

Powerful and Powerless Fuzzy Graphs and it's Application

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Abstract. In this article, we aim to enhance a fuzzy graph to achieve its optimal state and introduce a powerless or powerful fuzzy graph derived from it. To accomplish this, we will define a semi-strong arc and first determine modified fuzzy graphs that result from adjusting the value of an arc in the original fuzzy graph. Next, we will calculate the standard deviation of each fuzzy graph. To continue, we will identify the fuzzy graphs with the maximum and minimum standard deviations, designating them as the powerful and powerless fuzzy graphs, respectively. Finally, we will discuss an intriguing and significant application in identifying common injuries among high school students.

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1 Introduction

In classical set theory introduced by Cantor, the values of elements in a set can either be 0 or 1. This means that for any given element, there are only two possibilities: the element is either a member of the set or it is not. Consequently, Cantor's set theory cannot accommodate data that involves ambiguity or uncertainty. In 1965, Zadeh [1] 1965, proposed fuzzy set theory, which addresses these limitations. The most important feature of a fuzzy set is that it includes a class of objects that satisfy one or more specific properties. In a fuzzy set, each element is assigned a membership value selected from the interval $[0, 1]$, known as the grade of membership in the set.

The origins of graph theory trace back to the Knigsberg bridge problem in 1735. This problem led to the development of the concept of the Eulerian graph. Euler analyzed the Knigsberg bridge problem and devised a structure that solved it, which is now referred to as an Eulerian graph. In 1840, Mbius introduced the ideas of the complete graph and the bipartite graph, and Kuratowski later proved that these types of graphs can be planar by using recreational problems. Today, graph theory is extensively applied within computer science, particularly in research fields such as data mining, image segmentation, clustering, and networking (see [2]).

In 1975, Rosenfeld [3] introduced the concept of a fuzzy graph by merging the theories of fuzzy sets and graph theory. Fuzzy graph theory is increasingly being applied to model real-time systems where the levels of information have varying degrees of precision. Fuzzy models are useful because they aim to bridge the gap between traditional numerical models used in engineering and sciences and the symbolic models employed in expert systems. The field of fuzzy graph theory has expanded rapidly and now serves numerous applications

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across various domains. In 2012, Nagoor Gani and Latha [4] introduced the concept of irregular fuzzy graphs, including the ideas of total degree and totally irregular fuzzy graphs. The strong neighborhood of a node and the distance between two nodes in a fuzzy graph are useful for determining the relationships among nodes. Borzooei and other researchers [5, 6, 7, 8, 4, 9, 10, 11, 12] have conducted numerous studies on fuzzy graphs. Additionally, Yeh and Bang [13] 1975 introduced various concepts related to connectedness in fuzzy graphs. The initial studies on graph connectivity began in 1975 when Rosenfeld [3] introduced the concept of distance in fuzzy graphs. Based on this notion of distance, Bhattacharya [14] 1987 introduced the concepts of eccentricity and center in fuzzy graphs. Furthermore, Meenal [15] 2019, defined antipodal and eccentric fuzzy graphs, respectively, based on the definition of distance. Akram et al. ([16]-[18]) examined various aspects, including length, distance, eccentricity, radius, and diameter of bipolar fuzzy graphs, and introduced the concept of self-centered bipolar fuzzy graphs.

In this paper, we aim to enhance a fuzzy graph to its optimal state and introduce powerful and powerless fuzzy graphs derived from it. To achieve this, we first define the semi-strong arc in a fuzzy graph and determine modified fuzzy graphs that result from adjusting the value of an arc in the original fuzzy graph. We then calculate the standard deviation of each fuzzy graph. Finally, we categorize the fuzzy graphs with the maximum standard deviation as powerful and those with the minimum standard deviation as powerless. Moreover, recognizing that one of the most prevalent threats in society is the interest and inclination of young people and students toward drug use, we will demonstrate a significant application of powerless fuzzy graphs in identifying and controlling this issue. Specifically, we aim to illustrate the damage caused among high school students and propose methods to prevent its spread.

2 Preliminaries

In this section, we present definitions and results of fuzzy graphs that are utilized in this paper (see [19]-[21],[3]).

A *fuzzy graph* on simple graph $G^* = (V(G), E(G))$ is a pair $G = (\sigma, \mu)$, where $\sigma : V(G) \rightarrow [0, 1]$ and $\mu : E(G) \rightarrow [0, 1]$ are fuzzy sets such that for all $xy \in E(G)$,

$$\mu(xy) \leq \sigma(x) \wedge \sigma(y).$$

Notation. From now on, let $G = (\sigma, \mu)$ or G be an fuzzy graph on simple graph $G^* = (V(G), E(G))$, unless otherwise stated.

The *neighbourhood* of node $v \in V(G)$ is defined by $N(v)$ and defined as follows;

$$N(v) = \{u \in V(G) \mid 0 < \mu(uv)\}.$$

The arc $uv \in E(G)$ is called an *effective arc* whenever $\mu(uv) = \sigma(u) \wedge \sigma(v)$. Also, the *order* and *size* of G denoted by $O(G)$ and $S(G)$, respectively, and defined as below

$$O(G) = \sum_{v \in V(G)} \sigma(v) \quad , \quad S(G) = \sum_{uv \in E(G)} \mu(uv).$$

The *degree* of node $u \in V(G)$ is called $d(u)$ and define as below

$$d(u) = \sum_{uv \in E(G), u \neq v} \mu(uv)$$

G is called *complete*, if $\mu(uv) = \sigma(u) \wedge \sigma(v)$, for all $u, v \in V(G)$, is called *complete bipartite*, if $V(G) = V_1 \cup V_2$ where $V_1 \cap V_2 = \emptyset$ and for every $u, v \in V_1$ (or V_2), $\mu(uv) = 0$ and for every $u \in V_1$ and $v \in V_2$, $\mu(uv) =$

$\sigma(u) \wedge \sigma(v)$.

A *path* ρ in G is a sequence of distinct nodes $v_0, v_1, v_2, \dots, v_k, \in V(G)$ such that for any $i = 1, 2, \dots, k$, $\mu(v_{i-1}v_i) > 0$, then k is called the *length* of the path ρ .

For every $u, v \in V(G)$ and every path $\rho : u = u_0, u_1, \dots, u_{k_\rho} = v$, δ -distance from u to v is

$$\delta(u, v) = \bigwedge_{\rho} \left(\sum_{i=1}^{k_\rho} \left(\frac{1}{\mu(u_{i-1}u_i)} \right) \right).$$

The *eccentricity* of a node v is defined as $e(v) = \max_{u \in V(G)} \delta(u, v)$. The *diameter* and *radius* in G are denoted by $diam(G)$ and $r(G)$ and are defined as follows;

$$diam(G) = \bigvee_{v \in V(G)} e(v) \quad , \quad r(G) = \bigwedge_{v \in V(G)} e(v).$$

Let $u, v \in V(G)$. If u and v are connected by means of a path $\rho : u = v_0, v_1, \dots, v_k = v$ of length k in G , then $\mu_\rho^k(u, v)$ is defined as

$$\mu_\rho^k(u, v) = \mu(uv_1) \wedge \mu(v_1v_2) \wedge \mu(v_2v_3) \wedge \dots \wedge \mu(v_{k-1}v).$$

the *strength of connectedness* between u and v is

$$\mu^\infty(u, v) = \sup \{ \mu_\rho^k(u, v) \mid \rho \text{ is a path from } u \text{ to } v \text{ with length } k \}.$$

An arc uv in G is said to be a *strong arc*, if $\mu(uv) \geq \mu^\infty(u, v)$. The *strong neighbourhood* of node v in G is denoted by $N_s^*(v)$ and defined as follows;

$$N_s^*(v) = \{u \in V(G) \mid uv \text{ is strong}\}.$$

3 Powerful and powerless fuzzy graphs

In this section, we introduce the concept of standard deviation in a fuzzy graph. Following this, we present powerful and powerless fuzzy graphs and finally, we discuss its application in high school education. In this section, it is assumed that all fuzzy graphs are finite and simple.

Definition 3.1. The arc $uv \in E(G)$ is called *semi-strong* if

$$\mu(uv) = \frac{1}{\delta(u, v)}. \quad (1)$$

Example 3.2. Let G be a fuzzy graph as Figure 1:

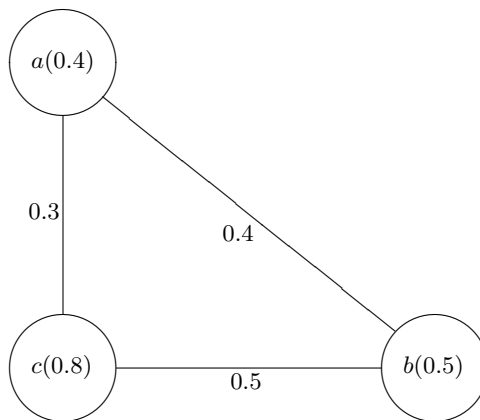


Figure 1: Fuzzy graph G

Then for arc ac , we have,

$$\delta(a, c) = \frac{1}{0.3} \quad \rightarrow \quad \mu(ac) = \frac{1}{\delta(a, c)} = 0.3.$$

Hence uv is a semi-strong arc.

Theorem 3.3. *Every strong arc in G is semi-strong.*

Proof. Suppose $uv \in E(G)$ is a strong arc. Then for every path $\rho : u = v_0, v_1, \dots, v_k = v$, we have

$$\mu(uv) \geq \mu^\infty(u, v)$$

such that

$$\mu^\infty(u, v) = \sup\{\mu_\rho^k(u, v) \mid \rho \text{ is a path from } u \text{ to } v \text{ with length } k\}$$

where

$$\mu_\rho^k(u, v) = \sup\{\mu(u, v_0) \wedge \mu(v_0 v_1) \wedge \mu(v_1 v_2) \wedge \mu(v_2 v_3) \wedge \dots \wedge \mu(v_{k-1} v)\}.$$

Hence, for a strong arc uv and every path with length $1, 2, \dots, k, \dots$, we have

$$\mu(uv) \geq (\mu_\rho^1(u, v)) \vee (\mu_\rho^2(u, v)) \vee (\mu_\rho^3(u, v)) \vee \dots \vee (\mu_\rho^k(u, v)) \vee \dots$$

Therefore,

$$\begin{aligned} \mu(uv) &\geq \mu_\rho^1(u, v), \\ \mu(uv) &\geq \mu_\rho^2(u, v), \\ &\vdots \end{aligned}$$

So by definition of $\mu_\rho^k(u, v)$ we get,

$$\begin{aligned} \mu(uv) &\geq \mu(uv_1) \wedge \mu(v_1 v), \\ \mu(uv) &\geq \mu(uv_1) \wedge \mu(v_1 v_2) \wedge \mu(v_2 v), \\ &\vdots \end{aligned}$$

We see that $\mu(uv_1) \wedge \mu(v_1 v) = \mu(uv_1)$ or $\mu(uv_1) \wedge \mu(v_1 v) = \mu(v_1 v)$. Therefore

$$\frac{1}{\mu(uv_1) \wedge \mu(v_1 v)} = \frac{1}{\mu(uv_1)} \quad \text{or} \quad \frac{1}{\mu(uv_1) \wedge \mu(v_1 v)} = \frac{1}{\mu(v_1 v)}.$$

And this shows that

$$\frac{1}{\mu(uv_1) \wedge \mu(v_1 v)} < \frac{1}{\mu(uv_1)} + \frac{1}{\mu(v_1 v)}.$$

So, if $\mu(uv) \geq \mu(uv_1) \wedge \mu(v_1 v)$ then

$$\frac{1}{\mu(uv)} \leq \frac{1}{\mu(uv_1) \wedge \mu(v_1 v)} < \frac{1}{\mu(uv_1)} + \frac{1}{\mu(v_1 v)} \quad \rightarrow \quad \frac{1}{\mu(uv)} < \frac{1}{\mu(uv_1)} + \frac{1}{\mu(v_1 v)}$$

Similarly,

$$\begin{aligned} \frac{1}{\mu(uv)} &< \frac{1}{\mu(uv_1)} + \frac{1}{\mu(v_1v_2)} + \frac{1}{\mu(v_2v)} \\ \frac{1}{\mu(uv)} &< \frac{1}{\mu(uv_1)} + \frac{1}{\mu(v_1v_2)} + \frac{1}{\mu(v_2v_3)} + \frac{1}{\mu(v_3v)} \\ &\vdots \end{aligned}$$

Then

$$\frac{1}{\mu(uv)} < \bigwedge_{\rho} \sum_{i=1}^n \left(\frac{1}{\mu(v_{i-1}v_i)} \right).$$

Hence, by the definition of δ -distance, we have

$$\delta(u, v) = \frac{1}{\mu(uv)} \rightarrow \mu(uv) = \frac{1}{\delta(u, v)}.$$

□

Note. The converse of Theorem 3.3, is not correct in general. For example, let G be fuzzy graph as in the Example 3.2. We see that $\mu(ac) = \frac{1}{\delta(a,c)}$, but ac is not a strong arc.

Definition 3.4. Let $E_s(G)$ be a set of all strong arcs in G . Then the standard deviation of G denoted by $sd(G)$ and defined as follows,

$$sd(G) = \frac{\sum_{uv \in E_s(G)} \left(\mu(uv) - (\sigma(u) \wedge \sigma(v)) \right)^2}{|E_s(G)|} + \frac{\sum_{uv \notin E_s(G)} \left(\mu(uv) - \frac{1}{\delta(u,v)} \right)^2}{|E(G)| - |E_s(G)|}. \quad (2)$$

Note. It should note that, in this definition, we have defined the standard deviation by the difference between the value of a strong arc and the value of the effective arc (which is the most optimal case in this definition) and also the difference between a non-strong arc and the semi-strong arc (which is the best case in this definition of the non-strong arc). In other words, we obtain that if the standard deviation is the lowest, then the arcs are effective or the non-strong arcs are semi-strong. As a result, a fuzzy graph in which all strong arcs are effective or all non-strong arcs are semi-strong is more important to us.

Definition 3.5. (i) Let $uv \in E(G)$. If we change the value of arc uv , meaning we increase or decrease $\mu(uv)$, then the resulting fuzzy graph is called a modified fuzzy graph and is denoted by G^{uv} .

(ii) The powerful and the powerless fuzzy graphs of G denoted as G^P and G_P and defined as follows, respectively,

$$G^P = \left\{ G^{uv} \mid \bigwedge_{uv \in E(G)} \left(sd(G^{uv}), sd(G) \right) = \{ sd(G^{uv}) \text{ or } sd(G^{u'v'}) \}, E_s(G^{uv}) \geq E_s(G^{u'v'}) \right\}$$

$$G_P = \left\{ G^{uv} \mid \bigvee_{uv \in E(G)} \left(sd(G^{uv}), sd(G) \right) = \{ sd(G^{uv}) \text{ or } sd(G^{u'v'}) \}, E_s(G^{uv}) \leq E_s(G^{u'v'}) \right\}$$

Note. By this definition, we see that G^P and G_P may not be unique.

Example 3.6. Let G be a fuzzy graph as Figure 2:

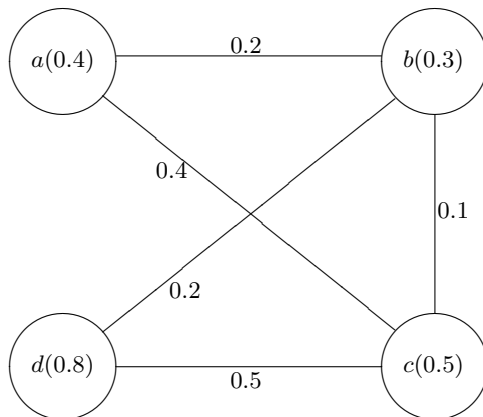


Figure 2: Fuzzy graph G

In fuzzy graph G , we see that

$$E_s(G) = \{ab, bd, ac, cd\} \quad , \quad \delta(b, c) = \left(\frac{1}{0.1}\right) \wedge \left(\frac{1}{0.2} + \frac{1}{0.5}\right) \wedge \left(\frac{1}{0.2} + \frac{1}{0.4}\right) = 7,$$

Therefore we have,

$$\begin{aligned} sd(G) &= \frac{1}{|E_s(G)|} \left(\left(\mu(ab) - (\sigma(a) \wedge \sigma(b)) \right)^2 + \left(\mu(bd) - (\sigma(b) \wedge \sigma(d)) \right)^2 + \left(\mu(ac) - (\sigma(a) \wedge \sigma(c)) \right)^2 \right. \\ &\quad \left. + \left(\mu(cd) - (\sigma(c) \wedge \sigma(d)) \right)^2 \right) \\ &\quad + \frac{1}{|E_s(G)| - |E(G)|} \left(\left(\mu(bc) - \frac{1}{\delta(b, c)} \right)^2 \right) \\ &= \frac{1}{4} \left((0.2 - (0.4 \wedge 0.3))^2 + (0.2 - (0.3 \wedge 0.8))^2 + (0.4 - (0.4 \wedge 0.5))^2 + (0.5 - (0.5 \wedge 0.8))^2 \right) \\ &\quad + \left(0.1 - \frac{1}{7} \right)^2 = 0.007 \end{aligned}$$

Consider G^{ab} and G^{bc} be two modified fuzzy graphs from G (resulting from the change of $\mu(ab) = 0.2$ and $\mu(bc) = 0.1$ in G to $\mu(ab) = 0.3$ and $\mu(bc) = 0.2$, respectively) as Figure 3:

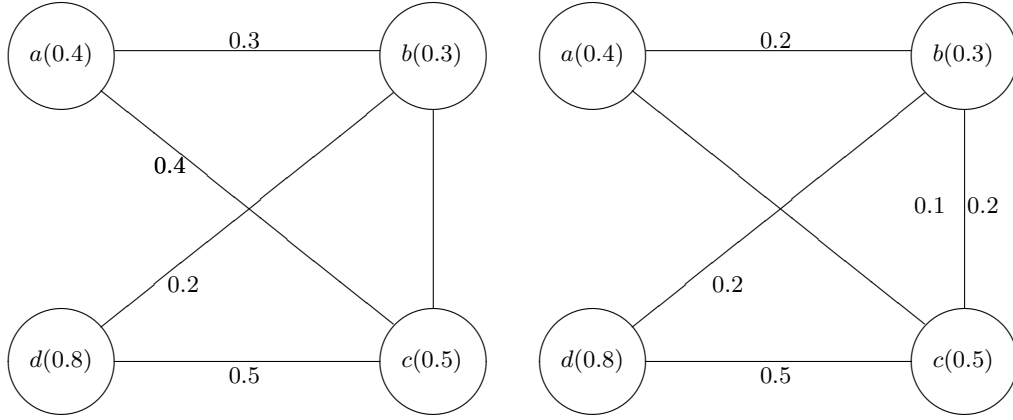


Figure 3: Modified fuzzy graphs of G (left is graph G^{ab} , right is graph G^{bc})

Now, since $E_s(G^{ab}) = \{ab, ac, dc\}$, we have G^{ab} we have,

$$\begin{aligned}
 \delta(b, d) &= \frac{1}{0.2} \wedge \left(\frac{1}{0.1} + \frac{1}{0.5}\right) \wedge \left(\frac{1}{0.3} + \frac{1}{0.4} + \frac{1}{0.5}\right) = 5 \wedge 12 \wedge 7.8 = 5 \\
 \delta(b, c) &= \frac{1}{0.1} \wedge \left(\frac{1}{0.2} + \frac{1}{0.5}\right) \wedge \left(\frac{1}{0.3} + \frac{1}{0.4}\right) = 10 \wedge 7 \wedge 5.8 = 5.8 \\
 sd(G^{ab}) &= \frac{1}{|E_s(G)|} \left(\left(\mu(ab) - (\sigma(a) \wedge \sigma(b)) \right)^2 + \left(\mu(ac) - (\sigma(a) \wedge \sigma(c)) \right)^2 + \left(\mu(dc) - (\sigma(d) \wedge \sigma(c)) \right)^2 \right) \\
 &+ \frac{1}{|E_s(G)| - |E(G)|} \left(\left(\mu(bd) - \frac{1}{\delta(b, d)} \right)^2 + \left(\mu(bc) - \frac{1}{\delta(b, c)} \right)^2 \right) \\
 &= \frac{1}{3} \left((0.3 - (0.4 \wedge 0.3))^2 + (0.4 - (0.4 \wedge 0.5))^2 + (0.5 - (0.5 \wedge 0.8))^2 \right) \\
 &+ \frac{1}{2} \left(\left(0.2 - \frac{1}{5} \right)^2 + \left(0.1 - \frac{1}{5.8} \right)^2 \right) = 0.003
 \end{aligned}$$

Since $E_s(G^{bc}) = \{ab, bc, cd, ac, bd\}$ and all arcs in G^{bc} are strong, then we have

$$\begin{aligned}
 sd(G^{bc}) &= \frac{1}{5} \left((0.2 - (0.4 \wedge 0.3))^2 + (0.4 - (0.4 \wedge 0.5))^2 + (0.2 - (0.3 \wedge 0.5))^2 + (0.5 - (0.5 \wedge 0.8))^2 \right. \\
 &\quad \left. + (0.2 - (0.3 \wedge 0.8))^2 \right) = 0.006
 \end{aligned}$$

So, we see that

$$sd(G) = sd(G) \vee sd(G^{ab}) \vee sd(G^{bc}) \quad , \quad sd(G^{ab}) = sd(G) \wedge sd(G^{ab}) \wedge sd(G^{bc}).$$

Therefore, G and G^{ab} are powerless and powerful fuzzy graphs of G , respectively. In other words,

$$G_P = G \quad , \quad G^P = G^{ab}.$$

Theorem 3.7. If $sd(G) = 0$, then every arc $uv \in E(G)$ is effective or semi-strong.

Proof. Since $sd(G) = 0$, we have;

$$\frac{\sum_{uv \in E_s(G)} \left(\mu(uv) - (\sigma(u) \wedge \sigma(v)) \right)^2}{|E_s(G)|} + \frac{\sum_{uv \notin E_s(G)} \left(\mu(uv) - \frac{1}{\delta(u,v)} \right)^2}{|E(G)| - |E_s(G)|} = 0. \quad (3)$$

If $E_s(G) = E(G)$, that is all arc's are strong, since;

$$\frac{\sum_{uv \in E_s(G)} \left(\mu(uv) - (\sigma(u) \wedge \sigma(v)) \right)^2}{|E_s(G)|} = 0 \quad (4)$$

then for all $uv \in E(G)$, $\mu(uv) - (\sigma(u) \wedge \sigma(v)) = 0$ and so $\mu(uv) = (\sigma(u) \wedge \sigma(v))$. Then all arc's are effective. If there exists $u_0v_0 \in E(G)$ such that $u_0v_0 \notin E_s(G)$, since

$$\frac{\sum_{uv \notin E_s(G)} \left(\mu(uv) - \frac{1}{\delta(u,v)} \right)^2}{|E(G)| - |E_s(G)|} = 0. \quad (5)$$

then $\mu(u_0v_0) - \frac{1}{\delta(u_0,v_0)} = 0$ and so $\mu(u_0v_0) = \frac{1}{\delta(u_0,v_0)}$. This shows that u_0v_0 is semi-strong arc in G . $\square$

Lemma 3.8. [22] G has at least one strong arc.

Corollary 3.9. Let G has no any effective arc or no any semi-strong arc, then $sd(G) \neq 0$.

Proof. If G has no effective arc, so by Lemma 3.8, G has at least one strong arc. Suppose that uv is an arc in G such that is strong. According to the assumption of the theorem we have $\mu(uv) < \sigma(u) \wedge \sigma(v)$ and so $(\mu(uv) - \sigma(u) \wedge \sigma(v))^2 > 0$. Hence

$$\sum_{uv \in E_s(G)} \left(\mu(uv) - (\sigma(u) \wedge \sigma(v)) \right)^2 > 0.$$

Therefore, $sd(G) \neq 0$. Similar to the above process and according to the definition of a semi-strong arc, we consider that if G has no semi-strong arc, then $sd(G) \neq 0$. $\square$

Remark 3.10. G has at least one powerful or powerless fuzzy graph.

Lemma 3.11. [22] Every effective arc in G is strong.

Theorem 3.12. Let $sd(G) = 0$. Then every strong arc in G is effective.

Proof. Let $sd(G) = 0$. Then

$$\frac{\sum_{uv \in E_s(G)} \left(\mu(uv) - (\sigma(u) \wedge \sigma(v)) \right)^2}{|E_s(G)|} + \frac{\sum_{uv \notin E_s(G)} \left(\mu(uv) - \frac{1}{\delta(u,v)} \right)^2}{|E(G)| - |E_s(G)|} = 0. \quad (6)$$

and so

$$\frac{\sum_{uv \in E_s(G)} \left(\mu(uv) - (\sigma(u) \wedge \sigma(v)) \right)^2}{|E_s(G)|} = 0, \quad \frac{\sum_{uv \notin E_s(G)} \left(\mu(uv) - \frac{1}{\delta(u,v)} \right)^2}{|E(G)| - |E_s(G)|} = 0. \quad (7)$$

If

$$\frac{\sum_{uv \in E_s(G)} \left(\mu(uv) - (\sigma(u) \wedge \sigma(v)) \right)^2}{|E_s(G)|} = 0 \quad (8)$$

then for every $uv \in E_s(G)$, $\mu(uv) - (\sigma(u) \wedge \sigma(v)) = 0$, this shows that uv is effective. In other words, for every strong arc in G as uv , we see that

$$\mu(uv) = \sigma(u) \wedge \sigma(v).$$

Then every strong arc in G is effective arc. $\square$

Theorem 3.13.

$$sd(G) \leq \frac{\sum_{uv \in E(G)} \left(\frac{1}{\delta(u,v)} - (\sigma(u) \wedge \sigma(v)) \right)^2}{|E(G)|}$$

Proof. By Corollary 3.9, if G has no effective or semi-strong arc, then $sd(G) \neq 0$. Suppose that every arc in G is neither effective nor semi-strong. Let uv be an arbitrary arc in G , if $uv \in E_s(G)$ then $\mu(uv) - (\sigma(u) \wedge \sigma(v)) \neq 0$ and so $\mu(uv) \neq \sigma(u) \wedge \sigma(v)$. Hence $\mu(uv) < \sigma(u) \wedge \sigma(v)$. Now, by Theorem 3.3, we see that;

$$\mu(uv) = \frac{1}{\delta(u,v)}.$$

Therefore, for every $uv \in E_s(G)$, $\mu(uv) = \frac{1}{\delta(u,v)} < \sigma(u) \wedge \sigma(v)$. Thus we get

$$\frac{\sum_{uv \in E_s(G)} \left(\mu(uv) - (\sigma(u) \wedge \sigma(v)) \right)^2}{|E_s(G)|} = \frac{\sum_{uv \in E_s(G)} \left(\frac{1}{\delta(u,v)} - (\sigma(u) \wedge \sigma(v)) \right)^2}{|E_s(G)|}$$

On the other hand, if $uv \in E(G) \setminus E_s(G)$, then $\mu(uv) - \frac{1}{\delta(u,v)} \neq 0$ and so $\mu(uv) \neq \frac{1}{\delta(u,v)}$. Hence

$$\frac{1}{\delta(u,v)} < \mu(uv) < \sigma(u) \wedge \sigma(v).$$

Now for every $uv \in E_s(G)$, we have

$$\mu(uv) - \frac{1}{\delta(u,v)} < (\sigma(u) \wedge \sigma(v)) - \frac{1}{\delta(u,v)}$$

and so

$$\left(\mu(uv) - \frac{1}{\delta(u,v)} \right)^2 < \left((\sigma(u) \wedge \sigma(v)) - \frac{1}{\delta(u,v)} \right)^2.$$

Therefore we get

$$\frac{\sum_{uv \in E_s(G)} \left(\mu(uv) - \frac{1}{\delta(u,v)} \right)^2}{|E(G)| - |E_s(G)|} < \frac{\sum_{uv \notin E_s(G)} \left(\frac{1}{\delta(u,v)} - \sigma(u) \wedge \sigma(v) \right)^2}{|E(G)| - |E_s(G)|}$$

So by Definition 3.4, we get

$$\begin{aligned} sd(G) &= \frac{\sum_{uv \in E_s(G)} \left(\mu(uv) - (\sigma(u) \wedge \sigma(v)) \right)^2}{|E_s(G)|} + \frac{\sum_{uv \in E_s(G)} \left(\mu(uv) - \frac{1}{\delta(u,v)} \right)^2}{|E(G)| - |E_s(G)|} \\ &\leq \frac{\sum_{uv \in E_s(G)} \left(\frac{1}{\delta(u,v)} - (\sigma(u) \wedge \sigma(v)) \right)^2}{|E_s(G)|} + \frac{\sum_{uv \notin E_s(G)} \left(\frac{1}{\delta(u,v)} - \sigma(u) \wedge \sigma(v) \right)^2}{|E(G)| - |E_s(G)|} \\ &= \frac{\sum_{uv \in E(G)} \left(\frac{1}{\delta(u,v)} - \sigma(u) \wedge \sigma(v) \right)^2}{|E(G)|}. \end{aligned}$$

This shows that

$$sd(G) \leq \frac{\sum_{uv \in E(G)} \left(\frac{1}{\delta(u,v)} - \sigma(u) \wedge \sigma(v) \right)^2}{|E(G)|}.$$

□

Theorem 3.14. *If every arc in G is effective, then G has a unique powerful fuzzy graph, in other words, $G = G^P = G_P$.*

Proof. Let every arc in G is effective. By Lemma 3.11, every arc in G is strong. So we see that $E_s(G) = \{uv \mid uv \in E(G)\}$ and since for every $uv \in E(G)$, $\mu(uv) = \sigma(u) \wedge \sigma(v)$ thus we have,

$$\begin{aligned} sd(G) &= \frac{\sum_{uv \in E_s(G)} \left(\mu(uv) - (\sigma(u) \wedge \sigma(v)) \right)^2}{|E_s(G)|} \\ &= \frac{\sum_{uv \in E_s(G)} \left((\sigma(u) \wedge \sigma(v)) - (\sigma(u) \wedge \sigma(v)) \right)^2}{|E_s(G)|} \\ &= 0. \end{aligned}$$

This shows that G is only powerful fuzzy graph in G and $G^P = G = G_P$. □

Note. If every arc in G is effective, then $sd(G) = 0$. But the converse is not correct in general. For example let G be a fuzzy graph in the Example 3.2, we see that $E_s(G) = \{ab, bc\}$ then,

$$sd(G) = \frac{(0.4 - (0.4 \wedge 0.5)) + (0.5 - (0.5 \wedge 0.8))}{2} + (0.3 - 0.3) = 0.$$

But ac is not a strong arc.

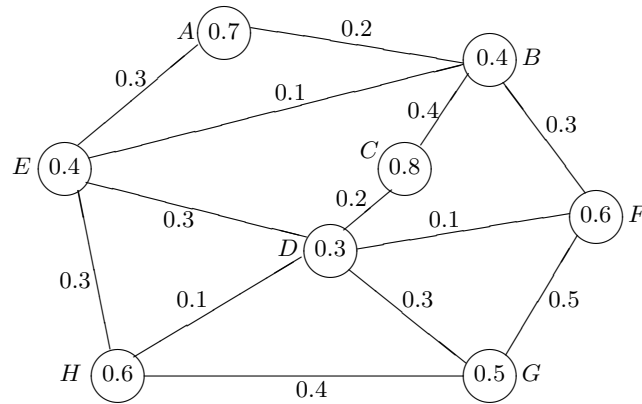
Corollary 3.15. *If G is complete or complete bipartite fuzzy graph, then $G = G^P$.*

4 Application of powerless fuzzy graphs

We are examining a high school with 100 students. Recently, a new drug has been discovered among the student body, and field research has shown that 90 percent of the students have tried this drug at least once. Our objective is to eliminate drug use from the school without the students being aware of our actions.

To achieve this, we have identified ten students who have been using these substances or whose appearance suggests they may be users. We will create a graph featuring these ten students, with each student represented as a vertex. Their relationships with one another will be illustrated as arcs. Each vertex will reflect various values, such as psychological security, morals, the emotional dynamics of their family relationships, parental education, overall well-being, comfort, and the sense of peace within each student's family.

Additionally, each arc will represent the degree of communication between two students, evaluated based on factors such as the length of their acquaintance, family connections, and family interaction. Using this information, we will create a fuzzy graph to illustrate our findings (see Figure 4).

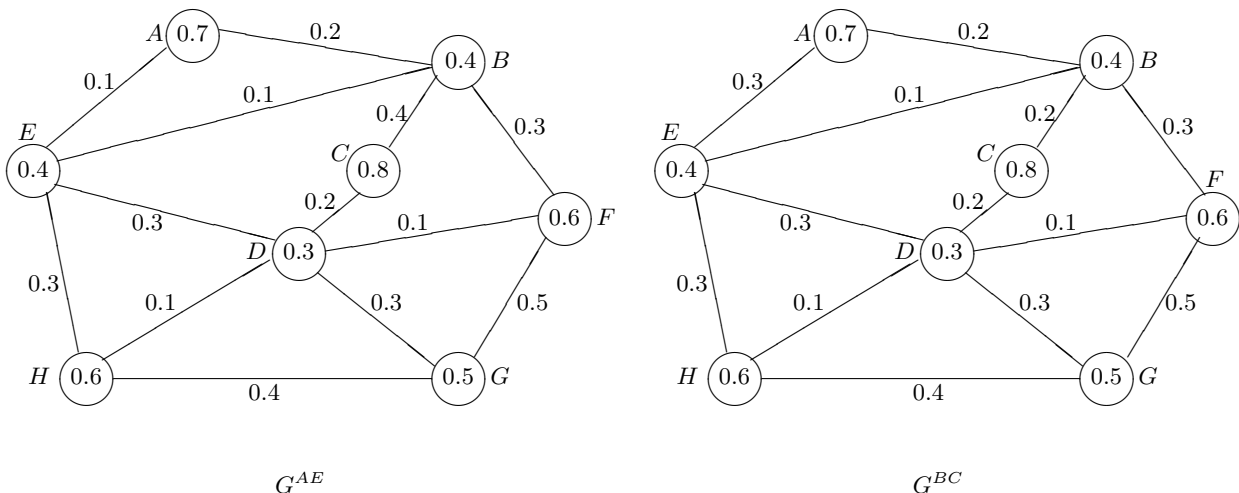
**Figure 4:** Fuzzy graphs G of students

$$E_s(G) = \{AE, BC, BF, ED, EH, DG, FG, GH\},$$

$$\delta(A, B) = 5 \quad , \quad \delta(B, E) = 8.3 \quad , \quad \delta(C, D) = 5 \quad , \quad \delta(F, D) = 5.3 \quad , \quad \delta(D, H) = 5.8,$$

$$\begin{aligned} sd(G) &= \frac{1}{8} \left((0.3 - (0.4 \wedge 0.7))^2 + (0.4 - (0.4 \wedge 0.8))^2 + (0.3 - (0.4 \wedge 0.6))^2 + (0.3 - (0.4 \wedge 0.3))^2 \right. \\ &\quad + (0.3 - (0.4 \wedge 0.6))^2 + (0.3 - (0.3 \wedge 0.5))^2 + (0.5 - (0.6 \wedge 0.5))^2 + (0.4 - (0.6 \wedge 0.5))^2 \Big) \\ &\quad + \frac{1}{5} \left((0.2 - \frac{1}{5})^2 + (0.1 - \frac{1}{8.3})^2 + (0.2 - \frac{1}{5})^2 + (0.1 - \frac{1}{5.8})^2 + (0.1 - \frac{1}{5.3})^2 \right) = 0.008 \end{aligned}$$

In this graph, our goal is to significantly reduce the relationships between individuals, which leads us to seek a powerless fuzzy graph. To achieve this, we aim to minimize the val of strong arcs. Consequently, we have defined modified fuzzy graphs G^{AE} , G^{BC} , G^{BF} , G^{ED} , G^{EH} , G^{DG} , G^{FG} and G^{HG} (Figures 5, 6, 7 and 8) by modifying the arcs $\mu(AE)$, $\mu(BC)$, $\mu(BF)$, $\mu(ED)$, $\mu(EH)$, $\mu(DG)$, $\mu(FG)$ and $\mu(HG)$, accordingly



$$E_s(G^{AE}) = \{AB, BC, BF, ED, EH, DG, FG, GH\},$$

$$\delta(A, E) = 10 \quad , \quad \delta(E, B) = 10 \quad , \quad \delta(C, D) = 5 \quad , \quad \delta(F, D) = 5.3 \quad , \quad \delta(D, H) = 5.8,$$

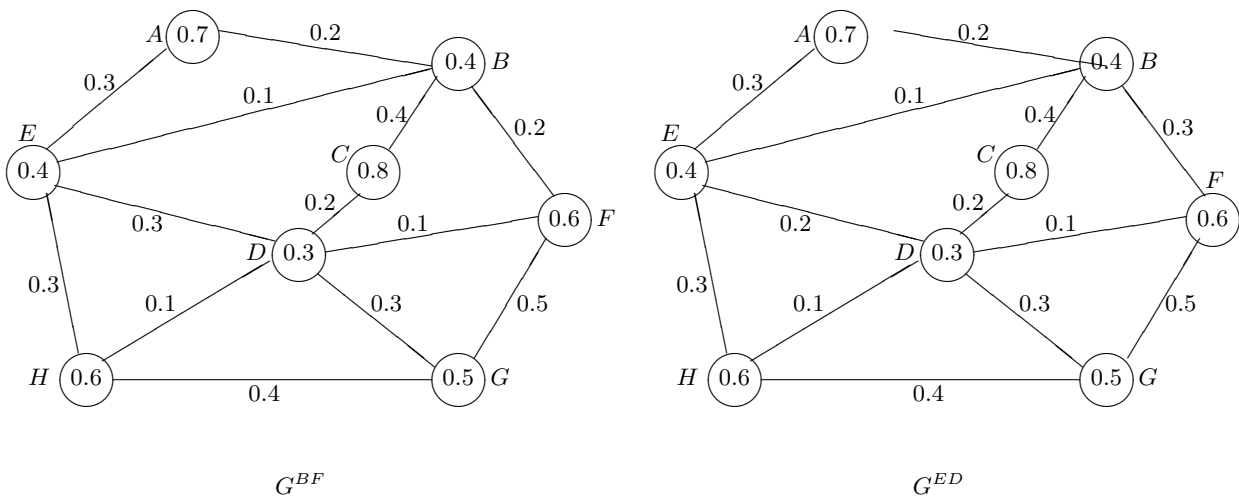
Figure 5: Modified fuzzy graphs G of students

$$\begin{aligned}
sd(G^{AE}) &= \frac{1}{8} \left((0.2 - (0.4 \wedge 0.7))^2 + (0.4 - (0.4 \wedge 0.8))^2 + (0.3 - (0.4 \wedge 0.6))^2 + (0.3 - (0.4 \wedge 0.3))^2 \right. \\
&\quad + (0.3 - (0.4 \wedge 0.6))^2 + (0.3 - (0.3 \wedge 0.5))^2 + (0.5 - (0.6 \wedge 0.5))^2 + (0.4 - (0.6 \wedge 0.5))^2 \Big) \\
&\quad + \frac{1}{5} \left((0.1 - \frac{1}{10})^2 + (0.1 - \frac{1}{10})^2 + (0.2 - \frac{1}{5})^2 + (0.1 - \frac{1}{5.3})^2 + (0.1 - \frac{1}{5.8})^2 \right) = 0.012.
\end{aligned}$$

$$E_s(G^{BC}) = \{AE, BC, BF, DC, ED, EH, DG, FG, GH\},$$

$$\delta(A, B) = 3.3, \delta(E, B) = 8.3, \delta(H, D) = 5.8, \delta(F, D) = 5.3,$$

$$\begin{aligned}
sd(G^{BC}) &= \frac{1}{9} \left((0.3 - (0.4 \wedge 0.7))^2 + (0.2 - (0.4 \wedge 0.8))^2 + (0.3 - (0.4 \wedge 0.6))^2 + (0.2 - (0.3 \wedge 0.8))^2 \right. \\
&\quad + (0.3 - (0.4 \wedge 0.3))^2 + (0.3 - (0.4 \wedge 0.6))^2 + (0.3 - (0.3 \wedge 0.5))^2 + (0.5 - (0.6 \wedge 0.5))^2 \\
&\quad + (0.4 - (0.6 \wedge 0.5))^2 \Big) + \frac{1}{4} \left((0.2 - \frac{1}{3.3})^2 + (0.1 - \frac{1}{8.3})^2 + (0.1 - \frac{1}{5.8})^2 + (0.1 - \frac{1}{5.3})^2 \right) \\
&= 0.016.
\end{aligned}$$

**Figure 6:** Modified fuzzy graphs of G of students**Figure 6:** Modified fuzzy graphs of G

$$E_s(G^{BF}) = \{AE, AB, BC, BF, DC, ED, EH, DG, FG, GH\},$$

$$\delta(B, E) = 8.3, \delta(F, D) = 5.3, \delta(D, H) = 5.8,$$

$$\begin{aligned}
sd(G^{BF}) &= \frac{1}{10} \left((0.3 - (0.4 \wedge 0.7))^2 + (0.2 - (0.4 \wedge 0.7))^2 + (0.4 - (0.4 \wedge 0.8))^2 + (0.2 - (0.4 \wedge 0.6))^2 \right. \\
&+ (0.2 - (0.3 \wedge 0.8))^2 + (0.3 - (0.4 \wedge 0.3))^2 + (0.3 - (0.4 \wedge 0.6))^2 + (0.3 - (0.3 \wedge 0.5))^2 \\
&+ (0.5 - (0.5 \wedge 0.6))^2 + (0.4 - (0.5 \wedge 0.6))^2 \Big) \\
&+ \frac{1}{3} \left((0.1 - \frac{1}{8.3})^2 + (0.1 - \frac{1}{5.3})^2 + (0.1 - \frac{1}{5.8})^2 \right) = 0.016.
\end{aligned}$$

$$E_s(G^{ED}) = \{AE, BC, BF, EH, DG, FG, HG\},$$

$$\delta(A, B) = 5, \delta(B, E) = 8.3, \delta(C, D) = 5, \delta(E, D) = 5, \delta(F, D) = 5.3, \delta(H, D) = 5.8,$$

$$\begin{aligned}
sd(G^{ED}) &= \frac{1}{7} \left((0.3 - (0.4 \wedge 0.7))^2 + (0.3 - (0.4 \wedge 0.6))^2 + (0.4 - (0.4 \wedge 0.8))^2 + (0.5 - (0.5 \wedge 0.6))^2 \right. \\
&+ (0.3 - (0.3 \wedge 0.5))^2 + (0.3 - (0.4 \wedge 0.6))^2 + (0.4 - (0.5 \wedge 0.6))^2 \Big) \\
&+ \frac{1}{6} \left((0.2 - \frac{1}{5})^2 + (0.1 - \frac{1}{8.3})^2 + (0.2 - \frac{1}{5})^2 + (0.1 - \frac{1}{5.3})^2 + (0.1 - \frac{1}{5.8})^2 + (0.2 - \frac{1}{5})^2 \right) \\
&= 0.012
\end{aligned}$$

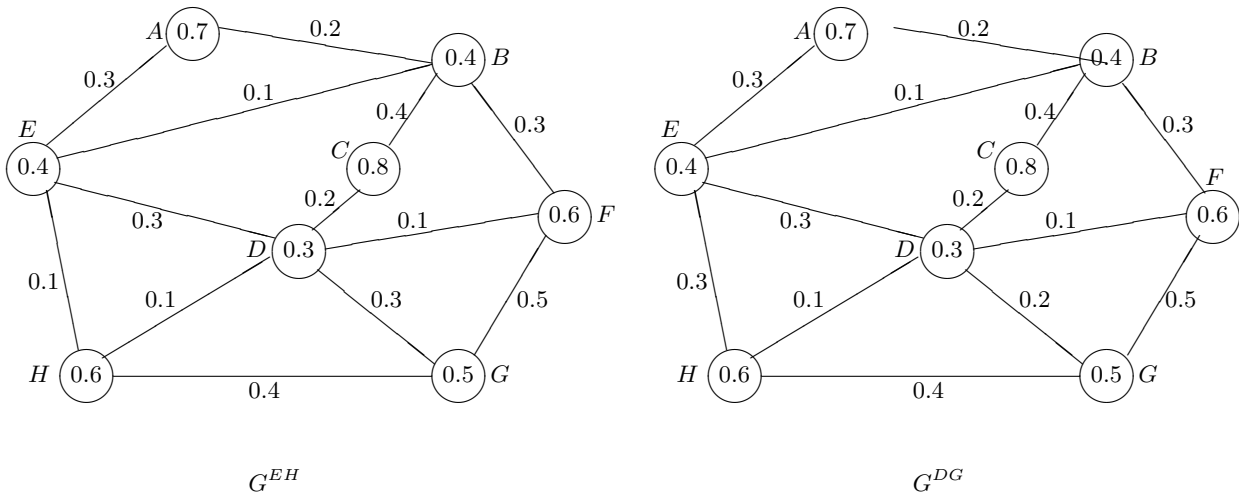


Figure 7: Modified fuzzy graphs G of students

$$E_s(G^{EH}) = \{AE, BC, BF, ED, DG, FG, GH\}$$

$$\delta(A, B) = 5, \delta(B, E) = 8.3, \delta(D, C) = 5, \delta(D, F) = 5.3, \delta(D, H) = 5.8, \delta(E, H) = 9.1$$

$$\begin{aligned}
sd(G^{EH}) &= \frac{1}{7} \left((0.3 - (0.4 \wedge 0.7))^2 + (0.4 - (0.4 \wedge 0.8))^2 + (0.3 - (0.4 \wedge 0.6))^2 + (0.3 - (0.4 \wedge 0.3))^2 \right. \\
&+ (0.3 - (0.3 \wedge 0.5))^2 + (0.5 - (0.5 \wedge 0.6))^2 + (0.4 - (0.5 \wedge 0.6))^2 \Big) \\
&+ \frac{1}{6} \left((0.2 - \frac{1}{5})^2 + (0.1 - \frac{1}{8.3})^2 + (0.2 - \frac{1}{5})^2 + (0.1 - \frac{1}{5.3})^2 + (0.1 - \frac{1}{5.8})^2 + (0.1 - \frac{1}{9.1})^2 \right) \\
&= 0.007.
\end{aligned}$$

$$E_s(G^{DG}) = \{AE, BC, BF, ED, EH, FG, GH\}$$

$$\delta(A, B) = 5, \delta(B, E) = 8.3, \delta(D, C) = 5, \delta(F, D) = 7, \delta(D, G) = 5, \delta(D, H) = 6.6$$

$$\begin{aligned} sd(G^{DG}) &= \frac{1}{7} \left((0.3 - (0.4 \wedge 0.7))^2 + (0.4 - (0.4 \wedge 0.8))^2 + (0.3 - (0.4 \wedge 0.6))^2 \right. \\ &\quad + (0.3 - (0.4 \wedge 0.6))^2 + (0.1 - (0.3 \wedge 0.6))^2 + (0.5 - (0.5 \wedge 0.6))^2 + (0.4 - (0.5 \wedge 0.6))^2 \Big) \\ &\quad + \frac{1}{6} \left((0.1 - \frac{1}{7})^2 + ((0.2 - \frac{1}{5})^2 + (0.1 - \frac{1}{8.3})^2 + (0.2 - \frac{1}{5})^2 + (0.2 - \frac{1}{5})^2 + (0.1 - \frac{1}{6.6})^2 \right) \\ &= 0.011. \end{aligned}$$

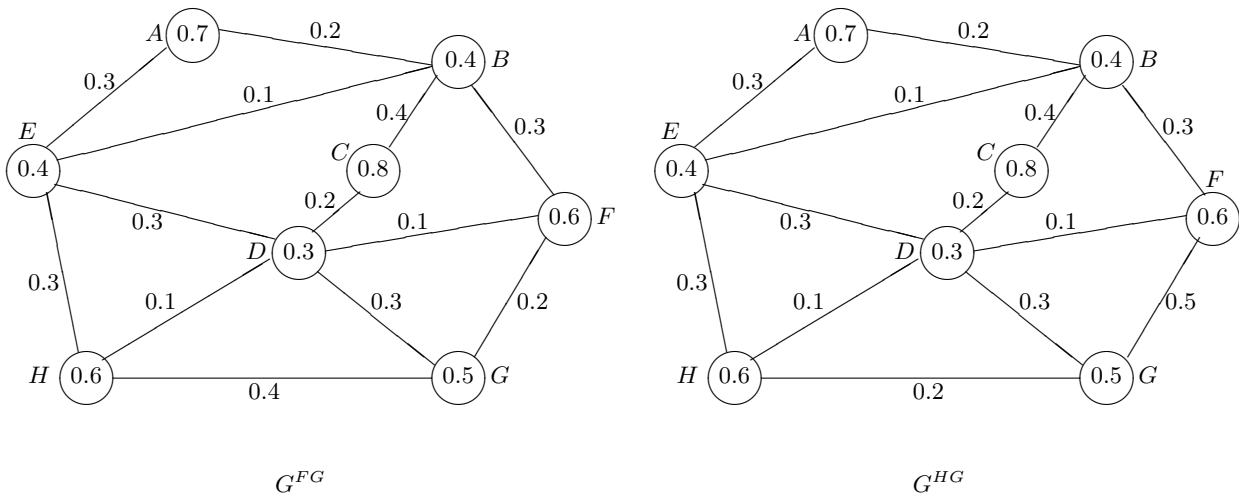


Figure 8: Modified fuzzy graphs G of students

$$E_s(G^{FG}) = \{AE, AB, BC, BF, DC, ED, EH, DG, FG, GH\},$$

$$\delta(B, E) = 8.3, \delta(D, F) = 8.3, \delta(D, H) = 5.8,$$

$$\begin{aligned} sd(G^{FG}) &= \frac{1}{10} \left((0.3 - (0.4 \wedge 0.7))^2 + (0.2 - (0.4 \wedge 0.7))^2 + (0.4 - (0.4 \wedge 0.8))^2 + (0.3 - (0.4 \wedge 0.6))^2 \right. \\ &\quad + (0.2 - (0.3 \wedge 0.8))^2 + (0.3 - (0.4 \wedge 0.3))^2 + (0.3 - (0.4 \wedge 0.6))^2 + (0.3 - (0.3 \wedge 0.5))^2 \\ &\quad + (0.2 - (0.5 \wedge 0.6))^2 + (0.4 - (0.5 \wedge 0.6))^2 \Big) \\ &\quad + \frac{1}{3} \left((0.1 - \frac{1}{8.3})^2 + (0.1 - \frac{1}{8.3})^2 + (0.1 - \frac{1}{5.8})^2 \right) = 0.02 \end{aligned}$$

$$E_s(G^{HG}) = \{AE, BC, BF, ED, EH, DG, FG\},$$

$$\delta(A, B) = 5, \delta(B, E) = 8.3, \delta(D, C) = 5, \delta(F, D) = 5.3, \delta(D, H) = 6.6, \delta(H, G) = 5,$$

$$\begin{aligned}
sd(G^{HG}) &= \frac{1}{7} \left((0.3 - (0.4 \wedge 0.7))^2 + (0.4 - (0.4 \wedge 0.8))^2 + (0.3 - (0.4 \wedge 0.6))^2 + (0.3 - (0.4 \wedge 0.3))^2 \right. \\
&\quad \left. + (0.3 - (0.4 \wedge 0.6))^2 + (0.3 - (0.3 \wedge 0.5))^2 + (0.5 - (0.6 \wedge 0.5))^2 \right) \\
&\quad + \frac{1}{6} \left((0.2 - \frac{1}{5})^2 + (0.1 - \frac{1}{8.3})^2 + (0.2 - \frac{1}{5})^2 + (0.1 - \frac{1}{5.3})^2 + (0.1 - \frac{1}{6.6})^2 + (0.2 - \frac{1}{5})^2 \right) \\
&= 0.006.
\end{aligned}$$

Now, Since for all G^{AE} , G^{BC} , G^{BF} , G^{ED} , G^{EH} , G^{DG} , G^{FG} and G^{HG} we get,

$$sd(G^{HG}) = sd(G^{AE}) \wedge sd(G^{BC}) \wedge sd(G^{BF}) \wedge sd(G^{ED}) \wedge sd(G^{EH}) \wedge sd(G^{DG}) \wedge sd(G^{FG}) \wedge sd(G^{HG}) = 0.006.$$

And we see that $sd(G^{HG}) \leq sd(G)$, then $G^P = G^{HG}$.

In other site

$$sd(G^{FG}) = sd(G^{AE}) \vee sd(G^{BC}) \vee sd(G^{BF}) \vee sd(G^{ED}) \vee sd(G^{EH}) \vee sd(G^{DG}) \vee sd(G^{FG}) \vee sd(G^{HG}) = 0.02.$$

And also $sd(G^{FG}) \geq sd(G)$, then $G_P = G^{FG}$. Therefore, according to the above modified fuzzy graphs, $G(HG)$ and $G(FG)$ are powerful and powerless fuzzy graphs of graph G , respectively. This means that if we decrease the value of the strong arc FG in fuzzy graph G , in total, we also reduce the relationship of the other high-risk students and get a powerless fuzzy graph. Then, by this, we could prevent the spread of new drugs in high school, by controlling and reducing the relationship between two students F and G .

5 Conclusion

Graph theory has wide applications in computer science and engineering, especially in genetics and economics. The importance of this field of mathematics is palpable and undeniable. Most of the time, the aspects of graph problems are uncertain and fuzzy, respectively. In this case, nice use of fuzzy sets. There are some interesting features for handling fuzzy data that are unique to fuzzy sets, such as allowing for a more intuitive graphical representation of fuzzy data, which facilitates significantly better analysis of data relationships, incompleteness, and similarity measures. The notion of fuzzy sets was initially incorporated into relations. So fuzzy graphs are more important than crisp graphs.

In this paper, we introduce the semi-strong arc in fuzzy graphs and define modified fuzzy graphs by decreasing and increasing the values of the function a arc. Then, we define a powerful and powerless fuzzy graph. We were shown one application of powerless fuzzy graphs in real life. Helpful when the fuzzy graphs are very large. The natural extension of this work is an exploration of the applications of a powerful and powerless fuzzy graph in database theory, computer networks, transport networks, neural networks, and all of the networks in the real world.

Conflict of Interest: "The authors declare that they have no conflict of interest."

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