**Journal of Optimization in Soft Computing (JOSC)**
Vol. 3, Issue 3, pp: (30-36), Autumn-2025
Journal homepage: https://sanad.iau.ir/journal/josc

**Research paper**

# Min-Max Multiple Fixed Watchman Routes in Minbar Polygons, with Non-domination Assumption

Rahmat Ghasemi[1], Alireza Bagheri[2,*], Faezeh Farivar[1], Fatemeh Keshavarz-Kohjerdi[3]

*1. Department of Computer and Mechatronics Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran*
*2. Department of Computer Engineering, Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran*
*3. Department of Computer Science, Shahed University, Tehran, Iran*

**Article Info**

**Article History:**
*Received: 2025/08/25*
*Revised: 2025/10/01*
*Accepted: 2025/10/26*

*DOI:*
*10.82553/josc.2025.202508251215973*

**Keywords:**
Multiple watchman routes, Staircase polygons, Minbar polygons, Art gallery problem,
Dynamic programming.

* Alireza Bagheri
ar_bagheri@aut.ac.ir

**Abstract**

In this paper, the problem of multiple watchman routes in Minbar polygons is studied, where every point in the given polygon must be visible from at least one point on some watchmen's route. The problem of multiple watchman routes is NP-hard even in simple polygons. However, some limited types of polygon have been shown to have polynomial-time solutions. We propose an algorithm based on the dynamic programming approach that requires $O(n)$ space and consumes $O(n \cdot \log n)$ time for min-max criterion, where $n$ is the number of polygon vertices. We assume that the starting points of watchmen do not dominate each other.

## 1. Introduction

The Watchman Route Problem (WRP) is an intriguing variant of the well-known Art Gallery Problem (AGP). The AGP, introduced by Victor Klee in 1973 during a conversation with Vasek Chvátal, to determine the minimum number of stationary guards required to cover all points in a polygonal gallery [1]. In contrast, the WRP focuses on a connected polygonal domain $P$, and aims to identify the shortest path that a mobile guard, called the "watchman," must follow to observe every point in $P$. When the starting point of the watchman is known, the problem is classified as a fixed or anchored watchman route [2]. On the other hand, if the starting point is unspecified, it is referred to as a float watchman route [2-4].

The WRP has common similarities with problems such as touring polygons [4], traveling salesman [5], safari and zoo-keeper [2]. The WRP has many practical applications, including security and monitoring, efficient simulation, and optimization of time and energy [6]. The $k$-watchman route problem involves finding a group of $k$ closed routes that cover the entire area while minimizing the length of the routes. There are two commonly used measures for minimizing the length: the min-max measure which aims to minimize the length of the longest route and the min-sum measure which minimizes the cumulative length of all routes [3].

This paper is organized as follows. In Section 2, we provide the necessary preliminaries and discuss related works. Section 3 delves into the problem of the single watchman route. In Section 4, we introduce our algorithm for multiple watchman routes aimed at optimizing the min-max criterion. Finally, we present our conclusions and outline future works in Section 5.

## 2. Preliminaries and related works

A *simple polygon*, $P$, having $n$ vertices, is a closed, simply-connected region whose boundary is a union of $n$ (straight) line segments (edges), whose endpoints are the vertices of $P$ [7]. A *rectilinear polygon* is a polygon whose edges are either horizontal or vertical. Let $a$ and $b$ be two points inside polygon $P$. Point $b$ is visible from or is guarded by point $a$ if the line segment $[a, b]$ lies entirely inside $P$ [8]. A *staircase polygon* also defined in [9] and shown in Figure 1, is a rectilinear polygon consisting of two vertical edges at the left and right and is bounded above and below by two staircase like connected to vertical edges.

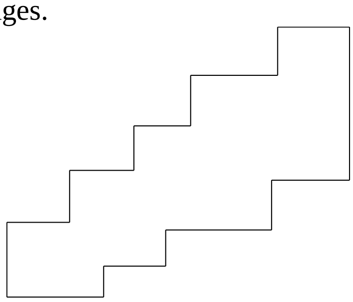

**Figure 1: A staircase polygon**

A *Minbar polygon*, illustrated in Figure 2, is classified as a staircase polygon [9]. It is composed of three distinct parts: a long horizontal line segment known as the *horizontal base*, a long vertical line segment referred to as the *vertical wall*, and a chain formed by alternating horizontal and vertical line segments. Notably, this chain is monotone with respect to both the x-axis and the y-axis. Assuming the vertical wall is connected to the right vertex of the horizontal base, the bottom-right corner of the polygon is defined as the *origin*. The vertices are numbered in a clockwise sequence, beginning at the origin, as illustrated in Figure 3 with the origin assigned an index of zero. A vertex $v$ of the polygon $P$ is called *reflex* if the internal angle at $v$ exceeds $180^o$ ; otherwise, it is designated as *convex*. We denote the vertex of $P$ with index $c$ as $P_c$ .

A polygon is called *star-shaped* if there exists a set of points from which all points of the polygon are visible. This set of points is known as the *kernel*, as shown in Figure 3 (highlighted in gray). When the edges of the kernel are extended, the polygon is divided into four sub-polygons: a rectangle located above the kernel, denoted as U; a rectangle to the left of the kernel, denoted as L; a Minbar sub-polygon situated in the upper-left corner of the kernel, denoted as M; and the kernel itself, denoted as K.

Any Minbar polygon is star-shaped. Taking into account a Minbar polygon with $n > 4$ vertices and $k > 1$ watchmen positioned inside it, the starting point for each watchman is specified (refer to Figure 2). Each watchman follows a route within the polygon and ultimately returns to its starting point, ensuring that every point of the polygon is visible from at least one of these routes. We define a *corner* as a convex vertex in $P$ whose index is even. Let $C$ represent the set of corner indices in , which can be expressed as $C = \{0,2, \dots, n-2\}$ . For any point , we denote the $x$-coordinate by $x(p)$ and the $y$ -coordinate by $y(p)$ .

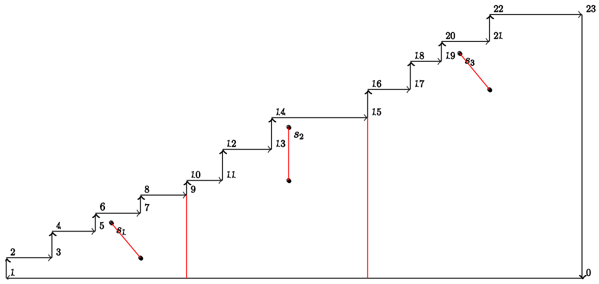


**Figure 2: A Minbar polygon with 24 vertices and 3 watchmen starting points $s_1, s_2, s_3$, each vertex takes an index in the clockwise order, and $c_1^- = 6$, $c_2^- = 14$, $c_3^- = 20$ , $C_1^- = \{8, 10, 12\}$, $C_2^- = \{16, 18\}$, $C_3^- = \{22\}$ and $Z_1 = \{6, 8, 10, 12\}$, $Z_2 = \{14, 16, 18\}$, $Z_3 = \{20, 22\}$ and the route of each watchman under min-max criterion.**

Let $s_i$ denote the starting point of watchman , and we will also use $s_i$ interchangeably to refer to the watchman itself. The set of all corner points on the chain that can be seen from watchman $i$ at the specific starting point $s_i$ is denoted as $(i)$ . It is assumed that for any two starting points $s_i$ and $s_j$ (where $1 \leq i, j \leq k$ ), the conditions $x(s_i) < x(s_j)$ and $y(s_i) < y(s_j)$ hold, and that $vp(i) \cap vp(j) = \emptyset$ , where $k$ represents the number of watchmen. A point $a$ is said to *dominate* point $b$ if $x(b) \leq x(a)$ and $(b) \geq y(a)$ . When point a dominates point , the condition $dom(a, b)$ is satisfied. For simplification, this paper assumes that the starting points of the watchmen do not dominate each other. Additionally, we use $h(c)$ and $v(c)$ to refer to the horizontal and vertical line segments that pass through the corner $P_c$ and are contained within the polygon $P$.

The WRP in polygons with holes has been proven to be NP-hard [10,11]. Similarly, the multiple WRP is also NP-hard even in simple polygons [3]. However, some limited types of polygon have been shown to have polynomial-time solutions [6,10,12-15].

Nilsson and Wood considered the multiple WRP in spiral polygons with the min-sum criterion, and provided a $\theta(n^2)$-time algorithm based on dynamic programming for this problem [13]. Nilsson and Packer exhibited a polynomial-time 7.1416 approximation algorithm for computing the min-max two-watchman route in simple polygons [3]. Nilsson and Schuierer gave an $O(n^2 \log n)$-time algorithm to compute the min-max optimum set of $m$ watchmen in a histogram polygon in the float version of problem [16]. In [6], Packer presented heuristics to compute multiple watchman routes in polygons possibly with holes.

Bagheri *et~al*. [17] investigated the multiple WRP in staircase polygons and proposed an $O(n^2 \cdot min\{m,n\})$-time algorithm under the min-sum criterion, where $m$ and $n$ are the number of watchmen and vertices, respectively. In [18], the authors proposed a $O(n^2 \cdot k^2 \cdot \log n)$-time greedy algorithm for the fixed multiple watchman routes problem in Minbar polygons under min-max criterion. In this study, a faster algorithm based on the dynamic programming method is proposed, which can find the optimal solution in min-max criteria.

The use of a Minbar polygon simplifies the geometry to highlight the main challenge: optimizing multiple watchman routes under non-domination constraints. This strategic choice emphasizes algorithmic complexity over geometric intricacy, laying a clear foundation for tackling more complex shapes later.

Recent works in semi-supervised generative modeling for medical imaging focus on identifying critical regions under uncertainty [19,20]. These challenges align with geometric coverage problems like watchman routes, suggesting that visibility-based path planning may offer transferable strategies for improving sample selection and structural consistency in imbalanced or partially observed domains.

## 3. Single fixed watchman route

First, we present an algorithm to address the single fixed watchman route problem. The value $\xi$ defined by Equation (3.1) formulates this problem specifically within a Minbar polygon. The watchman is required to go to the nearest point of the kernel from its starting point and then return back.

For the single fixed watchman route, we need the watchman starting point $s_i$ along with the indices of the first and last corners on the chain of the given Minbar polygon (or sub-polygon), denoted by $c$ and $c'$ , respectively. The horizontal and vertical cuts within $P$ that need to be covered by watchman $i$ are represented by $h(c)$ and $(c')$ . We formulate this problem as $(i,c,c')$ , which calculates the length of the minimum route for watchman $i$ to effectively guard the polygon. For simplicity, we compute half of the minimum length route, as illustrated in Equation (3.1) and shown in Figure 3.

$$\xi(i,c,c') = |[s_i, q_i]| , where\ q_i = \begin{cases} s_i, & s_i \in K \\ \big(x(s_i), y(P_c)\big), & s_i \in U \\ \big(x(P'_c), y(s_i)\big), & s_i \in L \\ \big(x(P_{c'}), y(P_c)\big), & s_i \in M \end{cases} \quad (3.1)$$

There are four possible cases to consider: **Case 1:** If the watchman is located within the kernel, denoted as sub-polygon K , the watchman does not need to move. **Case 2:** If the watchman is located above the kernel, indicated as a sub-polygon U , the watchman should move vertically down to reach the kernel. **Case 3:** If the watchman is positioned to the left of the kernel, represented as sub-polygon L , the watchman should move horizontally right towards the kernel. **Case 4:** If the watchman is located in the upper-left corner of the kernel, denoted as sub-polygon M , the watchman should move diagonally to access the kernel. Refer to Figure 3, where the guards $s_i^1$ , $s_i^2$ , $s_i^3$ , and $s_i^4$ correspond to cases 1, 2, 3, and 4, respectively.

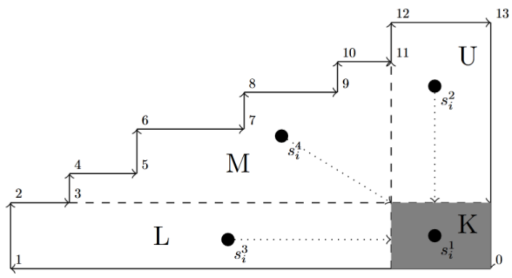


**Figure 3: Single fixed watchman route. The two essential cuts of a the Minbar polygon are shown with dashed lines and the gray area shows the kernel of the Minbar polygon, $c = 2$ and $c' = 12$ and four possible locations of $s_i$ marked as $s_i^1, s_i^2, s_i^3$ and $s_i^4$. Routes of each $s_i$ is shown by dotted lines.**

## 4. The proposed Algorithm

In this section, an efficient algorithm based on the dynamic programming method is presented to solve the fixed multiple watchman routes problem in Minbar polygons. The algorithm finds the optimal solution for min-max criterion.

For a watchman $i$, $c_i^-$ is the maximum index of a corner on the chain that $s_i$ can see (see Equation 4.1), and $\forall\, i,j, 1 \le i,j \le k, i \ne j$, we have, $c_i^- \ne c_j^-$ (see Figure 2).

$$c_i^- = max_{c \in C} \{c \mid dom(s_i, P_c)\} \tag{4.1}$$

For each watchman $i$ there are a number of corners, say $c$, where $x(s_i) < x(P_c) < x(s_{i+1})$ and $P_c$ is not visible from $s_i$ and $s_{i+1}$. We call this set of corners $C_i^-$ (see Equation (4.2) and we define $Z_i$ in Equation (4.3) as the union of $\{c_i^-\}$ and $C_i^-$, and we have $\forall\, i, j, 1 \leq i, j \leq k, i \neq j, Z_i \cap Z_j = \emptyset$.

$$C_i^- = \{ c \in C \mid x(s_i) < x(P_c) < x(s_{i+1}) \;\&\; \neg\, dom(s_i, P_c) \;\&\; \neg dom(s_{i+1}, P_c)\}, \quad 1 < i < k \tag{4.2}$$

$$Z_i = \begin{cases} \{c_i^-\} \cup C_i^-, & if\ i < k \\ \{ n-2 \}, & if\ i = k \end{cases} \tag{4.3}$$

**Lemma 4.1:** The number of elements in union of $Z_i$ for all watchmen, is less than $n$ (i.e: $|| \cup_{i=1}^{k} Z_i| < n$).
**Proof:** The proof follows directly from the fact that for all $i, j$, where $1 \leq i, j \leq k$ and $i \neq j$, the sets $Z_i$ and $Z_j$ are disjoint, i.e., $Z_i \cap Z_j = \emptyset$.

■

**Lemma 4.2:** Let $c$ be a corner of Minbar polygon $P$ that is dominated by watchman $s$. If $r$ is a route of the watchman $s'$ (where $s \neq s'$) in an optimal solution for min-max, then it does not intersect the horizontal extension $h(c)$ or the vertical extension $v(c)$.
**Proof:** If we assume that $y(s') > y(P_c)$ and the line segment $r$ intersects $h(c)$, the only explanation is that $s'$ is trying to see an unguarded corner, like $z$, that lies below the corner $c$. Therefore, watchman $s$ cannot move while watchman $s$ is closer to $h(z)$, because $y(z) < y(s) < y(s')$. On the other hand, since we want to minimize the maximum route length, we can assign guarding of corner $z$ to watchman $s$ so that watchman $s'$ does not need to intersect the horizontal cut of $c$, and $r$ will be shorten. Thus, $r$ cannot intersect $h(c)$ and similarly $v(c)$.

■

It implies from Lemma 4.2 that, in the optimal solution for the min-max, an initially unguarded corner should be guarded by the watchmen whose starting points are immediately before or after it.

**Lemma 4.3:** If $r$ is a route in the optimal solution for min-max and $|r| > 0$, then the endpoint of $r$ lies on a horizontal or vertical cut of a corner that was not visible initially by any watchman. By $| r|$, we mean the length of $r$.

**Proof:** Assuming that $x$ is the intersection point of $r$ with a horizontal (or vertical) cut of the unguarded corner and $q$ is the endpoint of $r$ other than $x$, no new corner can be guarded by any point on line segment $[x, p]$. This means that $r$ can be shortened in such a way that $x$ becomes its endpoint, so $r$ lies on a cut.

■

**Lemma 4.4:** If $c$ is an initially unguarded corner of $P$, then there is exactly one route (such as $r$) in the optimal solution, which intersects or touches either $h(c)$ or $v(c)$.
**Proof:** Firstly, in order to see or guard the corner $c$, at least one of the cuts must be meet by a route. Secondly, if we assume that one of the cuts intersects or touches a route $r_i$ and the other cut intersects or touches another route $r_j$, then this is not optimal, because we can shorten either $r_i$ or $r_j$, so the lemma is proven.

■

As indicated in Equation (4.4), $\zeta(c')$ gives the minimum longest watchman route in a Minbar polygon. To solve the min-max problem, we split the polygon into sub-polygons, where each sub-polygon exactly contains one watchman. To do this, we can exclude the last watchman from the polygon by determining the left boundary of the last sub-polygon that contains the last watchman, where the last watchman guards that sub-polygon.

$$\zeta(c') = \begin{cases} \xi(1,2,c'), & \text{if } c' \in Z_1 \\ \min\limits_{c \in Z_{i-1}} \{ \max( \zeta(c), \xi(i, c+2, c')) \}, & \text{if } c' \in Z_i \text{ and } i > 1 \end{cases} \tag{4.4}$$

If a polygon contains only one watchman, the problem can be addressed as a single fixed watchman route using Equation (3.1), as outlined in the first part of Equation (4.4). If a polygon contains more than one watchman, the last watchman (let us call it watchman i ) in a given sub-polygon needs to be able to see corner $c'$ on the right side and corner $c + 2$ on the left side, where $c$ belongs to $Z_{i-1}$ and is the right corner visible to watchman $i - 1$ . To achieve this, we must calculate $\zeta(c)$ for all $c$ in $Z_{i-1}$ recursively (as indicated in the second part of Equation (4.4), and then select the optimal value. The function $\xi$ calculates the single fixed watchman route in a specified sub-polygon, as expressed in Equation (3.1), Additionally, $Z_i$ can be determined through a pre-processing step that runs in $O(n)$ time, for all watchmen.

**As illustrated in Figure 4, we have constructed the flowchart corresponding to Equation (4.4). Although Equation (4.4) is defined recursively, the flowchart evaluates it through an iterative procedure. Specifically, it computes $\zeta(c')$ for all corners in $Z_i$, for each watchman $i$, proceeding sequentially up to the final watchman. We assume that the polygon contains at least one watchman. At each iteration, the computed value of $\zeta(c')$ is cached to enable reuse in subsequent steps. Ultimately, the value obtained for the last corner—corner $n-2$—represents the solution to the problem.**

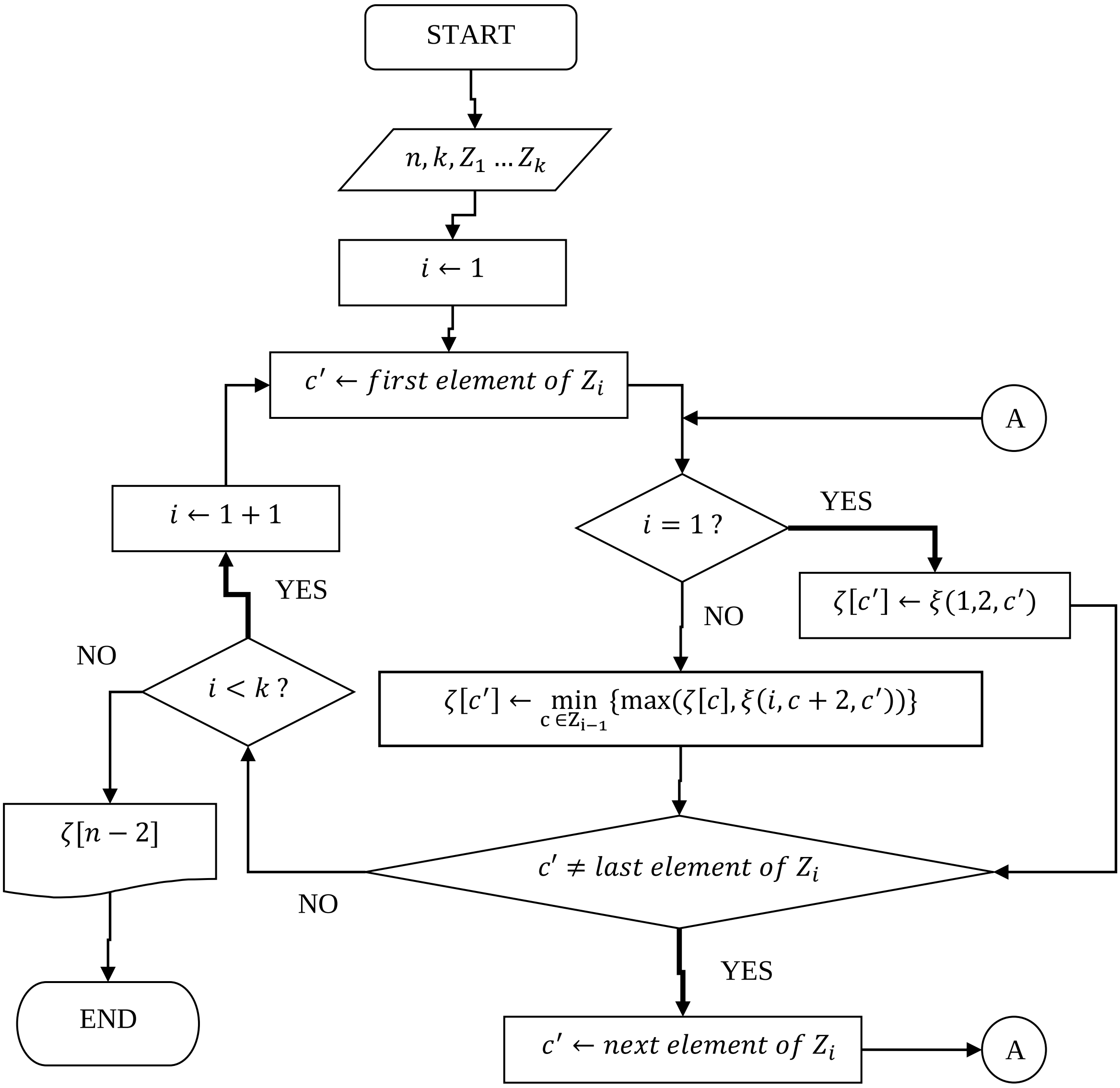


**Figure 4 :** Flowchart corresponding **to Equation (4.4)**

**Theorem 4.5:** The min-max recursion provided in Equation (4.4), finds a solution for fixed multiple watchman routes problem.

**Proof:** We call the recursion by $\zeta(n-2)$ to find the min-max solution. The value of $\zeta$ is calculated for all the corners in $Z_i, i \in [1, k]$. Each unguarded corner on the left side of $s_1$ will be covered by $s_1$, since corner number 2 must be guarded by the watchman starting at $s_1$. Each unguarded corner on the right side of $s_k$ will be covered by $s_k$ and each unguarded corners in $Z_i, i \in [1, k-1]$ will be covered by watchman $i$ or $i+1$. Hence all the vertices on the chain are guarded by some watchmen and the solution visits the entire polygon.

■

**Theorem 4.6:** The min-max recursion provided in Equation (4.4), finds an optimal solution for the min-max criterion.

**Proof:** We need an array $\zeta$ as the storage. The value of $\zeta$ is calculated for all the corners in

$Z_i, i \in [1,k]$. So, the desired criterion value is calculated in all the smaller Minbar sub-polygons where for a corner $c' \in Z_i$, $v(c')$ is a vertical essential cut of the sub-polygon. Thus, every potential condition has been addressed. On the other hand, in the second part of Equation (4.4), the optimal value will be created on one of the corners between the last two watchmen, and the desired criterion is optimized here. So, the given recursion finds the optimal solution.

■

**Theorem 4.7:** The min-max recursion provided in Equation (4.4), finds the solution in $O(n^2)$ time and requires $O(n)$ space.

**Proof:** According to Lemma 4.1, to store $\zeta$ values, a memory of $O(n)$ space is enough. On the other hand, $\xi$ function in Equation (3.1) has $O(1)$ complexity. The time complexity of the $min$ function in the second part of Equation (4.4) is $O(n)$. Therefore, the time complexity of the whole recursion is $O(n^2)$.

■

## 4.1. Improving the time-complexity

In this section, we improve the time-complexity of the algorithm. To find the minimum point in the second part of Equation (4.4), instead of using a linear search method, we can use a binary search method, which will improve the time complexity of the algorithm.

**Theorem 4.8:** The $max$ function in Equation (4.4), can be divided into two parts, such that the first part is decreasing and the second part is increasing.

**Proof:** Considering Equation (4.4), the first input of the $max$ function is the value of $\zeta(c)$, that changes in the interval between two last watchmen ($c \in Z_{i-1}$). When $c$ increases, the value of $\zeta(c)$ is either constant or increases. Also, the second input is the value of the $\xi$ function, which calculates the route length of the last watchman. It is strictly decreasing because the distance between the horizontal cut (i.e. $h(c+2)$) of the right sub-polygon that contains the last watchman, and the starting point of the last watchman is strictly decreasing.
At first, the $max$ value may be selected from the second input and then selected from the first input. Therefore, as shown in Figure 5, at first the value of the $max$ function is strictly decreasing and then it is increasing.

■

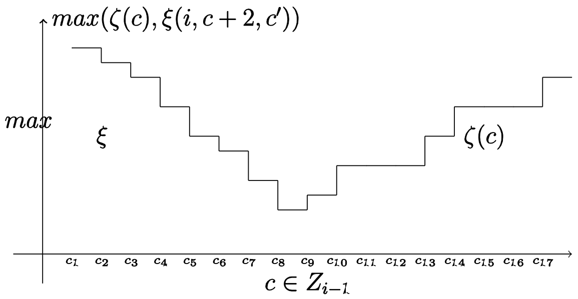


**Figure 5: The value of the $max$ function in the second part of Equation (4.4).**

Theorem 4.8 shows that we can use a binary search to find the minimum value, and improve the time-complexity.

**Theorem 4.8:** The improved min-max algorithm, computes the solution in $O(n \log n)$ time and requires $O(n)$ space.

## 5. Conclusion and future works

In this paper, we studied the min-max multiple fixed watchman route problem in Minbar polygons, which is a specific case of staircase polygons. The algorithms presented find the optimal solution using a dynamic programming approach. Our proposed algorithm has a time complexity of $O(n \cdot \log n)$ and requires $O(n)$ space, offering an improvement over the runtime reported in [17,18]. Future research directions include addressing this problem in the min-sum scenario and exploring the possibility of domination among watchmen. Additionally, examining other types of polygons could provide further insights into the problem.

## 6. Acknowledgments

The author would like to thank the anonymous referee who provided useful and detailed comments on a previous/earlier version of the manuscript.
During the preparation of this work the author(s) used Grammarly and Writefull in order to improve the readability and language of the manuscript. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication.

## References

[1] Mark de Berg, Otfried Cheong, Marc van Kreveld, and Mark Overmars. "Computational Geometry: Algorithms and Applications". Springer-Verlag TELOS, Santa Clara, CA, USA, 3rd ed. edition, 2008.

[2] Ning Xu. "On the watchman route problem and its related problems", dissertation proposal. 2015.

[3] Bengt J. Nilsson and Eli Packer. "An approximation algorithm for the two-watchman route in a simple polygon". In EuroCG 2016, 2016.

[4] Moshe Dror, Alon Efrat, Anna Lubiw, and Joseph S. B. Mitchell. "Touring a sequence of polygons. In Proceedings" of the Thirty-Fifth Annual ACM Symposium on Theory of Computing, STOC '03, page 473–482, New York, NY, USA, 2003. Association for Computing Machinery.

[5] V. Černý. "Thermodynamical approach to the traveling salesman problem": An efficient simulation algorithm. Journal of Optimization Theory and Applications, 45:41–51, 1985.

[6] Eli Packer. "Computing multiple watchman routes". In Catherine C. McGeoch, editor, Experimental Algorithms, pages 114–128, Berlin, Heidelberg, 2008. Springer Berlin Heidelberg.

[7] J. S. Mitchell, "Chapter 15 - geometric shortest paths and network optimization". In J.-R. Sack and J. Urrutia, editors, Handbook of Computational Geometry, pages 633–701. North-Holland, Amsterdam, 2000.

[8] H. El Gindy, D. Avis, "A linear algorithm for computing the visibility polygon from a point". Journal of Algorithms, 2(2):186–197, 1981.

[9] Mireille Bousquet-Melou, Anthony J Guttmann, William P. Orrick, and Andrew Rechnitzer. "Inversion relations, reciprocity and polyominoes". Annals of Combinatorics, 3:223–249, 1999.

[10] W. pang Chin, S. Ntafos, "Optimum watchman routes". Information Processing Letters, 28(1):39 – 44, 1988.

[11] Adrian Dumitrescu and Csaba D. Tóth. "Watchman tours for polygons with holes". Computational Geometry, 45(7):326 – 333, 2012.

[12] Wei-Pang Chin and Simeon Ntafos. "Shortest watchman routes in simple polygons". Discrete & Computational Geometry, 6(1):9–31, Mar 1991.

[13] Bengt J. Nilsson and Derick Wood. " Optimum watchmen routes in spiral polygons". In Proceedings of the Second Canadian Conference in Computational Geometry, pages 269–272, Ottawa, Ontario, 1990.

[14] Xuehou Tan and Bo Jiang. "An improved algorithm for computing a shortest watchman route for lines". Information Processing Letters, 131:51 – 54, 2018.

[15] X. Tan, Q. Wei, "Improved exploration of unknown polygons". Theoretical Computer Science, 922:424–437, 2022.

[16] B. J. Nilsson and S. Schuierer. "Shortest m-watchmen routes for histograms": the minmax case. In Proceedings ICCI '92: Fourth International Conference on Computing and Information, pages 30–33, 1992.

[17] Alireza Bagheri, Anna Brötzner, Faezeh Farivar, Rahmat Ghasemi, Fatemeh Keshavarz-Kohjerdi, Erik Krohn, Bengt J. Nilsson, and Christiane Schmidt. "Minsum m watchmen's routes in Stiegl polygons". pages 41–44. XX Spanish Meeting on Computational Geometry, 2023.

[18] Rahmat Ghasemi, Alireza Bagheri, Fatemeh Keshavarz-Kohjerdi, and Faezeh Farivar. "Fixed k-watchman routes under the min-max criterion in staircase polygons". AUT Journal of Mathematics and Computing, 2024.

[19] Iraji, Mohammad Saber. "A novel wavelet-transformer discriminator for semi-supervised GANs with controlled regularization and ensemble techniques". Multimed Tools Appl, (2025).

[20] Iraji, Mohammad Saber. "Semi-supervised generative adversarial networks for imbalanced skin lesion diagnosis with an unbiased generator and informative images". Engineering Applications of Artificial Intelligence, 159: pages 11643, (2025).