modeling	No	Explored delamination effects on free vibration; no optimization
Present study	Cylindrical & conical, laminated	Nonlinear	Yes	Donnell theory + Galerkin method + NSGA-II	Yes	Multi-objective optimization considering delamination and large-amplitude vibrations

Table 1 – Summary of representative studies on nonlinear vibration and optimization of laminated composite shells

Shell Geometry

Figures 1 and 2 illustrate the multi-ply conical and cylindrical shells considered in this study. Both have symmetric and balanced layups with throughout-circumference delamination located in the middle layers.

- **Cylindrical shell:** Length L, thickness h, mid-surface radius R, and density ρ
- **Conical shell:** Length L, thickness h, vertex radii Ro and R_2, mid-surface radius R_1, density ρ_0, and vertex half-angle α.

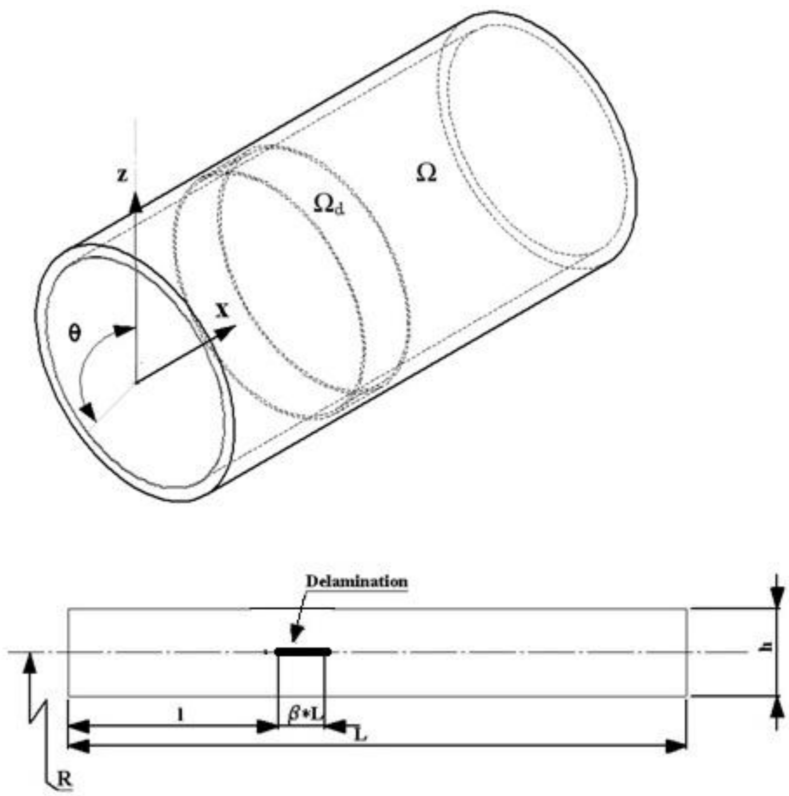

Fig. 1: Geometry of multi-ply cylindrical composite shell with throughout circumference delamination

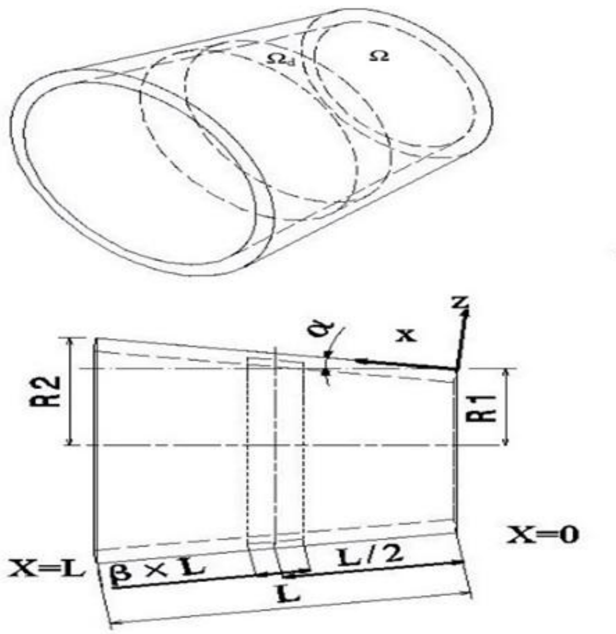

Fig. 2: Geometry of multi-ply conical composite shell with throughout circumference delamination

Delamination is modeled as throughout circumference delamination in two adjoining layers, length $\beta\cdot L$ ($0<\beta<1$).

Mathematical areas:

$$\Omega = \{(x, \theta)|0 \leq x \leq L\,, 0 \leq \theta \leq 2\pi\}$$

$$\Omega_d = \{(x, \theta)|l \leq x \leq l + \beta.L\,, 0 \leq \theta \leq 2\pi\}$$

In this study, shells are divided into three areas, namely the whole cone, the upper area, and the lower area of delamination which are respectively indicated by superscripts (0) and k(k=1,2); here, the upper area of delamination is indicated by a superscript (1) and the lower area is shown by (2).

Applying the above said definitions and Hamilton's principle for shell geometry as in the references (Kamaloo et al., 2019a), Kamaloo et al. computed the governing equations of motion. In order to simplify the solutions, they converted the order of governing equations from nonlinear partial differential equations to ordinary differential equations. By using the structural relation and the kinematic among resultants and stress couplings with strain, curvature, and in turn, the displacement equations, they calculated the governing equations in the form of second-order nonlinear ordinary differential equations in terms of displacement equations.

Equations of motion are derived using Hamilton’s principle and reduced to nonlinear second-order ODEs.

Boundary conditions and numerical solution:

Firstly, the fixed-bearing boundary conditions are used at both ends of the shell as follows:

$$w^0 = 0, \qquad (w^0)_{,xx} = 0\,, \qquad N_x = 0\,, \qquad v^0 = 0$$

Continuity at delamination boundaries:

$$u^{(k)}|_{x=l,l+\beta.L} = 0\,, v^{(k)}|_{x=l,l+\beta.L} = 0\,, w^{(k)}|_{x=l,l+\beta.L} = 0\,, (w^{(k)})_{,x}|_{x=l,l+\beta.L} = 0$$

Displacement expansions are defined for axial wave number m and circumferential wave number n. The system reduces to coupled nonlinear ODEs (Eq. 7–8), solved by 4th-order Runge-Kutta with initial conditions.

For axial half-wave number m and circumferential wave number n, the displacements are expanded as

$$U^0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} u_{mn}^0(t)\cos\frac{m\pi x}{L}\cos n\theta \qquad (1)$$

$$V^0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} v_{mn}^0(t)\sin\frac{m\pi x}{L}\sin n\theta \qquad (2)$$

$$W^0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn}^0(t)\sin\frac{m\pi x}{L}\cos n\theta \qquad (3)$$

$$U^k = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} u_{mn}^k(t)\sin\frac{m\pi(x-l)}{\beta.L}\cos n\theta \qquad (4)$$

$$V^k = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} v_{mn}^k(t)\sin\frac{m\pi(x-l)}{\beta.L}\sin n\theta \qquad (5)$$

$$W^k = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn}^k(t)\cos\frac{m\pi(x-l)}{\beta.L}\cos n\theta \qquad (6)$$

Following Kamaloo et al. (2019a), $U^i(t)$ and $V^i(t)$ in terms of $W^i(t)$ (i=0,k) are expressed in terms of $W^i(t)$,yielding a pair of coupled nonlinear ODEs (constants ai fixed):

$$(\rho_0^0+\rho_0^k)\frac{d^2w^0(t)}{dt^2}+\rho_0^k\frac{d^2w^k(t)}{dt^2} = a1w^0(t)+a2(w^0(t))^3+a3w^0(t)w^k(t)+a4w^0(t)(w^k(t))^2 \qquad (7)$$

$$\rho_0^k\left(\frac{d^2w^0(t)}{dt^2}+2\frac{d^2w^k(t)}{dt^2}\right) = a5(t)+a6\left(w^k(t)\right)^2+a7\left(w^k(t)\right)^3 \qquad (8)$$

The system is decoupled and integrated with a **fourth-order Runge–Kutta** scheme. Initial, dimensionless amplitudes:

Table 1: Initial conditions

$W^0t(0)/h$	2.5
$W^kt(0)/h$	2.5

Multi-objective optimization

A multi-objective optimization problem (MOP) seeks to optimize multiple, possibly conflicting objectives. The solution set is characterized by the Pareto front: solutions for which improving one objective necessarily degrades at least one other.

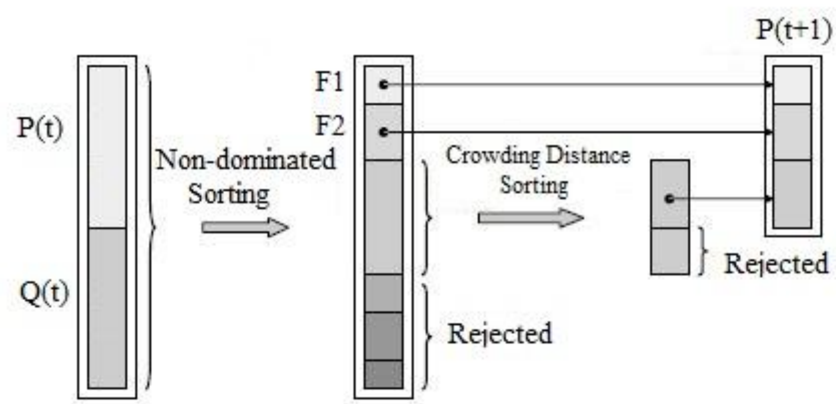

Fig. 3: Stages of the Multi-objective Non-dominated Sorting Genetic Algorithm

We optimize two conflicting objectives: minimize total thickness (f1) and maximize nonlinear free-vibration frequency (f2). NSGA-II uses binary tournament selection by rank then crowding distance. For a front, the crowding distance of member i is:

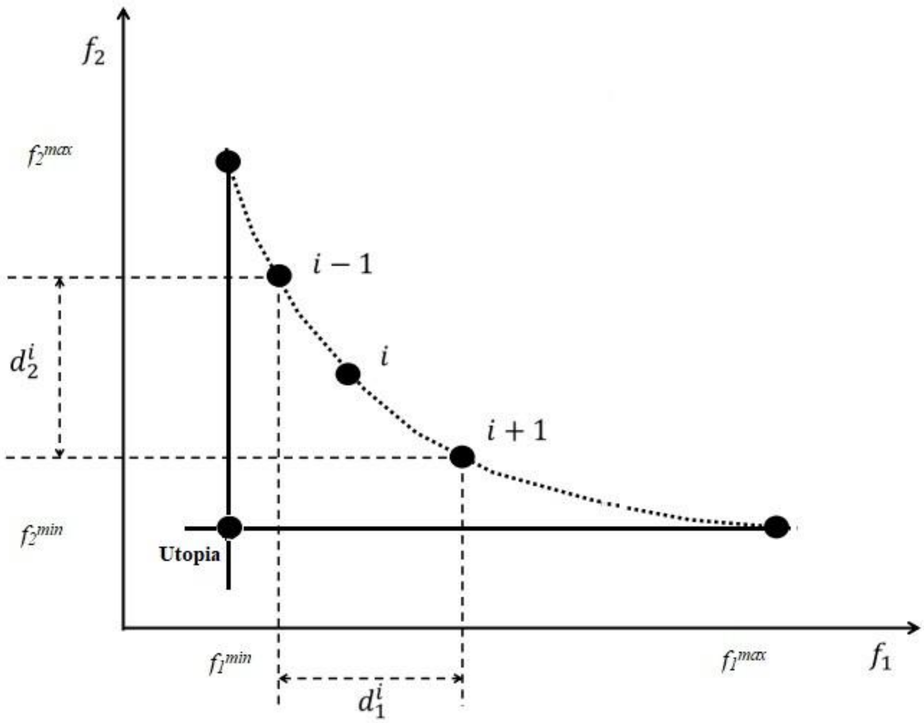

Fig. 5: Definition of crowding distance

The crowding distance is indicated by d_i and defined as below:

$$d_i^1 = \frac{\left|f_1^{i+1} - f_1^{i-1}\right|}{f_1^{max} - f_1^{min}}$$

$$d_i^2 = \frac{\left|f_2^{i+1} - f_2^{i-1}\right|}{f_2^{max} - f_2^{min}}$$

$$d_i = d_i^1 + d_i^2$$

Results and discussion:

As mentioned in the introduction, one of the main[1]capabilities of NSGA-II is the multi-objective optimization of two or more conflicting objective functions. Accordingly, this section deals with the optimization of thickness and frequency of nonlinear free vibrations in conical and cylindrical shells with throughout circumference delamination employing multi-objective genetic algorithm in order to obtain a shell with minimum thickness and maximum nonlinear free vibration frequency. Defining an optimization problem depends on considering design constraints, design variables, and objective functions. Considering the type of layups, this research assumes the following constraints and variables:

[1] Modified Non-Dominated Sorting Genetic Algorithms

Design variables

- Ply thicknesses.
- Fiber orientation angles (off-axis plies).

Constraints

- Fiber angles: Range of fiber orientation: (At 5-degree intervals) [-90:5:90]
- Range of layers thickness (Inch): [0.001:0.0001:0.009]

Objectives

- f1: minimize total shell thickness (inch).
- f2: maximize nonlinear free-vibration frequency (Hz).

Model Conditions

Table 1. Cylindrical shell

l	0.3
m	1
n	2
R	2in
L	2in

Table 2. Conical shell

β	0.2
m	1
n	2
R_0	5in
L	5in
α	10^0

To validate the formulation, results are compared with Kamaloo et al. (2019a) for 10-ply graphite/epoxy cylindrical and conical shells (cross- and angle-ply). As shown in Figures 6 and 7, **cross-ply** yields higher nonlinear free-vibration frequencies than **angle-ply** for both shell types.

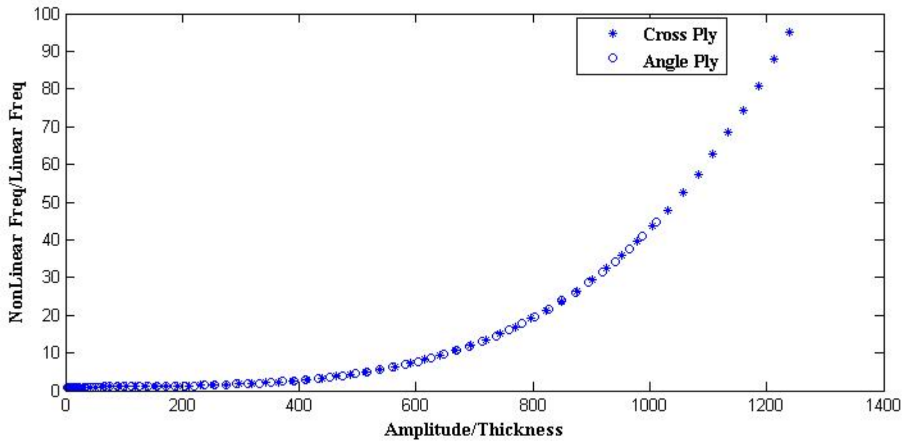

Fig. 6: Effect of variation in the orthotropic properties of delaminated conical shells on their nonlinear behaviors

(β=0.2, l=0.3in, R=5in, $\alpha=10^0$, L=2in, m=1,n=2, h=0.025in)

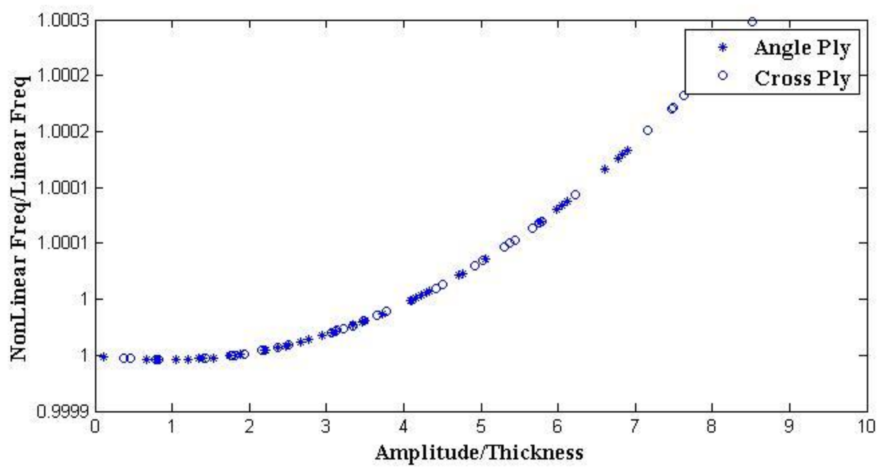

Fig. 7: Effect of variation in the orthotropic properties of delaminated conical shells on their oscillating motion.

(β = 0.2, l = 0.3 in, R= 2in, L = 2 in, m = 1, n=2, h = 0.025 in).

As could be understood from Figures 6 and 7, the nonlinear free vibration frequency, in both delaminated conical and cylindrical shells, is higher in cross-ply layup compared to angle-ply layup under identical conditions.

Cylindrical Shells (10 plies)

Angle-ply**:** population = 5, iterations = 15.

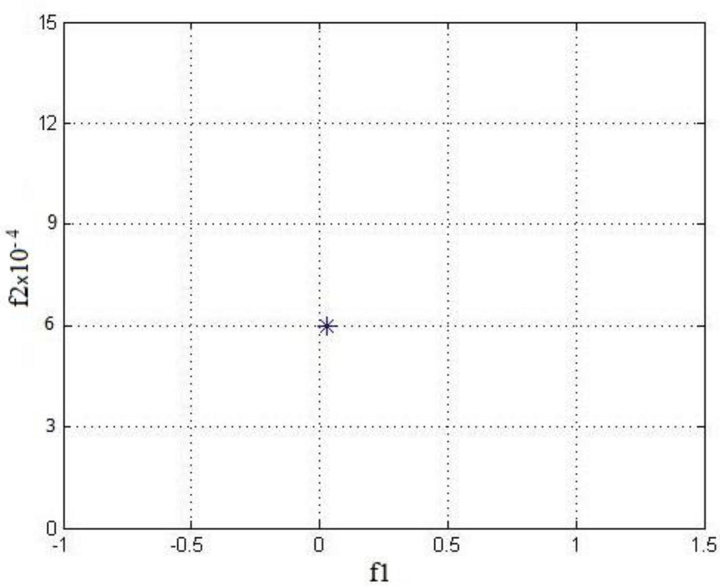

Figure 8. First Pareto front (one member).

Best solution:

t = $[0.0062 \quad 0.0021 \quad 0.0025 \quad 0.0036 \quad 0.0046]_s$

$\phi = [45 \quad -45 \quad 45 \quad -45 \quad 45]_s$

Nonlinear Frequency =

6.0014 x 10^{-4} HZ

Thickness =

0.038 in

Cross-ply: population = 5, iterations = 15.

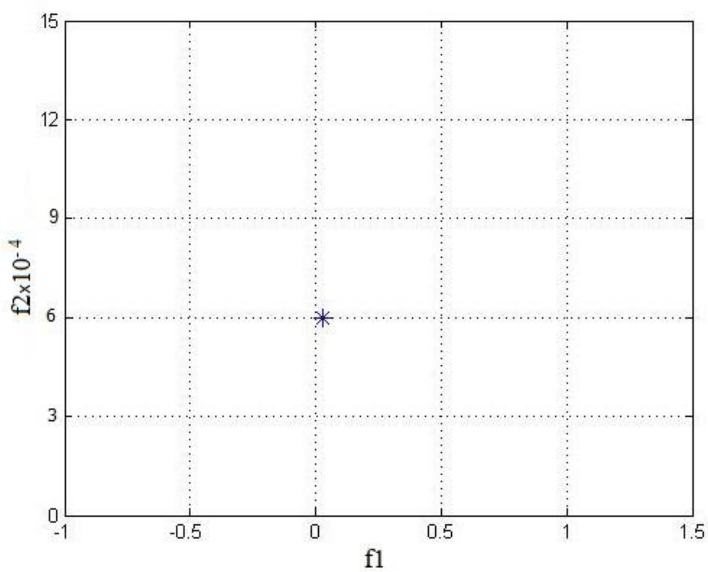

Figure **9.** First Pareto front (one member).

Best solution:

t = [0.0067 0.0017 0.0013 0.0012 0.0032]$_s$

ϕ = [0 90 0 90 0]$_s$

Nonlinear Frequency =

6.0015 x 10^{-4} HZ

Thickness =

0.0282 in

Observation: Cross-ply achieves a higher frequency at lower thickness than angle-ply.

Conical Shells (10 plies)

Angle-ply: population = 5, iterations = 15.

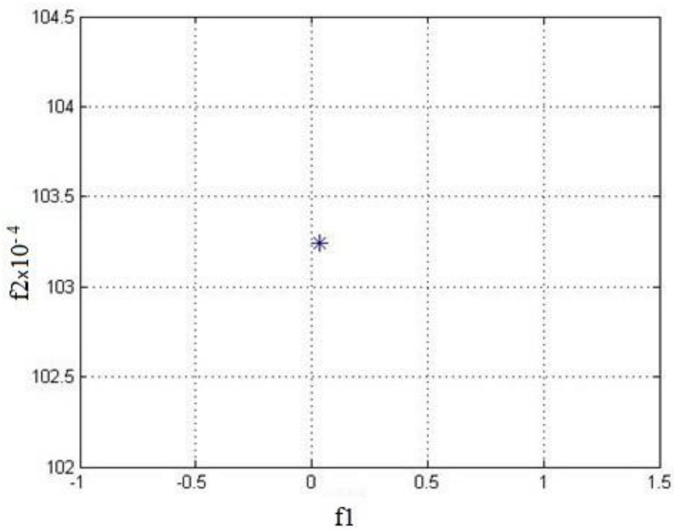

Figure 10. First Pareto front (one member).

Best solution:

t = [0.0013 0.0033 0.0080 0.0042 0.0012]$_s$

ϕ = [45 -45 45 -45 45]$_s$

Nonlinear Frequency =

103.2389 x 10^{-4} HZ

Thickness =

0.036 in

Cross-ply: population = 5, iterations = 15.

As shown in Figure 11, there is one member in the first front of Pareto frontiers in this optimization

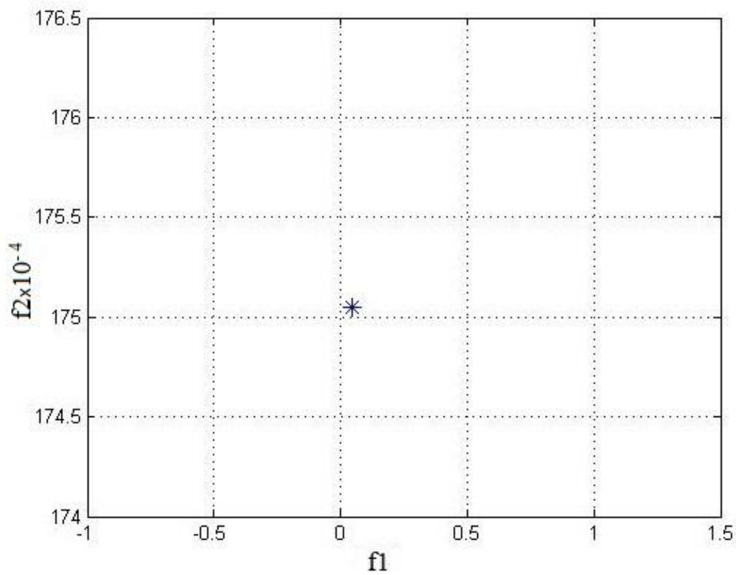

Figure 11. First Pareto front (one member).

Best solution:

t = [0.0083 0.0037 0.0040 0.0052 0.0019]$_s$

ϕ = [0 90 0 90 0]$_s$

Nonlinear Frequency =

175.0511 x 10^{-4} HZ

Thickness = 0.023 in

Observation: As with cylinders, cross-ply outperforms angle-ply for conical shells.

Off-Axis Studies

Cylindrical Shells

10-ply: population = 5, iterations = 10.

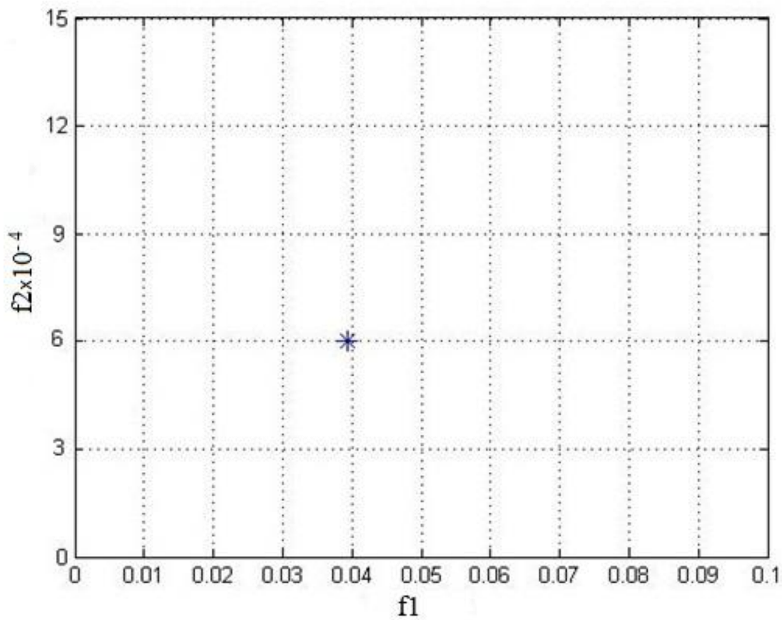

Figure 12. First Pareto front (one members)

Number of populations: 5

Number of iterations: 10

$t = [0.0044 \quad 0.0049 \quad 0.0019 \quad 0.0028 \quad 0.0057]_s$

$\phi = [-30 \quad -45 \quad 90 \quad 60 \quad 45]_s$

Nonlinear Frequency =

6.0013 x 10^{-4} HZ

Thickness =

0.0394 in

20-ply: population = 5, iterations = 15.

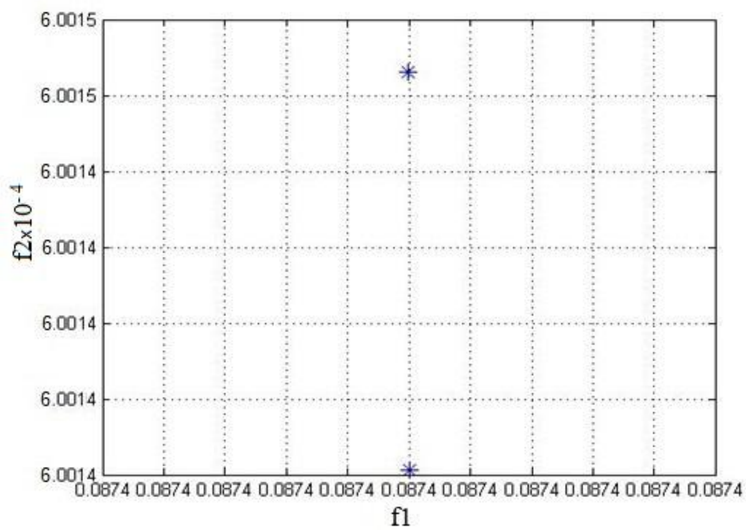

Figure 13. First Pareto front (two members). Higher-frequency member selected:

t = [0.0059 0.0037 0.0012 0.0024 0.0048 0.0073 0.0068 0.0069 0.0025 0.0022]$_s$

ϕ = [45 -45 -60 -50 10 35 40 -15 -5 -90]$_s$

Nonlinear Frequency =

6.0015 x 10^{-4} HZ

Thickness =

0.0874 in

30-ply: population = 5, iterations = 15.

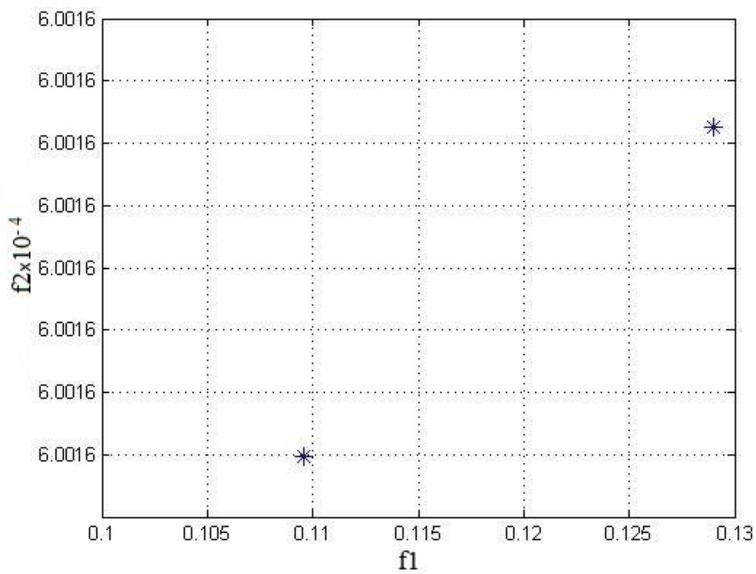

Figure 14. First Pareto front (two members); best by frequency:

Number of populations: 5

Number of iterations: 15

t = [0.0040 0.0017 0.0064 0.0068 0.0045 0.0031 0.0022 0.0014 0.0027 0.0058 0.0038 0.0051 0.0011 0.0034 0.0028]$_s$

ϕ = [15 -50 35 -45 -25 -10 25 -80 50 75 70 20 80 -20 -55]$_s$

Nonlinear Frequency =

6.0016 x 10^{-4} HZ

Thickness =

0.011 in

Conical Shells

10-ply: population = 5, iterations = 15.

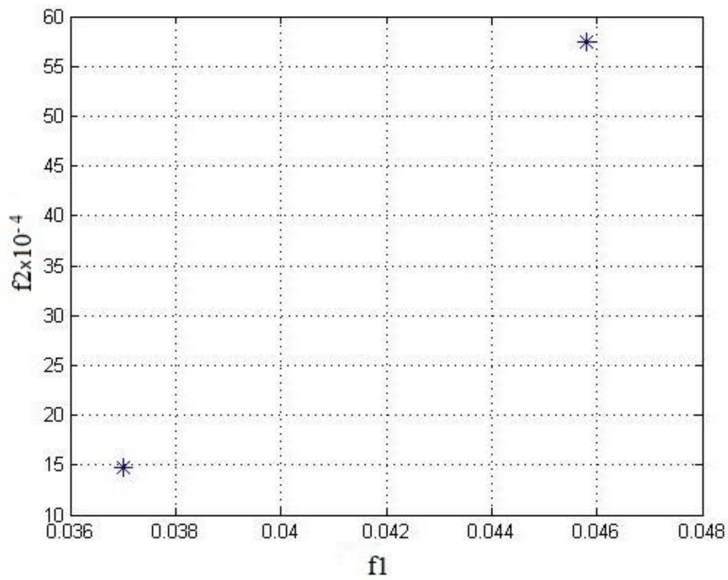

Figure 15. First Pareto front (two members); best by frequency

t = [0.0023 0.0071 0.0010 0.0053 0.0072]$_s$

ϕ = [-45 5 70 50 75]$_s$

Nonlinear Frequency =

57.3730 x 10^{-4} HZ

Thickness =

0.0458 in

20-ply: population = 5, iterations = 15.

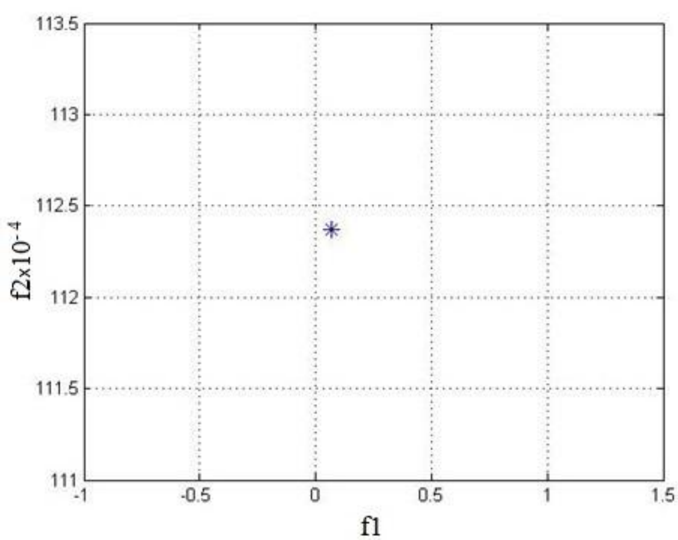

Figure 16. First Pareto front (one member).

t = [0.0082 0.0015 0.0029 0.0014 0.0058 0.0020 0.0045 0.0027 0.0011 0.0034]$_s$

ϕ = [-15 -85 -10 45 -60 -65 70 -40 75 25]$_s$

Nonlinear Frequency =

112.3670 x 10^{-4} HZ

Thickness =

0.0670 in

Summary of layup effects: For both shell types, cross-ply consistently yields higher nonlinear free-vibration frequencies than angle-ply and off-axis for comparable thicknesses.

30-ply: population = 5, iterations = 15.

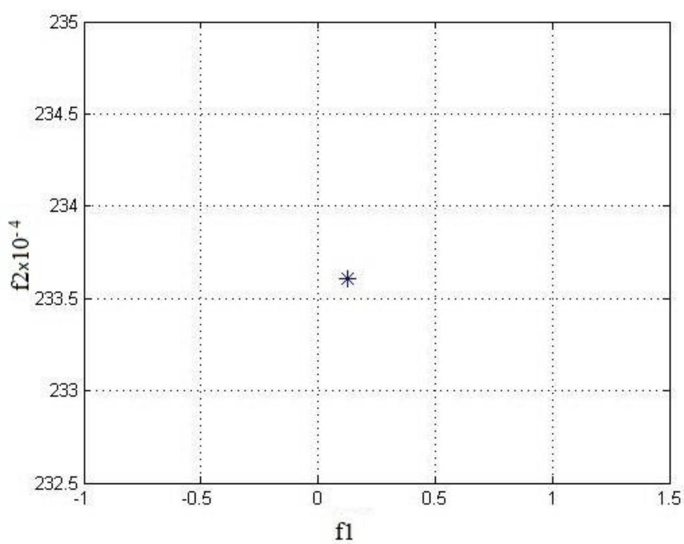

Figure 17. First Pareto front (one member).

t = [0.0028 0.0041 0.0087 0.0020 0.0035 0.0061 0.0012 0.0055 0.0047 0.0044 0.0034 0.0037 0.0019 0.0039 0.0086]$_s$

ϕ = [60 -80 -55 -35 15 40 85 -30 50 -40 55 70 65 -5 90]$_s$

Nonlinear Frequency =

233.6061 x 10^{-4} HZ

Thickness =

0.0129 in

Effect of Population and Iterations (NSGA-II Settings)

Conical Shells (cross-ply, 10-ply)

Population = 10, Iterations = 40:

t = $[0.0069 \quad 0.0015 \quad 0.0045 \quad 0.0033 \quad 0.0016]_s$

$\phi = [0 \quad 90 \quad 0 \quad 90 \quad 0]_s$

Nonlinear Frequency =

510.4673 x 10^{-4} HZ

Thickness =

0.0356 in

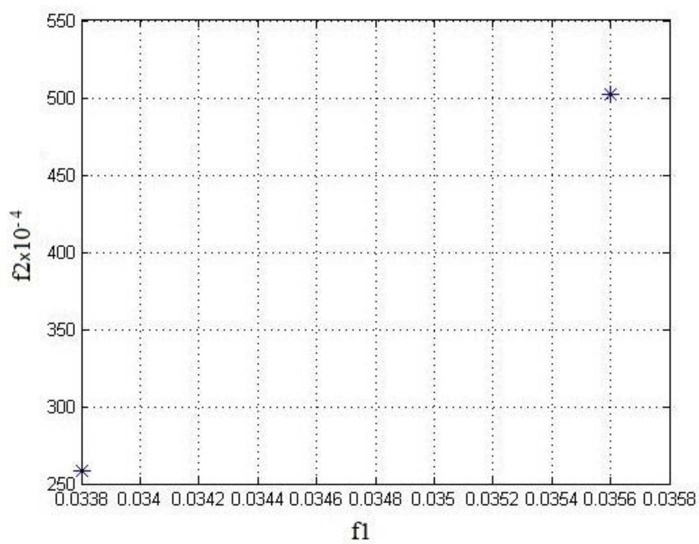

Figure 18. First Pareto front.

Population = 10, Iterations = 60 (geometry/delamination fixed):

t = $[0.0061 \quad 0.0018 \quad 0.0057 \quad 0.0043 \quad 0.0020]_s$

$\phi = [0 \quad 90 \quad 0 \quad 90 \quad 0]_s$

Nonlinear Frequency =

238.95 x 10^{-4} HZ

Thickness =

0.0398 in

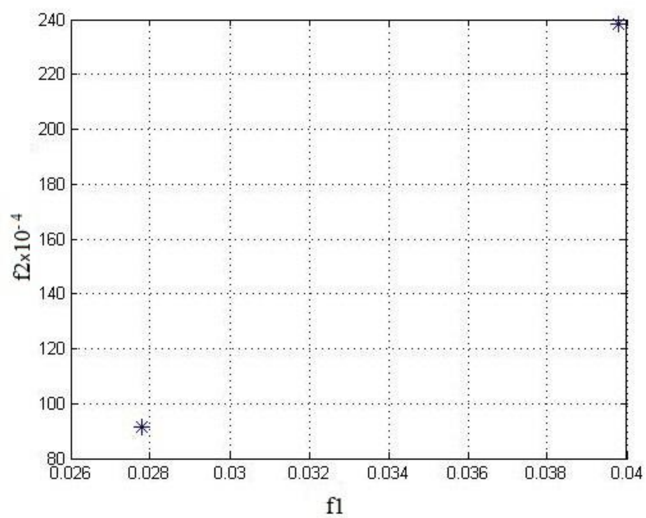

Figure 19. First Pareto front (two members).

Observation: More iterations worsened conical-shell results under these settings.

Cylindrical Shells (cross-ply, 10-ply)

Population = 10, Iterations = 40:

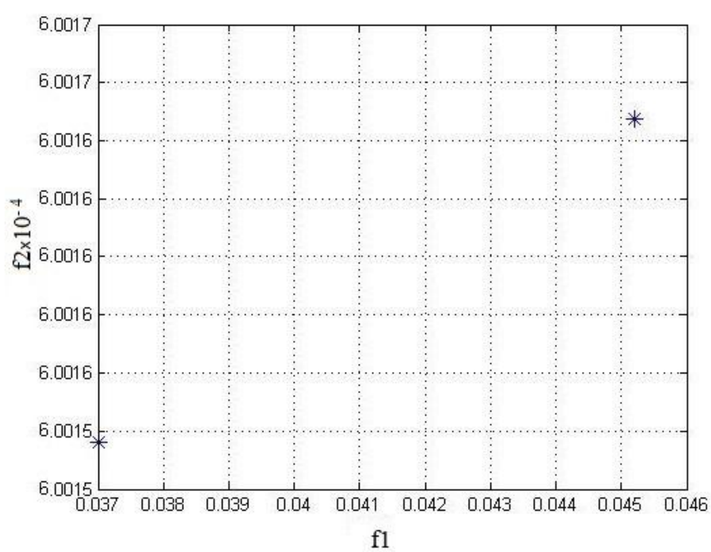

Figure 20. First Pareto front (two members).

Best member:

t = $[0.0056 \quad 0.0036 \quad 0.0050 \quad 0.0024 \quad 0.0060]_s$

$\phi = [0 \quad 90 \quad 0 \quad 90 \quad 0]_s$
Nonlinear Frequency =

6.0016×10^{-4} HZ

Thickness =

0.0452 in

Observation: More iterations **improved** cylindrical-shell results (higher frequency, lower thickness).

Overall: Increasing **initial population** helps both shell types; increasing **iterations** helps **cylindrical** but **hurts conical** shells under the tested conditions

Conclusion

This study successfully applied the NSGA-II multi-objective optimization algorithm to design delaminated cylindrical and conical composite shells with maximized strength and minimized thickness. The key findings are summarized as follows:

1. Cross-ply layups were found to be superior to angle-ply and off-axis configurations, consistently providing higher nonlinear free vibration frequencies for a given thickness.
2. The NSGA-II algorithm effectively identified a set of Pareto-optimal solutions, providing designers with a range of high-performance designs that balance the conflicting objectives of strength and weight.
3. The study of algorithmic parameters revealed that while a larger initial population is universally beneficial, the optimal number of iterations is geometry-dependent. Cylindrical shells benefit from more iterations, whereas conical shells show degraded performance, a critical consideration for future optimization studies.

The presented framework provides valuable insights for the design of damage-tolerant composite structures subjected to demanding dynamic environments.

References:

Akbari, M., Kiani, Y., Aghdam, M., & Eslami, M. (2014). Free vibration of FGM Lévy conical panels. Composite Structures, 116, 732-746.

Akbulut, M., & Sonmez, F. O. (2008). Optimum design of composite laminates for minimum thickness. Computers & Structures, 86(21-22), 1974-1982.

Almeida Jr, J. H. S., Ribeiro, M. L., Tita, V., & Amico, S. C. (2017). Stacking sequence optimization in composite tubes under internal pressure based on genetic algorithm accounting for progressive damage. Composite Structures, 178, 20-26.

Amabili, M. (2012). Nonlinear vibrations of angle-ply laminated circular cylindrical shells: Skewed modes. Composite Structures, 94(12), 3697-3709.

Bagheri, H., Kiani, Y., & Eslami, M. (2017a). Free vibration of conical shells with intermediate ring support. Aerospace Science and Technology, 69, 321-332.

Bagheri, H., Kiani, Y., & Eslami, M. (2017b). Free vibration of joined conical-conical shells. Thin-Walled Structures, 120, 446-457.

Chen, J. C., & Babcock, C. D. (1975). Nonlinear vibration of cylindrical shells. AiAA Journal, 13(7), 868-876.

Chu, H.-N. (1961). Influence of large amplitudes on flexural vibrations of a thin circular cylindrical shell. Journal of the Aerospace Sciences, 28(8), 602-609.

Civalek, Ö. (2013). Vibration analysis of laminated composite conical shells by the method of discrete singular convolution based on the shear deformation theory. Composites Part B: Engineering, 45(1), 1001-1009.

Deb, K. (2001). Multi-objective optimization using evolutionary algorithms (Vol. 16): John Wiley & Sons.

Dey, T., & Ramachandra, L. (2017). Non-linear vibration analysis of laminated composite circular cylindrical shells. Composite Structures, 163, 89-100.

Do, Q. C., Vu, T. T. A., & Nguyen, D. D. (2018). Vibration and nonlinear dynamic response of eccentrically stiffened functionally graded composite truncated conical shells surrounded by an elastic medium in thermal environments. Acta Mechanica.

Duc, N. D. (2013). Nonlinear dynamic response of imperfect eccentrically stiffened FGM double curved shallow shells on elastic foundation. Composite Structures, 99, 88-96.

Duc, N. D., & Quan, T. Q. (2013). Nonlinear postbuckling of imperfect eccentrically stiffened P-FGM double curved thin shallow shells on elastic foundations in thermal environments. Composite Structures, 106, 590-600.

Duc, N. D., & Thang, P. T. (2014a). Nonlinear buckling of imperfect eccentrically stiffened metal–ceramic–metal S-FGM thin circular cylindrical shells with temperature-dependent properties in thermal environments. International Journal of Mechanical Sciences, 81, 17-25.

Duc, N. D., & Thang, P. T. (2014b). Nonlinear response of imperfect eccentrically stiffened ceramic–metal–ceramic FGM thin circular cylindrical shells surrounded on elastic foundations and subjected to axial compression. Composite Structures, 110, 200-206.

Ertas, A. H., & Sonmez, F. O. (2010). Design of fiber reinforced laminates for maximum fatigue life. Procedia Engineering, 2(1), 251-256.

Friswell, M. I. (2006). Damage identification using inverse methods. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 365(1851), 393-410.

Fritzen, C. P. (2005). Vibration-based structural health monitoring–concepts and applications. Paper presented at the Key Engineering Materials.

Fu, Y.-M. (2013). Nonlinear analyses of laminated plates and shells with damage: WIT Press.

Fu, Y., & Chen, C. (2001). Non-linear vibration of elastic truncated conical moderately thick shells in large overall motion. International Journal of Non-Linear Mechanics, 36(5), 763-771.

Hammami, M., El Mahi, A., Karra, C., & Haddar, M. (2016). Nonlinear behaviour of glass fibre reinforced composites with delamination. Composites Part B: Engineering, 92, 350-359.

Hirano, Y. (1989). Nonlinear vibrations of composite material shells.

Hu, H.-T., & Ou, S.-C. (2001). Maximization of the fundamental frequencies of laminated truncated conical shells with respect to fiber orientations. Composite Structures, 52(3-4), 265-275.

Ikeya, K., Shimoda, M., & Shi, J.-X. (2016). Multi-objective free-form optimization for shape and thickness of shell structures with composite materials. Composite Structures, 135, 262-275.

Irie, T., Yamada, G., & Muramoto, Y. (1984). Free vibration of joined conical-cylindrical shells. Journal of Sound and Vibration, 95(1), 31-39.

Jin, G., Ma, X., Shi, S., Ye, T., & Liu, Z. (2014). A modified Fourier series solution for vibration analysis of truncated conical shells with general boundary conditions. Applied Acoustics, 85, 82-96.

Kachanov, L. (1976). Separation failure of composite materials. Mechanics of Composite Materials, 12(5), 812-815.

Kamaloo, A., Jabbari, M., Tooski, M. Y., & Javadi, M. (2019a). Nonlinear Free Vibrations Analysis of Delaminated Composite Conical Shells. International Journal of Structural Stability and Dynamics, 2050010.

Kamaloo, A., Jabbari, M., Tooski, M. Y., & Javadi, M. (2019b). Optimization of thickness and delamination growth in composite laminates under multi-axial fatigue loading using NSGA-II. Composites Part B: Engineering, 106936.

Kessler, S. S., Spearing, S. M., Atalla, M. J., Cesnik, C. E., & Soutis, C. (2002). Damage detection in composite materials using frequency response methods. Composites Part B: Engineering, 33(1), 87-95.

Khan, N., Goldberg, D. E., & Pelikan, M. (2002). Multi-objective Bayesian optimization algorithm. IlliGAL Report(2002009), 114-125.

Koide, R. M., & Luersen, M. A. (2013). Maximization of fundamental frequency of laminated composite cylindrical shells by ant colony algorithm. Journal of Aerospace Technology and Management, 5(1), 75-82.

Lam, K., & Hua, L. (1999). On free vibration of a rotating truncated circular orthotropic conical shell. Composites Part B: Engineering, 30(2), 135-144.

Najafov, A., Sofiyev, A., & Kuruoglu, N. (2014). On the solution of nonlinear vibration of truncated conical shells covered by functionally graded coatings. Acta Mechanica, 225(2), 563-580.

Nanda, N., & Sahu, S. (2012). Free vibration analysis of delaminated composite shells using different shell theories. International Journal of Pressure Vessels and Piping, 98, 111-118.

Ohta, Y. (2010). Optimal Stacking Sequence for Nonlinear Vibration of Laminated Composite Circular Cylindrical Shells. Paper presented at the 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference 18th AIAA/ASME/AHS Adaptive Structures Conference 12th.

Qu, Y., Chen, Y., Long, X., Hua, H., & Meng, G. (2013a). A modified variational approach for vibration analysis of ring-stiffened conical–cylindrical shell combinations. European Journal of Mechanics-A/Solids, 37, 200-215.

Qu, Y., Chen, Y., Long, X., Hua, H., & Meng, G. (2013b). A variational method for free vibration analysis of joined cylindrical-conical shells. Journal of vibration and control, 19(16), 2319-2334.

Sasidhar, G., Dayan, M. G., & Dora, M. H. (2013). Multi-objective optimization of laminated composite plate using a non-dominated sorting genetic algorithm. International Journal of Engineering Science and Technology, 5(4), 844.

Shi, J.-X., & Shimoda, M. (2015). Interface shape optimization of designing functionally graded sandwich structures. Composite Structures, 125, 88-95.

Shimoda, M., & Liu, Y. (2014). A non-parametric free-form optimization method for shell structures. Structural and Multidisciplinary Optimization, 50(3), 409-423.

Sofiyev, A. (2012). The non-linear vibration of FGM truncated conical shells. Composite Structures, 94(7), 2237-2245.

Sofiyev, A., Kuruoglu, N., & Halilov, H. (2010). The vibration and stability of non-homogeneous orthotropic conical shells with clamped edges subjected to uniform external pressures. Applied Mathematical Modelling, 34(7), 1807-1822.

Tong, L. (1993a). Free vibration of composite laminated conical shells. International Journal of Mechanical Sciences, 35(1), 47-61.

Tong, L. (1993b). Free vibration of orthotropic conical shells. International Journal of Engineering Science, 31(5), 719-733.

Tornabene, F., Fantuzzi, N., Bacciocchi, M., & Reddy, J. (2017). A posteriori stress and strain recovery procedure for the static analysis of laminated shells resting on nonlinear elastic foundation. Composites Part B: Engineering, 126, 162-191.

Tornabene, F., Fantuzzi, N., Bacciocchi, M., Viola, E., & Reddy, J. (2017). A numerical investigation on the natural frequencies of FGM sandwich shells with variable thickness by the local generalized differential quadrature method. Applied sciences, 7(2), 131.

Tripathi, V., Singh, B., & Shukla, K. (2007). Free vibration of laminated composite conical shells with random material properties. Composite Structures, 81(1), 96-104.

Van Dung, D. (2013a). Nonlinear buckling and post-buckling analysis of eccentrically stiffened functionally graded circular cylindrical shells under external pressure. Thin-Walled Structures, 63, 117-124.

Van Dung, D. (2013b). Research on nonlinear torsional buckling and post-buckling of eccentrically stiffened functionally graded thin circular cylindrical shells. Composites Part B: Engineering, 51, 300-309.

Wei, R., Pan, G., Jiang, J., Shen, K., & Lyu, D. (2019). An efficient approach for stacking sequence optimization of symmetrical laminated composite cylindrical shells based on a genetic algorithm. Thin-Walled Structures, 142, 160-170.

Wilkins Jr, D., Bert, C., & Egle, D. (1970). Free vibrations of orthotropic sandwich conical shells with various boundary conditions. Journal of Sound and Vibration, 13(2), 211-228.

Xu, C., Xia, Z., & Chia, C. (1996). Non-linear theory and vibration analysis of laminated truncated, thick, conical shells. International Journal of Non-Linear Mechanics, 31(2), 139-154.

Yang, J., & Fu, Y. (2006). Delamination growth of laminated composite cylindrical shells. Theoretical and applied fracture mechanics, 45(3), 192-203.

Yang, J., Fu, Y., & Wang, X. (2007). Variational analysis of delamination growth for composite laminated cylindrical shells under circumferential concentrated load. Composites science and technology, 67(3-4), 541-550.

Yang, Z., Chen, X., Yu, J., Liu, R., Liu, Z., & He, Z. (2013). A damage identification approach for plate structures based on frequency measurements. Nondestructive Testing and Evaluation, 28(4), 321-341.

Zou, Y., Tong, L., & Steven, G. P. (2000). Vibration-based model-dependent damage (delamination) identification and health monitoring for composite structures—a review. Journal of Sound and Vibration, 230(2), 357-378.