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Research Article

Designing a Robust Approach to Resolve the Tehran Metro Trains Scheduling Problem in Uncertain Conditions

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ABSTRACT

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Abstract:
Reliability and punctuality in the traffic supervision of the metro network are the key performance indicators for the evaluation of this industry's efficiency which can lead to the participants' satisfaction. A common technique to improve the level of the Metro traffic control system's reliability and performance increasing the rate of train receives and dispatches from special position determined, particularly in the different periods of designing train timetables, which can be out-of-the-way or lead to a stable level. Therefore, to increase the permanency level of the metro timetables, the traffic control center's operation in the metro network considers fixed time values as Buffer times between rail events with a high probability of flaws using normal procedures in the timetables. In this respect, the first delay in the metro network is the possibility of its spread in all of the lines and its effect that will be realized on all of the entities and the being of subsequent luckless consequences that will be prevented. Adding to this, Buffer time in the train schedule in some cases may cause the capacity of the metro network to be reduced, which could be the basis of traffic special effects for eventful lines such as Tehran Metro's line one. However, the ability of time tables to add fixed times values as a Buffer time between two conflicting events in the metro network can be whispered subsequently in this case, it is necessary to allocate Buffer times with sufficient accuracy and according to the traffic priorities of dispatching and receiving trains. Some important measures in Metro passenger activities (such as unpredicted train dispatches) take place properly and prevent certain incidents. In current study, the objective of cultivating and improving the stability of train running schedule tables was accomplished by assigning optimum Buffer in the network which, times has been studied utilizing innovative methods. So, the resources for allocating optimum time have been modeled via combinatorial optimization algorithms and particle swarm optimization methods. In this algorithm, each Buffer time according to the priority of the events as well technical, economic, and operational criteria is shown to one of the prevailing assignments of the schedule tables which results and its effects in terms of time units (seconds). Ultimately, the validity of the proposed model that is presented applying the Tehran Metro line one is verified.

Keywords:
Particle Swarm Method,Combinations Optimization, Robustness Approach, Scheduled Timetable and Tehran Metro

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1. Introduction

Metro is one of the urban transportation systems modes playing an effective role in the citizen's mobility, especially in great cities, due to the value of travel time (Headway) and the increasing ratio in passenger demands (Jamili & Aghaee, 2018). Planning and evolving train timetables are some of the most important and also routine issues in the Metro industry, which has been of illimitable interest to researchers in the field of transportation planning and design in recent years (Kang et al. al., 2018). Overall, there are three steps in the train movement planning and scheduling in the Metro: first, the process of train scheduling. second, the process of assigning train fleets to rail lines, and third, assigning crews to rail fleets (Larsen et al., 2017). The lifetime of the metro industry in Iran is a little more than two decades, therefore, from this time forth, the technologies related to the train running scheduling are considered a new phenomenon.

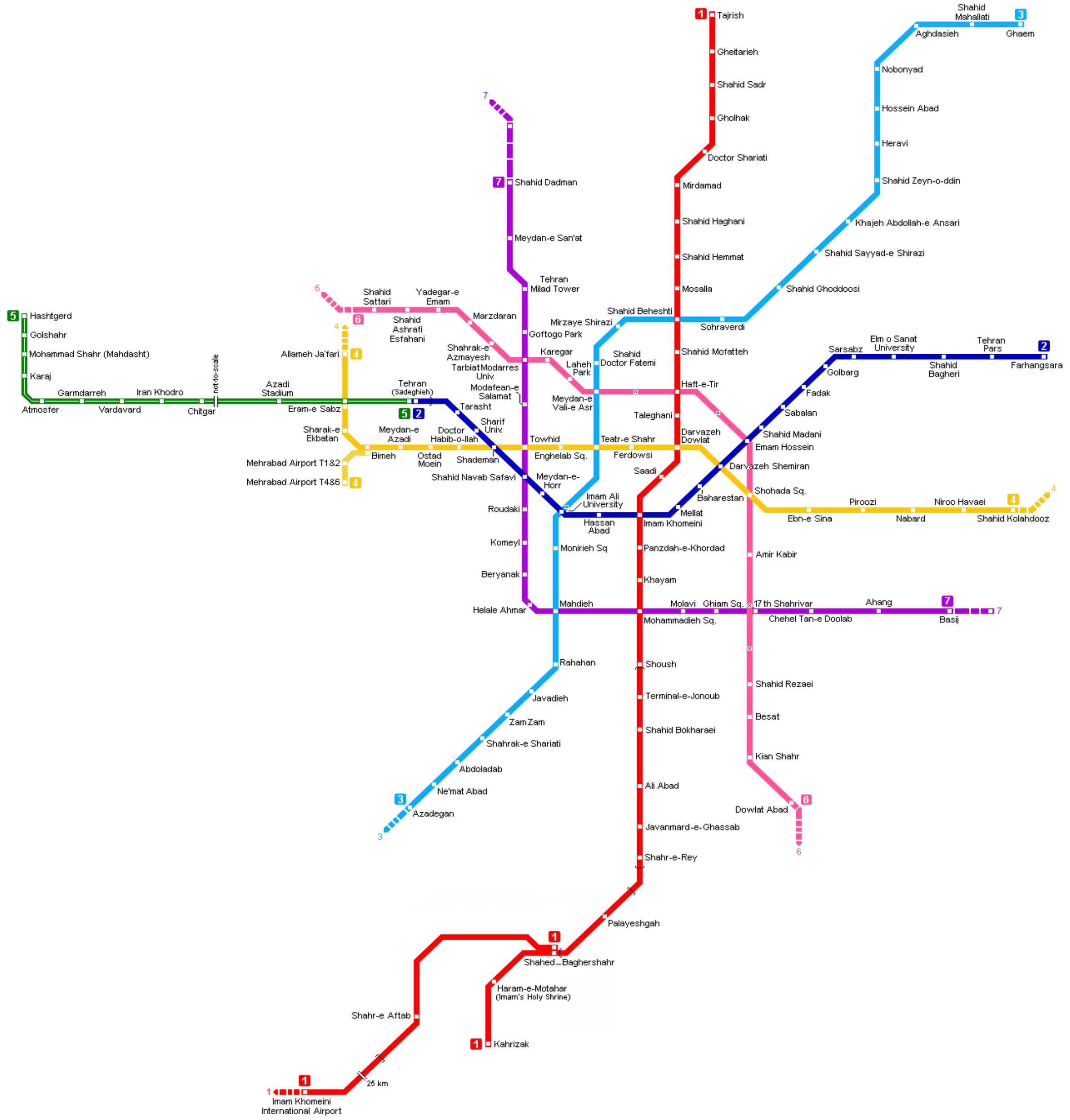

Figure 1. Comprehensive map of Tehran Metro lines

Source: Tehran Metro technical documents and central archive

In recent years, new approaches have been developed for planning Metro timetables known as the robust programming approach. Robust design is one of the divisions of systematic and structured scheduling, the purpose of which is to respond to uncertainties in metro Network scheduling (Mao et al., 2017). Metro is one of the industries that face ambiguity and uncertainty in its state space for various aims (Parbo et al., 2019). For all intents and purposes, in real conditions, the subway operation and train fleet movement in metro lines are different from the pre-planned nominal timetable. It causes the complete implementation of the nominal schedule. In addition to the mentioned case, there are many other reasons for this issue, some of which could be caused by the delay in the train's dispatching from the origin stations or the point of the rail route, as well as the disaster of fixed and mobile rail equipment in the process of running trains. A delay in the initial scheduling plan causes the dispatching of trains not to be carried out according to the expectations of the Metro operating companies and passengers, and as a result, its implementation is different from the initial plan (Schmöcker et al., 2015). occurrence and spreading delays in the Metro network is a problem that imposes relatively great and sometimes irremediable costs on the Metro operating companies and its benefactors. One of the solutions to reduce these costs is routine dynamic programs that are resilient to uncertainties, which is called robust planning (Nachtigall & Voget, 2006). In recent years, through the increase in demand caused by the tension in the metro industry to improve the service level, dedication has been paid to the issues of Metro traffic robustness and distortions control. the advances accomplished in the computer microprocessors' speed and operations research algorithms have made it possible to achieve steady programs and develop table retrieval tools based on optimization and integration (Puong & Wilson, 2011). There are three general approaches to solve the running and movement of Metro trains planning problem:

- Integrated Approach
- Robustness Approach
- Delay Management Approach (Tamannaei et al., 2019).

Some of the Metro operators have solved the running and movement of trains' planning problems in their rail network through integrated approaches or delay management. Therefore, the operation of Metro lines is subject to some important uncertainties as the Metro network disturbances are often due to unscrupulous climate conditions, special unpredicted cases related to the corrective maintenance and emergency repairs of fixed and mobile equipment, safety controls that are applied via traffic centers, system considerations through Train and station safety, security cases or other similar circumstances occur (Kang et al., 2018). The objective of robust planning in the metro network is to produce a comprehensive and barrier-free timetable that is sensitive to metro network conflicts. The type of regulating train timetables is known as demand-based strategic functions in the rail network, which also includes the dimensions of events and network uncertainty (Lidén & Joborn 2020). Studying the research literature shows that two major approaches have been defined for the program, stable planning in the rail network and that, the stochastic programming model is used to reflect and explain the uncertainties. In this method, disturbance modeling is used to define and verify the conditions that consider the stochastic nature of the problem. For example, the research by Tirachini et al (2014) showed that the conventional models of assigning trains and crews to lines and fleets by constructing less stretch in the hubs and circulation loops of the rail track show better performance in punctuality. Also, the fact that robust planning models try to prevent the happening and spreading of delays and disturbances caused by this despite it in a preventive manner. Robust recovery models are based on the fact that in the event of a disturbance in the metro network according to the conditions, the best response indicates the possibility that one from the set of answers so that the effects of distortion on the network are reduced to the minimum potential and the spread of delay in the entire network is prevented (Ulusoy et al., 2013). The second approach is to use robust planning to set up Metro train timetables. In this method, train running and scheduling are done in such a way that the whole system is flexible and resilient to probable disruptions and delays and that is less affected by negative consequences (Van Aken et al., 2020). For that reason, the use of time Buffers (mediators) as a solution that could be used in the recovery phase to reschedule train timetables and train running in the metro network will appeal in the direction of reports and prevent the occurrence of cumulative and dispersion effects as well as the spread of probable delays. (Wang et al., 2017). Instead, since the use of these reserve times in case of no delay event means the waste of metro resources and the relative decline of the network capacity, therefore, one of the main challenges of stable planning is the being of unused Buffer time values during the scheduling of the journey and the mobility of Metro lines. Too, the objective function of train planning models is used to determine the initial scheduling method and minimize costs through linear programming, not the direct reduction of operational costs of train traffic control, as a result of these models which certainly leads to the fact, that such programs will not logically have the lowermost amount of Buffer time between dispatch rows in the timetable of sequential trains, which causes major problems such as described below in the metro operation lines:

- Increasing the prospect of disaster in the initial schedule and the uncertainty of the entire network in the face of possible long-term disruptions
- Reducing the coefficient of the table retrieval options utility
- Increasing the cumulative effects probability and delays spread (Xiong et al., 2018).

According to what mentioned above, reliability criteria and punctuality index in metro traffic management are midst the key performance tools for measuring the effectiveness of the train dispatch and reception systems affecting significantly the passenger experience attraction in the metro network (Yue et al., 2019). In the metro network, the train movement is based on an accurate and initial schedule including several components such as the scheduled time for dispatching and receiving trains at the origin, route length, and destination stations (ADENEY, 2004). An unambiguous rail route for tracking the trains' movement is founded on a written timetable as a sequence of departure and stop times at stations and railway sections. Important limitation of metro traffic planning is the necessity of trains' spatial separation in a similar infrastructure, such as rail blocks between two signals to prevent accidents among rail vehicles by keeping safety margins (Zhao et al., 2019). In metro operations management, this obligation is made automatically through control and signaling systems, such as intelligent monitoring, which can prevent possible accidents, unnecessary braking, and frequent train accelerations and as a result, unsafe activities.Therefore, in a dynamic and appropriate schedule, it is necessary to take into account the physical separation of trains and the minimum Headway in the network, microscopic inferences, and through the theory of time sequences, the journey length is calculated and achieved values would be added to the train travel times on the rail route (De-Los et al., 2015). Sometimes, unexpected events and obvious changes during the Metro trains' headway may cause delays in dispatching trains from their origins or receiving points at destinations, and as a result, the precise implementation of a pre-planned timetable is impossible (Abbink et al. 2014). Additionally, in an eventful and high-traffic metro network, often a delayed load of the train fleet from one line could be moved to train passengers in other rail lines, especially at connection stations that have the similar infrastructure and their schedule for passengers moving continuously used for the crew and planned facilities are shifted. So, to improve the level of rail network stability and prevent the delays spread in the metro network and effect on the operational processes, a Buffer time is considered in the interval time of train movement in the form of numerical sequences (Barrena et al., 2017). In terms of Buffer time, the objectives are to absorb the possible disturbances of the metro network and as a result, the deviation of the train's running time from the initial timetable, which would prevent spreading delays to the next trains that run consecutively on the metro routes (Cacchiani et al., 2015). In this research, an approach based on "optimizing combination " has been used to add supplement time to high-traffic timetables in Tehran Metro. For any certain sequence of train fleets in the metro network, the time intervals formed in the schedule would be adjusted through the complements included in the planning process, which are distinguished as time residuals after integration calculations and employing Buffer times. It could be stored in the rail tracks for normal conditions and considered to manage and control distortion situations (Fischetti & Monaci, 2012). Decreasing the time reserves in Metro schedules is often done to increase the minimum Headway of running trains by setting specific amounts of supplement times by traffic controllers so that its effects are small for the metro network scale (Ghaemi et al., 2019). Therefore, the main question of the present study is how to allocate the Buffer times toward the interval times of the train fleet in the schedule tables so that the stability of the line schedule reaches its maximum value. This is followed by modeling the optimizing combinational algorithm. In this way, a specific Buffer time is assigned to each interval time in the train dispatching, which has a weight value equivalent to the timetable accumulative. The Buffer times values in the studied tables as well as the size of the deviations resulting from it to coup with the unexpected events of the metro network through parameters such as the delay effect, its spread in the metro network, and the analyzing sensitivity process in the effect delays logic on other trains and the stations are decorated. Therefore, in the present study, a challenge has been made to formulate the optimum allocation of supplement times utilizing the particle swarm algorithm. Therefore, the used tools which compared focusing on the alternative's quality and the computational solutions complexity. The case study of this research is the operational space in Tehran Metro line one, which mainly tries to study the appropriate allocation of Buffer times according to the traffic conditions of the Tehran Metro. So, in the current research, the scheduling framework of the train movements is explained as one of the important and effective subdivisions in the subway processes, which can verify the new approaches used in this part of the Metro industry for stable planning and delay management. Metro experts have mainly considered the delay as an inseparable part of the subway operation, which the operating companies in different subways aspect according to the type of passenger movements (in terms of fast or normal). Therefore, to resolve the delay problem in the metro network, various solutions have been proposed and developed, one of the most important and effective approaches is based planning, which can respond to ambiguous conditions and uncertain situations. This approach can identify the most important factors that cause delays in running trains and non-operationalization of the initial schedule and provide sustainable solutions to do away with limitations and optimize the schedule.

2. Literature Review

Xu et al. (2019) proposed three approaches to procurement of the amount of noise (distortions) in Metro routes, as: the first, a simple estimation method through comparing with a pre-planned timetable (for example, a schedule Delay in receiving the train at the destination up to three minutes caused by congestion and track conditions); the second, with mathematical expectation or average weighted of the delays that occurred for the trains fleet in the train schedule program, and,the third, using the data delay collection of the train fleet recorded in the traffic control centers database (Xu et al., 2019). The "distortion threshold" parameter is considered as the average non-negative delay acquired from the traffic data recorded in the train fleet database (Yan et al., 2013). The Buffer times for each train is the amount of time that is calculated schedule among that specific train and the other trains before it in the running metro track sequence. If this time is less than the distortion threshold time, the risk of conflict on the railway track increases. Therefore, the use of robust methods causes small improvements in terms of Buffer times in the rail route (Abkowitz & Tozzi, 2004).

Uncertainty in linear programming problems or optimization systems is one of the important issues in the metro industry. As a result, various approaches such as stochastic or fuzzy planning have been developed to deal with uncertainty in the subway through mathematical modeling. The robust planning approach is one of the most up-to-date methods of dealing with uncertainty in the Metro, which has been widespread due to its special capabilities in identifying limitations and optimizing the objective function. Therefore, the current research objectives are to analyze different planning methods and their applications in metro management network disturbances. The robust decision-making approach is a method in which the focus is on stability promoting against environmental ambiguities, and the method that has the least possible variations against unstable conditions is a resolution to the optimization problem in the general state spaces formal. They could have stability and optimum robustness (ADENEY, 2004). Possible stability means that the acquired solution should remain feasible for all situations or at most for parameters with uncertainty. Also, optimum stability is a situation in which the value of the objective function remains stable for the proposed solution, and therefore, for all or most of the parameters with uncertainty, it is neighboring to its optimum value and has the minimum possible deviation from the optimum value (Birge & Louveaux, 2014). CADARSO et al. (2016) have provided a framework for robust optimization, which includes two main concepts: "stable solution" and "stable model". This optimization method is related to approaches in which the data type is scenario-oriented. The results of this research state that the use of operational research in mathematical planning models leads to uncertainty through uncertain and ambiguous data, so opposite this type of data using methods of sensitivity analysis stochastic planning is problematical. It deals with implementation, which includes the structural (fixed) and the control parts. In this case, the linear programming model for optimization is written as follows:

$$
\begin{aligned}
&Minmize\ c^T x + d^T y \\
&\text{Subject to: } Ax=b \\
&Bx+Cy=e \qquad (1) \\
&x, y \geq 0 \\
&x \in R^{n_1}, y \in R^{n_3}
\end{aligned}
$$

In the above relationship, x represents the decision variables for the deterministic parameters and y represents the decision variables of the control sector.

The approach designed by Aharon Ben and Nemirovski (2012) has a high coefficient validity in the functional state and is precise attentive and detailed in theory, so it could provide an answer through sensitivity analysis though considering all problem dimensions. The objective Model is much better than the nominal state. To solve the conventional optimization model problems, Burggraeve & Vansteenwegen (2020) have offered a robust optimization framework that can control the conditions and consider the problem dimensions considerately, but considering that the robust model resulting from This approach is a non-linear problem of the second-order conic type, so it cannot be used for discrete optimization problems, and therefore it increases the structural complexity of the problem. Bertsimas & Sim (2014) offered a new method for data modeling uncertainty states, which solved many problems of previous stable approaches to a large level. To understand the dimensions of this framework, the following linear optimization problem is assumed:

$$Max\ \acute{C}x$$
$$s.t.AX \leq b \qquad (2)$$
$$I \leq X \leq u$$

The above equal apprehends the constraints optimization for the stability of the solution space by crucial the objective function and the constraints matrix (Bertsimas & Sim, 2014). Many recent studies in the field of optimization problems mathematical modeling have referred to the research of Bertsimas & Sim (2014), in which the researchers' main focus is on providing optimization frameworks in conditions of uncertainty despite strict limitations. It should be mentioned that the conventional procedures of operations research, moreover to having hard limitations, also include a soft limitations variety (CARRARESI et al., 2006). In robust approaches, the destruction of soft constraints is defined under the uncertainty through a penalty function. So, the use of slackness variables is significant for structural simplification. Also, robust models formulate problems by emphasizing the integration of two-stage ideal and probabilistic planning concepts (CORTÉS et al., 2013). In such approaches, uncertainty is necessary and its demonstration method is mainly scenario-oriented. In robust approaches, the penalty area is to determine the set of justified answers and, as a result, to choose solutions (DAUZÈRE-PÉRÈS et al., 2018). In the model emphasized by DAUZÈRE-PÉRÈS et al. (2018), A and B are matrices with definite data and E, C, and B are the uncertain and ambiguous parts of the model. Therefore, the mentioned model consists of two categories of decision variables, the first category represents the design variables of the model ($x \in R^{n_1}$), which is generally independent of the scenarios that occur, and alternatively, the optimal value of the control variables ($y \in R^{n_2}$), depending on the amount of design variables and which scenarios are probable to occur. This is shown in the following equation:

$$Minimize\ f(x,y) = c^T x + d^T y$$
s.t
$$Ax=b \qquad (3)$$
$$Bx+Cy=e$$
$$x, y \geq 0, x \in R^{n_1}, y \epsilon R^{n_2}$$

Here, the scenarios are shown as a set of $\Omega = \{1,2, \dots, \omega\}$.

In the present research, a two-stage stochastic approach for designing stable scheduling tables is offered, In the first phase, an initial solution is presented with the scenario of solving stochastic disturbances. Probable problems of the initial solution would be corrected in the second step. Therefore, it is necessary to use a separating algorithm concerning upper and lower restrictions for investigative searches to assess the complexities of the problem. Also, in this method, the beginning time of the dispatch and the train route are considered as decision variables in the model based on the principles and limitations such as the priority of the trains based on the priority of the route In the current process of planning the timetables of Tehran Metro trains, a timetable is set up at the beginning of each year, which could be altered according to the conditions. In this procedure, the feedback is analyzed through the information acquired from the performance evaluation indicators of Metro lines, and new requests are applied to consider the line and passenger conditions of congestion for assigning rail routes to trains in the timetable. So, to generate optimum sequences in the structure and mobility of Metro trains, the capacity of necessary calculations could be reduced to the lowest possible by minimizing the life cycle of periodic scheduling programs. However, traffic planners in Tehran's metro network are dealing with challenges by setting up preparation tables to achieve optimum value based on predetermined traffic requirements or unexpected cases, which are caused by conventional approaches and assigning specific amounts of Buffer times with Some partial deviations from the designed time tables and increase the stability of the tables.

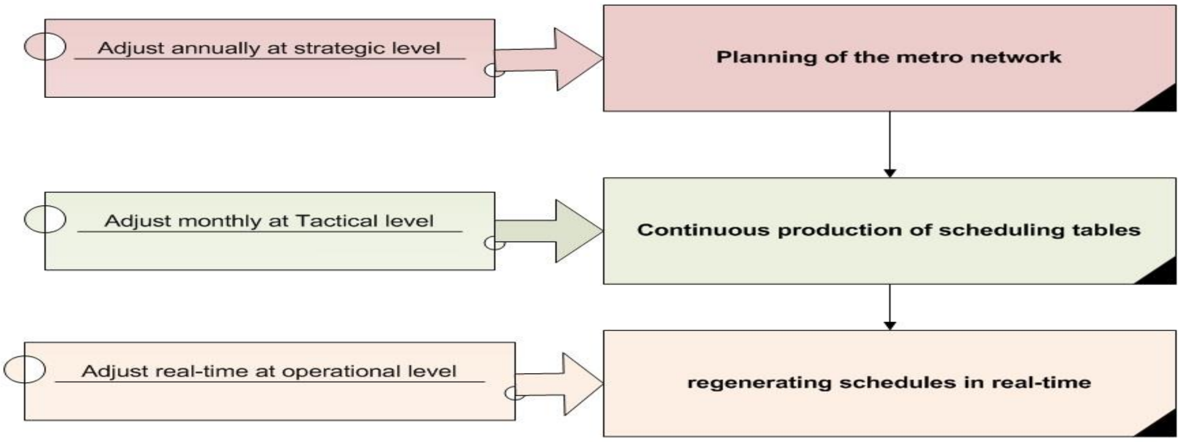

Figure 2. The phases of designing and recovering timetables of Tehran Metro trains

Source: Tehran Metro technical archives and central archive

Generally, Buffer times are considered based on planned and unplanned demands in train scheduling issues. The use of practical programs in organizing scheduling tables consists of stochastic optimization approaches and innovative methods that use stable principles and concepts in their framework to regularly adjust the tables in a lagrangian method that Makes its implementation and recovery possible. Here, retrievable robustness is a proposed alternative option for expanding the technical features of the robust optimization approach to reduce the computational complexities in scheduling problems and in a localized structure, the historical information in the database to improve the stability of the table. Then, firstly, it is obligatory to calculate the Buffer times and then the problem turns into a linear program, and in this way, positive and negative deviations from the initial values of the table are removed. Here, the critical points of the table could be considered as candidates for the application of Buffer times, and in the form of lane exits and emergency overtaking, they could be an indicator to measure the stability of the tables.

In a study directed by Luan et al. (2020), the researchers have developed an optimization model in terms of limitations related to Buffer times at extreme points. In the study mentioned, by using the buffer times included in the timetables and reallocating them, the value of some corner points in the acceptable answer area has been reinforced. The computational outputs of the method offered by Luan (2020) show that the mentioned approach has abundant effectiveness for the traffic scheduling of two-way lines (such as the subway), which originates from the inherent characteristics of the corner points.

In another research directed by Budai et al (2011), the researchers studied the problem of unexpected deviations in the field of train schedules. In the mentioned study, the researchers described an optimal framework for reducing the delay spreading in rail lines through a random model. Also, by distributing Buffer times in a compound, congested, and large-scale network in a specific framework that has a minimum risk of delay propagation, they have estimated the traffic status of the lines and predicted possible solutions in the future. However, the use of this technique may affect other parts of the Metro network, especially in common stations, due to changes and insertion of many Buffer times among continuous traffic events, which have not been considered in the mentioned approach. Therefore, in the present research, an approach based on optimization in terms of the distribution of Buffer times according to the available capacity in the Metro network for congestion conditions has been presented, which covers the problems of previous research. This approach has the advantage of presence able to be used in different traffic situations in one-way or two-way subway lines. Also, to form stability in the traffic plans for a wide spectrum in the scale of the Tehran subway network, particularly in the conditions of traffic congestions and disturbances of the Metro lines, it is focused on the restructuring of time reserves in the table rows of the traffic plan. Considering the computational complexities in scheduling problems for applying complementary times in the process of train running and movement. The proposed approach of an organized and specific time structure (precedence between trains in dispatching and the sequence of their movement in journey and movement) to separate Traffic outlines is used. therefore, the parameters of the optimization model could be calculated and evaluated in terms of all current limitations using the traffic pattern of the Metro lines. Therefore, the proposed approach guarantees that every possible option in the optimization problems could be implemented without irreverent the limitations of the problem through routine traffic approaches. Therefore, the problem itself will include several sub-problems that can answer the following questions:

- What are the optimal quantity and total Buffer times for distribution and deployment in the timetables of Tehran Metro trains?
- What is the effect of putting special Buffer times on the total times stored in the timetable of Tehran subway trains?
- What is the required amount for each Buffer time in the minimum Headway of a train running in the Tehran metro network?
- What effect does the use of Buffer Times have on the reliability of Tehran Metro timetables?

The challenge of responding to the above questions shows the necessity of developing an optimum model for assigning Buffer times through the development of an innovative algorithm. In this approach, the decision variables are nominated based on the Buffer times to enter the model. The separation of traffic events from the dimensions of the optimization problem variations it possible to increase the calculation speed by assigning the superlative Buffer times to the scheduled table rows.

3. Research methodology

In this study, the metro network traffic model is clarified firstly. The train schedule in Tehran Metro is ordinarily organized through a spherical and directional procedure in two lines. This diagram is often known as the "event-activity" network and is shown as nodes with N= (E, A) coordinates. This approach is a common method for metro modeling network traffic at the macroscopic level. Nodes (stations) are represented by N, traffic events are represented by E, and activity (journey and movement processes) is represented by A. In this case, only the events occurring in the metro stations (nodes) and the edges leading to them (rail sections) will have the possibility encompassed in the model. This framework distinguishes in the middle of the arrival and departure events of the train and other situations such as non-stop passing through the nodes with equality $\mathrm{E} = E_{departure} \cup E_{arrival} \cup E_{threshold}$ and The equation $A = A_{running} \cup A_{dwell} \cup A_{connection} \cup A_{headway}$ All the different states encompassed in the metro traffic system show the Headway between the trains and the routes among the stations for setting the timetables, such as the train movement of the railway route, the train stopping at the stations or dispatching them from other stations along the route to the destination. A specific event such as $i \in \mathrm{A}$ is signified using an ordered multiple in the general form (t_i, d_i, i), where each of the mechanisms respectively represents the number of trains or stations, the type of activity (arrival and departure) the train or the train passing through the station without stopping), the planned time for a specific event or unplanned events in the real state or disturbance in the rail route. Also, an ordered pair such as $(i, j) \in A$ with the event i starts and ends with event j. Therefore, i-j is the sign of a process in which planned time is $A_{i,j}$ and the minimum time of the process is $A_{i,j}^{minimum}$, which is the difference It shows between the planned times and the minimum time for the process of running the train in the real world of the lines to include time reserves (such as time supplements and Buffer time) in the Metro timetables. However, to set up the limitation and formulate the model, it is necessary to determine the passengers arriving at the station space. This is shown in diagram number 3:

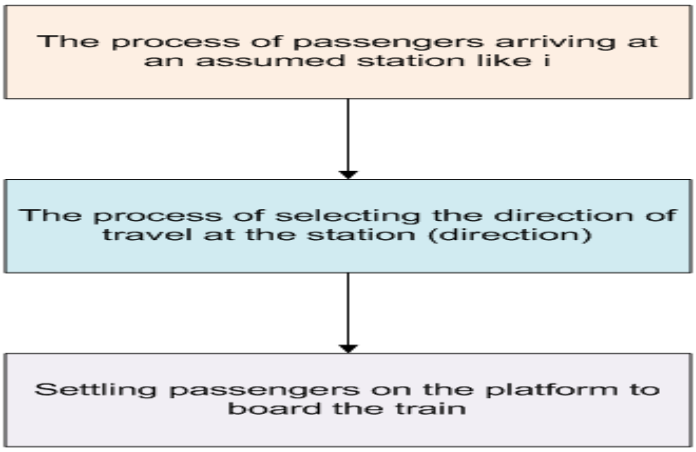

Figure 3. The space of passengers entering the station and standing in line to board the train

source: jafari et al, 2024

The state space of passengers boarding in the assumed train i and changing the capacity of the station and the train is shown in Figure number 4:

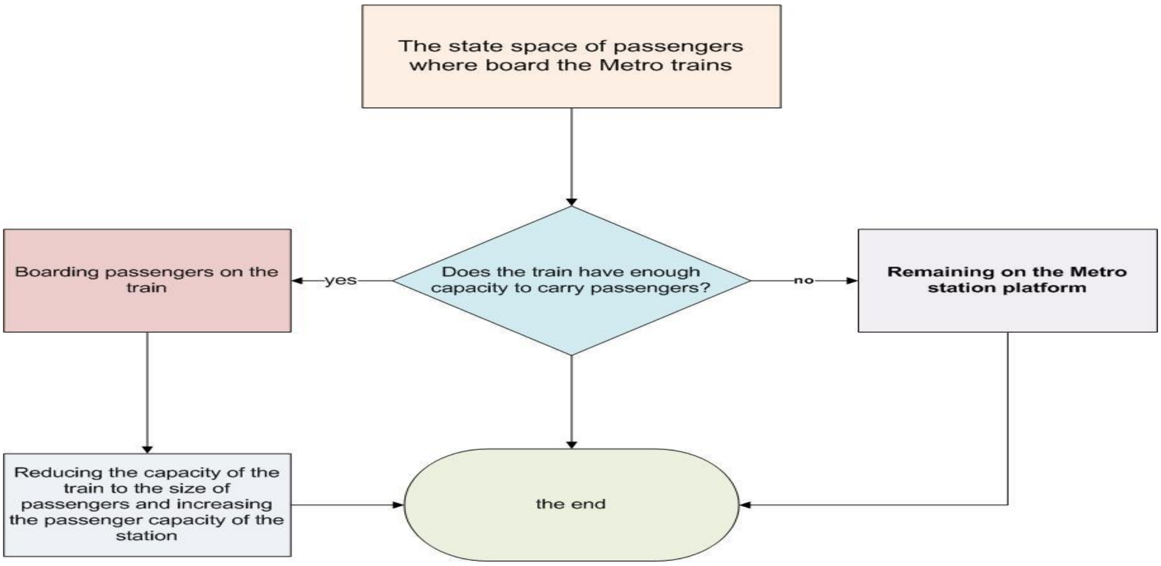

Figure 4. The state space of passengers boarding the train and changes in train and Metro station capacity

source: jafari et al, 2024

The state space of passengers alighting off the train at the station of the assumed destination j or continuing the trip by the passenger and changing the capacity of the train or staying the same is shown in Figure number 5:

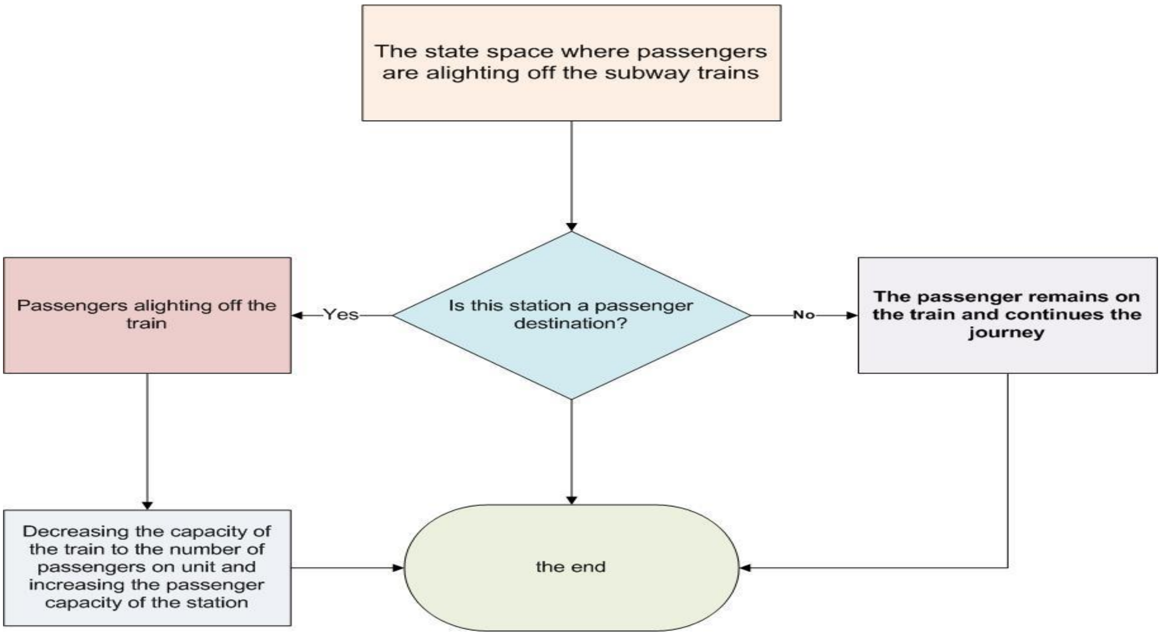

Figure 5. The state space of passengers alighting the train and the change in the capacity of the train and metro station

source: jafari et al, 2024

The process of the train entering a certain station and its departure from the same station, besides the passengers boarding and alighting, is shown in picture number 6.

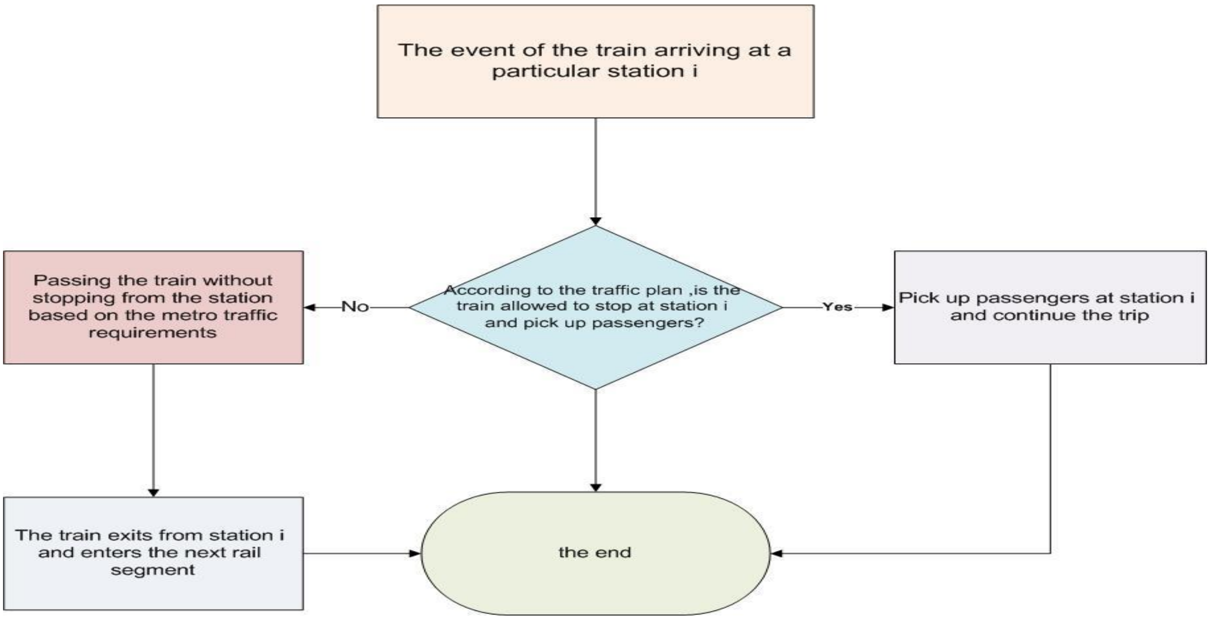

Figure 6. The process of a train entering a certain station and departure the same station as a journey

source: jafari et al, 2024

Based on this, the conceptual model of the current research is shown in diagram number 7:

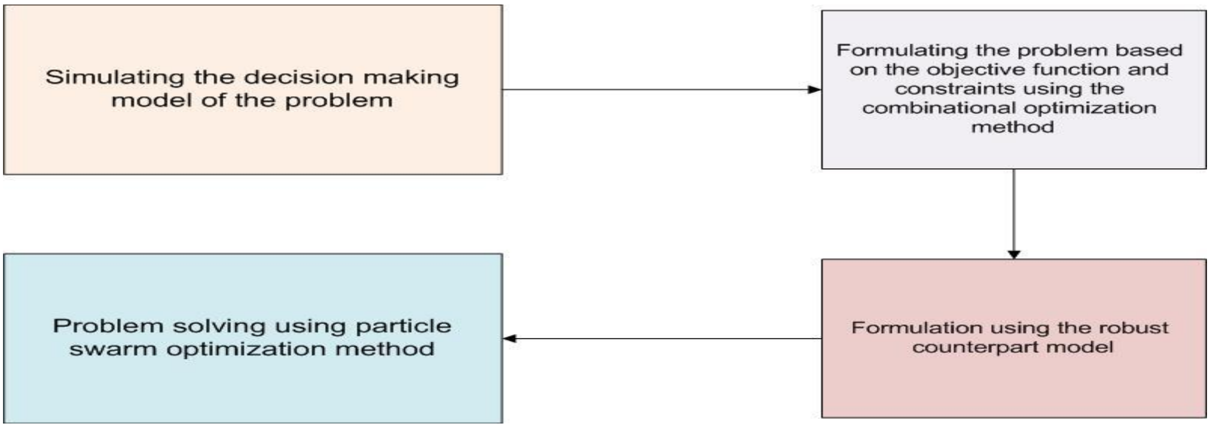

Figure 7. Conceptual model of research

source: jafari et al, 2024

3-1. Symbols related to indexes, parameters, and model variables

Table 1 shows the most important symbols used in the modeling process. It is necessary to clarify some activities such as dispatching the train, the minimum time obligatory for passengers to board the train and alight in it, the transfer of passengers, the boarding of passengers on the train, the minimum time to board the train, and so on is one of the most important traffic processes for scheduling in the metro network, which is mapped according to the events of disturbances, that include the cancellation of the train from the origin, unnecessary stops at the station, extended travel time, and useless times that cause delays in arrival train at the destination.

Table 1. Symbols related to indexes, parameters, and variables of the model

symbol	Descriptions
t_R	The termination time of the disturbance in the Metro and the start time of the timetable retrieval period
k	Index related to the dispatch of trains from the origins
i,j	Index of stations
λ_{it}	Passenger arrival rate at station i at time t
$OD^{[t]}$	The percentage matrix of travel demand in the destinations received by the train for time t
m_k	The set of stations along the rail route of dispatching and receiving trains from k origins after time t_R
$T_A^{(u)}$	The set of dispatches after the end of the rail disruption for trains whose journey has not yet started after time t_R.
$T_A^{(d)}$	The set of dispatches after the termination disturbance must be received at the travel destinations and have not yet arrived at the destination after time.
$T_B^{(u)}$	The set of dispatches that occurred before the termination disturbance, which specify the train travel time in the direction of the dispatch route from u stations of the metro route.
$T_B^{(\mathrm{d})}$	The set of dispatches before the termination disturbance for the trains that are going back to destinations d at the mentioned time.
T_A	The total set of transmissions that are sent from the origin after the disruption time is calculated through the contradictory relationship: $T_A = T_A^{(u)} \cup T_A^{(d)}$
T_B	The total set of trains that have been dispatched from the origin before the time of disruption and the amount is obtained from the following relationship:

symbol	Descriptions
	$T_E = T_E^{(u)} \cup T_E^{(d)}$
π_k^{max}	The maximum number of stations that the dispatch train can pass without stopping at time k.
δ_{ij}	The number of passengers present at station i going to destination j at the time of disturbance t_R
L_{kj}	The number of passengers who were present in the dispatch train k at the time t_o and intend to reach the destination j.
α	The time required to increase or decrease the acceleration of the train is assumed to be the same for all the railway parts of the running route, rendering the average slope of the line.
h	The index related to the number of active trains on the Metro route
t	The index related to the row number of the Headway in the timetable
d	The index related to the direction of the running train on the two-way Metro track, which has an appearance value d=1,2
m	The number of metro stations
k	Indexes related to the number of metro stations
n	The number of active trains on the rail sections
T	The number of interval times counted in in the Metro tables
V_{ave}	The average speed of trains on the railway track
V_{max}	The maximum speed of trains on the railway track
$Dis_{k,\ k+1}$	The distance between two consecutive stations k and k+1 in an assumed Metro line
$C^{maximum}$	Maximum capacity of a train on metro lines for peak passenger hours
$d\omega_{minmum}$	The minimum duration of train stops at metro stations
$rest_{minimum}$	The minimum duration of metro trains stopping at each metro route stations
CD	Duration of delay caused by passenger congestion at each station (in seconds)
η_{kd}^t	The arrival rate of passengers to station k in direction d for interval times 't' in the Metro network
φ_{kd}^t	The rate of passengers getting off at station k in direction d for the Headway t in the Metro network
TP	The total number of subway passengers on a particular Metro line for a typical day
TT	The total number of train passengers dispatched for a normal day in the metro network
$Q(t)_{kd}$	The number of passengers waiting at station k in direction d for the assumed interval time t in the Metro network
$b(t)_h$	The number of passengers on the train h for the time interval t.
$dep(t)_{lkd}$	The departure time of the lth train from station k for interval times t in direction d in the metro network
p_{lkd}	The number of passengers of the lth departure from station k in direction d for the metro network
P_{ljd}	The number of passengers in the lth train movement who have entered station k in direction d.
b_{lkd}	The number of passengers on the Metro train in the lth departure at station k and in direction d
FR	Passenger transportation rates in the metro network
$R(t)_k$	The duration of measuring of rail segments between station k and k+1 in the interval times t of the metro network
$d\omega_{lkd}$	The stopping time of the lth train from station K in direction d for the metro network
H_{ld}	interval times between l and l+1th running train in direction d for metro network

3-2. Assumptions of the model

To develop the optimization model in this research, the following assumptions are considered:

Assumption 1: According to the timetable established, in normal mode, no additional train enters the network and no train is canceled before the emergency occurs.

Assumption 2: the train travel time (without acceleration or deceleration) between any two consecutive stations has a constant value independent of the conventional pattern of stopping and passing, but in the case that a train passes through a certain station without stopping, it is programmed from the travel time. It is reduced by the amount of time of non-stopping acceleration and braking caused by it.

Assumption 3: If a passenger is on a train at the time t_R (disruption) and the train does not stop at the destination station of this passenger, before reaching the destination station, this passenger gets off at the nearest station and waits for a new train. It remained able to take him directly to his destination. This interrupts the balance of capacity in the metro network.

Assumption 4: For passengers who were present at the station platform during t_R and did not board a train, the opportunity to change the train is not considered.

Assumption 5: If the disturbance is over after the interval time t_R, the rescheduling of the trains and the retrieval of the schedule will continue until an ordinary situation is reached.

Assumption 6: In case of distortion in the Metro network, a passenger will wait for two consecutive trains to arrive and therefore, in the termination, he will board the third train to complete his trip. Therefore, if the first and second trains do not stop at the passenger's destination station, this passenger will probably board the third train.

Assumption 7: Passengers who are waiting for the arrival of the train at a certain station, will only board the train that stops at their destination (inter-route or terminal).

3.3-Limitations of the proposed model

In this section, the limitations related to the restriction of train timetables and the most important relationships interrelated to passenger flow under the influence of metro travel optimization patterns are offered. The stopping limit for the running time and train movement is determined according to the established timetable in such a technique that if the train does not stop at two consecutive stations, the running time of the train between these two stations is equal to the continuous running time. The relationship related to this limitation is determined by the following equation:

$$a_{k,i+1} - d_{k,i} - R_i = \langle x_{k,i} + x_{k,i+1} \rangle . \alpha \qquad k \in T_A \cup T_B \, , i, i+1 \in m_k \tag{4}$$

The stop time limitation is shown in the following equation:

$$stop_{min} . x_{k,i} \leq d_{k,i} - a_{k,i} \leq stop_{max} . x_{k,i} \qquad k \in T_A \cup T_B \, , i \in m_k \tag{5}$$

If the dispatching train k has a stop at station i, as much as the minimum amount of stops at the station $Dw_{minimum}$ will stop. Then, the stop time will be zero. According to the minimum interval time between dispatching consecutive trains $headway_{min}$ is written by the following limitations:

$$a_{k+1,i} - d_{k,i} \geq headway_{min} \qquad k, k+1 \in T_A \cup T_B \, , i \in m_k \tag{6}$$

The restrictions related to the flow of the train fleet in the metro stations and the shunt situation to separation time for each of the round-trip lines are summarized in the following equation. The train that arrives at the destination station stops or the destination terminal at least $t_{minimum}$and formerly is ready to be dispatched on the return route:

$$a_{k,1} - d_{p_k,2m} \geq t_{min} \qquad k \in T_A^{(u)} \tag{7}$$

$$a_{k,m+1} - d_{p_k,m} \geq t_{min} \quad k \in T_A^{(u)} \tag{8}$$

The limitation related to stop and movement patterns is defined in the following equation Based on this relationship, the total number of stations where the k dispatch and train breaks are determined based on the maximum number of allowed transit stations o_k^{max}.

$$\sum_{i \in m_k} x_{k,i} + o_k^{max} \geq m_k \quad k \in T_A \cup T_B \tag{9}$$

The main reason for the above limitation is that the failure of one or several trains to stop consecutively at some stations of the metro network is the basis of passenger dissatisfaction and crew complaints, As a result of this issue for those passengers who are members of B trains and Therefore, the special importance of some stations makes the passengers have to act early at a station to change trains and board another train that stops at the station of their desired destination.

In total, the value of $o_k^{max}$is calculated based on the rules, principles, requirements, and instructions of the Metro operation or the estimations of the traffic management. If an applicable traffic optimization pattern is not acquired aimed at the rail network, some passengers will have to wait for a long time to board the train so that the train can take them directly to their destinations. To prevent this situation, the following restrictions have been considered for transit stations in the metro network:

$$x_{k,i}.x_{k,j} + x_{k+1,i}.x_{k+1,j} + x_{k+2,i}.x_{k+2,j} \geq 1\ ;\ k, \ldots, k+n \in T_A \cup T_B \quad, i,j \in m_k \tag{10}$$

The above restriction guarantees that for each trip between stations i and j, at least one train stops at both stations i and j in all three consecutive dispatches. In other words, passengers who propose to travel between stations i and j experience at most two non-stop departures before a train arrives to reach their destination. Therefore, it is necessary to point out that the number of trains that could pass without stopping would be easily changed and altered in the proposed model. With a slight change in the above number limitation, similar variations can be obtained with different numbers of barriers from the stations along the rail route to the destination. So, the traffic dynamics of the Metro network can be limited by the following limitations:

$$t_i + a_{i,j}^{minimum} \leq t_j \ , \forall\, i,j \in E \ , (i,j) \epsilon\, A \tag{11}$$

The above relation shows the development and delay of the Metro mobility as well as the events time between these sequences in an orderly pair that describes the traffic inequalities.

$$t_j \geq d_j \ , \forall j \in E_{departure} \tag{12}$$

The above relationship shows the limitations of scheduling tables for events such as unplanned train departures from metro stations. Therefore, considering the time Buffer makes it necessary that in the schedule tables, it is necessary to guarantee the movement of the trains in the order of occurrence of the planned traffic events, as a result of which the following limitation could also be mentioned:

$$l_{i,j} = a_{i,j} - a_{i,j}^{min} \ , \forall\, (i,j) \in B_{headway} \tag{13}$$

However, what can affect the travel time and the rate of receiving the train at the destination is the number of stations along the route and the stopping time in them according to the congestion of the station, which of course will also affect the passenger carrying capacity. This is shown in figure number eight.

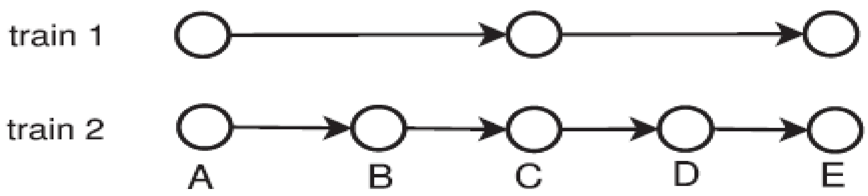

Figure 8. An example of the capacity and duration of passenger movement for two trains with the same distance and different numbers of stations

Through the above details of the research background of this study, it can be clarified that according to the metro timetables which are defined by the event and activity network in the form of a node like T, how to produce a stable table like T' so that $\acute{b}_{i,j} = \ \forall\,(i,j) \in B\backslash B_{headway}$. In this case, to increase the stability of timetables by altering the train's headway, $\forall\,(i,j)\ \in B\backslash B_{headway}$ could be made possible. So, by keeping the Headway and running time of each train constant and taking into account that the train interval time includes a minimum and constant value as well as a Buffer times value, $\forall\,(i,j)\ \in B_{headway}$ In the timetable, T' could be completed more stable than T in contrast to disturbances. An important principle in such problems is to focus on appropriate interval times, which are normally included in timetables. For example, the life cycle for a given time window in non-periodic timetables needs to remain unchanged. In other words, timetable stability could be used through the redistribution of cumulative time reserves and the placement of Buffer times. at that point, the timetable stability could be improved by redistributing time reserves and adjusting Buffer time. It should be mentioned that the entire time reserve in a secondary table is used through table compression to calculate the value of $T^{Complement}$. Subsequently, the timetable compression technique is a standard and common approach to improving the stability and the potential of a pre-designed timetable. In some research, more complex methods have been presented, which mainly depend on the absolute capacities of the pre-determined timetables, and in which the combined relations, development, and delay of the trains are investigated. In this way, the primary optimization problem for scheduling table compression is formulated as follows:

$$Min\ T^{Complement} \tag{14}$$

The above objective function is used to minimize the time window of the expression$T^{Complement}$.

s.t

$$t_i^{complement} \leq T^{Complement} \tag{15}$$

The above constraint guarantees that the occurrence time of a specific event called t $t^{complement}$ is less than the sum of all scheduled events in the compact schedule.

$$b_{i,j}^{Complement} = ab_{i,j} \tag{16}$$

The above constraint guarantees that all methodical couples associated with running, fixed, or stopped trains at the station and track have a fixed value in the table.

$$b_{i,j}^{complement} \geq b_{i,j}^{min} \tag{17}$$

The above constraint guarantees that all the ordered combines observing the running trains on route i-j are greater than or equal to the minimum Headway determined.

$$t_j^{complement} = t_i^{complement} + a_{i,j}^{complement} \tag{18}$$

The above limitation in this issue guarantees that the vinterval times could be reduced as much as possible Based on this, Figure number nine shows the capacity diagram - minimum passenger - which increases the minimum number of passengers by increasing the capacity of each train.

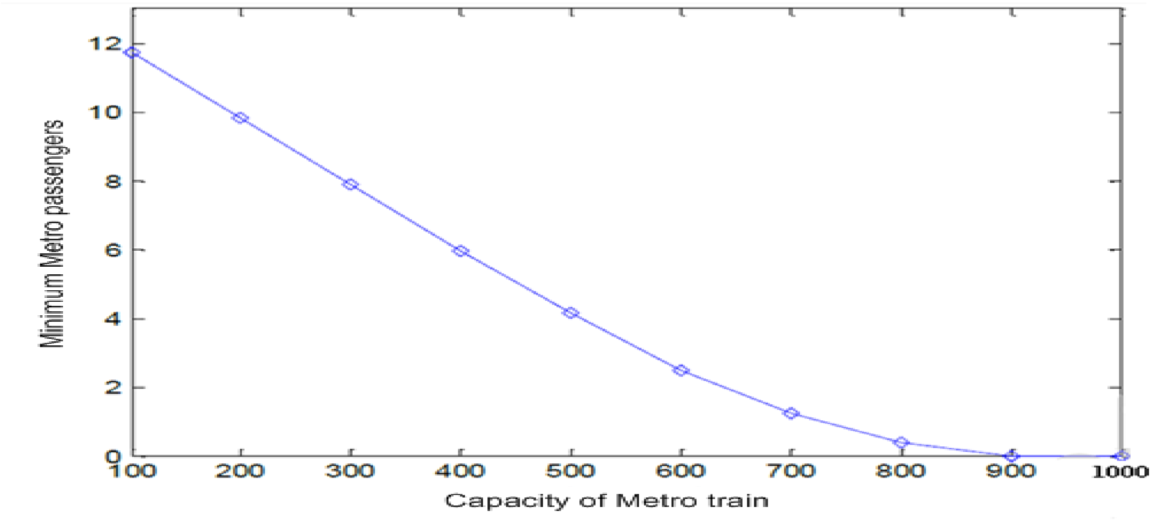

Figure 9. The Capacity diagram - the minimum standard amount of passenger movement in Tehran Metro

source: comprehensive Tehran Metro traffic system

In a rail section between two assumed stations A and B, to calculate the total time, reserve and obtain Buffers time in the same section based on the compacted timetable, at which the time of the first event in the time k is equal to $T = t_{first}(k-1) - t_{first}(k)$ and $T = t_{first}(k-1) - t_{first}(k)$ In this case, the total Buffer time for the assumed station is acquired from the relationship$B = T - T^c$. This method applies to two-lane railway lines (such as Tehran Metro lines) and one-lane (such as newly operated metro lines) or longer routes such as S networks. In this case, modeling requires different approaches. In the metro network, due to the essential interdependence of different traffic events, applying buffer time in a specific schedule can affect or possibly disrupt feasibility calculations in other parts of the schedule. Hence, in setting timetables and applying Buffer, it is necessary to check and calculate the total amount of time reserves for each section separately. Consequently, the following relationship is presented:

$$B_{compact} = t_{first}(k+1) - t_{last}^{c}(k) - t_{minmum} \tag{19}$$

In the above relation, $t_{last}^{c}$is the occurrence time of the last event in the rail segment dense in the compact timetable and the modified state ofT_c. It should be renowned that the time saving of each section is noticeably reduced with the interval time between the last event in a time window and the first event in the next time window usingt_{min}. If this restriction is observed for each part of the table, then the proposed approach for one-way, two-way lines and other sections of the metro network will have sufficient validity.

3-4. Formulating the proposed model

The occurrence of primary and secondary delays in the metro network indicates an increase in the travel time and train running on the Metro track. The spread and expansion of early delays are the consequence of a disturbance or a set of conflicts in the main processes of train movement (Yue et al., 2019). Predicting initial delays in the Metro network is sometimes very difficult and occasionally impossible, and its probability distribution function is often acquired from traffic data records and used as input in the modeling of Metro scheduling tables (Caprara et al., 2011). In the actual environment of the metro network, principally the initial delays occur in multiple events, probable and independent from each other in the state space, hence, to evaluate the timetables stability in a certain period of consecutive time windows and improve the level of network coverage of the events The trains arrangement and movement can theoretically generate a virtual initial delay for each line of the timetable, which is called the process of disruption generating scenario in the Metro network (ALBRECHT, 2016).

Hence, it is possible to calculate the secondary delay values released as an implementation result of a disruption management scenario in the metro network based on the "event-activity" network framework and through the normal stochastic optimization equations (Alfieri et al., 2016). In complex metro networks, delay spreading would be effectively measured through deterministic or stochastic algorithms (Brännlund et al., 2001). So, to compare and find the answer to this question, whether the schedule T' is more stable than the schedule T? The research process directed to a pairwise comparison between the performance and outputs of two timetables in terms of the propagation of secondary delays in the metro network. Hence, the validation assessment for the disturbance management scenario and the calculation of delays for each of the scheduling tables based on each robustness indicator such as the Relative sum of secondary delays, average delays per event, etc... For different situations of the metro network Estimated. Next, to increase the stability of train timetables, the approach of redistributing Buffer times was used. It should be mentioned that the inclusion of Buffer times in the rows of the schedule tables has a different effect on the tables' stability depending on the type of event and the rail lines conditions (Cadarso & Marín, 2017). Consequently, the importance of prioritizing events according to the happening time of the event in the schedule tables depends on factors, some of which are as follows: the amount and scope of the event in the "event-activity" network, the priority type of train traffic (passenger, shunting or unexpected events), the difference in the totals time supplements assigned to each row of the table in the train movement process, etc... Thus, it is possible to use the "event-activity" network structure in terms of the detecting priority of dispatching trains and time supplements to the purpose of controlling disturbances in the Metro network leads to the assignment of Buffer values $b_{i,j}, \forall (i,j) \in B_{headway}$in the tables (Caprara et al., 2019). Therefore, in the first phase, it is necessary to assign Buffer values to all the events of the primary schedule. In the second phase, the Buffer time value is used as an upper limit between two consecutive events in the metro network, which increases the flexibility and stability of timetables compared to unexpected events. This could finally lead to the calculation of the maximum return from each Buffer time through the analysis of Buffer times. The consequences of interleaving a Buffer time among the events occurring between the assumed nodes i and j, by reducing the amount of disturbance spread in the network from node i to j, leads to a comparative change in the stability of the table (Corman et al., 2020).

The problem of assigning buffer times to possible events in the metro network timetables can be formulated through meta-heuristic algorithms. The purpose of such an assignment is to calculate and allocate buffer times in such a way as to increase the overall benefit of the objective function to the maximum possible value (Han et al., 2014). The general relationship of the allocation problem in the Metro network could be described in this way the proposed options are numerical sequence c = 1, . . ., C is shown and, in this series, each alternative with a value of W_c and the causing profit P_c is located in a set by capacity D so that the total profit reaches the maximum possible value without exceeding the limitation of the metro network capacity (CHANG, C. & THIA, B. 2006). After the process of streamlining the train schedules, all the interval times are reduced to the minimum possible value to include the buffer time values. Therefore, the desired value of the buffer time $Baf_{i,j}$ could be calculated for each minimum interval time on schedule taking place on a small scale. In the combinational optimization algorithm, it is possible to assume an ordered couple such as $C = (W_c, P_c)$ through which the profit of the objective function can be obtained through sufficient detail through hierarchical priority methods (Salido et al. ., 2015). In this technique, the weight of the option (W_c) essentially has the same number that edges (distance among nodes) in the network. Consequently, according to the difference in the technical characteristics of the lines and the train fleet, the allocation of buffer times lined by consecutive events in different parts of the metro network is different and as a result, the total reserve times are reduced to different spaces. Nevertheless, one of its assumptions is that exceeding the upper limit of the reserve time of each line is not allowed (Goverde, 2010). Therefore, for each alternate c∈C, it is necessary to calculate the vector (W_c) for each entity in the set S so that $R_c \geq W_{c,s} \geq 0$. In this difference, $W_{c,s}$is a sentence through which the desired buffer value $R_c = Buf_{i,j}$is determined on the reserves time of Bs set. In this circumstance, every planned or unplanned event could be demonstrated as a member of s∈S and it is patterned by implanting the buffer times R_c in the altered schedule$T^{correct}$. And In this situation, $b_{i,j} = b_{i,j}^{minimum} + R_c$ could be adjusted to update the schedule tables and in this mode, the $T^{corr\acute{e}ct}$ value could be obtained. The entity value C in the calculations mentioned above by determining the distance journeyed by each train in the ultimate event for each part of the Metro timetable with the equation $W_{c,s} = t'_{last_s} - t_{last_{s'}}$ calculated separately for each prospect (Johnson et al., 2008). Because of the above details, the problem dimensions could be shown by the combination optimization technique as follows:

$$Max\ P_c.x_c \tag{20}$$

The above relationship shows the benefit of improving the running and moving capacity in the objective function of the model concluded the detriment parameter.

s.t

$$\sum_{c \in C} W_{c,s}.x_c \tag{21}$$

$$x_c \in \{0,1\}$$

The above relationship shows the limitation of the train movement route in the Metro network according to the weight of each edge (distance between stations) using a zero and one variable.

One of the important features of combinational optimization algorithms and meta-innovative approaches derived from them is that they fully incorporate an entity and use it in a heuristic method. From a mathematical point of view, a clear example of this is that if 5 minutes of reserve time is considered in the timetable of Metro trains, then the movement pattern cannot be applied through only a value of 3 or 4 minutes, and 3 And 4 minutes cannot be applied to the table at the same time. So, in a modified problem based on the combinational optimization method, a scheduling pattern for the movement of Metro trains is separated into many interval times in size$t_{i,j}$. In other words, for each decision, there will be two components $R_{c,}$ andl_c, where l_c indicates the length of interval time so that$l_c = R_c$. In the timetable of running trains, for each c∈C there is an ordered pair such as (c,f) where $0 \leq f \leq l_{c,}$ and hence the upper limit of each row in the timetable for each specific sample in the rail block C_E is equal to $\sum_f R_{c,f} = R_{c_E}$. In this case, $R_{c,f}$ is assigned to a specific rail route through the weight $W_{c,f}$ according to the inputs of the set S. Consequently, the following relationship can be attained:

$$Max \sum_c \sum_f P_{c,f} . x_{c,f}$$

s.t

$$\sum_c \sum_f W_{c,f} . x_{c,f} \leq A_s \tag{22}$$

$$\sum_c \sum_f R_{c,f} . x_{c,f} \leq R_c$$

$$x_{c,f} \in \{0,1\}$$

Equation 22 shows that for k+1 samples in the Metro network, the variable C can cover a numerical series of sentences through the last proposition f, which guarantees that the allocation of complementary times can increase the tables' stability. For example, in the time sequences of the Tehran Metro trains running for the fifth minute, the increase of the supplementary time for the fourth minute will have no effect, so finding the efficiency of $P_{c,f}$ seems more complicated. In the following, to measure the efficiency of the entire metro lines and complications caused by other factors in the metro network scheduling, non-passenger dispatch, and preservation breaks are also acquired from the larger ones in the table, and smaller numbers are derived from the damages caused by it. Passenger congestion and non-closing of doors can be partially mitigated through scheduling algorithms. Combination optimization algorithms are set in a dual manner, which connects the productivity of an entity in the "Activity- event" network to another entity in the same network according to the constraints (Yang et al., 2019). The use of combinational optimization techniques allows anticipating techniques to be used before calculating the total marginal profits to prevent the spread of disturbance in the rail network (Yang et al., 2018). The marginal efficiency of this n-th term for a specific capacity of type c is determined by the equation $n, p_{c,n} = g(n). p_c$to be meaningful and evaluate the performance. Therefore, if the relative sum of the normal value in terms of capacity c is equal to$\sum_{n=1}^{m_c} g(n) = 1$, then $g(n+1) \leq g(k)$ will be acquired (Li et al., 2017). In this case, despite the perfect conditions in the metro lines, and remaining assumptions for l_c According to the allowed capacity c for the metro network, the possibility of a real return to R_c in the network will be guaranteed. In other words, the efficiency of different parts of an entity in an indiscriminate network depends on the assumptions of the issue. Therefore, using equal variables and assigning value to the variable $p_{c,n}$in the numerical series n=1... l_c For both active entities the case that the weights are equal leads to the results of slow operation.

Approaches based on combinational optimization for allocating time reserves in a scheduling program can be seen as a comprehensive framework. Thus, in the first step, it is necessary to acquire the desired value for the travel time and train running between two stations i and j, and the benefits obtained from it according to the technical components of train i and train j. In the second step, the parameters of the problem are determined according to the effects of delay spread on all the events of the schedule table. The proposed values for train scheduling and movement depend on the amount of passenger demand. Among these, the highest priority is given to the safety of the train fleet in such a way that the greatest number of calculations for adjusting the timetables is spent to take safety considerations into account. Some of the most important rules for regulating timetables in the metro network are as follows (Kierzkowski & Haładyn, 2022):

- The maximum amount of time in a table for dispatching a particular train is when the next train in the dispatch sequences has a higher priority dispatch from the beginning points of the lines.

- The minimum interval time for the train's movement by a specific train in a schedule when departing from the origin or departing from stations along the rail route is set when the next train has a lower priority to be dispatched.
- The average travel time between two trains is equal in the timetable in the conditions where the dispatch priority is the same for both.

For passenger demands with dimensions variable according to capacity c, the scheduling of train movements between nodes i and j in the rail network can be determined by defining the value of R_c based on the technical components of the train. As an example, if we consider the maximum, average, and minimum values of 5, 4, and 3 separately for the timetable of Tehran Metro trains in terms of time interval, in this case, to evaluate the importance and priority of each chosen rail section in the timetable, three composite indicators are considered that can evaluate the priority of demand for the train's movement between nodes i and j, which is described as follows (De Martinis & Gallo, 2016).

- Classification of trains from the point of view of prioritization after the first dispatch for the second and later trains in the metro network is done according to the values determined for the intervals (5, 4, and 3) using the time interval adjustment relationships.
- The time required to complete the trip and movement on the railway route and to receive a specific train such as i at the destination before any unexpected event occurs, due to the existence of buffer times without increasing the speed of the train, has a relatively extensive amount of time. The maximum travel time for train j can be determined by the following sequences in the timetable. In this case, the lowest priority is determined in the timetable, and trains with higher priority are also determined.
- For the next trains in the progression of Metro train movement, if the Metro network suffers disturbance in the form of$\bar{J}$, in this case, it is assumed that the conflict condition in train movement is valid for scheduling, in this case, the buffer time to resolve the $\bar{J}$ an event can affect the next one as well. And so, if an assumed train in the Metro network experiences disturbance$\bar{J}$, then the next events will be the amount of delay$\delta_{c,o}$. In most cases, recovering and returning the network to normal conditions may cause delays in dispatching trains from the origin station.

To calculate the corner's stability and critical points in the metro network, the use of those operational standards that consider the highest industrial safety levels for trains is developed (Shang Guan et al., 2018). Using the locality value theory can be an appropriate reflection of the infrastructure manager's demands in the metro while covering the stability criteria. Although it seems difficult to monitor the quality of timetable entries for the Tehran Metro operator, however, other measurement indicators to determine the time priority of trains in the timetable's regulation require taking into account the amount of demand and the volume of passengers entering the stations and also the calculations are related to the metro line infrastructure and the conditions of crews and employees. Therefore, the regulation of Metro train timetables largely depends on the type of standard that is used to evaluate its stability (Samà et al., 2020). A look at the conducted studies shows that thermodynamic measurement units are one of the common tools to consider in multi-criteria decisions to determine the priority of train movement in the metro network, which is used to define the relative importance of each prioritization criterion. (Liang et al., 2019). Frequently, by using a balanced combination of criteria, they use a measure to calculate the efficiency of the plans and determine alternative times for the train's movement. So, in the first step, the value of c is determined for each m sample option, and the selected criteria of the table rows as c=1... C. It should be noted that based on the relations governing the theory of matrices, the values m=1... M is determined in column M.

The value of M in the scheduling matrix can be obtained by using the following equation:

$$\varrho_{c,s} = \frac{\alpha_{c,n}}{\sum_{c\in C}\delta_{c,n}} \tag{23}$$

The thermodynamic indicators of the Metro network for each criterion can be determined by using the following equality:

$$e_n = -\frac{1}{\ln C} \cdot \sum_{c\in C}\delta_{c,o} \cdot \ln \delta_{c,o} \tag{24}$$

The weight acquired for each criterion is determined as the normal coefficient of the mean using the equation number 25:

$$e_n = -\frac{1}{\ln C} \cdot \sum_{c\in C}\delta_{c,o} \cdot \ln \delta_{c,o} \tag{25}$$

The acquired weight for each criterion is determined as the normal coefficient of each passenger movement average using the following relationship:

$$W_q = \frac{1-e_n}{\sum_{n\in N} 1-e_n} \tag{26}$$

Finally, to determine the train travel time i on the rail route, the sum of the weights of the obtained values for each criterion can be obtained using the following equation:

$$o_c = \sum_{n\in N} W_n \cdot \delta_{c,qn} \tag{27}$$

So, we will have:

$$Z(x_0) = \sum_{i=1}^{N} W_i \mu_i(x_0) \tag{28}$$

where W_i or $\overline{W}_i$are non-deterministic parameters of the model according to the objective function values and μ_i are uncertain and deterministic functions of the type dependent on x_i (decision variables) Therefore, Bertsimas and Sim's approach can be used to form a stable counterpart problem (Wentges, 2022). Therefore, since the uncertain values are within the confidence interval $\sigma \pm 3\sigma$ from the cruel, the uncertainty set for the parameters of the objective function $\overline{W}_i$ can be obtained from the following equation (Bertsimas & Sim, 2004):

$$V_i = \left[\overline{W}_i - 3\hat{T}(\overline{W}_i), W_i + 3\hat{T}(\overline{W}_i)\right], \quad i = 1,2,\dots,n \tag{29}$$

The above relation $\hat{T}(\overline{W}_i)$ is obtained from the following equation:

$$\hat{T}^2(\overline{W}_i) = \frac{1}{R}\frac{1}{R-1}\sum_{j=1}^{R}\left(W_{ij} - \overline{W}_i\right)^2, \; i = 1,2,\dots,n \tag{30}$$

The above equation $\hat{T}(\overline{W}_i)$ is obtained from the following equation:

$$\hat{T}^2(\overline{W}_i) = \frac{1}{R}\frac{1}{R-1}\sum_{j=1}^{R}\left(W_{ij} - \overline{W}_i\right)^2, \; i = 1,2,\dots,n \tag{31}$$

The values of $\sum M$ and $\sum \theta$ can be simulated using input/output data; Therefore, their values will be known and definite, so only β is a function of x0, hence the structure of the objective function depends on the conditions and status of β, which is determined as follows:

$$\beta = \varphi^2 R_M(\vartheta, x_0) \tag{32}$$

In the above relationship, $R_M(\vartheta, x_0)$ is the correlation function, and $\varphi^2$is the variance of the simulated points. Now, considering m and as the lower and upper limits of the decision variables, the nominal planning model will be as follows:

$$\begin{aligned} &\text{Min } C_0 \\ &\text{s.t} \\ &\sum_{i=1}^{N} \widetilde{W}_{ie}\mu_{ie}(x_0) \le C_{ie}, e = 0,1 \\ &m \le x_0 \le v \end{aligned} \tag{33}$$

The index e is used to display the relationship between the objective function (e=0) and the constraint (e=1) and$C_1 = FR$. According to Bertsimas and Sim's method, the stable counterpart of the model is written as follows:

Min C_0

s.t

$$\sum_{i=1}^{N} \widehat{W}_{ie}\, \mu_{ie}\,(x_0) + \max T \cup \{t\}\ |T \subseteq J|\,, |T| = \Delta, t \in J\backslash T \left\{\sum_{i=1}^{N} W_{ie} z_{ie} + (\Delta_e - \lfloor\Delta_e\rfloor)\widehat{W}_{ie} y z_{ie}\right\} \leq C_e\,, e = 0,1 \quad (34)$$

$$-z_{ie} \leq \mu_{ie}(x_0) \leq z_{ie}\,, i = 1,2,\dots,n\,, e = 0,1\,, m \leq x_0 \leq v \quad (35)$$

$$z_{ie} \geq 0\,, i = 1,2,\dots,n\,, e = 0,1 \quad (36)$$

In the above, $\widehat{W}_i$ Is the maximum value allowed to alteration in the limited variable equal to $3\hat{T}(\overline{W}_i)$ and J is the set of non-deterministic coefficients of the objective function. A parameter Δ is not necessarily expressed as an integer and therefore can take values in the range $[0, \lfloor J \rfloor\}$. The role of this parameter is to transform the robustness of the approach used in the research against a certain level of conservatism of the justified set of answers. Considering the opposition level$\Delta = (\Delta_0, \Delta_1)$, the second part of the first constraints of the problem will be equal to the following optimization problem:

Min C_0

S.t

$$\sum_{i=1}^{N} \begin{matrix} o_{ie} + \Delta_{eze} \leq C_e\,, e = 0,1 \\ y_e + o_{ie} \geq \widehat{W}_{ie}\,|\mu_{ie}\langle x_0\rangle| \end{matrix}, i = 1,2,\dots,N\,, e = 0,1 \;\; o_{ie} \geq 0\,, i = 1,2,\dots,n\,, e = 0,1 \;\; z_e \geq 0, e = 0,1 \quad (37)$$

Therefore, according to preliminary principles and based on Dugan's theory, the optimal solution of the problem is equal to the optimal solution of the main problem. Considering $z_{ie}|\mu_{ie}\langle x_0\rangle|$ The final model of the problem will be as follows:

Min C_0

S.t

$$\sum_{i=1}^{n} \overline{W}_{ie}\big(\mu_{ie}(x_0)\big) + \sum_{i=1}^{n} \;\; o_{ie} + \Delta_e z_e \leq C_0 \; e = 0,1 \;\; y_e + o_{ie} \geq \widehat{W}_{ie}\, z_{ie}\,, \qquad \leq z_{ie}\,, i = 1,2,\dots,N \quad (38)$$

$$i = 1,2,\dots,n\,, e = 0,1 \; - z_{ie} \leq \mu_{ie}\,(x_0)$$

$$o_{ie} \geq 0, \qquad i = 1,2,\dots,n \;\; e = 0,1$$

$$z_{ie} \geq 0, \qquad i = 1,2,\dots,n \;\; e = 0,1$$

$$y_e \geq 0 \qquad e = 0,1$$

Finally, the structure of the obtained problem depends on the type of objective function and the intensity of correlation between the problem elements. For the correlation function of the presented model, which is linear, and for Gaussian and exponential correlation functions, this structure will be non-linear. Since the Gaussian correlation function was used for stochastic programming models in the case study, the structure of the obtained mathematical programming problem will be non-linear. Also, according to the findings of previous studies, the computational complexity of the mathematical programming model obtained from the random pattern such as n is the number of points sampled from the acceptable solution space. Therefore, a meta-heuristic approach is used to solve the robust problem obtained. For this purpose, the meta-initiative approach of particle swarming has been used due to its simplicity, high accuracy, fast convergence, and dependence on a small number of parameters.

4. Case Study

The method described in the present research, which was formed based on the combination optimization approach, can be subjectively used in the metro network. Line 1 of Tehran Metro is an intra-city rail network that, with 29 stations and 38 kilometers, connects Tajrish station in the north of Tehran to Kahrizak station and Shahre Aftab in the south of Tehran. A part of this route between the stations of Tajrish to Shush is designed underground and other stations between Shush to Kahrizak and Shahr Aftab in the south are designed on the ground. More than 90% of the metro network traffic is located between the stations of Tajrish and Shahe Rey. Also, in some sections of Line 1 between Panzdeh Khordad station and Shahid Haqqani, the traffic load and passenger demand are much greater than in other sections. During the last decade, the timetables of the Tehran Metro network have been studied many times by researchers. Some of the results of these studies show that the peak passenger hours were between 17:00 and 19:00. In the overriding time, one of the operational strategies of Tehran Metro to meet passenger demand has been to plan to reduce the interval times of trains by 3 minutes for peak hours in the timetable, especially towards Kahrizek. Trains enter the Fathabad terminal after passing the rail sections along the rail route (50 to 350-meter intervals according to the signals and signaling of the line) and unloading passengers at the destination stations. Therefore, the scheduling tables should be designed in such a way as to ensure the correct flow of trains based on the time sequence and priority of dispatch from the station of Tajrish to Kahrizek. Usually, a subway network consists of several single-track blocks, as shown in the following image. Rail lines are divided into sections called blocks. To maintain the safety margin and prevent accidents, no two trains must be present in the same rail segment at the same time. In the Tehran Metro network, usually, the distance between two stations or both signals at certain distances of less than 500 meters is considered as a block. Tehran metro network has several terminals. Terminals in the Tehran Metro are usually located at the beginning and termination of Metro lines and can receive and park several trains at the same time. Based on the topology of Tehran Metro, for inner-city sections (except terminals), each station can accept only one train at a time in any direction and at any time. Trains are waiting in Tehran metro terminals, and this means that there is no systematic and comprehensive timetable to manage the capacity and navigate possible disturbances. However, predetermined stops are embedded in the tables, which can act as a buffer time to manage unexpected events. Also, it is not possible to overtake the inner-city lines of the Tehran Metro. The movement of trains in the Tehran Metro is coordinated by an operation control center in the command center section. The train movement is controlled by the ATS at any time and the inspection and maintenance stops at each terminal. A stock time refers to the minimum time required for each train at the terminal to check the correct operation of the train and return in the opposite direction. A train that reaches the Fathabad terminal in the Tehran Metro must alight all passengers at the previous station (Shahre-Rey) and park in the built-in sheds for maintenance and repairs. In the Tehran metro network, the first-in-first-out (FIFO) policy is used for dispatching at the terminals.

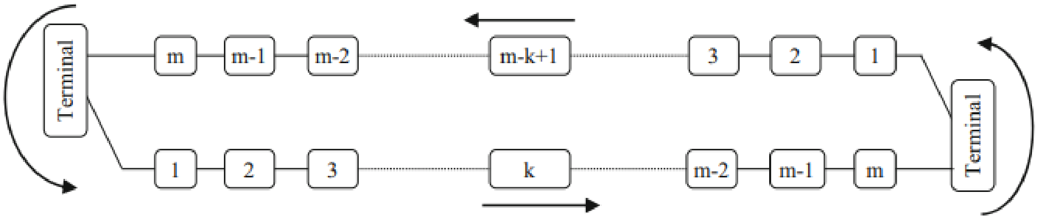

Figure 10. Circular movement Rounded trains in the Tehran Metro lines

The operation of metro lines by parts such as the dispatch time, the duration times of each block by the train, the duration times of the train stops at the station, the duration times of rest and parking of the train at the terminal, the process of passengers entering the stations and the process of boarding passengers, etc. It is explained. Trains start their trip according to the scheduled interval time. The movement process starts with the event of dispatching the train from the origin and trimmings with the event of reaching the destination. The received train stops at a station to carry out the process of boarding and alighting passengers at least for the mentioned period. Congestion of passengers at a Metro station may increase the duration times of the train stop. So, the length of the stopping time for subsequent stations may increase as a result of congestion or train capacity limitations. The stopping time of each train at the stations must be greater than a certain lower limit and less than a certain upper limit. The number of trains assigned to each station at any moment should not exceed the capacity of that station. Thus, the number of train services decreases during off-peak hours and increases again during peak hours. The demand for intra-city rail systems is considered time-dependent or dynamic with probabilistic distributions. Here, it is assumed that the scheduling interval times are divided into some Headways with a certain length. Therefore, t represents the period and T represents the total number of periods. Passengers start visiting the platform at the beginning of the period (t=0) and can enter the stations until the beginning of the last-mentioned period (t=T). Reducing the interval time leads to a reduction in the travel time of passengers on the one hand and a reduction in the train traffic rate oppositely, which is considered a positive effect and the second a negative effect. Finding the optimum distance to minimize the waiting time of passengers at the platform and to reach the predetermined transportation rate for working days of the week (Saturday to Wednesday) is a problem that needs to be investigated and solved by systematic methods. Here, the decision variable is determining the distance in different time intervals with different rates of passengers arriving at stations and trains. The random variables for the simulation model are the duration of a block by a train and the number of passengers arriving at a station in a certain time interval. These two random variables are the factors of uncertainty in the simulation model. The distribution function of train breakdowns is not included in the simulation model and it is assumed that train breakdowns or any event that leads to small delays in the scheduling of trains will affect the block time. As explained earlier, major disturbances lead to the re-planning of train models and the establishment of disturbance management methods in the Metro. As mentioned earlier, line 1 of the Tehran Metro has 29 stations and more than 350 passenger train departures during the day. All stations have two platforms. The first passenger dispatch is at 5:00 a.m. and the last dispatch is at 11:00 p.m., and the process of transporting passengers continues until noon. In the case study of this research, the capacity of each intermediate station is assumed to be numerically large. To determine the distribution function of non-deterministic parameters, the data provided by the Tehran Metro Operation Company was used, so Based on this, regarding the arrival and departure times of trains to the stations along the rail route for every 400 passenger trips, the normal probability distribution function for the travel time The blocks are satisfied at the confidence level of α=0.05 and regarding the passengers arrival to the stations, the information provided is the average number of passengers arriving the stations in different time frames. Also, based on the opinion of Tehran subway traffic control experts, the Poisson distribution function with a time-dependent slope has been used for passengers entering the stations on non-holiday days of the week for different periods. Therefore, some common assumptions are as follows:

- All trains and passengers must reach their destination within the framework of the usual schedule.
- The periods in a day are divided into classifications such as the continuation table.
- Planning intervals are divided into smaller intervals time in seconds.

The average speed of trains between consecutive stations along the railway track is constant.

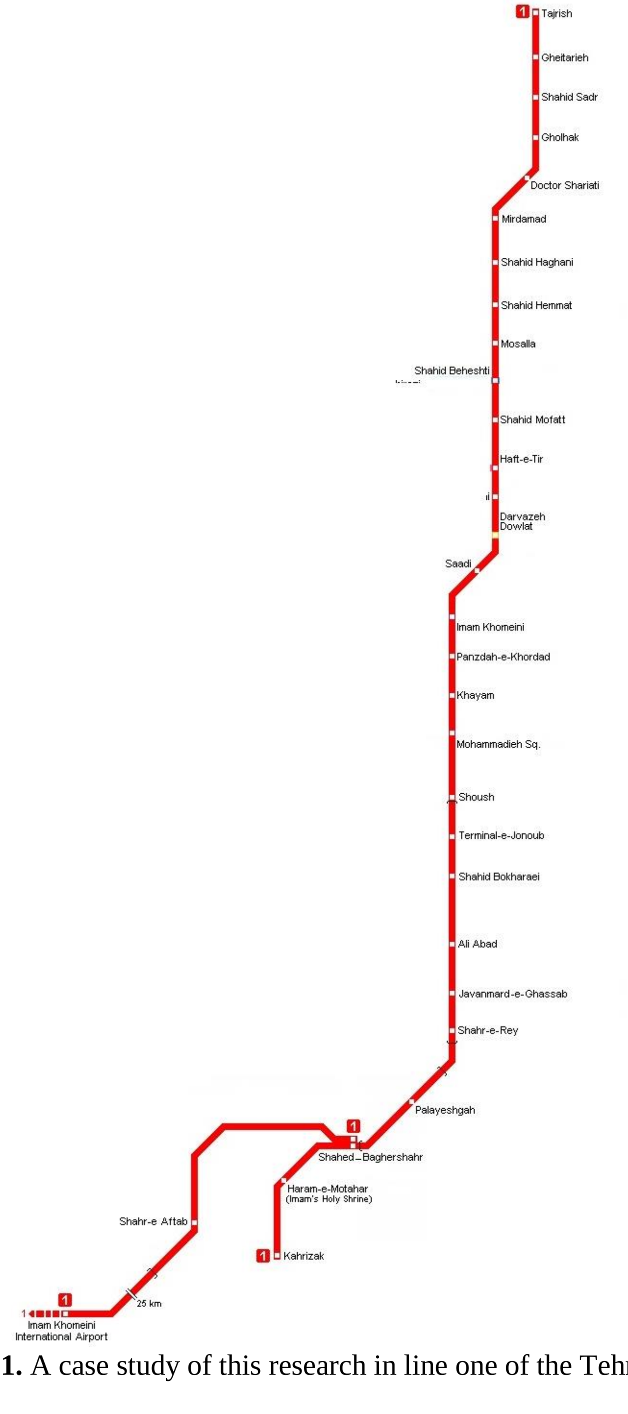

Figure 11. A case study of this research in line one of the Tehran metro

The average stopping time of trains at each station of Tehran Metro Line 1 is at least 30 seconds. The maximum operating speed of trains on line one of the Tehran metro is 80 km/h, which is reduced to 45 km/h due to the reduction of speed to stop at the stations between the rail route and the termination. The interval considered for the intervals is between 3 and 10 minutes. The simulation model has been developed and implemented in CPLEX software, and to verify its values, the research model has been developed in a modular way, and therefore the results have been evaluated by assigning fixed values for random variables.

Table 2. Classification of periods in the schedule tables of Tehran Metro line one

Row number	The time on a non-holiday working day	The Headway of the interval time in the relevant floor (minute)
1	5:00 – 7:00	$3 \leq x_1 \leq 5$
2	7:00 – 9:00	$3 \leq x_2 \leq 4$
3	9:00 – 11:00	$4 \leq x_2 \leq 6$
4	11:00 – 13:00	$3 \leq x_3 \leq 5$
5	13:00 – 15:00	$4 \leq x_3 \leq 7$
6	15:00 – 17:00	$5 \leq x_4 \leq 7$
7	17:00 – 19:00	$3 \leq x_4 \leq 4$
8	19:00 – 21:00	$5 \leq x_5 \leq 8$
9	21:00 – 23:00	$7 \leq x_5 \leq 10$
10	21:00 – 23:00	$10 \leq x_5 \leq 11$

To validate the simulation model, the T-test method was used in SPSS software. Also, the average delay time of each train in the total trips of a non-holiday working day has been used as one of the inputs of the simulator. Here, the zero assumption is the same as the outputs of the simulation model with the real data of the Tehran Metro, for this resolution, the simulation output was acquired from the number of 15 independent simulation runs for the distances used in the Tehran Metro and with the confidence coefficient α=0 05. The null hypothesis is accepted. Also, to perform the appropriateness test, a random simulation method was used for the average travel time of passengers and the traffic rate of trains on the Metro route with a value of 100 from the reasonable answer space using a simple random sampling method. Since random simulation algorithms often do not perform extrapolation with high accuracy, so, the corner points of the set of reasonable answers have been considered. The simulation model has been calculated up to 15 times for each point of the equitable set of answers, and consequently, the model has sufficient fit with a confidence interval of α=0.05. To find the optimal robust solution considering the conservatism level ζ, the linear programming model for the robust complement was obtained. To calculate the stable optimal solution, the particle swarm algorithm was used, and in this case, the number of particles was considered equal to 20. The coefficients of the parameters of the particle swarm method include the size of each particle (S) and the speed coefficients that mean moving towards the local optimal point's o_1 and the overall optimal point o_2. According to the previous studies regarding the implementation of the particle swarm algorithm for entities that have values above two, it shows a better performance. These studies also show that the value of each particle depends on the type of problem and often it is determined through trial and fault (Wu et al., 2019). Alternatively, the increase in the number of particles produces the possibility that they are caught in the local optimal trap, and as a result, the speed of the algorithm decreases. Therefore, in the scheduling problem of Tehran Metro lines, the selected value for coefficients o_1 and o_2 is approximately 3. Also, the investigations show that the value of 3 for the particle size, shows a positive performance and reduces the possibility of getting caught in the local optimal setup.

The considered values for each of the coefficients are shown in Table No. 3, which is related to the optimal solution and response variables. $Headway_1$ shows the distance of trains moving from south to north (Kahrizak to Tajrish) and $Headway_2$ also shows the distance of running trains from south (Kahrizak) to north (Tajrish). ζ_0Is the level of conservatism of the objective function and $\zeta_1$is the level of obscurantism of the restrictions related to passenger capacity. Z_1 is related to the first response variable for the average waiting time of passengers in seconds and Z_2 is related to the second response variable and the average rate of trains on the Metro route in percentage. The first line of this table, obtained for the level of conservatism (0, 0), represents the nominal answer to the problem. The next rows show the optimal values of train departure interval time and average passenger waiting time as well as traffic rate for different levels of reaction. As can be seen, with the increase in the level of conservatism, the values of the optimal distance and the average waiting time and shipment rate increase. The rate of modification in the values of passenger waiting time and transportation rate decreases with the increase in reaction level, and the answers are more stable in the face of possible alterations. Some of the advantages of the method presented in this research are as follows:

- Using the Minimax approach to improve the stable usage schedule, which is closer to the concepts of stable optimization than other methods.
- The method offered in the present study can be used for problems with limitations and it makes it possible to include stability in the feasibility of the problem along with stability in the optimality, which is considered an important advantage.
- Due to the high flexibility simulation tool to model the problem, the presented method needs to consider fewer simplifying assumptions and, in this respect, has an advantage over mathematical programming methods. Compared to model-based optimization methods, this method has the advantage of fewer executions of the simulation model and provides a favorable view of the structure of the objective function.

Table 3. Parameter values used in the case study

Problem parameters	**Number of stations**	**Number of trains**	v_{ave}	v_{max}	**T**	dW_{min}	rec_{min}	**FR**	C^{max}
Values	29	25	45 km/h	80 km/h	7	35	95	48%	1800

Table 4. The results related to the robust optimal values for the decision variables and the reaction variables for different conservatism

Optimal Headway (in seconds) $Headway_1(s)$, $Headwy_2(s)$	**The level of conservatism $\xi = (\xi_0, \xi_1)$**	**The mean of the reaction variables $Z_1(s)$, $Z_2(\%)$**	**The standard deviation Reaction variables**	**Minimum variables Reaction**	**Maximum variables Reaction**
H_2=(630, 180,270,195,490) H_1=(570,198,273,212,289)	$\xi = (0, 0)$	Z_1= 225 Z_2= 71	Z_1=6/2 Z_2=5	Z_1=201 Z_2=65	Z_1=239 Z_2=66
H_1=(740, 243,290,220,560) H_2=(600,220,359,298,635)	$\xi = (15, 15)$	Z_1= 254 Z_2= 72	Z_1=7/2 Z_2=9	Z_1=208 Z_2=68	Z_1=248 Z_2=73

Optimal Headway (in seconds) $Headway_1(s)$, $Headwy_2(s)$	**The level of conservatism $\xi = (\xi_0, \xi_1)$**	**The mean of the reaction variables $Z_1(s)$, $Z_2(\%)$**	**The standard deviation Reaction variables**	**Minimum variables Reaction**	**Maximum variables Reaction**
H_1=(702, 256,311,256,690) H_2=(502,216,395,253,793)	$\xi = (25, 25)$	Z_1= 267 Z_2= 79	Z_1=8/7 Z_2=6	Z_1=239 Z_2=72	Z_1=256 Z_2=77
H_1=(702, 283,259,261,786) H_2=(605,265,482,298,852)	$\xi = (35, 35)$	Z_1= 279 Z_2= 82	Z_1=8/5 Z_2=6	Z_1=240 Z_2=78	Z_1=266 Z_2=80
H_1=(725, 296,400,280,890) H_0=(700,256,489,385,814)	$\xi = (45, 45)$	Z_1= 290 Z_2= 84	Z_1=7/4 Z_2=4	Z_1=261 Z_2=80	Z_1=268 Z_2=80

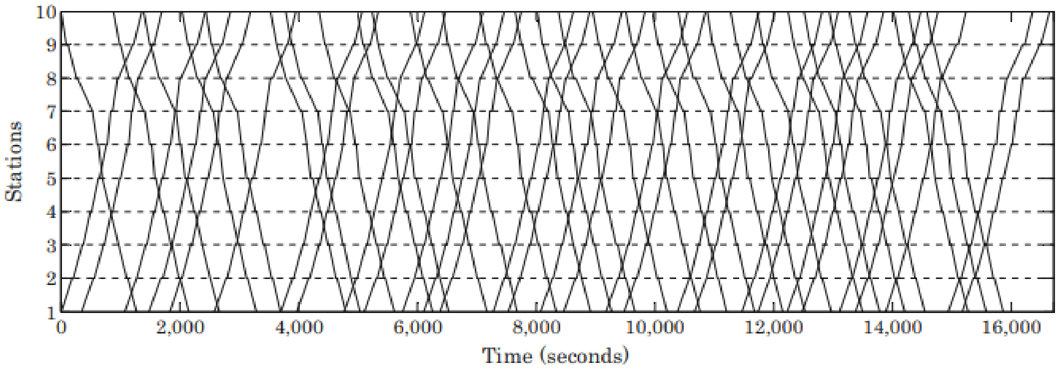

Figure 12. Diagram of the best solution obtained through the particle swarm algorithm

4-1. Case study number two, part of Tehran Metro Line 4 between "Kalahdoz" station and "Eram Sabz"

In the present research, to determine the problem and the second case study, a part of line 4 of the Tehran metro between "Kalahdoz" station and "Eram Sabz" was investigated. According to the traffic plan of line four with the current time interval, 31 trains have been allocated to this line. The minimum time interval of trains on this line is 2 minutes. The active trains in this line are of AC, DC, and DC+ series depending on the type of grade and elevation of the line and correspondingly operate at different speed profiles. In this line, the origin and destination stations have terminals. This issue is shown in Table 9. Also, in this line, the terminals are of the yard area type and the shunting of trains could be covered independently of the main line or continuously with it. On the other hand, due to the insufficient number of trains, it is not possible to fully cover extraordinary dispatches in the modeling of Tehran Metro line four. Thus, to evaluate the level and the frequency of disruptions and unpredictable changes in the timetables. It is assumed that the impact of the disturbance on the size of congestion and passenger behavior and the number of ready trains in areas other than the terminals of line four will be known. In the case study of the current research, a part of line 4 of the Tehran Metro has been investigated using AC100, DC, AC300, AC500, and DC+ trains. The detailed planning of the rail route between Shahid “Kalahdouz” and “BImeh” stations is shown in Figure 13. As seen in this figure, the intersection stations of line four are shown with lines six, two, three, one, and seven. At these intersecting stations, passengers could transfer between line four and other lines of the Tehran metro. More detailed information about intersection stations is given in Table 9. To optimize the model, the time windows took place at 9:00 –12:00. It is also assumed that a disturbance had happened between the evening hours of 9:50 and 10:20 during the signal blocks of “EbneSina” to “Ferdowsi” stations. To investigate more closely, it is assumed that the disruption led to the complete blockage of the route, and the routes were out of reach of the vehicles. However, it is presumptuous that the movement between the stations of the fourth line and other interchange stations in the intersecting lines will be possible until the end of the disruption.

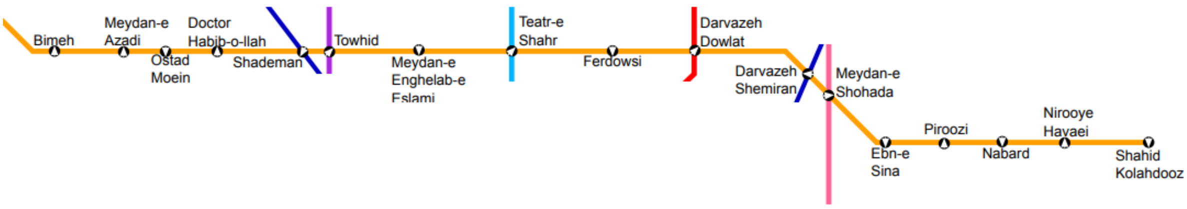

Figure 13. The metro network was included in the case study of “Klahdooz” to “Bimeh”

Table 5 shows the information and the optimized parameters of line 4.

Table 5. Optimization information in lines 4

Row	The title of the parameter	a numerical value (second)
1	The minimum interval time for trains	150
2	The maximum interval time of trains	600
3	Minimum stop time of the train at the passenger platform	30
4	The time required to retrieve the table	4200
5	Maximum delay time	2100

In this case, to solve the problems by using the mixed integer linear programming method, it is necessary to determine the passenger demand for all the events of the state space and finally check the limitations of the model. The data related to passenger demand is obtained based on the outputs of the Tehran Metro comprehensive traffic system. The peak hours of travel in Tehran metro line four are in the morning and evening hours, which for this study is a part of the service period between the hours of 9:00 and 12:00. Based on the priorities of the comprehensive traffic system, the traffic amount of some stations of the 4 line such as NirooyeHavaei and Ebneina is assumed to be of little importance. This is due to the volume of passengers and the waiting time of the line 4 for the ease of calculation of the congestion variable. However, the capacity of A passenger is assumed to have constant and fixed service delivery times. Some traffic parameters of Tehran Metro Line 4 for a normal month of the year are shown in Table 6:

Table 6. Some traffic parameters of Tehran Metro Line 4 for a normal month of the year

Average delay of each passenger trip (seconds)	Total delay time (in seconds)	Total number of delayed trips	Total number of canceled trips	Total number of extra passenger trips	Total number of planned trips	Total number of planned trips
1/03	5370	28	18	31	11133	11151

source: Tehran Metro Comprehensive Traffic System

Table 7 shows the demands of the origin, the stations along the route and the destinations of line 4 based on the outputs of the traffic system:

Table 7. Requests for point of origin, stations along the route, and destinations of Line 4 of Tehran Metro between the hours of 9 am and 12 pm

Statins \ hours	9-10	10-11	11-12
Shahid Kolahdooz	21773	28348	35798
Nirooye Havaei	35859	43816	53472
Nabard	36936	37106	48567
Piroozi	17166	20887	25451
Ebn-e Sina	24215	30308	40947
Meydan-e Shohada	36308	42282	52861
Darvazeh Shemiran	8733	8141	7621
Darvazeh Dowlat	16861	15358	13959
Ferdowsi	24136	31271	17504
Teatr-e Shahr	34389	25319	17577
Meydan-e Enghelab	67741	44894	32153
Towhid	31614	26683	21653
Shademan	25814	25481	25229
Doctor Habib-o-llah	26214	21210	20604
Ostad Moein	46263	28227	21480
Meydan-e Azadi	62577	71180	84321
Bimeh	78604	60807	51638

source: Tehran Metro Comprehensive Traffic System

The optimization of waiting time and passenger capacity was implemented in a case study of Tehran Metro Line 4 as a small-scale 296 km rail network. The simulation results show that just by reducing the number of trains and as a consequence passenger dispatches, the computational complexity is reduced to a significant extent. As a result, relatively favorable effectiveness is achievable in the conditions of line congestion, especially in intersection stations.

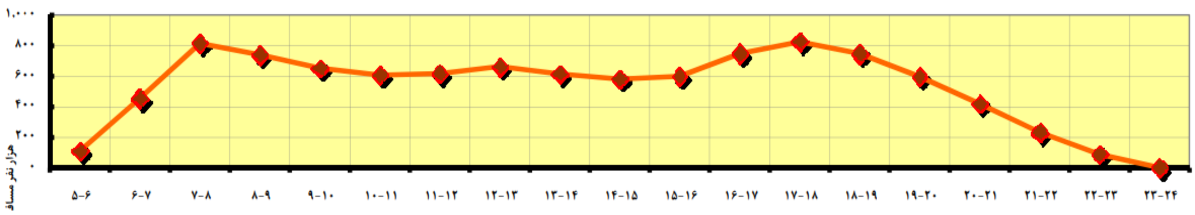

Figure 14. Distribution of passengers during the operation hours of Tehran Metro line 4 (Tehran Metro Comprehensive Traffic System, 2024)

Optimization passenger-oriented models were evaluated based on two objective functions with different weights. In these functions, cancellation of passenger movements due to disturbance is $\Omega_{canceal} = 100$, and for other situations such as $\Omega_{canceal} = 1$ delay and $\Omega_{sc} = 4200$.

Table 8 shows the process of canceling passenger dispatches and train delays in implementing schedules and train movements in Tehran Metro line 4.

Table 8. Train cancellation and delay process

delay	cancellation	
2100	18	$\Omega_{sc} = 0$
4208	18	$\Omega_{sc} = 4200$

Table 9 represents the maximum amount of passenger congestion over line 4. In the large number of stations, there's been a decreasing rate of traffic congestion. This is especially noticeable in Ferdowsi and Darvaza Dolat stations. On the other hand, it can be seen that there is a significant increase in passenger congestion at the "Tawhid" and "Shadman" stations. Table 13 depicts the schedule optimization for both objective functions in the model. The reduction of congestion value at $\Omega_{sc} = 4200$ level can be explained using additional DC trains. After the disruption; An AC500 and DC+ train will be injected into the line with a slight delay, thereby ensuring that the number of passengers at stations along the route is reduced in the shortest possible time.

Table 9. The maximum possible amount of crowding at each station according to the passenger load

The weight	Shahid Kolahdooz	Nirooye Havaei	Nabard	Piroozi	Ebn-e Sina	Meydan-e Shohada	Darvazeh Shemiran	Darvazeh Dowlat	Ferdowsi	Towhid	Shademan	Habib-o-llah	Azadi	Bimeh
Ω_{sc}=0	0/104	0/042	0/23	0/143	0/144	0/177	0/264	0/362	0/191	0/151	0/183	0/288	0/167	0/254
Ω_{sc}=4200	92	0/28	0/193	0/145	0/147	0/161	0/254	0/105	0/105	0/202	0/247	0/288	0/137	0/244

In Figure 15, each continuous line in the diagram shows the level of congestion of a station on line four of the Tehran Metro during the duration of operation. The dashed lines in the graph indicate the beginning and end of the disruption, according to the levels of rush hour traffic in the waiting time. Because many passengers cannot reach their destination due to the effects of the disruption, they use another means of transportation to their destination. For example, passengers with congestion caused to happen while waiting for a train at "Midan Azadi" station to reach "Eram Sabz" station should not continue their journey to get from their destination by train until the disruption ends.

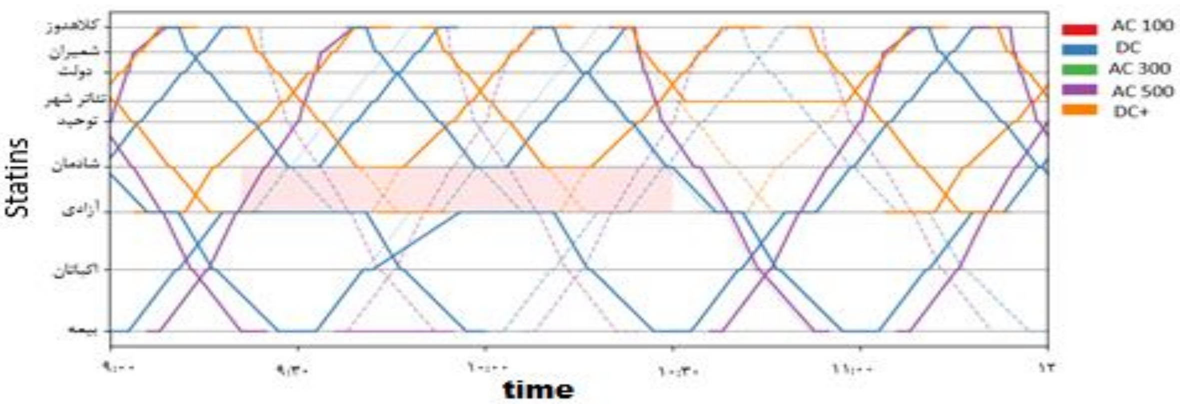

Figure 15. A timetable for traffic disruptions, in a case study ($\Omega_{sc} = 4200$)

A case study of Tehran Metro line 4 provided the possibility to evaluate the effectiveness and practicality of the rail transport fleet. Based on the direction of a comparison between the theoretical models for optimizing the travel and movement of trains in the conditions of disruption and practical models. The results show that passenger-orientated models for intersection stations lead to providing more favorable solutions. However, in terms of operation, timetables require a considerable amount of cancellation of passenger journeys due to insufficient exit points of trains and the obstruction of railway parts in the disturbed line. Therefore, providing models that can face the cancellation of passenger travel provides appropriate strategies to face the disruption. This model in which ready trains are located along the route and in appropriate buffer locations improves the overall performance of trains on subway lines during disruptions. It provides a basis for trains to be used for control measures at a later stage. By using this method, instead of dealing with a long delay for each station, especially switching between stations. It is possible to use the buffer values presented in the tables and intersecting stations, which seems sufficient to minimize the total delay. In this research, change levels of computational complexity, and optimization models have been implemented on a smaller scale. The simulation results show that the model presented in this research can significantly reduce the maximum level of traffic congestion in the timetable. Using the model presented in this research, a proportional balance can be generated between the deviation from the initial schedule and the level of passenger congestion at the stations. The achievements of the study show that using different integer programming approaches in passenger-orientated modes is effective enough for long computing times and small networks. Hence, it could occur that the integer programming may not have the time to retrieve the timetables and reorganize the efficiency. One of the alternative methods in this case can be the neighborhood search method. To optimize the efficiency of the many components included in the model. In this way, the neighborhood search algorithm could minimize the space for finding the most optimal solution. The assumption is that the recognition operator searches for the optimal solution in the neighborhood. The model needs an initial solution that, in addition to determining the sequence of events on the schedule, enables minimizing the deviation from the initial schedule. The acceptability of mathematical relationships in complex integer programming to cross-sectional robustness of events is one of the strengths of this method. The overall results of the case study show that the small-scale neighborhood search method is faster than the mixed integer programming method. The preliminary results of the case study show that the neighborhood search method could find an answer in 523 seconds in the short set. This has been done while the mixed integer programming method obtained a justified solution in the solution set in about 11 hours.

5. Conclusion

A look at the history of the research conducted in the field of the Metro shows that in the category of optimizing the characteristic uncertainty in the demand of Metro passengers, it has been underestimated. This issue may lead to overcrowding of the passengers and the occurrence of random disturbances and as a result cause delays in planning the train's schedule. In this study, the Tehran Metro was studied, taking into account the uncertainty in demand and the duration time of the blocks. To determine a stable schedule that can absorb small disturbances, a simulation optimization tool has been used. Metro model considers the limitations caused by the type of rails (single rail or multiple lines), the speed of the trains, the stop time limit at the stations, the limit related to the head distances, the limit of the number of lines in each station, and the safety limits and considerations. It was simulated in CPLEX software. Using the particle swarm algorithm, some points in the acceptable solution space were sampled, to estimate the relationship between the input (distances) and the output of the simulation model (passenger waiting time and average train rate) from random simulation, one for the objective function and the constraint were used and their suitability was evaluated. Also, Bertsmias and Sim's stable optimization method was used to formulate the stable counterpart problem for functions. The obtained stable peer model is solved using the particle swarm algorithm. For a case study, Line 1 of Tehran Metro was simulated, and stable optimal solutions for working days (Saturday to Wednesday) in 9 time periods with different demand rates for passengers were obtained. For future studies, the presented development is suggested to solve the scheduling problem in the entire network and to consider the connections and correlation between the lines. It is also suggested to use other pseudo-models such as regression pseudo-models' neural networks and other artificial intelligence techniques and compare the results obtained from the particle swarm method.

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Declaration of Competing Interest

The author has no conflicts of interest to declare that are relevant to the content of this article.

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