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Analyzing the Performance of Listed Companies with Specific Data, an Application of Data Envelopment Analysis (DEA) with a Network Structure

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Abstract
This paper explores the efficiency of listed companies using data envelopment analysis (DEA) with network structure. The DEA method, as a non-parametric technique, provides the possibility to evaluate the efficiency of decision-making units (DMU) based on multiple input and output data. In this paper, DEA approach with a network structure presents a more complex model that allows considering the internal structure of production processes and the transmission of variables during different stages. In this regard, specific data related to listed companies for a specific period are collected and used. This data includes various inputs such as total assets, total equity of capital owners, allocable profit, registered capital, total assets, equity to asset ratio and capital to asset ratio. A two-stage network is addressed in this paper. First, a two-stage network according to the production feasibility set is modelled, and then, according to the modified SBM structure, the efficiency of the first and second stages and its total efficiency are obtained. The obtained results contribute to a deeper understanding of the factors affecting the efficiency of companies, their weaknesses and strengths, and strategies to improve efficiency. This research can provide valuable insights to company managers, investors, and other stakeholders to make better decisions.

Keywords: Data Envelopment Analysis, Efficiency, Network Structure

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1. Introduction

In today's dynamic economic world, efficiency and productivity are among the most important determinants of the survival and success of companies. Performance measurement and improvement are of particular importance, especially for listed companies whose performance is continuously evaluated by investors and stakeholders. Traditional approaches to performance measurement often view companies as black boxes, considering only final inputs and outputs. However, in reality, companies are complex systems made up of multiple interconnected sections and activities. Ignoring the internal structure of the company and the relationships between different sections can lead to incomplete and misleading results in evaluating efficiency. For example, in listed companies with specific data, activities such as research and development or marketing may play a key role in creating values. However, traditional DEA methods are often unable to accurately assess the impact of these activities on the company's overall efficiency. Analyzing the efficiency of companies by considering their network structure can provide a more comprehensive view of the company's performance. This approach helps to identify the strengths and weaknesses in different sections of the company and provides the possibility to provide more targeted efficiency improvement solutions. The use of advanced analytical approaches such as DEA with a network structure can bring significant benefits, particularly for listed companies with specific data, which require accurate and transparent evaluation. The purpose of this paper is to present a comprehensive framework for analyzing the efficiency of listed companies with specific data, using data envelopment analysis (DEA) with a network structure. In this regard, while introducing appropriate network DEA models, it will be discussed how to apply these models to real data of listed companies and interpret the results obtained from it. Also, this paper will examine the advantages and limitations of using network DEA compared to traditional efficiency evaluation approaches. As a result, it emphasizes the need to replace traditional models with a new network framework that can analyze the efficiency of listed companies in a comprehensive and practical way, taking into account the various dimensions and existing complexities. This innovative approach can lead to greater transparency in performance evaluation and improved management decision-making processes. Evaluating the efficiency of listed companies is of great significant because the performance of these companies directly affects their stock value (or stock price), competitiveness, and financial sustainability. In the meantime, paying attention to "specific data" such as research and development or marketing costs, is necessary to accurately evaluate efficiency. The DMU interrelationship model allows us to see inside a complex DMU with multiple nodes. In general, systems in which there is more than one interconnected stage are called networks [1]. Many companies present in the stock exchange which play a key role in the country's economic development have a network structure [2]. Calculation of the efficiency of the resource mobilization stage and the resource allocation stage, as well as the overall efficiency of bank branches, is performed using a Network DEA model and a common set of weights approach by constructing a multi-objective program and solving it with fuzzy theory. In this method, the efficiency of all branches is evaluated with one weight, and their efficiencies are maximized simultaneously [3]. The evaluation and ranking of market risks in selecting investment projects is based on mathematical programming models with combined DEA/AHP techniques, so that according to the determined relative weights, the overall impact of market risks on the prioritization of projects is carried out [4]. To evaluate the performance of a four-stage supply chain in presence of uncontrollable data in cement industry, the SBM model in network data envelopment analysis is proposed to examine the performance of such chains [5]. After the presentation of CCR model by Charnes, Cooper, and Rhodes, this model became the foundation of a branch in operations research called data envelopment analysis [6]. After the introduction of the CCR model, other models such as the BCC model, SBM, collective model, etc. were introduced to support DEA [7]. Groskov and Fara (2000) presented a network data envelopment analysis approach for modeling general multi-stage processes with intermediate inputs and outputs [8]. In these systems, the outputs of the first stage or process are considered as inputs to the next stage or process, called intermediate data [9]. To calculate the efficiency of a network system, a network DEA model is required. The conventional DEA model is not a standard network DEA model, but its form depends on the structure of the target network [10]. For data envelopment analysis, it is a method for investigating optimization using linear programming and evaluating decision-making units that perform the same tasks [11]. Performance evaluation is the evaluation and measurement of the effectiveness of the decisions made regarding the optimal use of resources and facilities [12]. A two-stage network can display how an organization has used its resources to provide the best performance at a point in time [13]. It is possible to attain the efficiency of that model with nonlinear models using the weighted sum method and with multiple inputs and outputs [14]. Determining the cost allocation in bank branches has been implemented on 37 branches of Iranian banks. Fixed cost is allocated based on the set of inputs and outputs of bank branches and after the fixed cost allocation, all DMUs and sub-stages have achieved efficiency, which, in turn, has increased the efficiency of all branches [15]. The evaluation of relative and integrated efficiency of Tehran stock exchange has been achieved by using the network data envelopment analysis model, the production process and the financial production process [16]. The two-stage network data envelopment analysis model is employed to divide the operational process of internet banks into value operation stage and value creation stage [17]. The evaluation of efficiency values and volatility of the Nigerian stock market has been conducted using the data envelopment analysis method during 2010 to 2020 [18]. The main innovation of this research is the use of a two-stage data envelopment analysis model with a network structure to analyze the efficiency of listed companies. While past studies have mainly used traditional DEA models such as CCR and BCC, which consider companies as a black box, the current research separates the production process into two or more distinct stages using a network approach. This approach allows us to assess not only the overall performance of the company, but also the performance of each stage (e.g., the production stage and the marketing/sales stage) exclusively. Furthermore, using "specific data" related to listed companies (such as total debt, registered capital, and equity-to-asset ratio) as intermediate and final inputs and outputs in the network model provides a comprehensive and practical view of the financial and operational performance of these companies. This innovation leads to a more precise identification of the source of inefficiency in the internal processes of companies and the provision of targeted solutions for their improvement, which has received less attention in previous studies. This article is organized as follows, respectively: a review of the literature on the subject is provided, then the model and modeling process and its solution method are presented. And then, by introducing the real society and its data, tables and graphs are provided, and finally conclusions and suggestions are presented.

2. Literature Review

The optimal use of resources in organizations has always been of concern to managers. The reason for this is the limitation in resources and maximum use of them. Therefore, it is necessary to use a scientific method to evaluate the performance of units that use these resources. Several methods have been proposed in this field. All these methods are based on the estimation of a function called the production function. The production function is a function that produces the maximum possible output for various input vectors. It is possible to identify and estimate the production function by two methods, namely, parametric and nonparametric. With the advancement of technology, parametric methods failed to successfully deal with real problems and had some disadvantages. To solve the problems caused by parametric methods, nonparametric methods such as data envelopment analysis are applied, which include mathematical models to calculate the relative efficiency of decision-making units (DMUs) and provide appropriate patterns to inefficient units to improve their performance. Calculating the efficiency and determining the pattern in this method is possible with the help of the production function; however, the exact specification of this function is impossible. Therefore, defining a set called the production possibility set seems essential, so that part of its boundary will be an estimate of the production function.

Definition: Assume DMU_j, $(j=1,...,n)$ with input vector of $x_j=(x_{1j},...,x_{mj})$ and output vector of $y_j=(y_{1j},...,y_{sj})$. The production possibility set is a set of all producible DMUs, defined as follows:

$$T=\left\{\begin{pmatrix}X\\Y\end{pmatrix} \mid \text{Vector X can produce vector Y}\right\}$$

In which, x is the input vector and y is the output vector of the decision-making unit. The above definition specifies the production possibility set with respect to the returns to scale of production technology. In order to create the T set, principles that are the basis of the theory and construction of data envelopment analysis models are assumed.

Principle 1): Inclusion of observations: all observations belong to the set T, i.e.,

$$\begin{pmatrix}x_j\\y_j\end{pmatrix}\in T, \qquad j=1,...,n.$$

Principle 2): Convexity: This principle states convexity of the set.

$$\forall\begin{pmatrix}x'\\y'\end{pmatrix}\ \forall\begin{pmatrix}x''\\y''\end{pmatrix}\ \forall\lambda\left[\begin{array}{l}\begin{pmatrix}x'\\y'\end{pmatrix}\in T\ \&\ \begin{pmatrix}x''\\y''\end{pmatrix}\in T\\ \&\ \lambda\in[0,1]\Rightarrow\begin{pmatrix}\lambda x'+(1-\lambda)x''\\ \lambda y'+(1-\lambda)y''\end{pmatrix}\in T\end{array}\right]$$

Principle 3): Feasibility: This principle states that if x can produce y, then any input greater than x can also produce y, and x can also produce any output less than y, i.e.:

$$\forall\begin{pmatrix}x\\y\end{pmatrix}\ \forall\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}\left[\begin{array}{l}\begin{pmatrix}x\\y\end{pmatrix}\in T\ \&\ \bar{x}\geq x\\ \&\bar{y}\leq y\ \Rightarrow\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}\in T\end{array}\right]$$

Principle 4): Infinity of radiation (returns to constant scale): This principle states that if $\begin{pmatrix} x \\ y \end{pmatrix} \in T$, then for every $\lambda \geq \cdot$, there is:

$$\begin{pmatrix} \lambda x \\ \lambda y \end{pmatrix} \in T$$

Principle 5): Minimum interpolation: T is considered to be the smallest set that applies to the selected principles.

Set T in technology with returns to constant scale is represented by T_c and consists of:

$$T_C = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \middle| x \geq \sum_{j=1}^{n} \lambda_j x_j , y \leq \sum_{j=1}^{n} \lambda_j y_j , \lambda_j \geq 0, \; j = 1,\dots,n \right\}$$

The symbol C indicates that the production possibility T_c set is constructed by adopting the principle of returns to constant scale. Evaluating a DMU in this set leads to the construction of a model called CCR, as one of the basic models of data envelopment analysis. According to the definition of CCR model, the nature of input in model (1) is:

$$\begin{aligned} \theta_o^* &= Min\theta \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \quad i = 1,\dots,m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1,\dots,s, \\ & \lambda_j \geq 0, \qquad j = 1,\dots,n. \end{aligned}$$

Model (2) is designed to calculate the relative efficiency of the O-th decision-making unit. If $\theta_o^* = 1$, then DMU_o will be the relative efficiency. This value will be the relative efficiency of DMU_o . To determine whether the unit under consideration is strong or weak, other problems need to be solved. To solve this problem, the following non-radial model was developed based on auxiliary variables to calculate the relative efficiency of DMU_o [19].

To estimate $DMUp$ efficiency with input X_p and output Y_p, consider the following model in terms of variables λ_j and s_i^- and s_r^+ , called SBM (Slack Based Measure) model:

$$\begin{aligned} Min \;\; \rho_p &= \frac{1 - \frac{1}{m} \sum_{i=1}^{m} \frac{s_i^-}{R_i^-}}{1 + \frac{1}{s} \sum_{r=1}^{s} \frac{s_r^+}{R_r^+}} \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{ip} \quad , \; i = 1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{rj} \quad , \; r = 1,\dots,s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & S^- \geq 0 \;\&\; S^+ \geq 0 \;\&\; \lambda \geq 0 \end{aligned}$$

In which,

$$R_i^- = Max\{x_{ij} \mid j = 1,\dots,n\} - Min\{x_{ij} \mid j = 1,\dots,n\}, \quad i = 1,\dots,m$$

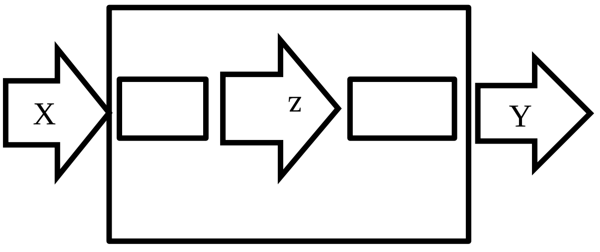
$$R_r^+ = Max\{y_{rj} \mid j = 1,\dots,n\} - Min\{y_{rj} \mid j = 1,\dots,n\}, \quad r = 1,\dots,s$$

If ρ_p^* is the optimal solution to the objective function (2), then ρ_p^* is called the relative efficiency of DMU_p . Obviously, if $\rho_p^* = 1$, then DMU_p is the relative efficiency; otherwise DMU_p is called the relative inefficiency. Note that in model (2), $X_p > 0 \;\&\; Y_p > 0$ is assumed. The SBM model is one of the advanced models in data envelopment analysis. The main objective of DEA is to assess the relative efficiency of a set of decision-making units (DMUs) that use similar inputs to produce similar outputs.

Unlike early DEA models such as CCR and BCC, which are based on efficiency ratios, SBM model directly measures inefficiency through slack variables. These variables represent the surplus value of inputs and the deficit value of outputs.

3. Modeling

In many issues of today's society, we are facing a two-stage structure. In this article, a two-stage network structure is considered.



Now suppose there are n decision-making units of (DMU), such that each DMU_j uses input vector $X_j = \left(x_{1j},...,x_{mj}\right)$ to produce output vector $Y_j = \left(y_{1j},...,y_{sj}\right)$ and $Z_j = \left(z_{1j},...,z_{kj}\right)$, which is $X_j \geq 0 \;\&\; X_i \neq 0 \;\&\; Y_i \geq 0 \;\&\; Y_j \neq 0 \;\&\; Z_k \geq 0$. Assuming the principles of inclusion of observations, convexity of return to the feasibility scale and minimum interpolation, the set of production possibilities will be as follows:

$$T_v = \left\{ (X,Y,Z) \left| \begin{array}{l} X \geq \sum_{j=1}^{n} \lambda_j^1 X_j \;\&\; \sum_{j=1}^{n} \lambda_j^1 Z_j \geq \sum_{j=1}^{n} \lambda_j^2 Z_j \\ \&Y \leq \sum_{j=1}^{n} \lambda_j^2 Y_j \;\&\; \sum_{j=1}^{n} \lambda_j^1 = 1 \\ \&\sum_{j=1}^{n} \lambda_j^2 = 1 \& \lambda_j^2, \lambda_j^1 \geq 0 \end{array} \right. \right\}$$

Now suppose that each decision-making unit has a two-stage structure. The efficiency DMU_p is obtained by solving the following problem:

$$Min \;\; \rho_p = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{R_i^-}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{R_r^+}}$$

$$\begin{array}{ll} s.t. \;\; \sum_{j=1}^{n} \lambda_j^1 x_{ij} + s_i^- = \theta x_{ip} & , \; i = 1,...,m \\ \sum_{j=1}^{n} \lambda_j^1 z_{kj} \geq \sum_{j=1}^{n} \lambda_j^2 z_{kj} & , \; k = 1,...,K \\ \sum_{j=1}^{n} \lambda_j^2 y_{rj} - s_r^+ = y_{rj} & , \; r = 1,...,s \\ \sum_{j=1}^{n} \lambda_j^1 = 1 & \\ \sum_{j=1}^{n} \lambda_j^2 = 1 & \\ S^- \geq 0 \;\&\; S^+ \geq 0 \;\&\; \lambda^1 \geq 0 \;\&\; \lambda^2 \geq 0 & \end{array}$$

If $\left(\lambda^{1*}, \lambda^{2*}, S^{-*}, S^{+*}\right)$ is the optimal solution of the above problem, the efficiency of each part is obtained as follows:

Efficiency of the first part: $E_p^1 = 1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^{-*}}{x_{ip}}$

Efficiency of the second part: $E_p^2 = \left(1 + \frac{1}{s}\sum_{y=1}^{s} \frac{s_r^{+*}}{y_{rp}}\right)^{-1}$

Total efficiency: $E_p^a = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^{-*}}{x_{ip}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^{+*}}{y_{rp}}}$

4. Findings

To carry out the study, processed data from the stock exchange titled “Top 48 Companies of Iran Stock Exchange in 1398 published every three months, was used. The information of these companies is reported as an indicator of efficiency and in order to be measured by other members of the stock exchange company community. The list of these companies is brought in Table 1.

Table 1: The companies' name

Symbol	Company's name	Symbol	Company's name
Hamrah 1	Mobile Communications of Iran	TAPPICO 1	Tamin petroleum & petrochemical investment
Betrans 1	Iran Transfo.	Khesapa 1	Saipa
Khodro 1	Iran Khodro	Vakharazm 1	Kharazmi Investment Group
App. 1	Asan Pardakht Persian	Vabank 1	National Development Investment Group
Vpars 1	Parsian Bank	Vasandoogh 1	National Pension Fund Investment
Vtejarat 1	Tejarat Bank	Vaghadir 1	Ghadir investment (Holding)
Vebsarder 1	Bank Saderat Iran	Fars 1	Persian Gulf Petrochemical Industries
Khepars 1	Pars Khodro	Befajr 1	Fajr Petrochemical
Shapna 1	Isfahan Oil Refinery	Fakhas 1	Khorasan Steel
Shepandar 1	Bandar Abbas Oil Refinery	Fakhouz 1	Khouzestan Steel
Shabriz 1	Tabriz Oil Refinery	Kave 1	South Kaveh Steel Co. (SKS)
Shatran 1	Tehran Oil Refinery	Foulad 1	Esfahan's Mobarakeh Steel Complex
Pars 1	Pars Petrochemical	Fasmin 1	Calcimine Company
Shapdis 1	Pardis Petrochemical	Hakashti 1	The Islamic Republic of Iran Shipping Line Group
Jam 1	Jam Petrochemical	Petrol 1	Iranian Petrochemical Group
Shakharak 1	Khark Petrochemical	Ramapna 1	MAPNA Group
Sharak 1	Shazand Petrochemical	Vamid 1	Omid Investment Group
Shafan 1	Fanavaran Petrochemical	Parsan 1	Parsian Oil and Gas Development Company
Mobin 1	Mobin Petrochemical	Da'abid 1	Dr. Abidi Pharmaceuticals
Shiraz 1	Shiraz Petrochemical	Akhaber 1	Iran Telecommunications Company
Sepp 1	Saman Electronic Payment	Kagol 1	Golgohar Mining and Industrial Company
Vama'aden 1	Mines and Metals Development	Kachad 1	Chadormalu Mining and Industrial Company
Ranfor 1	Informatics Services	Famelli 1	National Iranian Copper Industries Company
Hi-web 1	Asre dade novin Gostar	Shabharn 1	Behran Oil Company

The indicators affecting efficiency include total debts, total shareholders' equity, allocable profit, registered capital, total assets, equity-to-asset ratio, and capital-to-asset ratio. Before going into the details of the analysis, it is necessary to define these indicators carefully. This helps to better understand the concepts and their relationship with the data. Below, each indicator is explained to achieve complete transparency about the current study methodology.

Total Debts:

Total liabilities (debt) in a portfolio refers to the sum of all financial obligations and debts of a company. This amount includes all funds that the company is obligated to pay to other individuals or legal entities. Total liabilities can be used as a measure of a company's financial risk and financial condition. Total liabilities should be examined alongside total equity and total assets to get a comprehensive picture of the company's financial condition. This analysis can help determine financial and investment strategies.

Total Equity of Capital Owners

Total equity in the stock portfolio refers to total net value of a company's assets after deducting all liabilities. This value represents shareholders' share of the company's assets and is used as one of the key criteria in evaluating company's financial health and performance. Total equity can be affected by factors such as profits and losses, changes in investments and distribution of profits. This measure is very important for analysts and investors because it shows the company's financial stability and creditworthiness.

Allocable Profit:

Allocable profit in the context of stock portfolio refers to the net profit that can be distributed to shareholders and is also commonly known as distributable profit or retained earnings. Allocable profit is calculated after deducting expenses, taxes and other financial obligations and can be distributed to shareholders. Allocable profit can be affected by factors such as the company's financial policies and profit distribution, economic conditions, and market performance. This criterion is highly significant for investors and economic analysts because it reflects the company's ability to attract and maintain capital.

Registered Capital:

Registered capital refers to the capital that was officially registered and declared in the company registry at the time of the establishment of a company. This capital includes the total amount that the company's shareholders deposit as their initial capital and can be attained through the issuance of shares.

Total Assets:

Total assets refer to the total assets of a company, which includes all financial and non-financial resources that the company owns. This value represents the financial strength and capacity of the company to carry out commercial and production activities and is employed as one of the key criteria in analyzing the financial position of the company. The sum of total assets should be checked along with the sum of liabilities and the sum of shareholders' equity in order to obtain an accurate analysis of the company's financial situation. A thorough understanding of these three criteria can help investors and managers make better decisions.

Equity-to-Asset Ratio:

The equity-to-asset ratio is a financial ratio that indicates the share of equity in a company's total assets. This ratio can help assess the financial health of the company and its capital structure. In general, the equity-to-asset ratio should be analyzed alongside other financial ratios, such as the debt-to-equity ratio and the return on assets ratio, to get an accurate picture of the company's financial condition and performance.

Debt-to-Asset Ratio:

The debt-to-asset ratio is an important financial ratio that indicates the ratio of total liabilities to total assets of a company. This ratio helps analysts and investors assess the extent to which a company relies on debt financing and measures its repayment capacity. The debt-to-asset ratio should be considered alongside other financial ratios, such as debt-to-equity and liquidity ratios, to get an accurate picture of the company's financial health. A comprehensive assessment of these ratios can help provide a better understanding of the risks and opportunities within a company.

These indicators are as follows for these 48 companies.

Table 2. Data values of companies for efficiency calculation

Companies	Registered capital	Allocable profit	Total shareholders' equity	Total liabilities (debts)	Total assets	Equity to Asset Ratio	Capital to Asset Ratio
1	9600000	43059757	55176311	181061982	236238293	22.290	4.0636
2	3750000	3208767	7698058	19819256	27517314	25.288	13.627
3	15300000	-174543027	-150393506	436432381	286038875	55.671	5.348
4	2650000	2974241	5824381	3467840	9292221	60.526	28.518
5	23760000	-8451851	31707730	1272059805	1303767535	1.174	1.822
6	223926127	-108185169	116197796	1780960421	1897158217	6.100	11.803
7	175353972	-63600575	184963382	2354671942	2539635324	4.400	6.904
8	22720279	-29540971	-6615245	55243678	48628433	-14.026	46.722
9	20000000	56328014	91160308	38537530	129697838	58.850	15.420
10	20000000	34324365	58616126	44052862	102668988	52.912	19.480
11	20000000	18158624	24182434	17763310	41945744	90.971	47.680
12	20000000	39453519	64951327	23533670	88484997	67.190	22.602
13	20000000	61789949	68389949	22053451	90443400	90.432	22.113
14	20000000	26869425	33469425	31938277	65407702	71.657	30.577
15	20000000	39083887	55074788	35075557	90150345	65.539	22.185
16	20000000	29734778	31984778	12208508	44193286	112.539	45.255
17	20000000	14463618	23936637	14770455	38707092	89.036	51.670
18	20000000	16632541	17683358	7441046	25124404	145.804	79.603
19	20000000	24929116	40606316	11584449	52190765	86.086	38.320
20	20000000	165109	5738389	43736214	49474603	40.758	40.424
21	20000000	2330447	4305956	3120502	7426458	300.687	269.307
22	20000000	25968042	62850837	6312495	69163332	66.463	28.917
23	20000000	22965579	32254822	4109234	36364056	118.153	54.999
24	20000000	1641648	5766943	8044638	13811581	156.692	144.806
25	20000000	40606761	140963447	6587813	147551260	41.075	13.554
26	20000000	-184325155	-142261373	399990753	257729380	-63.758	7.760
27	20000000	2080225	17448085	7313108	24761193	89.172	80.771
28	20000000	19602754	47595749	12282890	59878639	66.138	33.400
29	20000000	17386491	51538052	14486590	66024642	56.625	30.291
30	20000000	32204582	120953064	10691627	131644691	39.655	15.192
31	20000000	120911290	175911290	259521596	435432886	32.361	4.593
32	20000000	10515366	17665366	10781262	28446628	107.272	70.307
33	20000000	4919218	15116681	14746473	29863154	83.444	66.972
34	20000000	33505318	61023330	80181427	141204757	37.892	14.163
35	20000000	6617385	17018133	40552639	57570772	46.234	34.739
36	20000000	174725148	339197399	201537282	540734681	36.011	3.698
37	20000000	6323558	9402807	3364528	12767335	206.178	156.649
38	20000000	5917949	23354306	142642520	165996826	15.613	12.048
39	20000000	10492515	31893651	44719870	76613521	39.800	26.105
40	20000000	116419581	269749350	350024037	619773387	22.011	3.226
41	20000000	15835870	53626881	53514086	107140967	33.447	18.666
42	20000000	57619907	111094455	11598459	122692914	63.263	16.300

43	20000000	1408671	3155871	4127234	7283105	293.949	274.608
44	20000000	44836415	116448756	295905176	412353932	15.723	4.850
45	20000000	51124930	105761857	97120771	202882628	35.057	9.857
46	20000000	30188326	76106797	28208262	104315059	48.112	19.172
47	20000000	59630100	147821412	73128895	220950307	36.0398	9.051
48	20000000	6939469	9725959	13175508	22901467	117.632	87.330

These data are for input, registered capital, and assets indicators. For intermediate indicators, they are total equity, and for output indicators, they are capital to asset ratio, equity to asset ratio and allocable profit. Now, all these values are inserted in the above modeling (Equation (4)) and the efficiencies have been obtained with the GAMS program and shown in the Table below.

Companies	First stage efficiency	Second stage efficiency	Efficiency of the entire course
1	0.96311	0.651895	0.42912
2	0.998715	0.777475	0.712499
3	1	1	1
4	1	1	1
5	0.892629	0.621873	0.284583
6	0.715541	0.611405	0.079965
7	0.685787	0.615844	0.061997
8	1	0.642308	0.443114
9	0.941259	0.677095	0.464362
10	0.962096	0.656331	0.438475
11	0.981814	0.660502	0.467815
12	0.960069	0.665313	0.457018
13	0.95144	0.696856	0.516424
14	0.975583	0.657909	0.455615
15	0.963347	0.664407	0.458246
16	0.97653	0.676501	0.498335
17	0.982711	0.658138	0.463273
18	0.9863	0.686445	0.52952
19	0.97452	0.662205	0.464414
20	0.989484	0.63742	0.420659
21	0.997482	1	0.997482
22	0.965953	0.655445	0.440271
23	0.977953	0.674845	0.496131
24	1	0.739862	0.648397
25	0.928381	0.656984	0.406274
26	1	1	1
27	0.986903	0.658524	0.468355
28	0.972997	0.651623	0.438366
29	0.971773	0.647077	0.426362
30	0.93965	0.650436	0.402219
31	0.823006	0.760043	0.50729
32	0.985485	0.665267	0.48233
33	0.987451	0.653705	0.457709
34	0.957039	0.650764	0.420383
35	0.984872	0.638778	0.419382
36	0.626061	1	0.626061

37	0.998543	0.744414	0.655204
38	0.970743	0.628145	0.378752
39	0.977997	0.638048	0.410716
40	0.77391	0.743169	0.428322
41	0.967298	0.638572	0.401305
42	0.934351	0.680172	0.464134
43	1	1	1
44	0.905965	0.652084	0.37242
45	0.929019	0.663475	0.421804
46	0.957381	0.651841	0.423264
47	0.908476	0.671527	0.419332
48	0.98851	0.67065	0.497418

5. Conclusion

This study was carried out with the aim of providing an in-depth and multifaceted analysis of the performance of 48 top companies selected from different industries of the stock exchange. The innovative approach of this research was the use of Data Envelopment Analysis (DEA) with a two-stage network structure, which enabled the evaluation of efficiency beyond the traditional perspectives. This structure not only allowed us to measure the overall performance, but also to examine the internal processes of the companies from the stage of inputs to intermediate variables and then from intermediate variables to outputs separately. For this analysis, seven key indicators such as capital to assets, allocable profit and other real and reliable financial variables were used, which provided a comprehensive picture of the companies' performance. The results of this study are of considerable value to a wide range of stakeholders, including investors, financial analysts, and especially company managers. Investors can rely on this method, beyond traditional financial ratios, to have a deeper understanding of companies' true capacities to generate value. For managers, this analysis provides a powerful tool to accurately identify strengths and weaknesses in each stage of internal operations, and to make strategic decisions for constant improvement of efficiency. From a theoretical point of view, this research contributes significantly to the management and financial literature by expanding the application of network DEA in the analysis of the efficiency of listed companies with real data. Despite the important achievements, this research also encountered limitations that can be a basis for future research. This study focused on the top 48 companies using specific financial indicators. Future research can enrich the results by expanding the statistical sample to more companies from more diverse industries, or by including non-financial variables (such as environmental, social, and governance indicators or innovation-related data). Also, investigating the impact of macroeconomic factors or external shocks on the efficiency of companies at each stage of the process, using a network approach, would lead to valuable insights.

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